$B \to X_s \gamma$ study
with the hadronic tagging method at *Belle*

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SAGA YONSEI 2014
Workshop on High-Energy Physics

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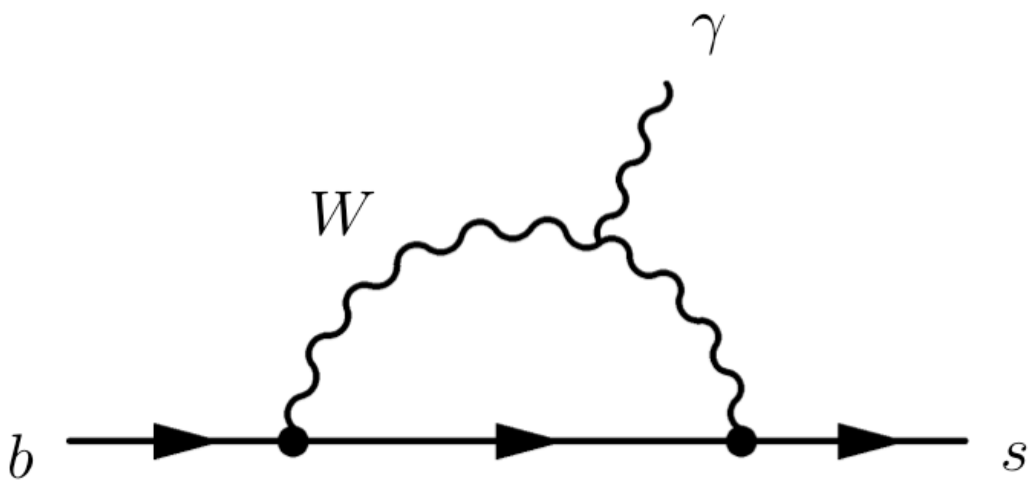
1. INTRODUCTION



As for the tree level decay of b→s**γ** is forbidden in SM, the decay takes place at least at the loop level with FCNC, which can be represented by a Electro-weak Penguin diagram.

The virtual W might be replaced by $H^{\pm}$ or non-SM particles, which leads to enhanced or suppressed branching fraction.

Next to Leading Order(NLO) prediction[1]

$B.F(B \rightarrow X_s\gamma) = (3.61^{+0.37}_{-0.49}) \times 10^{-4}$ **(2005)**[1]

The recent Next to NLO prediction[2]

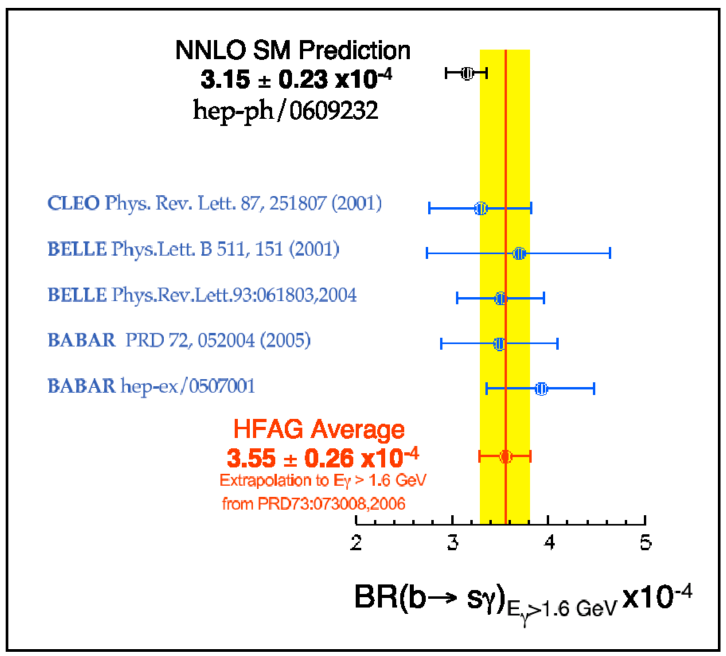
$B.F(B \rightarrow X_s\gamma) = (3.15 \pm 0.23) \times 10^{-4}$ **(2007)**[2]

HFAG Average

$B.F(B \rightarrow X_s\gamma) = (3.55 \pm 0.24 \pm 0.09) \times 10^{-4}$

(E_γ > 1.6 GeV)

This one has been taken for the signal B.F.



Measurements of the branching fraction and a comparison with the SM prediction at NNLO. A. Limosani, BN 1035

Agreement btw. experiment and theory has been degraded since the recent NNLO correction was done.

A good probe to search for the New Physics

[1] T. Hurth, E. Lunghi, and W. Porod, Nucl. Phys. B704, 56 (2005), hep-ph/0312260.
[2] M. Misiak et al., Phys. Rev. Lett. 98, 022002 (2007).

1. INTRODUCTION

This study will measure Eγ spectrum, isospin and CP asymmetry

The hadronic tagging method makes use of "EKP fullrecon"

This will let us analyze in the B-rest frame with the informations of fully-reconstructed tag side *B*

- **Isospin asymmetry can be directly measured**
  Provided the charge info. of tagged B, we can directly measure isospin asymmetry, A_I.

- **Improvement in the systematic uncertainty**
  Resolutions of signal is significantly improved! (As Yook said..)

- **Improvement in the statistical uncertainty**
  With about 4 times more data available, we can expected to be able to reduce the statistical uncertainty compared to the study by Babar using the same method.

Comparable study from Babar using "the recoil

$B.F(B \rightarrow X_s \gamma) = (3.97 \pm 0.97 \pm 0.64) \times 10^{-4}$

210 fb^{-1} used

Related studies and their results from *Belle*

Inclusive results

$A_{CP} = 0.002 \pm 0.050_{stat} \pm 0.030_{syst}$ — *S. Nishida(2004), PRL 93, 0318038*

$B.F = (3.55 \pm 0.32^{+0.30+0.11}_{-0.31-0.07}) \times 10^{-4}$ with $140 fb^{-1}$ — *P. Koppenburg(2004), PRL 061803*

$B.F = (3.45 \pm 0.15_{stat} \pm 0.40_{syst}) \times 10^{-4}$ with $605 fb^{-1}$ — *A. Limosani(2009), PRL 103, 241801*

2. SIMULATION

(1) Signal MC generation

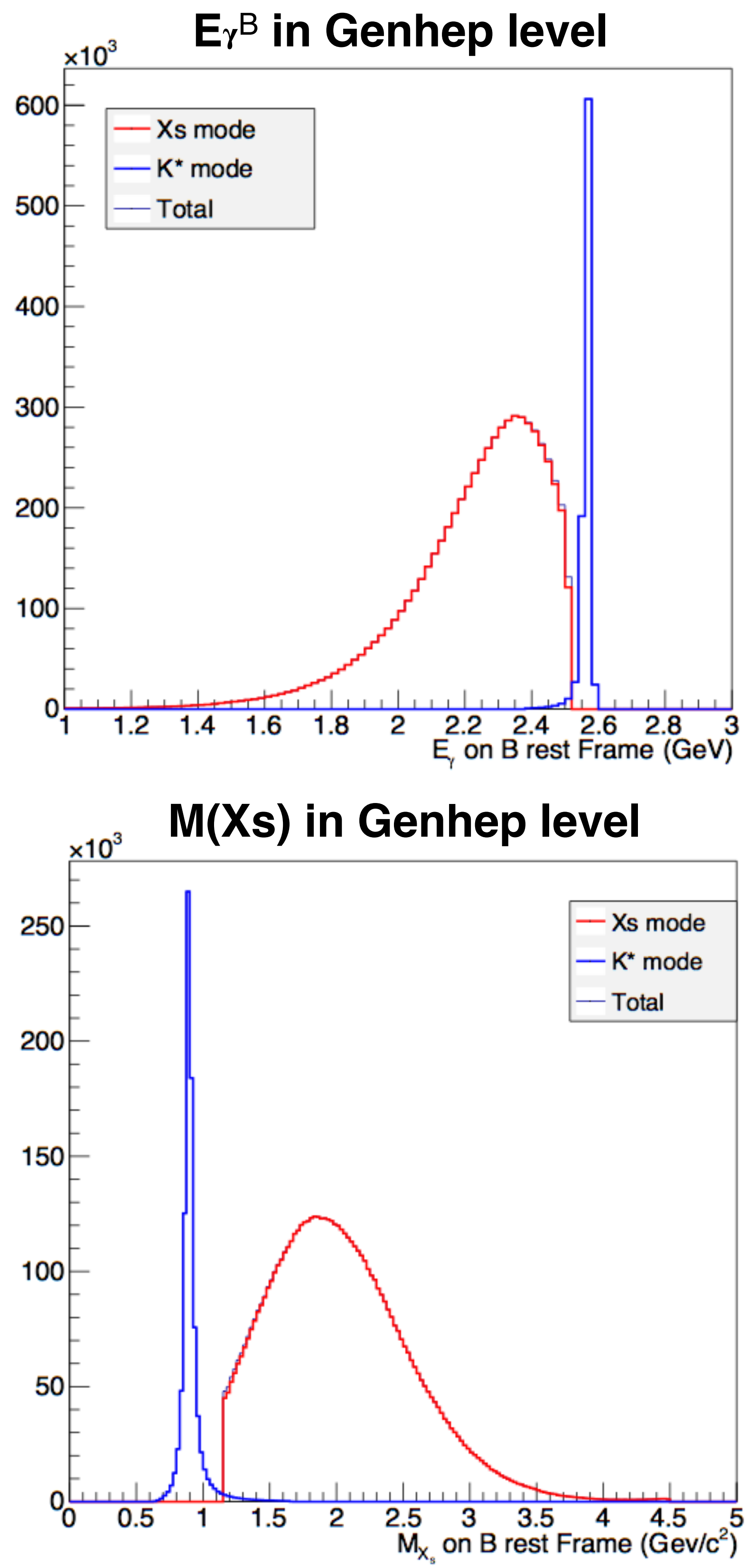


Figure 1. Signal event generation results

(1)24 mil. of events was generated using Evtgen and simulated using Gsim

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(2)$B \to X_s \gamma$ MC samples go through the random selection defined by the Kagan-Neubert model[1]

Heavy Quark Parameters are used for the K-N dist. parameters.

HFAG values $m_b = 4.574\ GeV,\ \mu_\pi^2 = 0.459\ GeV^2$

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(3)$B \to K^* \gamma$ MC samples are generated according to the ratio Xs : K* = 88 : 12, and simulated.

Generation Results		Gen & Gsim	Random selection	Eff.
Xs modes	Mixed	12 mil.	6369933	53.08%
	Charged	12 mil.	6370224	53.09%
K* modes	Mixed	868632		
	Charged	868674		

~33 streams of signal MC sample

Table 1. The number of signal MC events

[1] A. L. Kagan and M. Neubert, Eur. Phys. J. C7, 5 (1999), hep-ph/9805303.

2. SIMULATION

(2) Signal & Bkg. MC simulation, and Pre-selection

Pre-selection

Best-B_{tag} Selection

NBRank = 1
(if ∃ multiple choices, the one with higher NBout is chosen.)
$5.24 < M_{bc} < 5.29$ GeV/c^2
$|\Delta E| < 0.06$ GeV
NBout > 0.1

Candidate Selection

Most energetic (in B rest frame) gamma among sig-side gammas.
$E_\gamma{}^B{}_{candi} > 1.3$ GeV

Table 2. Pre-selection criteria

	Simulated	After fullrecon & pre-selection	Efficiency
Signal	14477463	25287	0.17%
Generic	5 streams	414812	
Continuum	5 streams	266364	

Table 3. The number of event before/after the fullrecon & pre-selection of sig/Generic/Continuum.

