Introduction to & current issues of string theory

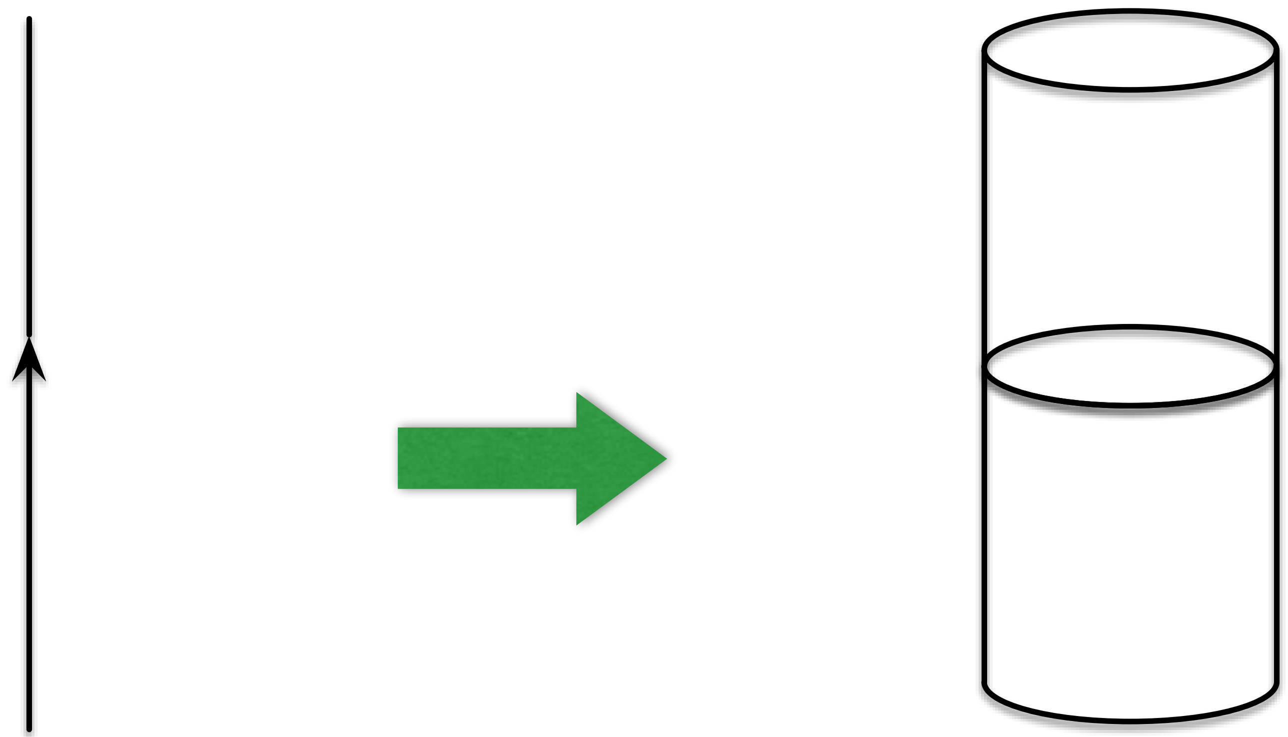
Hyeonjoon Shin (KIAS)

10th Saga-Yonsei Joint Workshop on High-Energy Physics

14 Jan. 2014

Contents

1. From particle to string



particle

string

$$S_{\rm ptl} = -m \int d\tau \sqrt{-\dot{X}^\mu \dot{X}^\nu \eta_{\mu\nu}}$$

$$S_{\rm NG} = -T \int d^2\sigma \sqrt{-\det(\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu})}$$

$$T = \frac{1}{2\pi\alpha'}$$

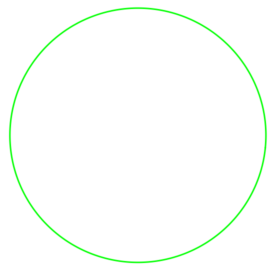
2. Bosonic string

- From quantum consistency of bosonic string,

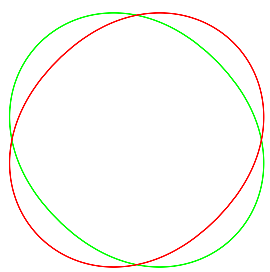
  conformal invariance on the worldsheet
  or
  Lorentz invariance in target space-time,

  target space-time dimension = 26.

- closed string excitations (particle spectra)



$$m^2 = -\frac{4}{\alpha'}$$ tachyon

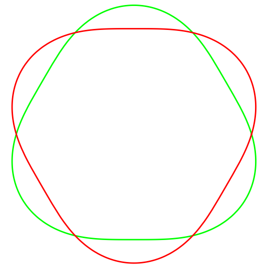


$$m^2 = 0$$ massless

$g_{\mu\nu}$ $B_{\mu\nu}$ Φ

symmetric

anti-symmetric



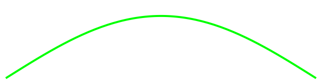
$$m^2 = \frac{4}{\alpha'}$$ massive

from the viewpoint of target space-time

$\vdots$

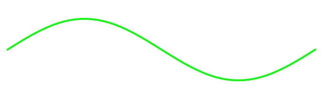
- open string excitations

$m^2 = -\frac{1}{\alpha'}$ tachyon



$m^2 = 0$ massless A_μ

gauge field



$m^2 = \frac{1}{\alpha'}$ massive

$\vdots$

- Low energy dynamics of string

At the energy scale much lower than the string scale, $E \ll \alpha'^{-1/2}$
only the dynamics of massless states or particles is important.

Then what is the action describing the dynamics of massless particles?

In other words, what is the identity of each massless particle?

There are two well known ways.

1. Compute the scattering amplitudes between massless states.
   Infer the action giving the amplitudes.

2. By requiring Weyl invariance on the string worldsheet, obtain
   the e.o.m. for each massless particle.
   Infer the action giving the e.o.m.

The low energy effective action :

$$S = \frac{1}{g_s^2(\alpha')^{12}} \int d^{26}x\sqrt{-g}e^{-2\Phi}\left(R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda}\right)$$

$$H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu}$$

$g_{\mu\nu}$: graviton

$B_{\mu\nu}$: two-form gauge field

Φ : dilaton

The string theory includes gravity in its structure,
and hence may be a candidate of the theory of quantum gravity.

3. D-brane

- Open string can have two types of boundary conditions.

  1. Neumann (free end)

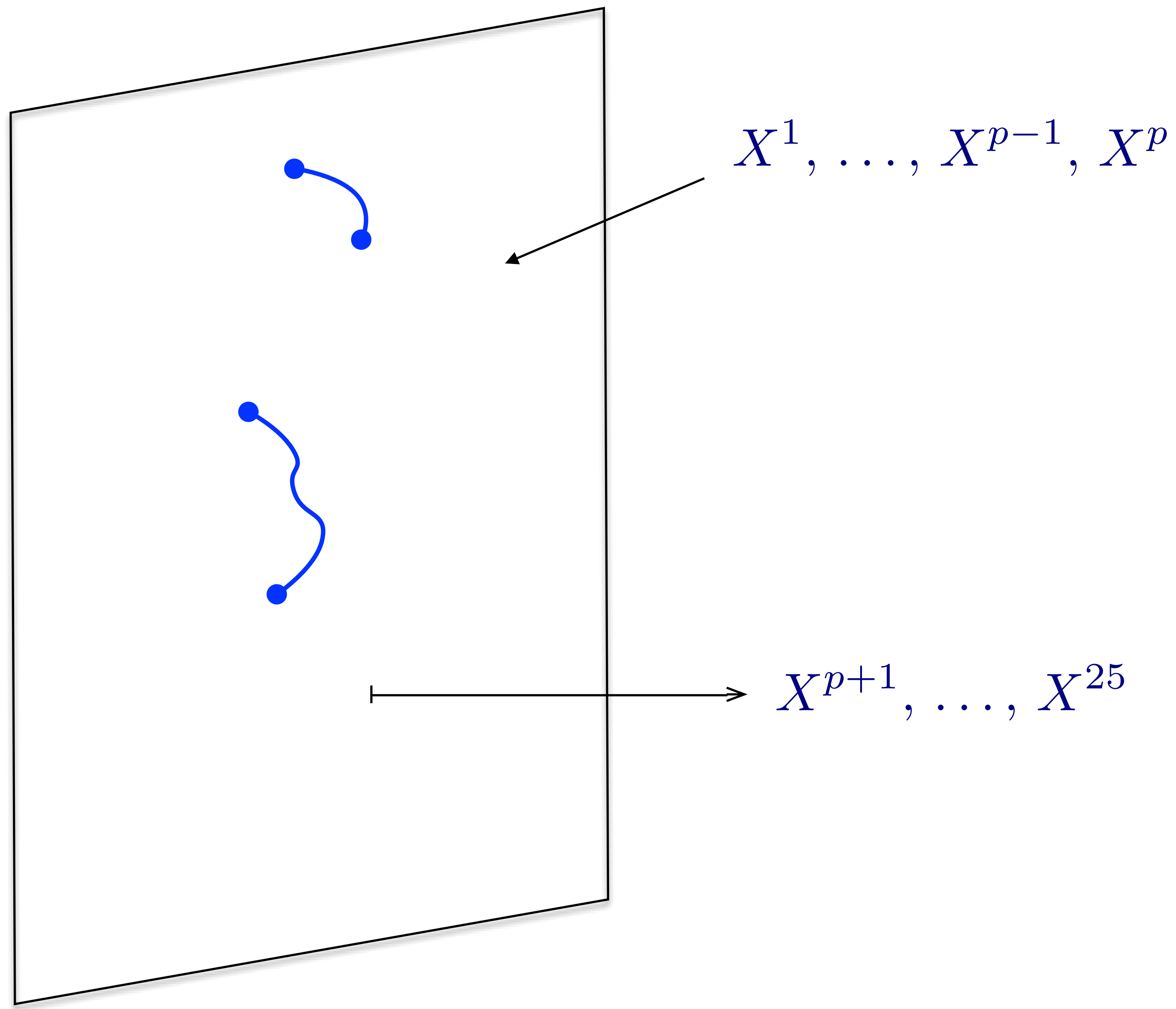
  2. Dirichlet (fixed end)