5 Streams of Generic & Continuum MC
50 streams of RareB, **20 streams** of Ulnu MC employed.

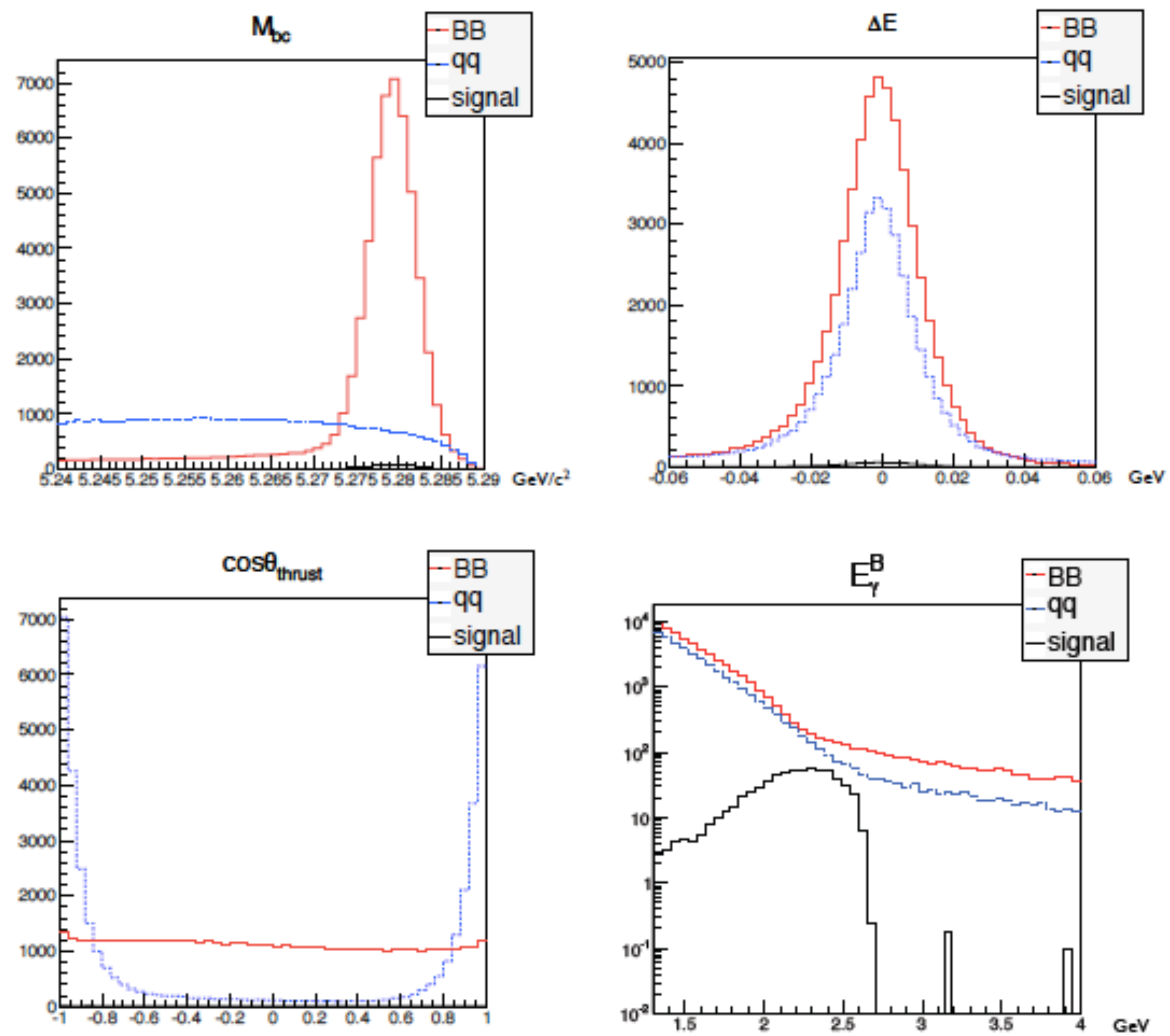
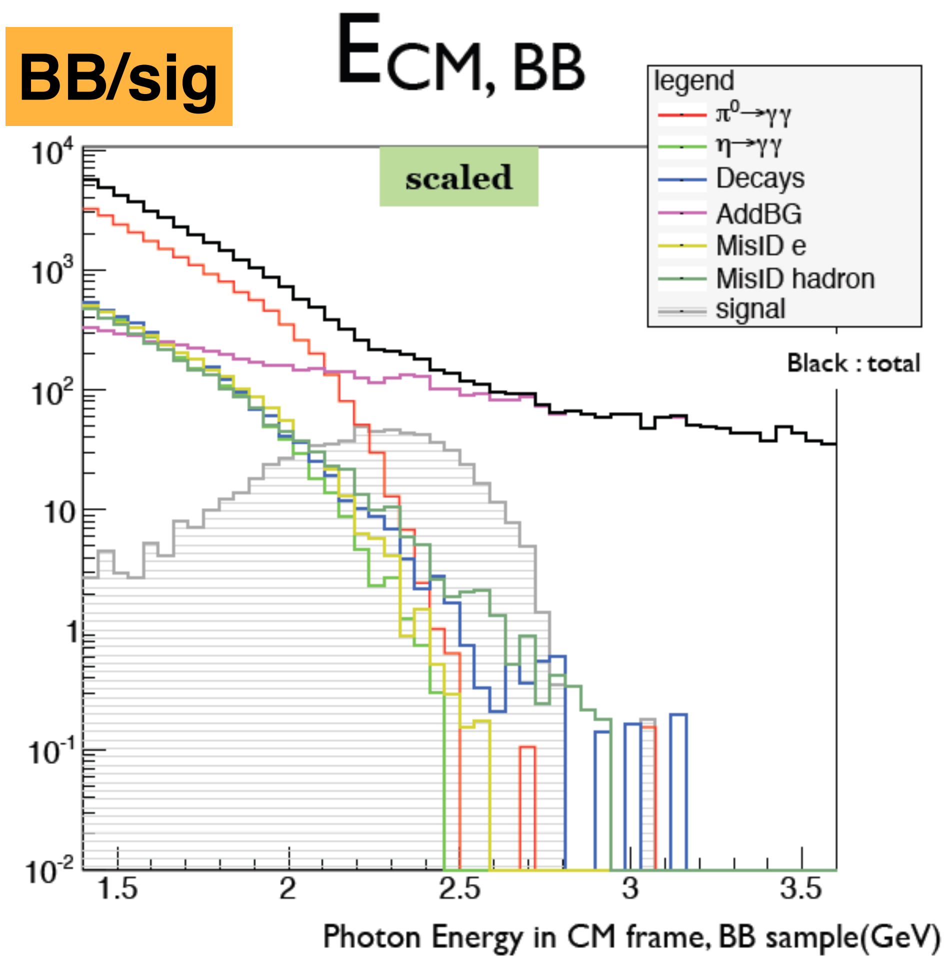


Figure 2. sig/bkg. simulation & pre-cut results

3. EVENT SELECTION

Selection Criteria



π^0&η and addbg(911, off-timing) particles are dominant.

Selection Criteria

1. idhep ≠ 911 for MC, off-timing for DATA
   - to veto off-timing particles
2. $P(\pi^0)<0.3$ & $P(\eta)<0.3$
   - to suppress π^0&η
3. $|\cos\theta_{thrust}| < 0.8$
   - to suppress the remaining continuum events
4. $PDERS_{ecl} > 0.2$
   - to suppress the hadronic shower events.
5. $\cos\theta e < 0.8$
   - to suppress the electron's radiative events.

✻ In each step, tag-correction is applied.

Analysis Regions

Region I : $1.3 < E_\gamma^B < 1.8$ GeV

Region II : $1.8 < E_\gamma^B < 2.0$ GeV (Optimization regions)

Region III : 2.0 GeV $< E_\gamma^B <$ 2.8 GeV

3. EVENT SELECTION

(1) Off-timing veto

911(addbg) particles in MC correspond to the off-timing events, overlay beam backgrounds(QED bkg) in DATA.
All the MC candidates with idhep=911 (isthep= -91) are vetoed.
All the off-timing on Triger Count is vetoed in the case of DATA.

(2) P(π^0)<0.3 & P(η)<0.3 Cut

Probability of the candidates to be a daughter of π^0 or η was calculated, according to the distribution of the mass combined by the candidate gamma paired with another gamma.
(Only signal-side pairs were considered since there was not a significant advantage in using the others)

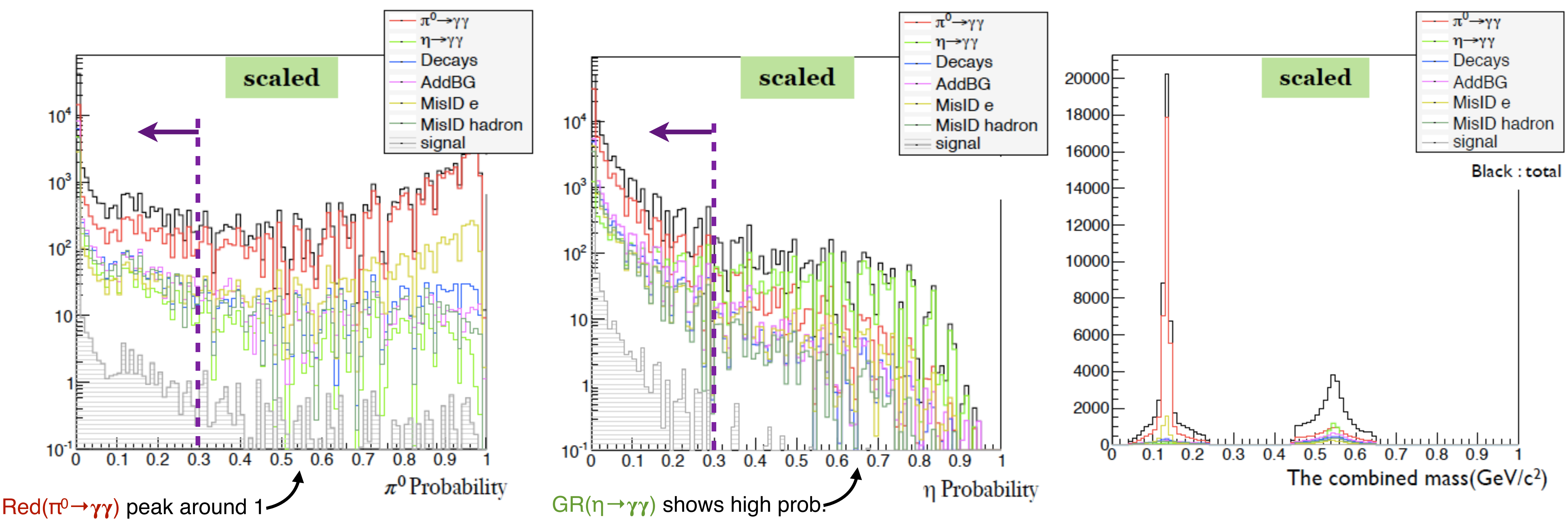


Fig 4. The related distributions to *P(π^0)<0.3 & P(η)<0.3 Cut*

3. EVENT SELECTION

(3) $|\cos\theta_{thrust}|$ < 0.8 Cut

To suppress the continuum events, thrust angle cut was employed.

$$\text{Thrust Axis} = \frac{\sum \vec{p}_i^{\,longitude}}{\vec{p}_i}$$

Angle btw. two thrust axis of sig-side and tag-side was calculated.

$$\cos\theta_{thrust} = \vec{A}_{sig} \cdot \vec{A}_{tag}$$

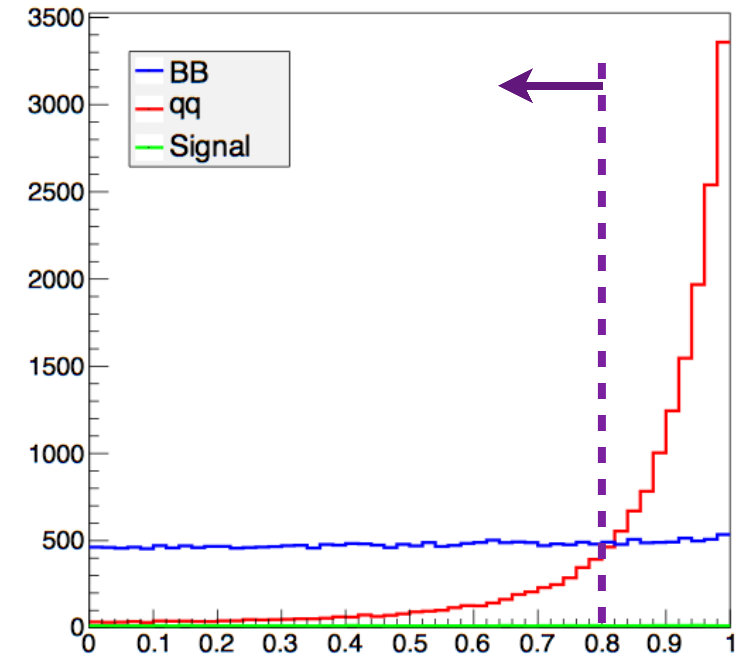


Fig 5. $|\cos\theta_{thrust}|$ distribution of sig/BB/qq

(4) ECL variables, MVA

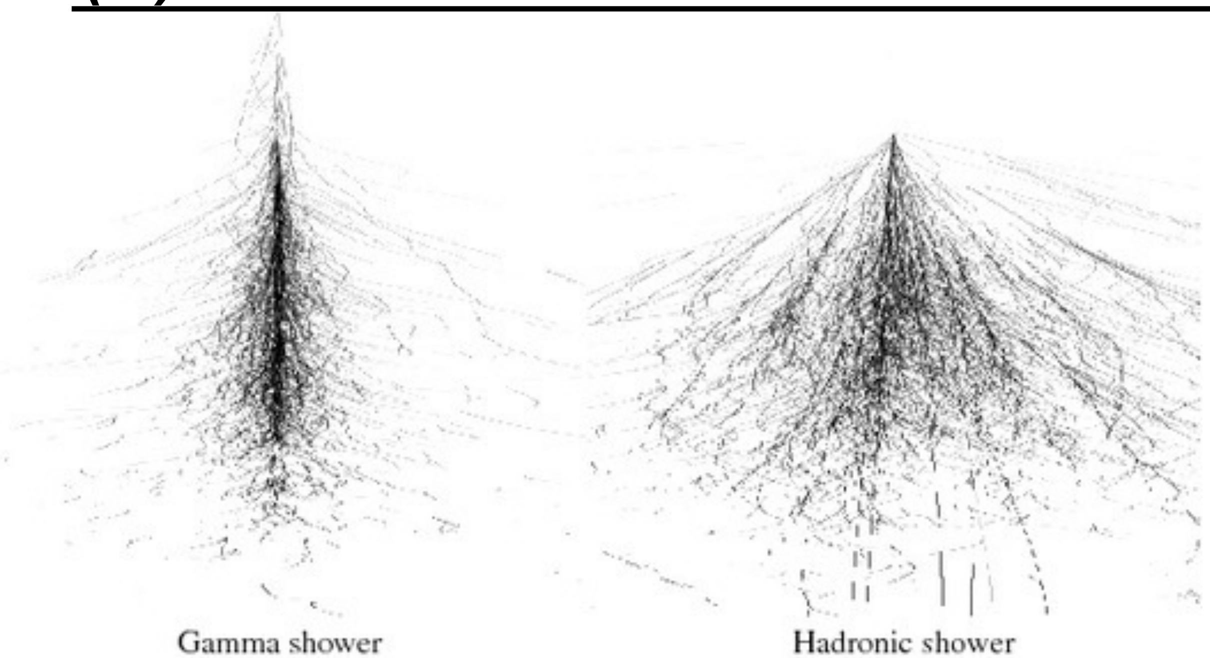


Fig 6. The different character of gamma showers and hadronic showers. (from K. Bernlöhr, Imaging very high energy gamma-ray telescopes)

Hadronic shower (e.g. anti-n, anti-p) shows widely spread shower shape compared to the real gamma shower inside calorimeters.
So we decided to make use of **ECL shower parameters** to veto hadronic shower events.

ECL shower parameters

- Shower width : The RMS width of shower shape
- Shower mass : The combined mass of showers
- E9/E25 : (E Deposited in 3X3 ECL cluster) / (E Deposited in 5X5 ECL cluster)

3. EVENT SELECTION

(4) ECL variables, MVA

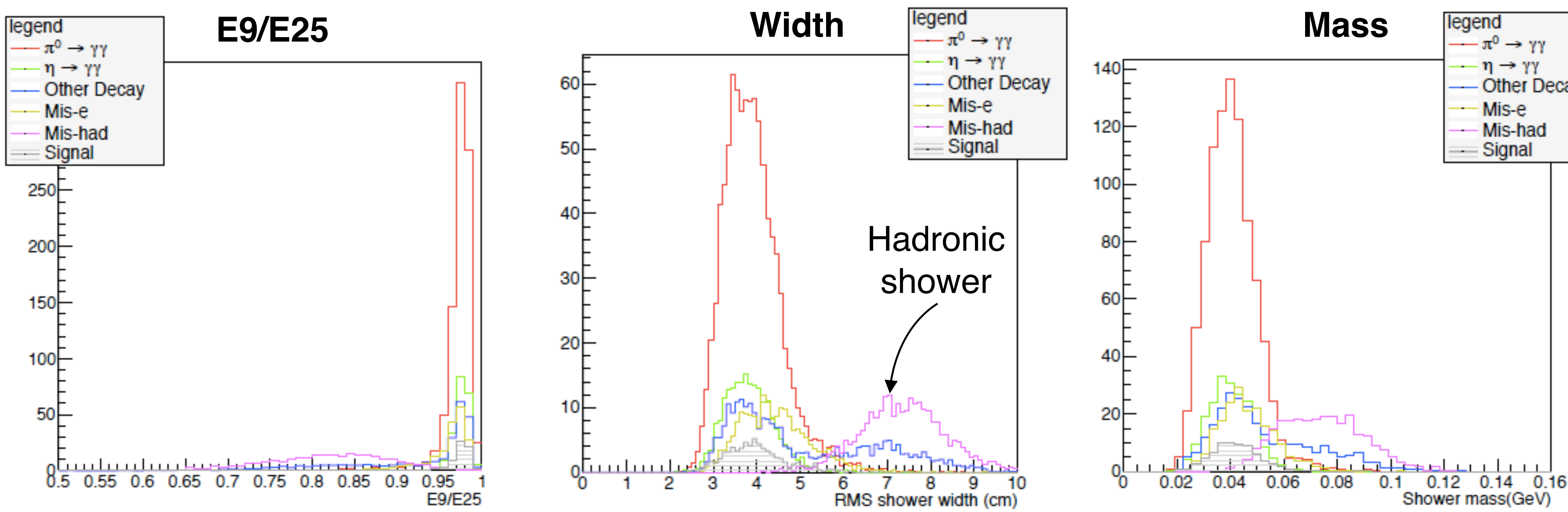


Fig 6. ECL shower parameters in Optimization region, SIG/BB (1.8~2.0 GeV)

These variables are correlated (correlation matrix in back-up), we tried Multi-Variate Analysis (MVA) using TMVA, the root built-in tool for MVA.