- Let's suppose that $X^0, X^1, \ldots, X^{p-1}, X^p \quad \in$ Neumann

  $X^{p+1}, \ldots, X^{25} \quad \in$ Dirichlet

  Then the open string describes a spatially p-dimensional extended object (not just a geometric surface), called **Dp-brane**.

- schematic view



- Massless fields on the Dp-brane

    1. open string oscillations along the Neumann directions give the gauge field propagating on the D-brane world volume:

$$A_a \quad (a = 0, 1, \ldots, p)$$

    2. open string oscillations along the Dirichlet directions give the scalar fields describing the fluctuations of D-brane in the target space-time:

$$\Phi^i \quad (i = p+1, \ldots, 26)$$

- The low energy effective action on a single Dp-brane :

Dirac-Born-Infeld (DBI) action

$$S_{\rm DBI} = -T_{{\rm D}p} \int d^{p+1}\xi e^{-\Phi} \sqrt{-\det\left(g_{ab} + B_{ab} + 2\pi\alpha' F_{ab}\right)}$$

$\xi^a \quad (a = 0, 1, 2, \ldots, p)$: world volume coordinates

$T_{{\rm D}p} = \dfrac{1}{g_s (2\pi)^p (\alpha')^{(p+1)/2}}$: Dp-brane tension

$F_{ab} = \partial_a A_b - \partial_b A_a$: world volume U(1) field strength

$$g_{ab} = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^\nu}{\partial \xi^b} g_{\mu\nu}$$

$$B_{ab} = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^\nu}{\partial \xi^b} B_{\mu\nu}$$