Training Set : 6 mil. of Signal / 1 stream of BB&qq

Tested method : Rectangular Cuts, Fisher(Linear) Discriminant, Multi-dimensional Likelihood(PDERS)

PDERS **shows a better ROC in general.**

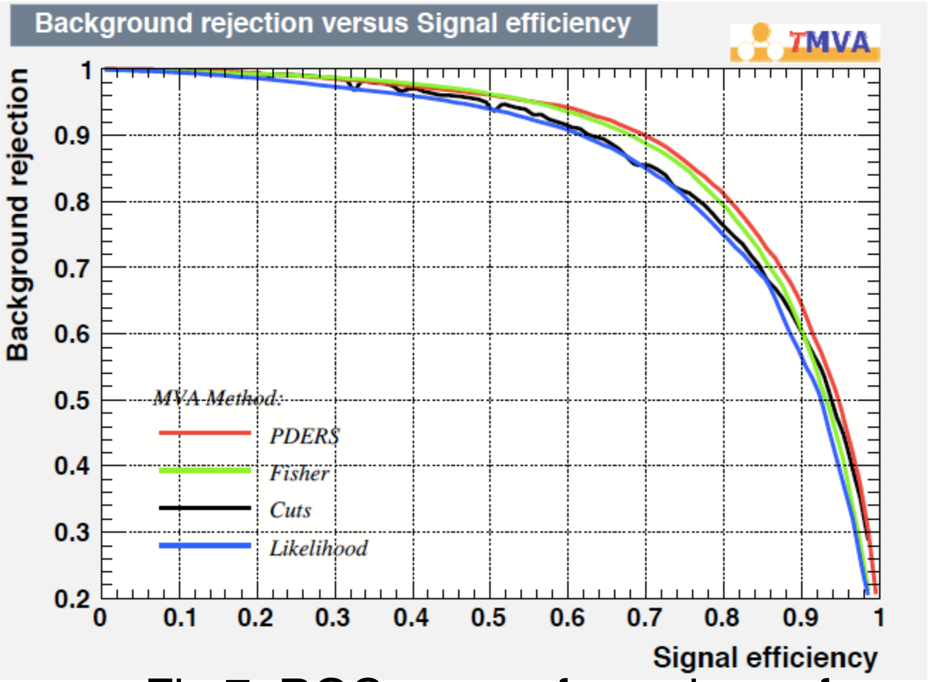


Fig 7. ROC curve of a various of methods

3. EVENT SELECTION

(4) ECL variables, MVA

After overtraining test and FoM test for optimization region, we decided the cut at PDERS > 0.2. (Details are shown in the BACK-UP)

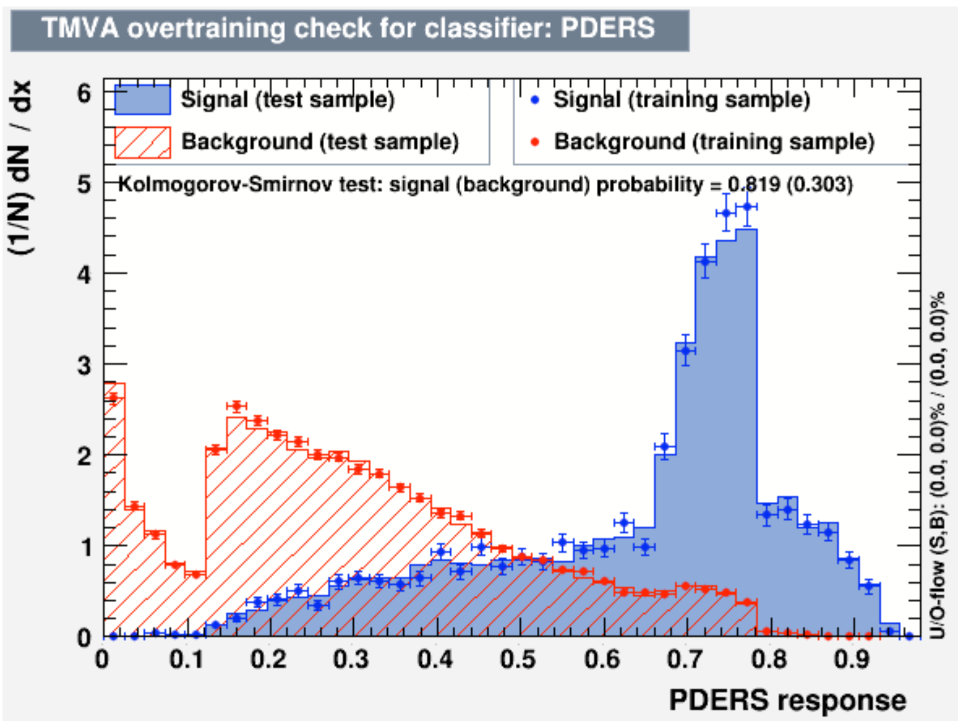


Fig 8. Overtraining test

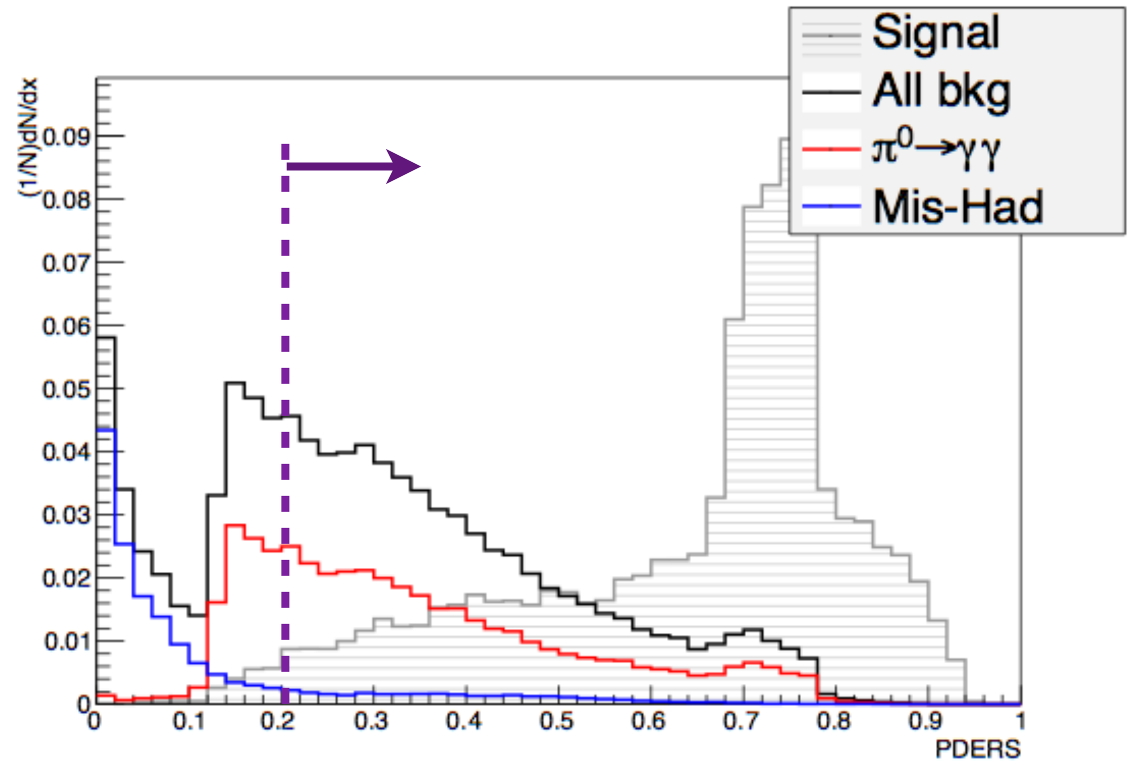


Fig 9. PDERS normalized dist. of specific bkg.

(5) $\cos\theta_e < 0.8$ cut

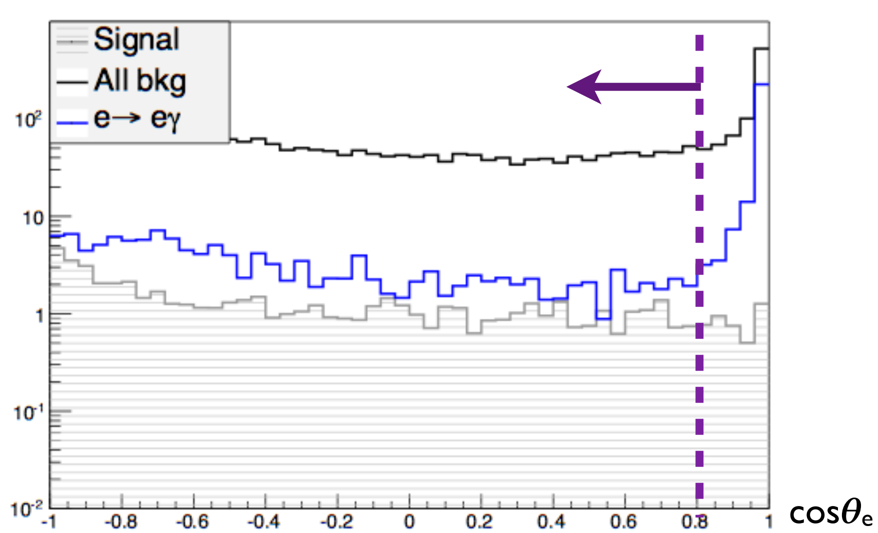


Fig 10. $\cos\theta_e$ dist. of specific bkg.

Angle btw. candidate gamma & the closest electron was tested to veto the events oriented by electron's emission. $e\rightarrow e\gamma$ events have a peak around **$\cos\theta_e$ = 1**

3. EVENT SELECTION

(6) Selection Efficiency

Region I , 1.3 < E_γ^B < 1.8 GeV

Region I	Sig	BB	qq	FoM
911 veto	99.48%	93.02%	95.74%	0.21
P(π0)&P(η)	87.10%	48.59%	50.32%	0.26
lcosθthrustl	80.27%	79.01%	22.79%	0.29
PDERS	86.56%	63.59%	64.22%	0.31
cosθe	97.46%	93.01%	96.24%	0.31
tot-efficiency	59%	21.12%	6.79%	
tot-cutoff	41%	78.88%	93.21%	

Region 2 , 1.8 < E_γ^B < 2.0 GeV

Region II	Sig	BB	qq	FoM
911 veto	99.48%	83.22%	89.16%	1.01
P(π0)&P(η)	95.17%	54.79%	59.68%	1.27
lcosθthrustl	77.35%	79.43%	20.45%	1.32
PDERS	96.87%	79.9%	79.13%	1.43
cosθe	99.17%	96.22%	97.38%	1.44
tot-efficiency	70.64%	27.84%	8.39%	
tot-cutoff	29.65%	72.16%	91.61%	

Region 3 , 2.0 < E_γ^B < 2.8 GeV

Region III	Sig	BB	qq	FoM
911 veto	99.48%	38.81%	62.94%	8.71
P(π0)&P(η)	97.78%	69.4%	68.35%	10.00
lcosθthrustl	79.27%	80.03%	18.43%	10.61
PDERS	99.39%	77.25%	80.67%	11.43
cosθe	99.26%	97.76%	97.78%	11.45
tot-efficiency	76.07%	16.28%	6.25%	
tot-cutoff	23.93%	83.72%	93.75%	

Table4. Signal Efficiency of selection criteria

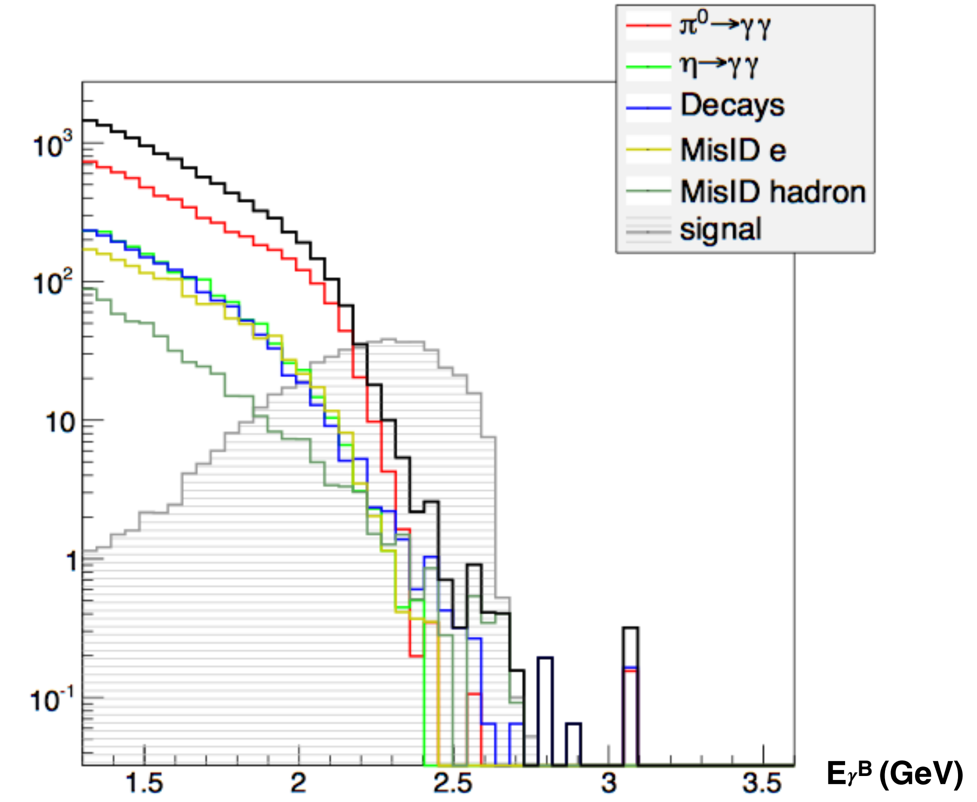
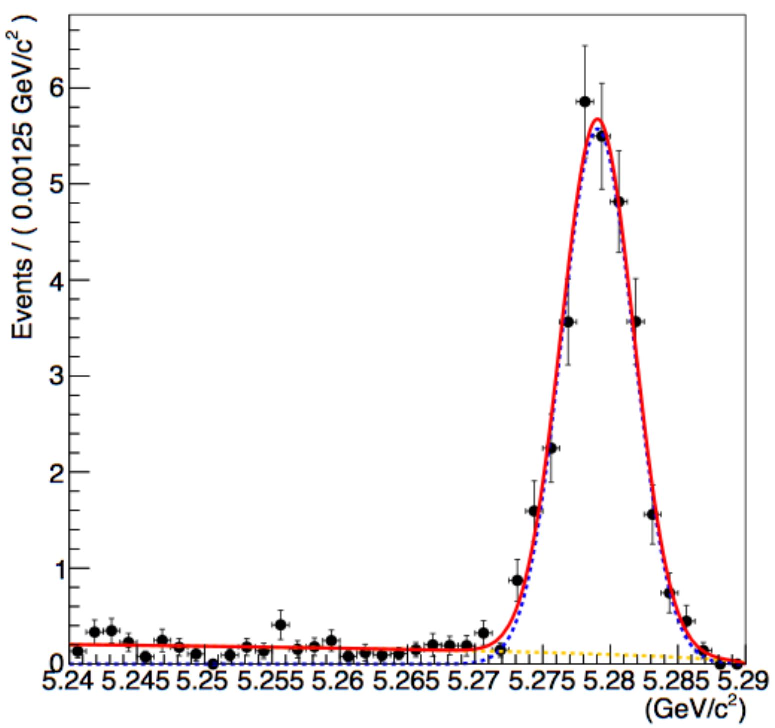


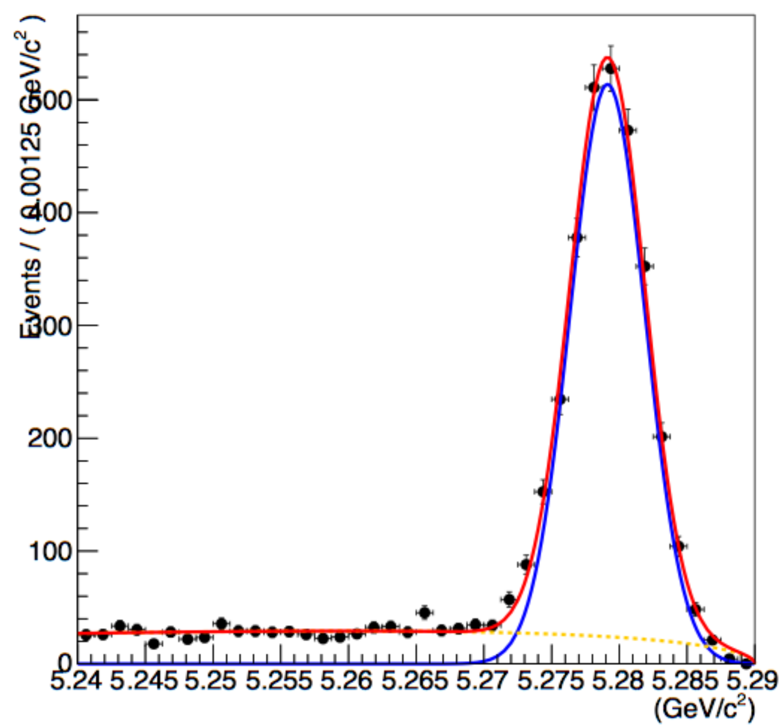
Fig 11. E_γ^B distribution after selection (sig/BB)

4. EXPECTED YIELD

Signal M_{bc} Fit



BB/qq M_{bc} Fit



Signal M_{bc} Fitting
- Crystal ball + ARGUS

BB/qq Fitting
- Crystal ball + ARGUS

Sig + BB/qq Fitting
- Crystal ball + ARGUS

sig + BB/qq M_{bc} Fit

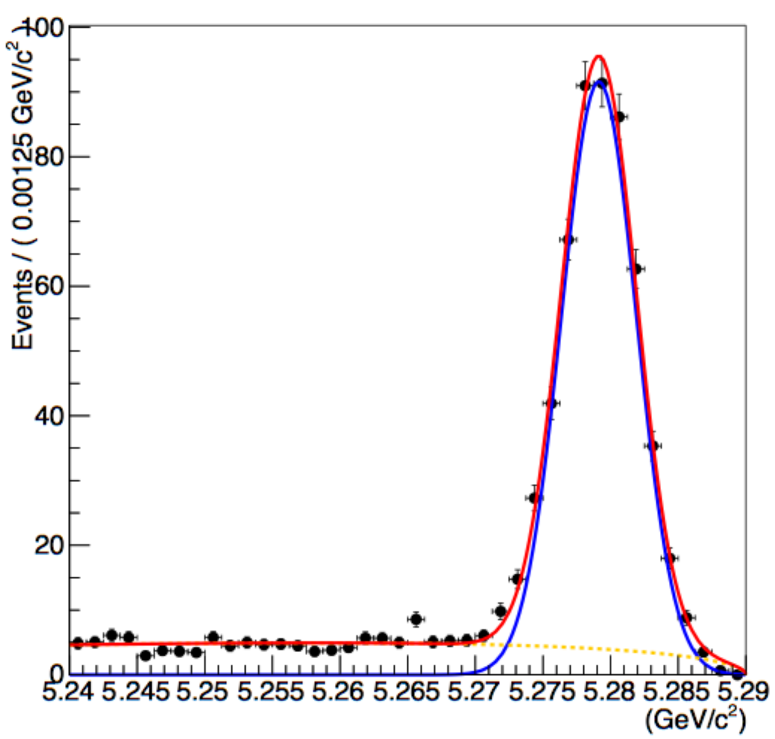


Fig 12. M_{bc} Fit result in the bin 1.9-2.0 GeV

- Bin-by-Bin M_{bc} fitting was performed to obtain bkg-subtracted yield for each E_{γ}^{B} bin of 0.1 GeV range
- 10 streams of BB & 5 streams of qq MC samples were used for bkg. estimation.

4. EXPECTED YIELD

Bin#	range	Subt. Yield	error	Signifi-cance	True Significance
0	1.6-1.7	8.70	45.12	0.193	0.212
1	1.7-1.8	15.36	38.77	0.396	0.410
2	1.8-1.9	23.82	32.85	0.725	0.774
3	1.9-2.0	37.84	27.78	1.362	1.447
4	2.0-2.1	51.73	22.56	2.293	2.427
5	2.1-2.2	69.00	16.70	4.131	4.321
6	2.2-2.3	82.68	12.35	6.693	6.917
7	2.3-2.4	81.37	10.31	7.889	7.981
8	2.4-2.5	60.93	8.87	7.276	7.349
9	2.5-2.6	40.18	6.59	6.096	6.104

Table 5. Results of signal ftting and subtracted yield for each bin btw $1.6 < E_\gamma^B < 2.6$ GeV