: pullback of bulk fields

In the flat empty space and weak field limit,

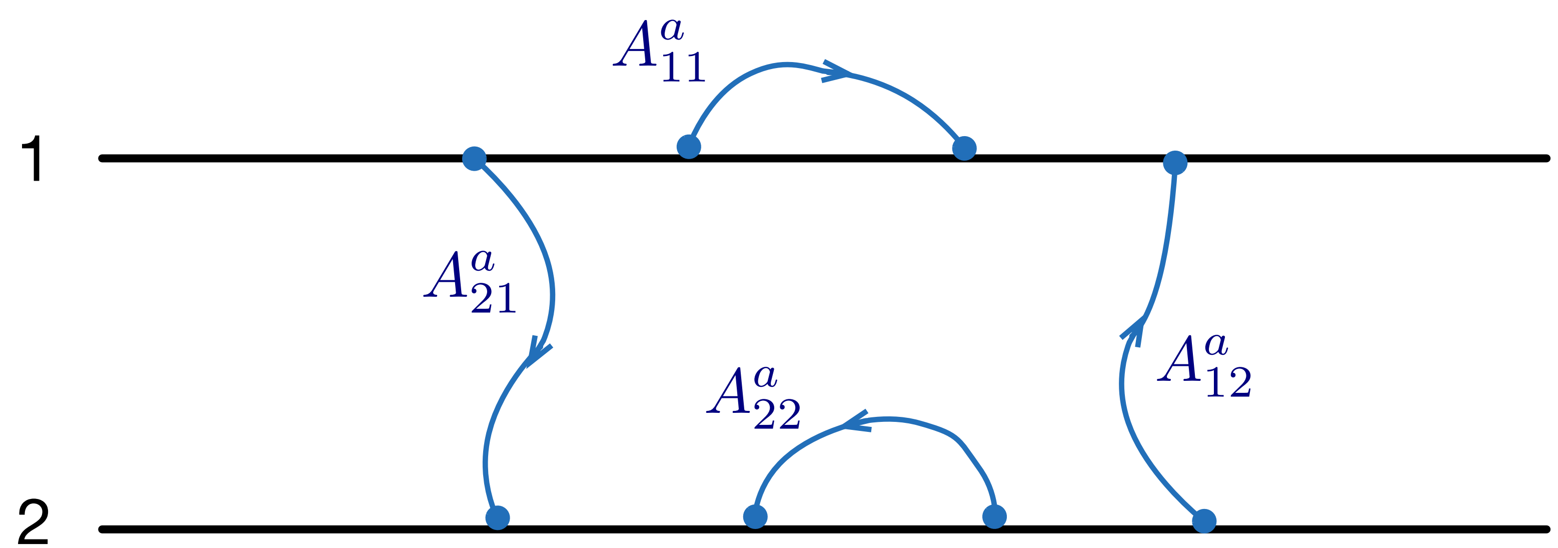
$$S_{\mathrm{DBI}} \sim -\frac{1}{4g_{\mathrm{YM}}^2}\int d^{p+1}\xi\,(F_{ab}F^{ab} + 2\partial_a\Phi^i\partial^a\Phi^i)$$

$\Phi^i \equiv \dfrac{X^i}{2\pi\alpha'} \quad (i = p+1, \ldots, 26)$: fluctuations of D-brane

$$g_{\mathrm{YM}}^2 = T_{\mathrm{D}p}^{-1}(2\pi\alpha')^{-2} = (2\pi)^{p-2}(\alpha')^{(p-3)/2}g_s$$

- Coincident multiple Dp-branes (Non-Abelian extension)

For coinciding two Dp-branes,



A^a_{ij} $(i, j = 1, 2)$: gauge fields in the adjoint representation of U(2)

Extending to the case of coincident N Dp-branes leads to
A^a_{ij} $(i, j = 1, \ldots, N)$: gauge fields in the adjoint representation of U(N)

Low energy effective action on multiple N Dp-branes :

in the flat empty space and weak field limit,

$$S_{\text{DBI}} \sim -\frac{1}{4g_{\text{YM}}^2} \int d^{p+1}\xi \, \text{Tr}\left(F_{ab}F^{ab} + 2D_a\Phi^i D^a\Phi^i + [\Phi^i, \Phi^j]^2\right)$$

$$F_{ab} = \partial_a A_b - \partial_b A_a + [A_a, A_b]$$

$$D_a\Phi^i = \partial_a\Phi^i + [A_a, \Phi^i]$$

All the fields are in the adjoint representation of U(N) and N x N matrices.

1. The action is simply the dimensional reduction of D=26 YM term to p+1 dimensions.

2. D-brane provides a geometric description of non-Abelian theory.

4. Superstring

- Bosonic string can not give fermionic states. We must have them in order to describe matter.

- One possible way of having fermions in target space-time is to introduce **supersymmetry (SUSY)**.
  (at present, it is the almost unique way of making fermions in the quantum mechanical description of string theory.)

- Two equivalent approaches:

  1. SUSY on the string worldsheet : Ramond-Neveu-Schwarz (RNS) formulation

  2. SUSY on the target space-time: Green-Schwarz (GS) formulation

- From quantum consistency of superstring,

  superconformal invariance on the worldsheet in the RNS formulation
  or
  super Poincare invariance in the target space-time in the GS formulation,

  target space-time dimension = 10.

- There are five consistent superstring theories.

  1. type I (open + unoriented closed) with gauge group SO(32)   ($\mathcal{N}$=1)
  2. heterotic (closed) with gauge group E_8 x E_8 ($\mathcal{N}$=1)
  3. heterotic (closed) with gauge group SO(32)   ($\mathcal{N}$=1)
  4. type IIA (closed)   ($\mathcal{N}$=2)
  5. type IIB (closed)   ($\mathcal{N}$=2)

- No tachyon in superstring theory.

- Massless particles from superstring

  One essential feature of SUSY: $N_B = N_F$ for physical d.o.f.