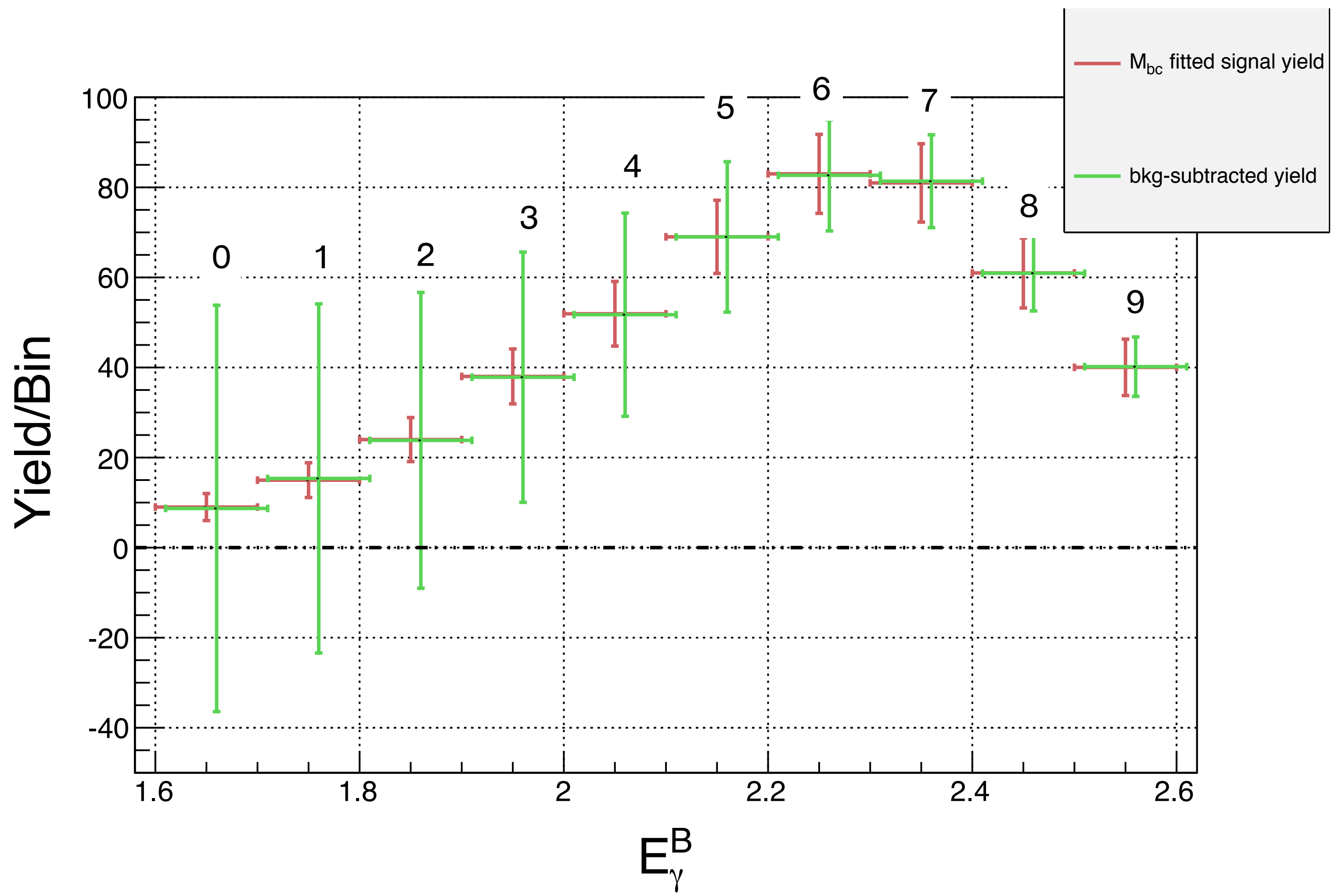


Fig 13. bkg-subtracted yield of each bin btw $1.6 < E_\gamma^B < 2.6$ GeV

4. EXPECTED YIELD

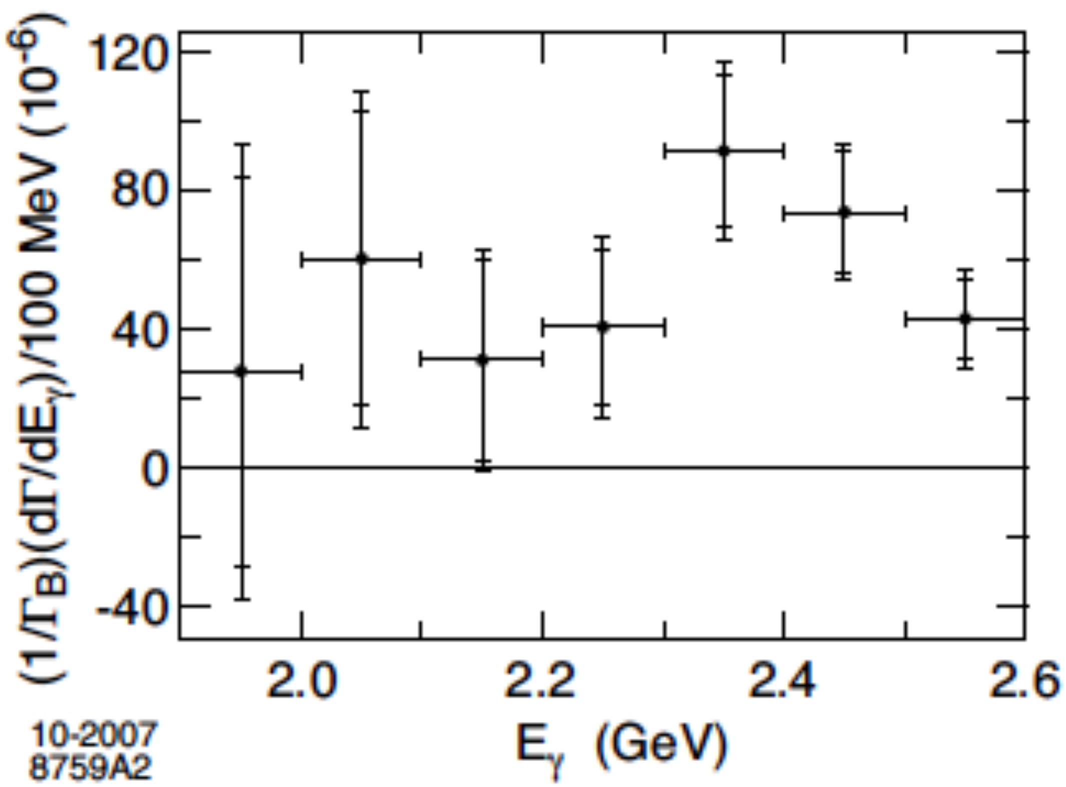


Fig 14. BaBar Result

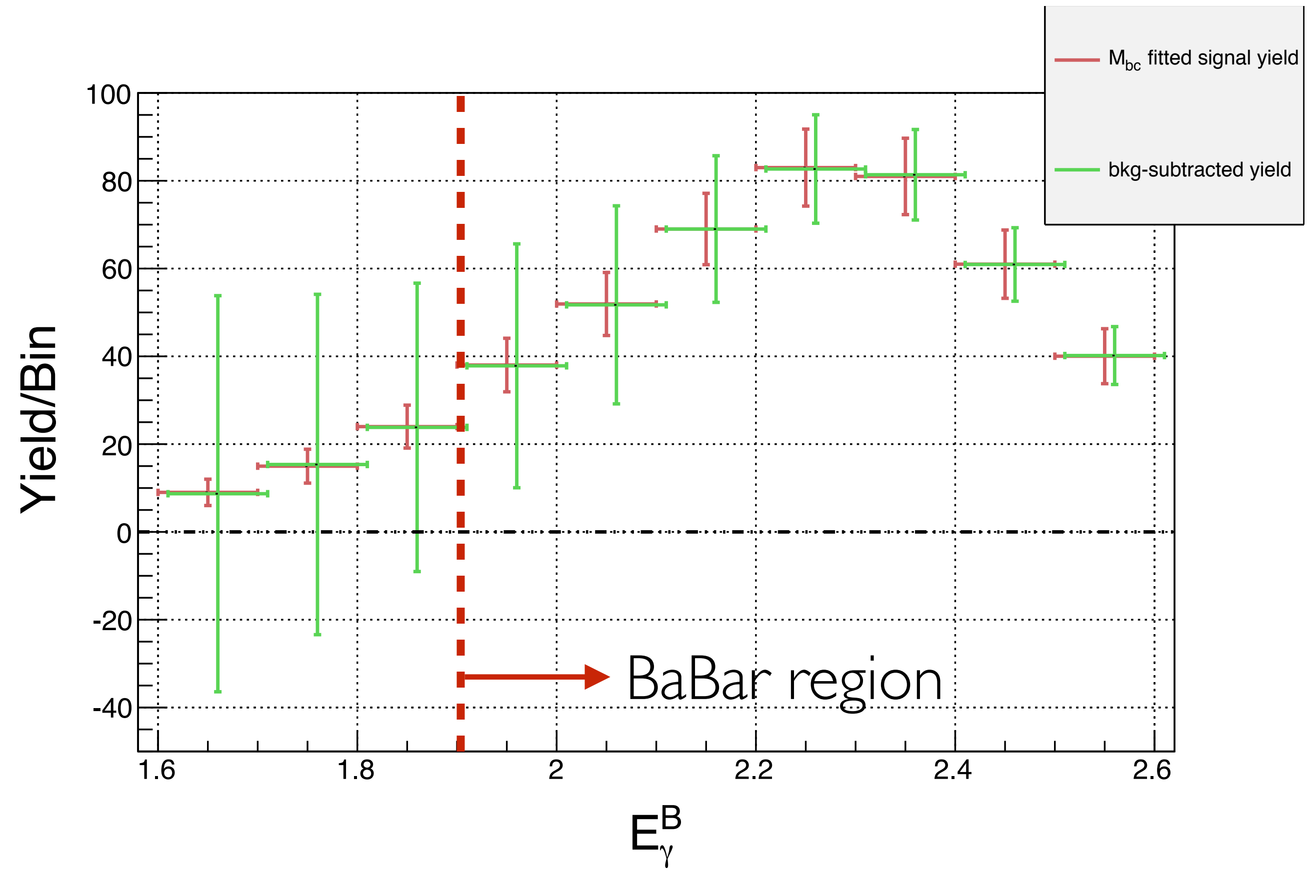


Fig 15. Our expectation (bkg-subtracted yield)

Significance in 1.9-2.0 Range

At Babar, ~$0.5\sigma_{stat}$
Our expectation, ~$1.4\sigma_{stat}$

We could take advantage of large statistics of Belle!
(If everything goes fine.)

5. SUMMARY & PLANS

1. Selection Criteria is decided.

2. Expected Yields is calculated. (will be updated with including RareB & Ulnu)
   We could expect a better result than that of Babar.

3. π^0&η (dominant) background study should be done.

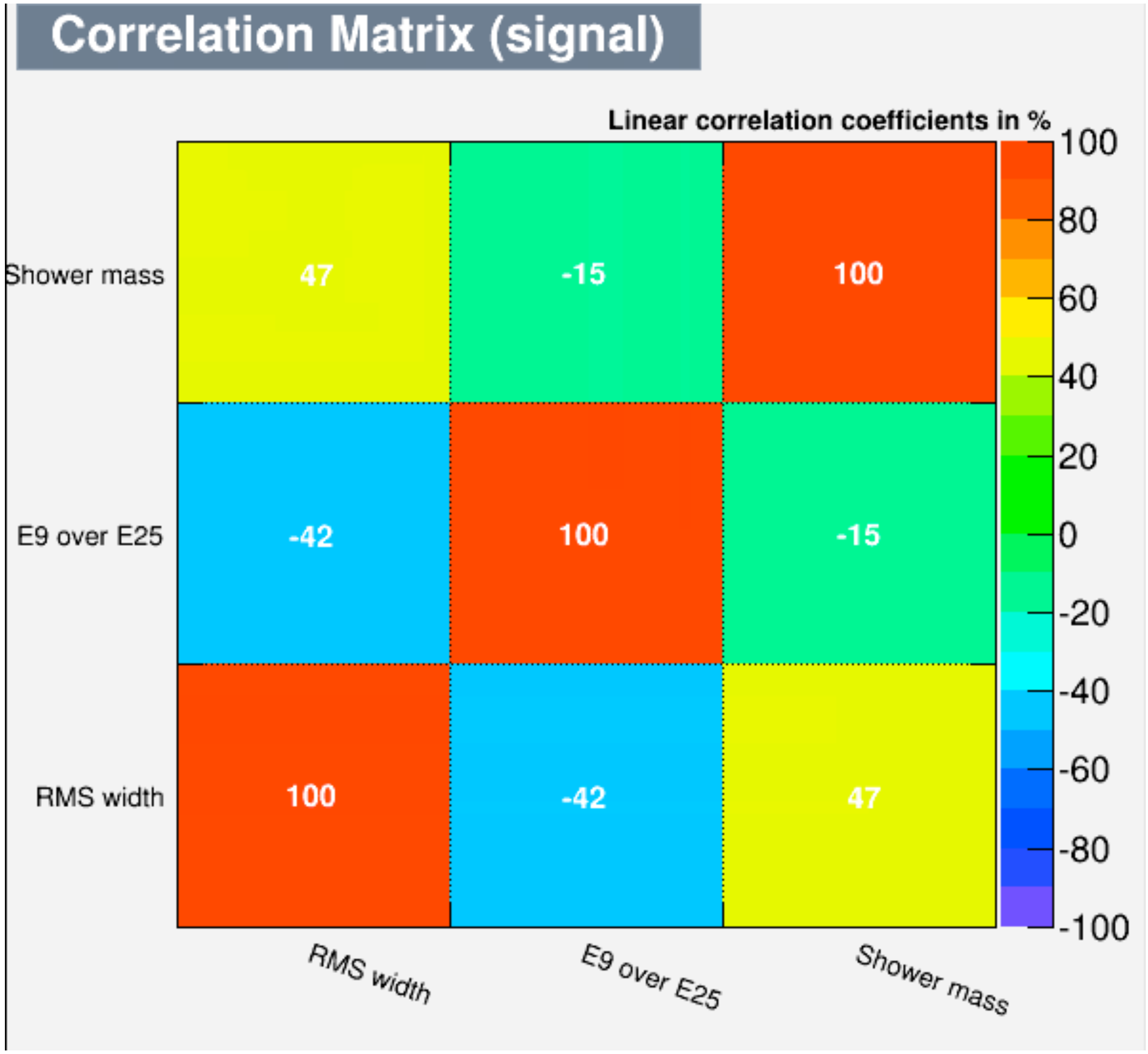
6. BACK UP

ECL variables

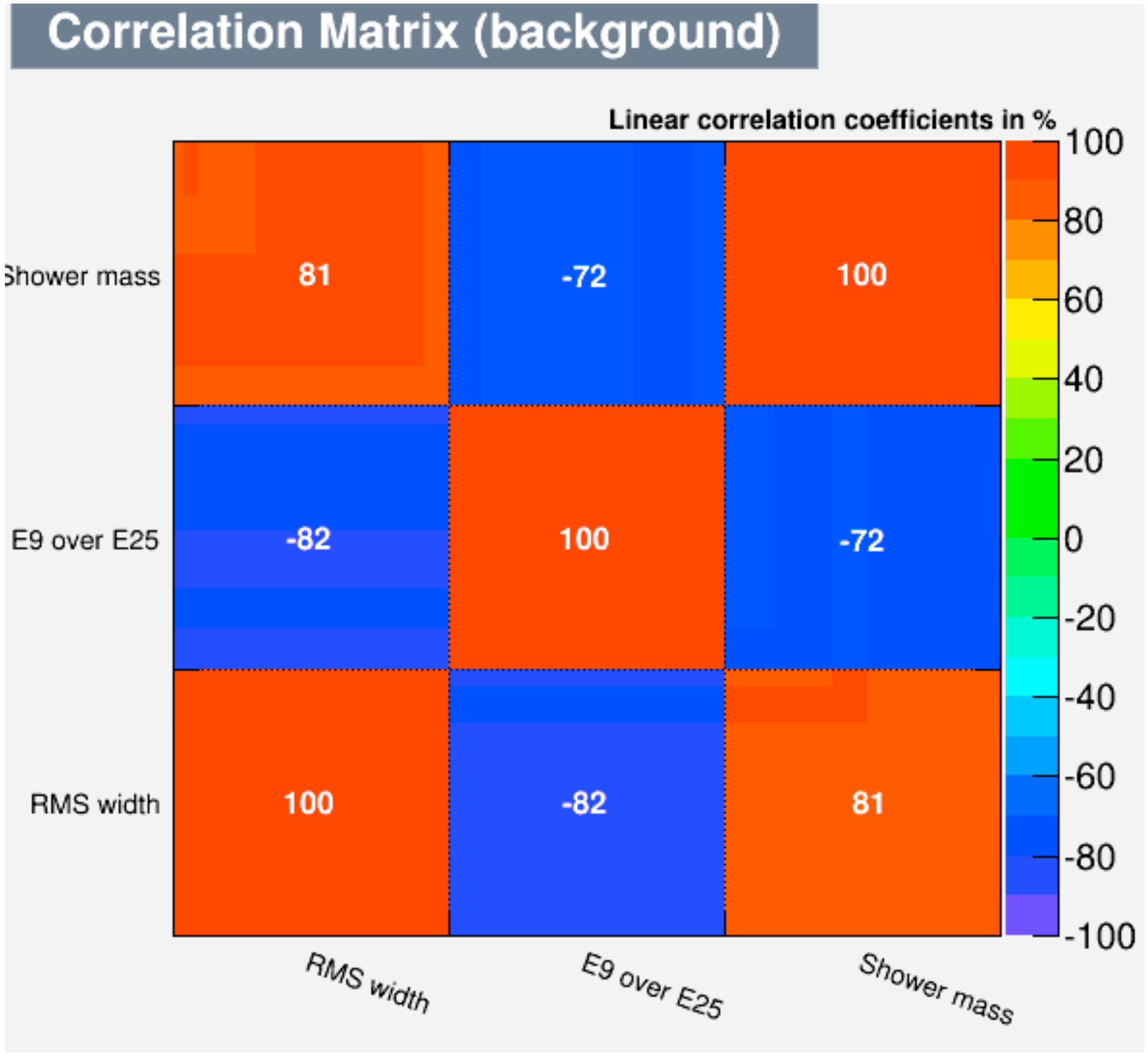
OK, apparently, it seems hard to generally apply MVA(distances, $\cos\theta_e$) cut to my samples of interest. (But I want to use this MVA somehow!)

So I checked the linear correlations btw ECL variables again, but this time, the whole MC samples were considered. (I only used samples in 1.8 ~ 2.0 GeV region in the last time)

Linearity(signal)

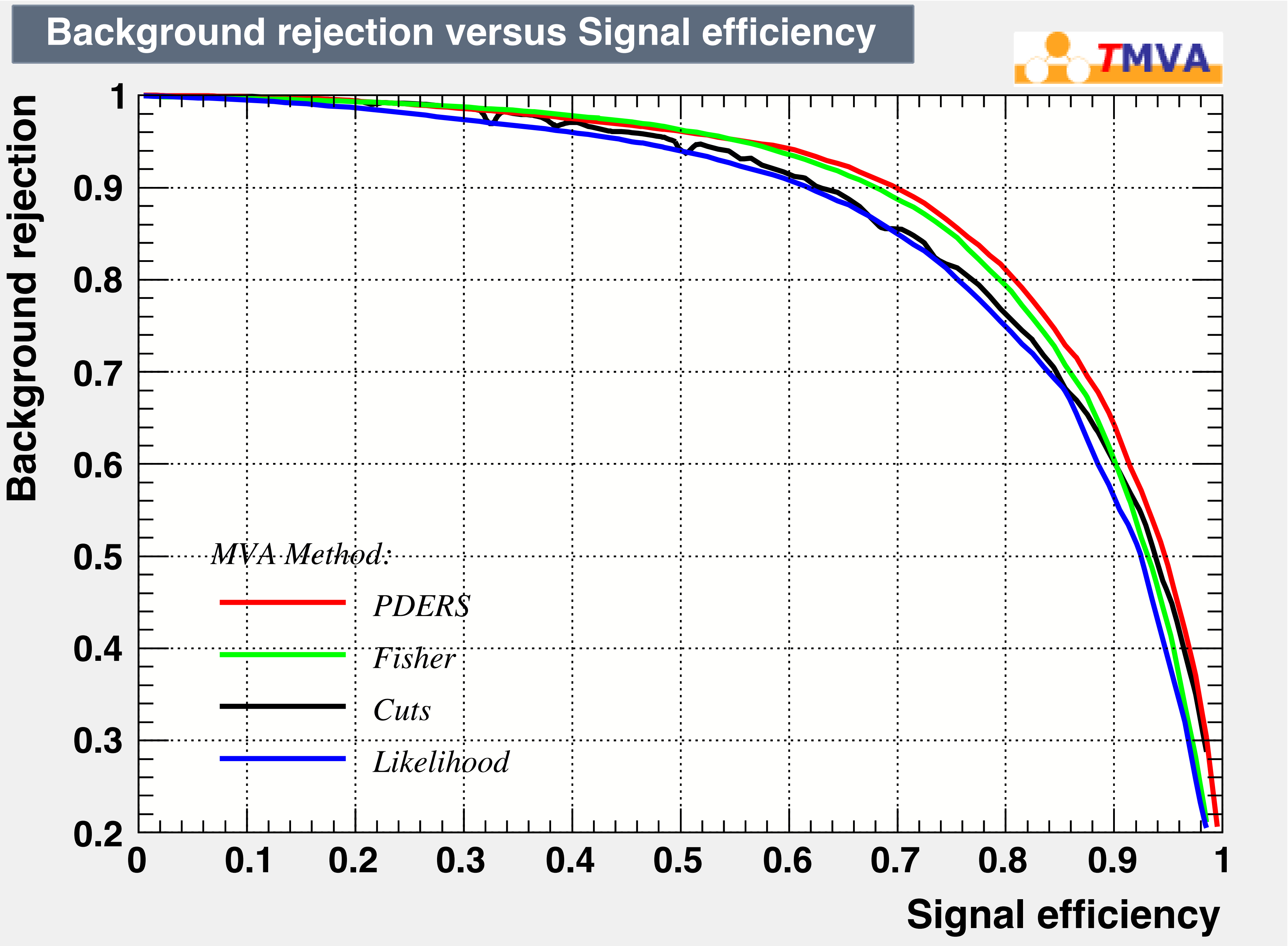


Linearity(BB+qq)



Wow, it seems hopeful!

ECL variables - MVA results



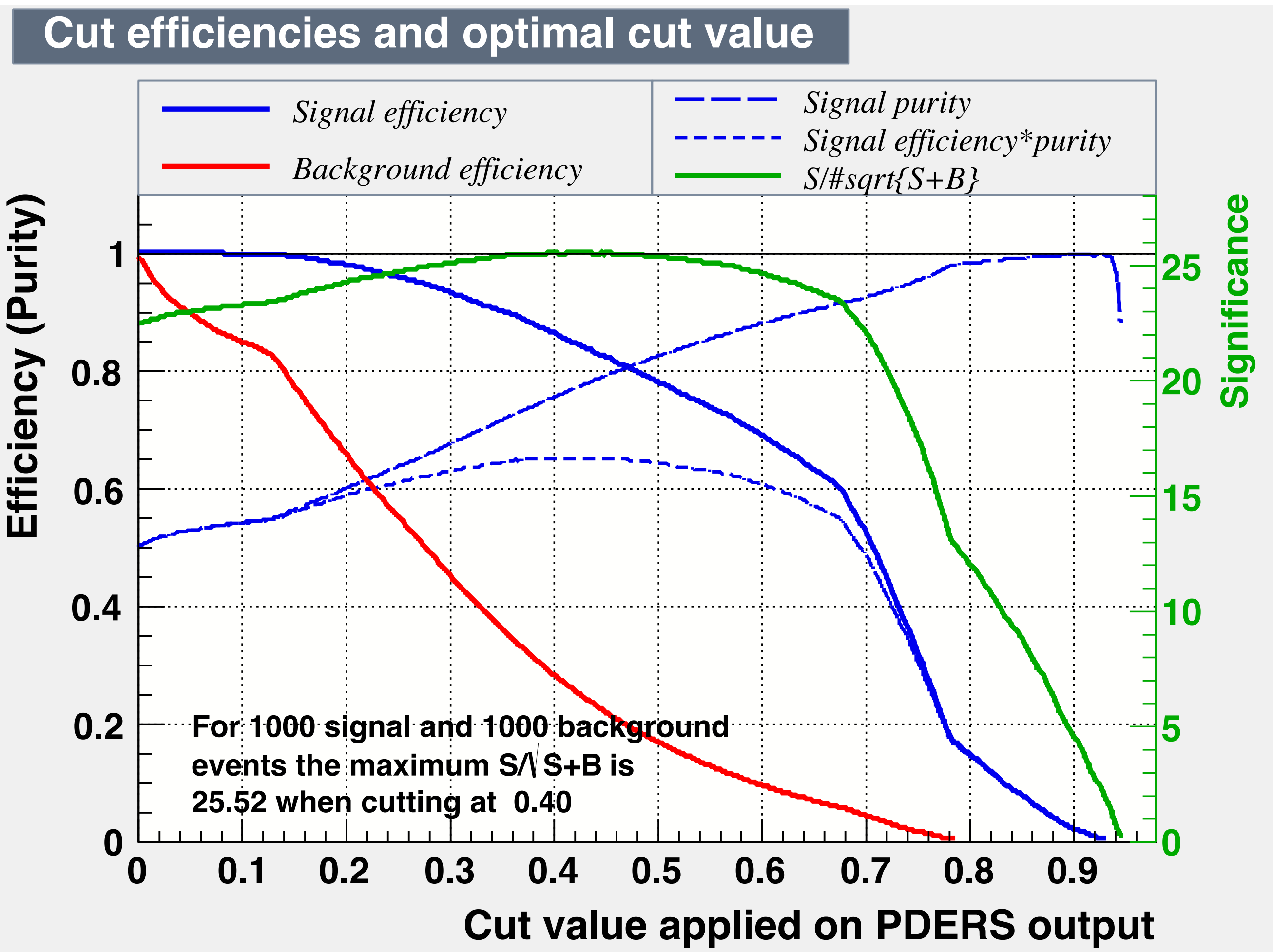
Fisher(linear discriminant) & PDERS(Multi-dimensional Likelihood) are generally better than just Rectangular Cuts.
And MVA can be a good choice for the optimization of rectangular cuts only.

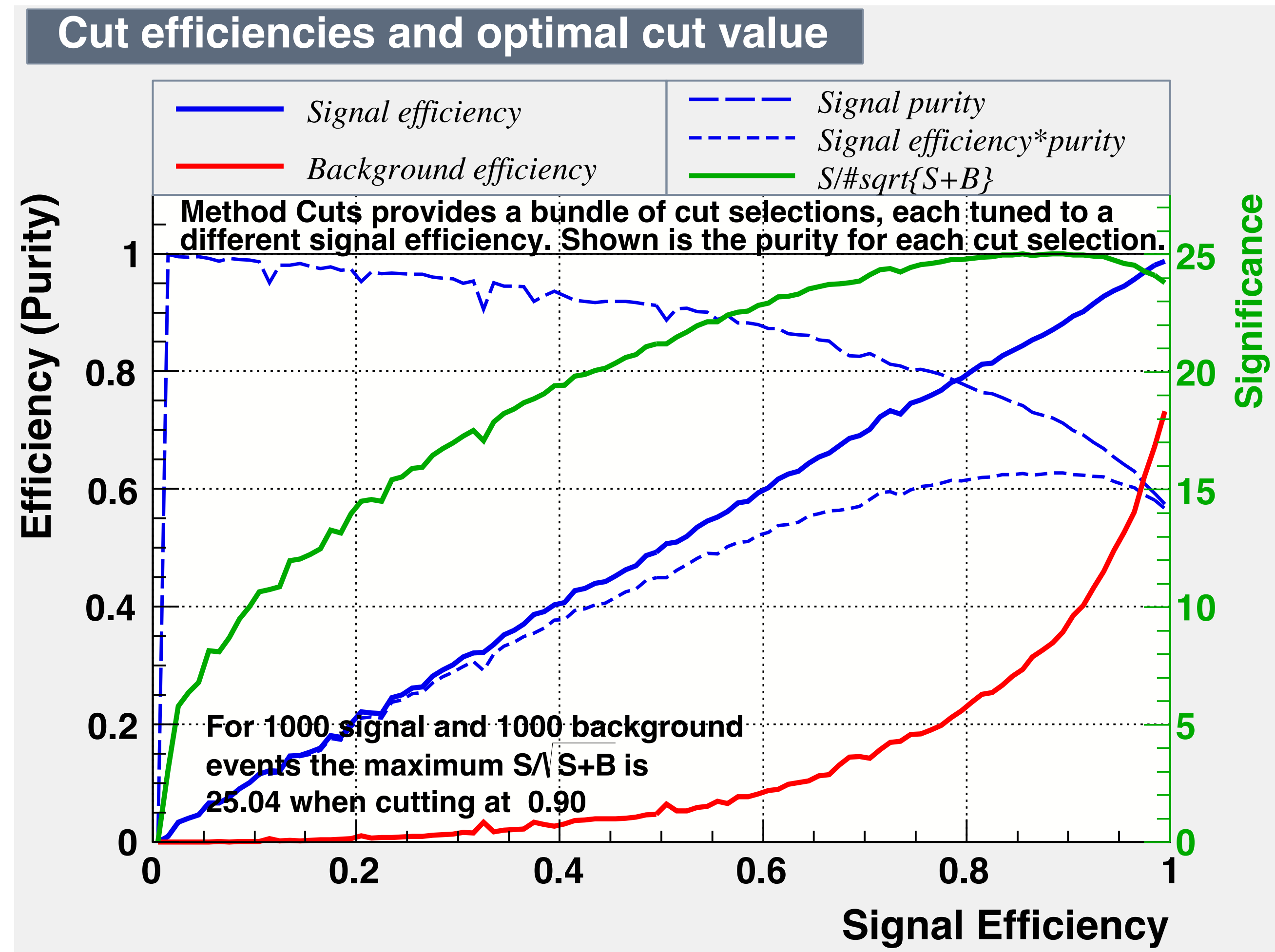
Fisher = -0.602*(width) -0.104*(e9/e25) - 0.534*(mass) +12.38

ECL variables - MVA results

FoM Comparison with Rectangular Cut

of random test set (# of sig , # of bkg) = (1000, 1000)