  The left (right)-moving superstring modes on the worldsheet give states in the target space-time in terms of SO(8) representation:

$$8_v + 8_s \qquad \text{or} \qquad 8_v + 8_c$$

  For closed superstring case, the tensor product of states from the left- and right-moving modes leads to the states in the target space-time.

$D=10$ $\mathcal{N}=1$: $8_V + 8_s = 8_B + 8_F = 16$ gauge multiplet

$8_V \times (8_V + 8_s) = (1+28+35)_B + (8+56)_F = 128$ supergravity multiplet

$\longrightarrow$ type I, heterotic supergravity

$$D=10 \quad \mathcal{N}=2: \quad (8_v + 8_s) \times (8_v + 8_c) = (1+28+35+8_v+56_t)_B + (8_s+8_c+56_s+56_c)_F$$

$$= 128_B + 128_F = 256 \quad \text{supergravity multiplet}$$

$\longrightarrow$ type IIA supergravity

$$(8_v + 8_c) \times (8_v + 8_c) = (1+28+35+1+28+35_+)_B + (8_s+8_s+56_s+56_s)_F$$

$$= 128_B + 128_F = 256 \quad \text{supergravity multiplet}$$

$\longrightarrow$ type IIB supergravity

$$(1+28+35)_B = \Phi + B_{\mu\nu} + g_{\mu\nu}$$

= Ramond-Ramond (RR) gauge fields

- D-brane

D-brane in superstring theory = **charged object under the RR gauge field**

Let us focus on the type II theories.

RR fields in IIA : $8_v + 56_t = C_1 + C_3$ (1- and 3-form gauge fields, thus D0 and D2.)

Actually, Dp with even p

RR fields in IIB : $1+28+35_+ = C_0 + C_2 + C_4$ (0-, 2-, 4-form gauge fields, thus D(-1), D1, D3.)

Actually, Dp with odd p

5. AdS/CFT

- A concrete realization of holography

  The physics of (d+1)-dimensional bulk is encoded in the dual d-dimensional gauge theory. (The notion itself came from the study of black hole entropy.)

- One example of gauge/string duality or gauge/gravity duality

  The gauge theory on the world volume of the branes describes the same physics as string theory in the warped geometry created by the branes.

- AdS/CFT correspondence provides an important framework for the current research topics in string theory.

- Any current topic dubbed as holographic XXXX is based on the AdS/CFT correspondence.

  - holographic RG
  - holographic Wilson loop
  - holographic entanglement entropy
  - holographic QCD (AdS/QCD)
  - holographic superconductor
  - holographic Schwinger effect

  …. etc

- AdS/CFT correspondence

(a typical example: AdS_5/CFT_4 from type IIB superstring theory)

(i) Let us consider N coincident D3-branes.
(ii) Take the decoupling limit $\alpha' \to 0$ while keeping fixed all dimensionless parameters including g_s , N.

Then, at finite energy,
[type IIB superstring theory on AdS_5 x S^5] = [$\mathcal{N}$=4 D=4 U(N) SYM theory]

Mathematically,

$$Z_{\text{string}}(\phi_0) = \left\langle \exp \int_{\partial(AdS_5)} \phi_0 \mathcal{O} \right\rangle_{\text{CFT}}$$

- relation between parameters

$$\frac{R}{\sqrt{\alpha'}} = (4\pi g_s N)^{1/4}$$: radius of both AdS_5 and S^5 in string unit.

$$g_{\mathrm{YM}}^2 = 4\pi g_s$$: YM coupling constant

$$\lambda = g_{\mathrm{YM}}^2 N = 4\pi g_s N$$: 't Hooft coupling constant

$$\Longrightarrow \quad \frac{R}{\sqrt{\alpha'}} = \lambda^{1/4}\,, \quad g_s = \frac{\lambda}{4\pi}\frac{1}{N}$$

- perturbative regime (supergravity approximation)

In the 't Hooft limit, $N \to \infty\,, \quad \lambda = \text{fixed}$

bulk string theory : $g_s \to 0$ no string loop correction (genus 0)
Supergravity approximation is valid for weak curvature, $R \gg \sqrt{\alpha'}$, i.e. $\lambda \gg 1$

(strong 't Hooft coupling suppresses also the stringy corrections.)

dual gauge theory : planar approximated in the 't Hooft limit (genus 0)
Perturbative calculation of planar graphs is valid for $\lambda \ll 1$

Therefore, the bulk supergravity describes the dual boundary large-N gauge theory at strong ’t Hooft coupling.

Conversely, the weakly coupled dual gauge theory describes the string theory in strongly curved geometry.

This gives a basic motivation for the study of holographic something.

Thank you