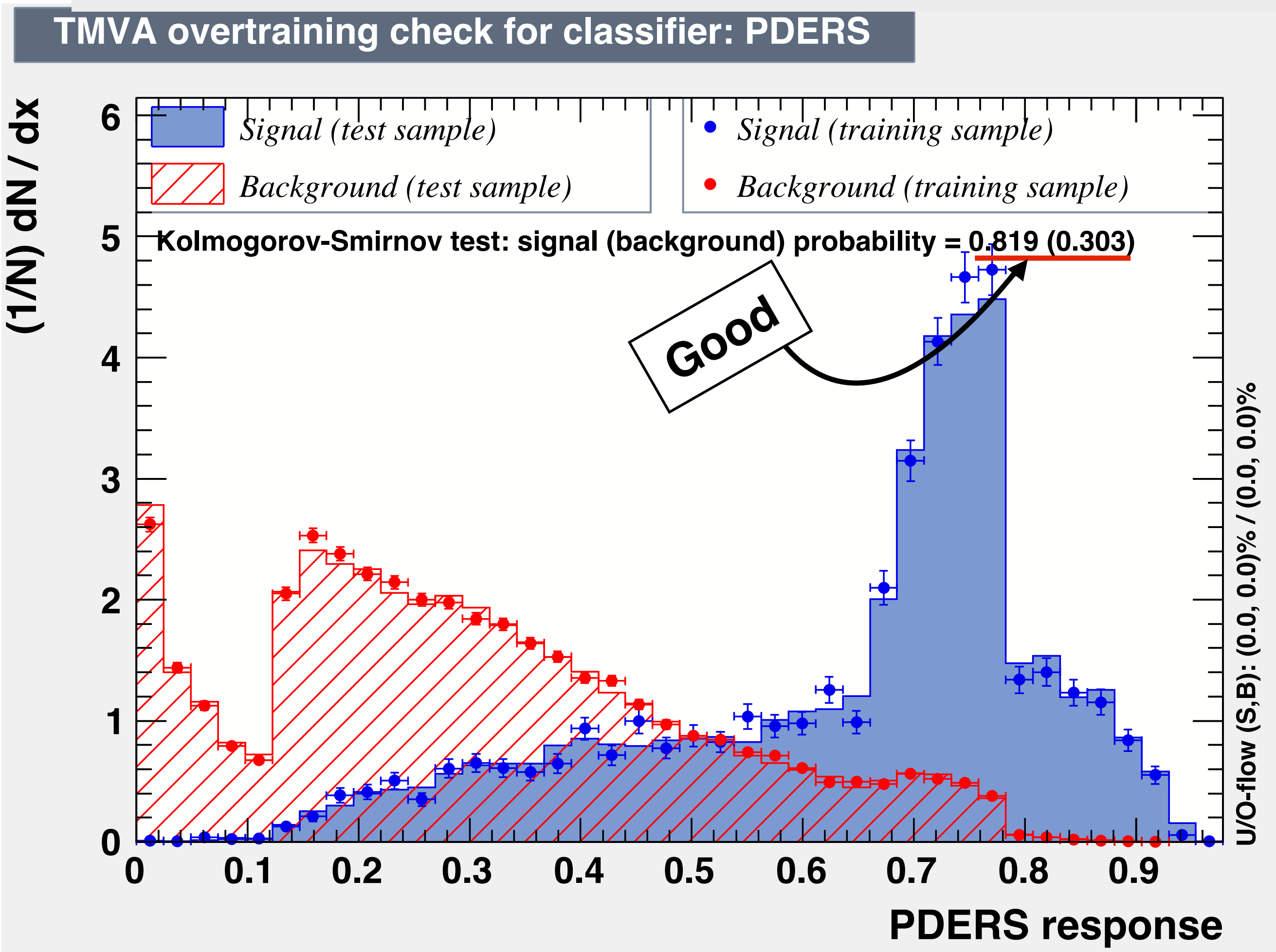


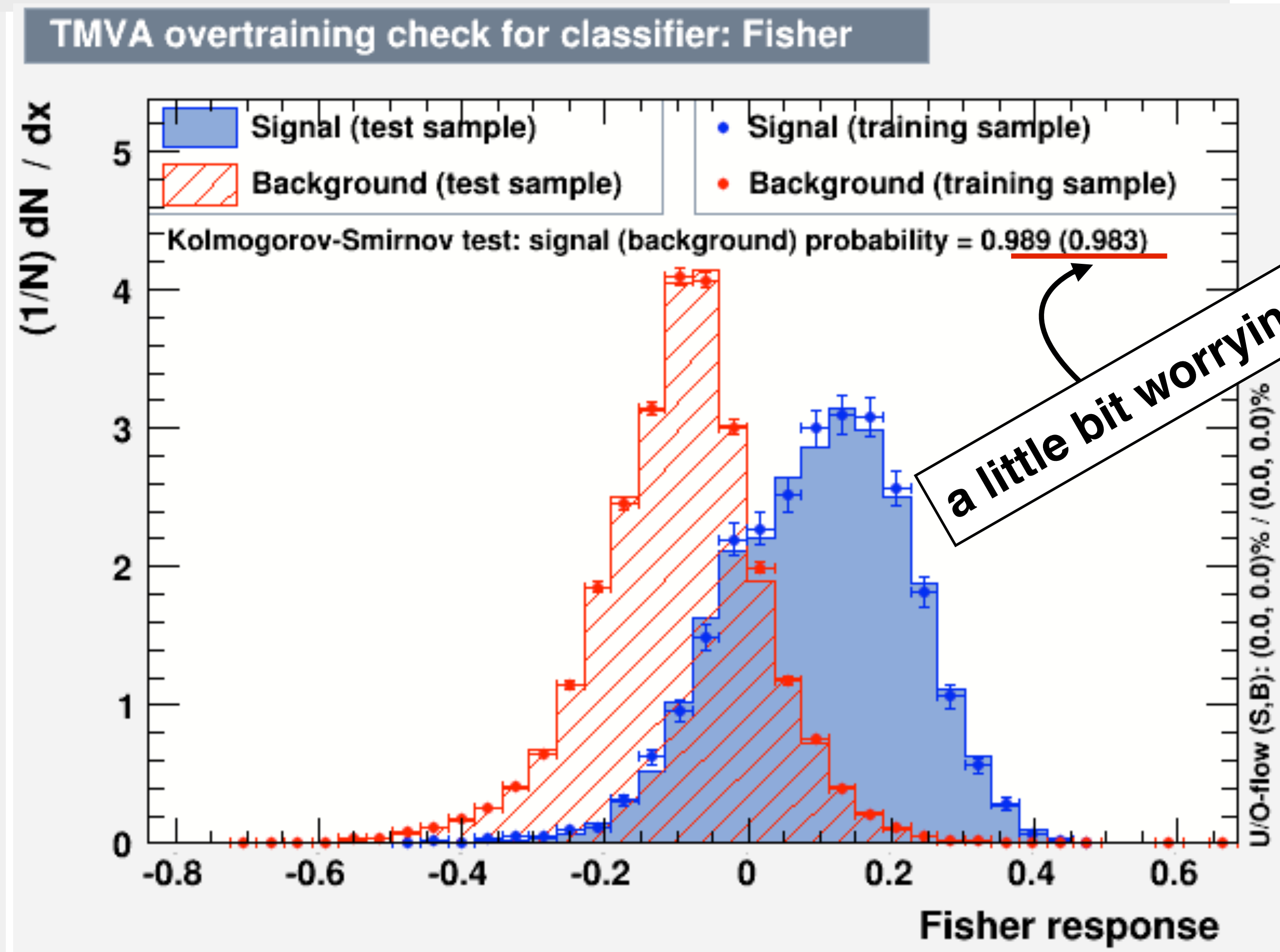
		PDERS	Rect. Cuts	Improvement(PDERS to Cuts)
FoM_{MAX}		25.5169	25.0428	1.89%
at FoM_{MAX}	Sig eff.	0.8641	0.8814	-1.96%
	Bkg Cut-off	0.7173	0.6426	11.62%

ECL variables - MVA results

Overtraining Check - Training Samples vs Testing Samples

(TMVA guide) **Proper training and validation requires three statistically independent data sets: one for the parameter optimi- sation, another one for the overtraining detection, and the last one for the performance validation. In TMVA, the last two samples have been merged to increase statistics. The (usually insignificant) bias introduced by this on the evaluation results does not affect the analysis as far as classification cut efficiencies or the regression resolution are independently validated with data.**





Kolmogorov-Smirnov test : A sort of comparison like χ^2 btw H_0(trained PDF) & H_1(testing PDF)
almost 1 : the same PDF(overtrained), almost 0 : no meaning of training, 0.1~0.9 : usual expectation

PDERS can be a better choice according to K-S test

Signal efficiency for all MC samples

Region 1 , 1.3 < $E\gamma^B$ < 1.8 GeV

Region 1	Sig	BB	qq	rare	ulnu	FoM
911 veto	99.48%	93.02%	95.74%	97.99%	95.65%	0.21
P(π0)&P(η)	87.1%	48.59%	50.32%	41.58%	47.4%	0.26
|cosθthrust|	80.27%	79.01%	22.79%	78.28%	80.39%	0.29
PDERS	86.56%	63.59%	64.22%	69.38%	75.44%	0.31
cosθe	97.46%	93.01%	96.24%	94.69%	78.97%	0.31
tot-efficiency	59%	21.12%	6.79%	20.95%	21.71%	
tot-cutoff	41%	78.88%	93.21%	79.05%	78.29%	

Signal efficiency for all MC samples

Region 11 , 1.8 < $E\gamma^B$ < 2.0 GeV

Region 11	Sig	BB	qq	rare	ulnu	FoM
911 veto	99.48%	83.22%	89.16%	97.28%	92.36%	1.01
P(π0)&P(η)	95.17%	54.79%	59.68%	46.30%	55.10%	1.27
lcosθthrustl	77.35%	79.43%	20.45%	79.29%	81.22%	1.32
PDERS	96.87%	79.9%	79.13%	86.59%	95.65%	1.43
cosθe	99.17%	96.22%	97.38%	96.33%	85.66%	1.44
tot-efficiency	70.64%	27.84%	8.39%	29.79%	33.87%	
tot-cutoff	29.65%	72.16%	91.61%	70.21%	66.13%	

Signal efficiency for all MC samples

Region 111 , 2.0 < $E\gamma^B$ < 2.8 GeV

Region 111	Sig	BB	qq	rare	ulnu	FoM
911 veto	99.48%	38.81%	62.94%	93.92%	79.85%	8.71
P(π0)&P(η)	97.78%	69.4%	68.35%	55.87%	61.45%	10.00
lcosθthrustl	79.27%	80.03%	18.43%	79.64%	79.73%	10.61
PDERS	99.39%	77.25%	80.67%	91.93%	98.33%	11.43
cosθe	99.26%	97.76%	97.78%	97.81%	93.55%	11.45
tot-efficiency	76.07%	16.28%	6.25%	37.58%	35.99%	
tot-cutoff	23.93%	83.72%	93.75%	62.42%	64.01%	

Bkg subtracted yield

Procedure for each bin

(1)

33 streams of signal MC $\xrightarrow{\text{Fitting}}$ PDF_{sig}

9 streams of BB $\xrightarrow{\text{Fitting}}$ PDF_{BB} $\longrightarrow$ $N_{BB,MC}$

5 streams of qq $\xrightarrow{\text{Fitting}}$ PDF_{qq} $\longrightarrow$ $N_{qq,MC}$

$N_{BB,MC}$, $N_{qq,MC}$ $\rightarrow$ $N_{bkg,MC}$

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(2)

PDF_{sig} $\longrightarrow$ generate $DATA\_SET_{sig}$

PDF_{BB}, PDF_{qq} $\longrightarrow$ generate $DATA\_SET_{bkg}$

DATA mimic
(1 stream amount)

---

(3)

$DATA\_SET_{sig}$ $\longrightarrow$ Fitting with Crystal Ball + ARGUS $\longrightarrow$ $N_{sig,true}$

$DATA\_SET_{bkg}$ $\longrightarrow$ Fitting with Crystal Ball + ARGUS $\longrightarrow$ $N_{bkg,true}$

Bkg subtracted yield

Procedure for each bin

(4)

$DATA\_SET_{sig}$ and $DATA\_SET_{bkg}$ —merge→ Fitting with Crystal Ball + ARGUS → $N_{whole,true}$

---

(5)

Subtracted yield = $N_{whole,true} - N_{bkg,MC}$ error = $\mathrm{sqrt}(\sigma_{whole,true}^2 + \sigma_{bkg,MC}^2)$

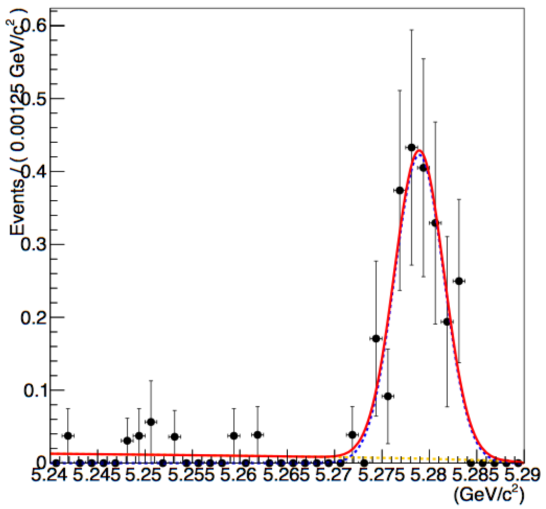
$$\text{True Significance} = \frac{N_{sig,true}}{\sqrt{\sigma^2_{sig,true} + \sigma^2_{bkg,true}}}$$

Due to the low bkg statistics in the ranges beyond 2.5 GeV,

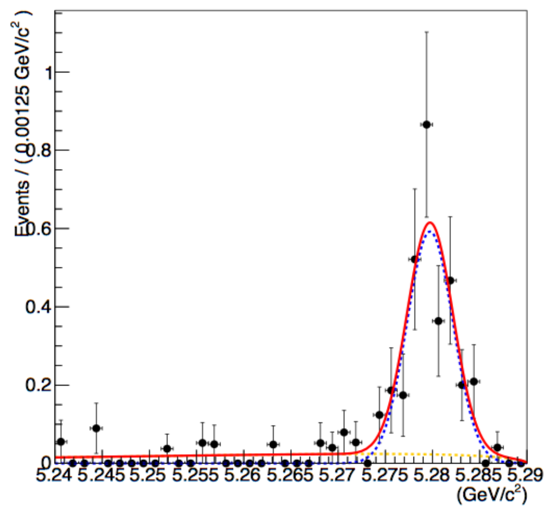
I employed $\text{True Significance} = \dfrac{N_{sig,true}}{\sigma_{whole}}$

M_{bc} Fit results - Signal

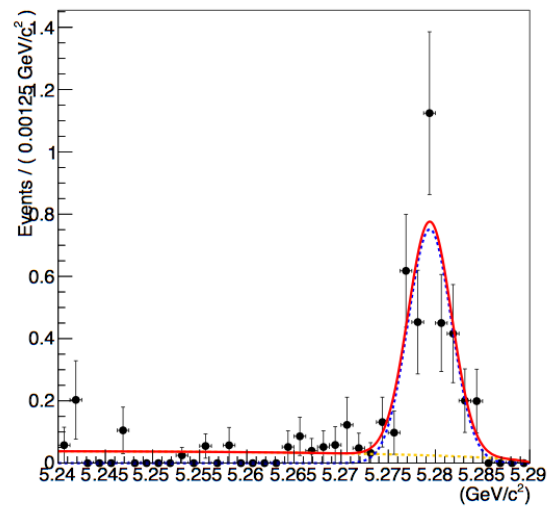
1.3-1.4



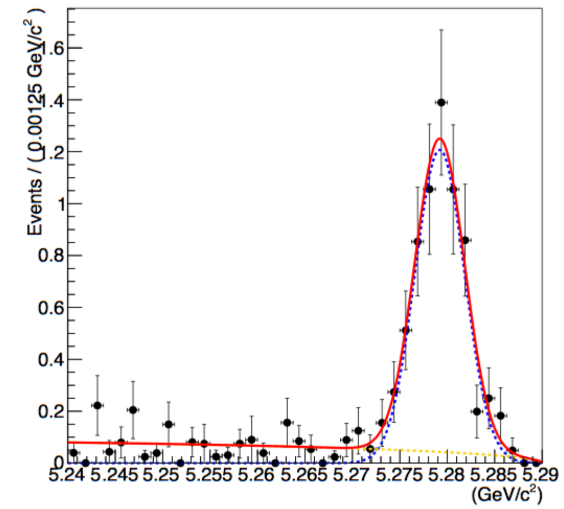
1.4-1.5



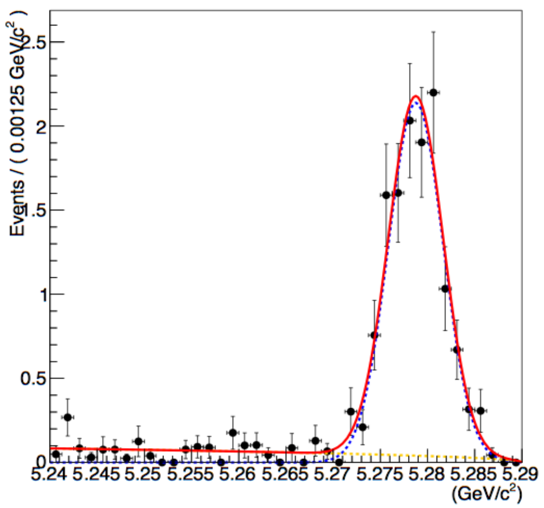
1.5-1.6



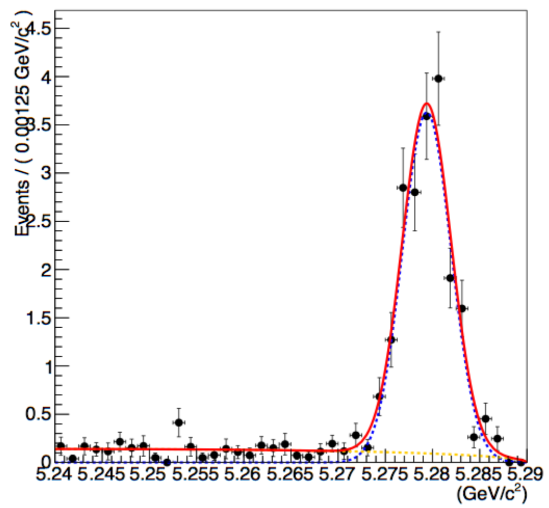
1.6-1.7



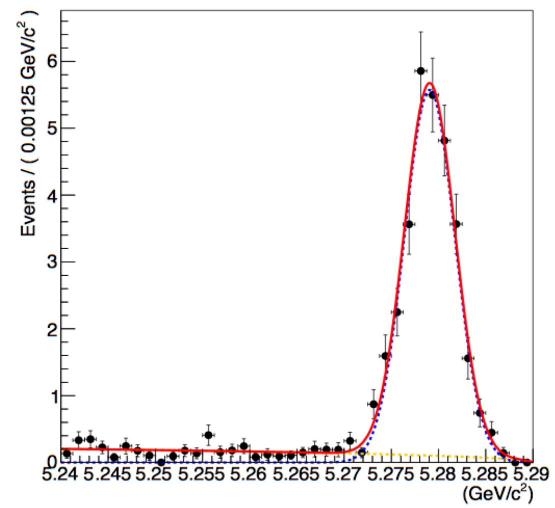
1.7-1.8



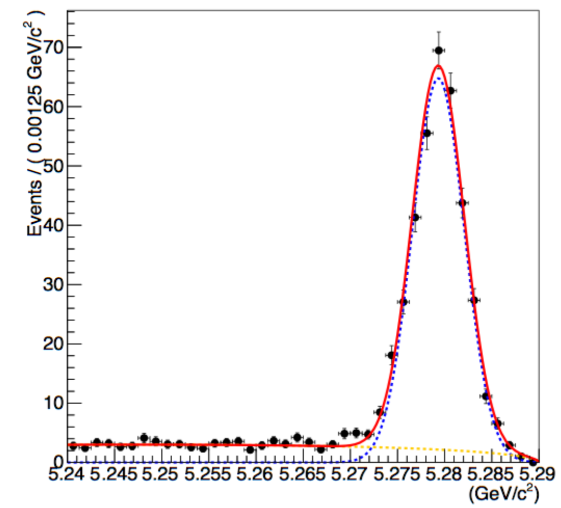
1.8-1.9



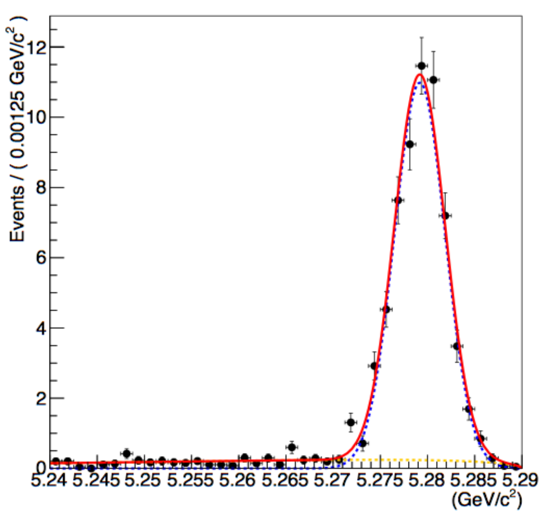
1.9-2.0



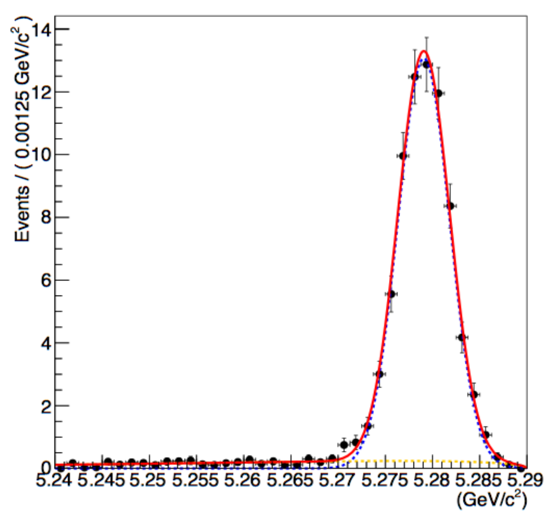
2.0-2.1



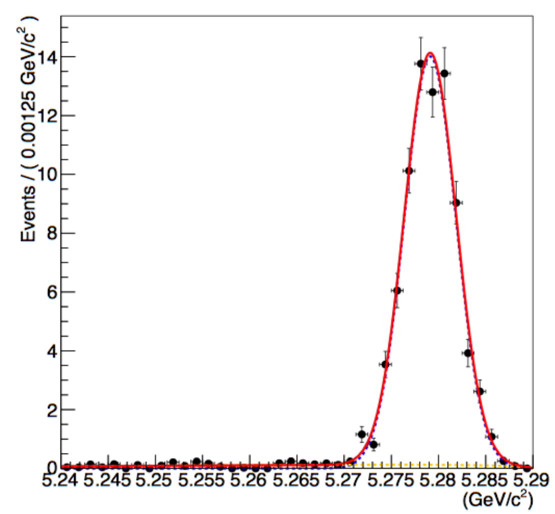
2.1-2.2



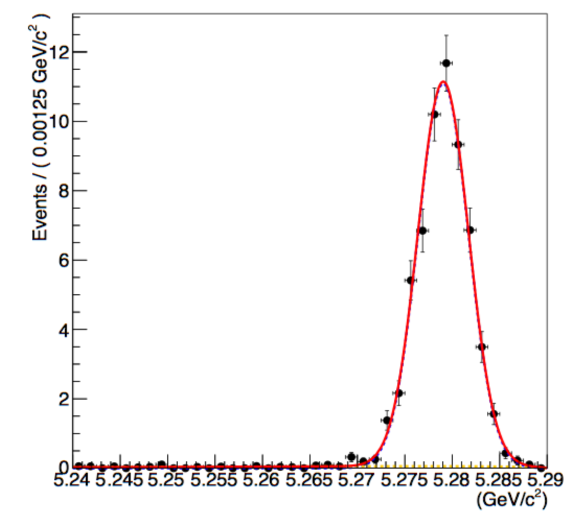
2.2-2.3



2.3-2.4

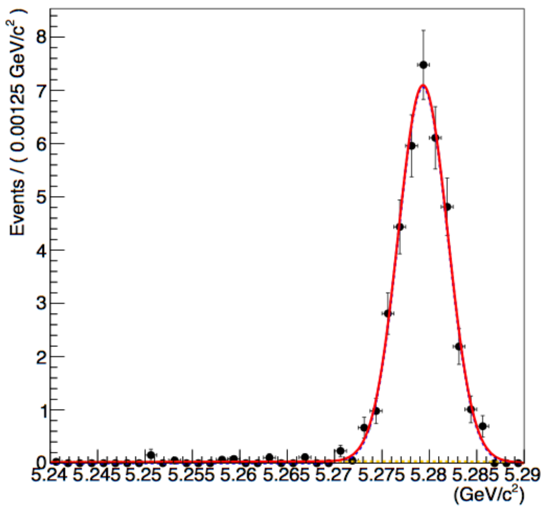


2.4-2.5

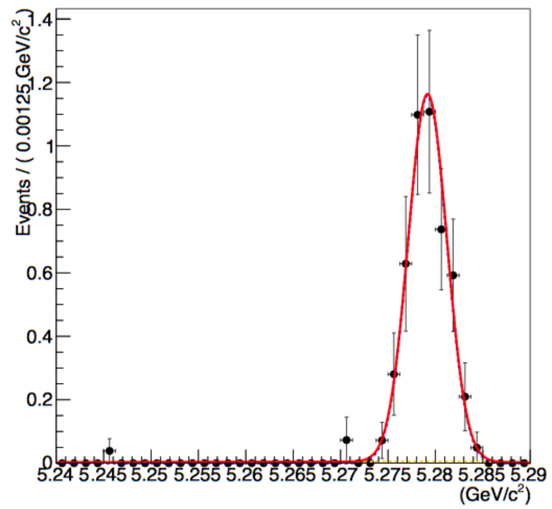


M_{bc} Fit results - Signal

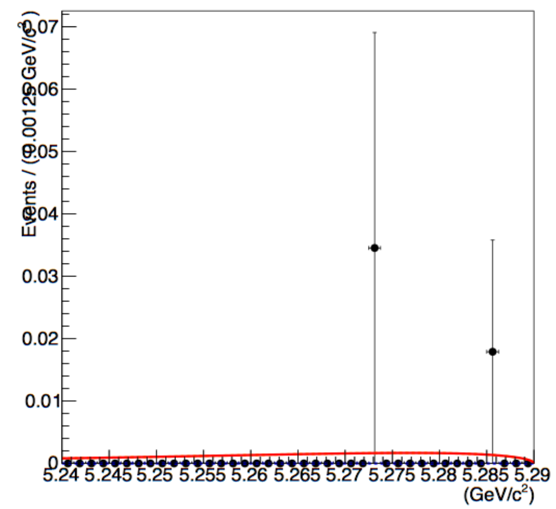
2.5-2.6



2.6-2.7

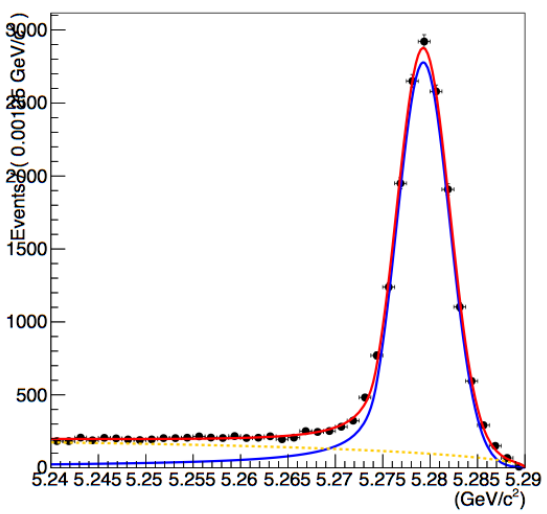


2.7-2.8

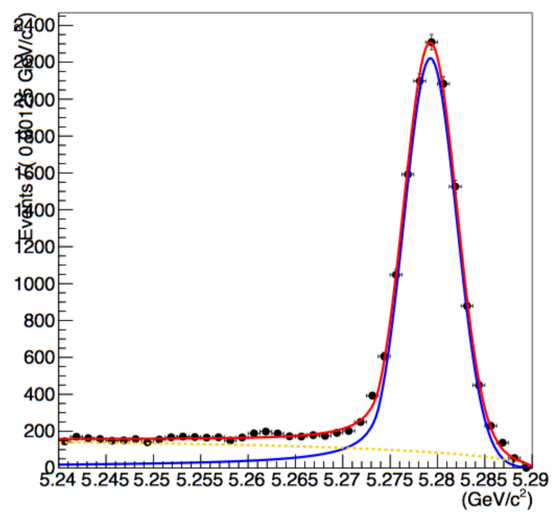


M_{bc} Fit results - BB/qq

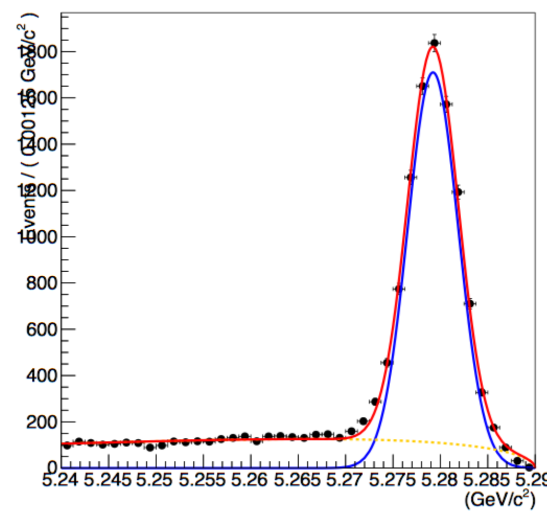
1.3-1.4



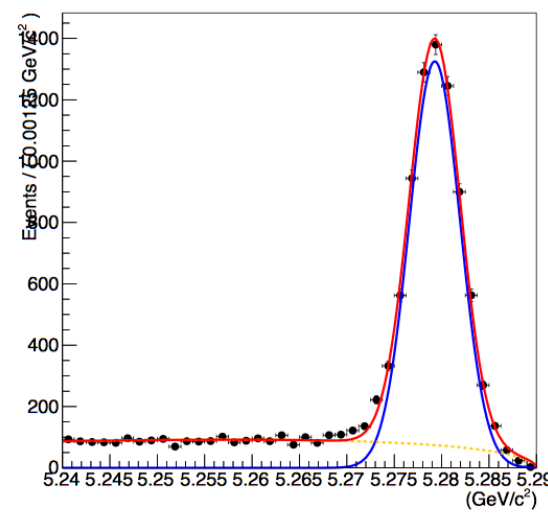
1.4-1.5



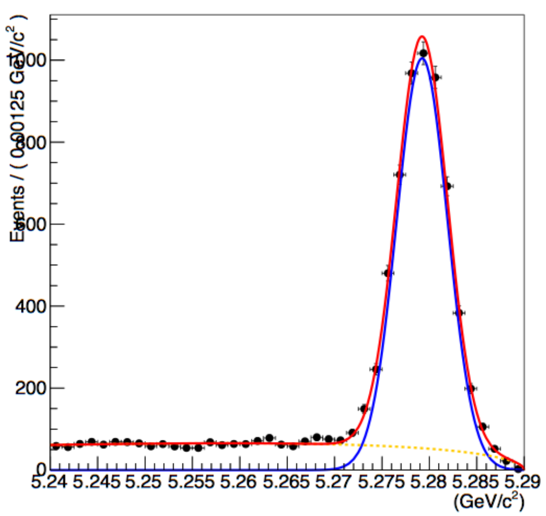
1.5-1.6



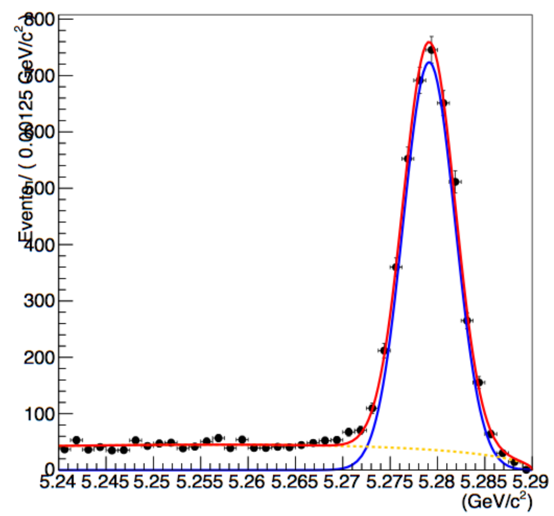
1.6-1.7



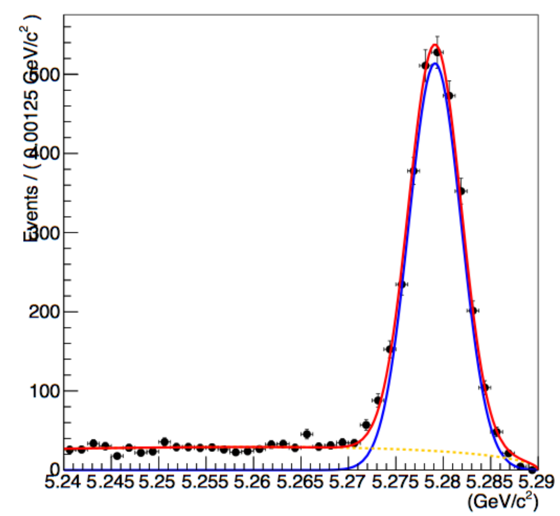
1.7-1.8



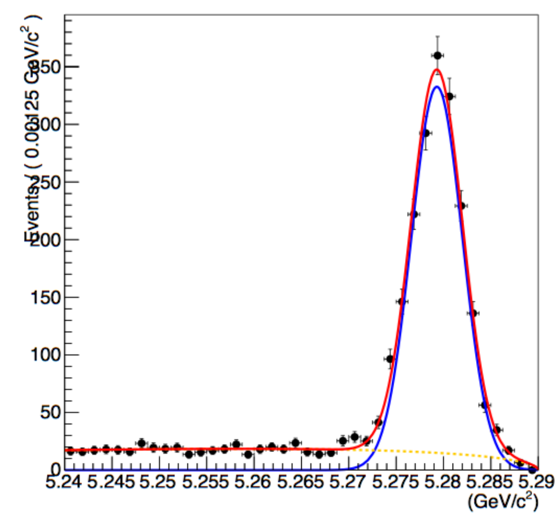
1.8-1.9



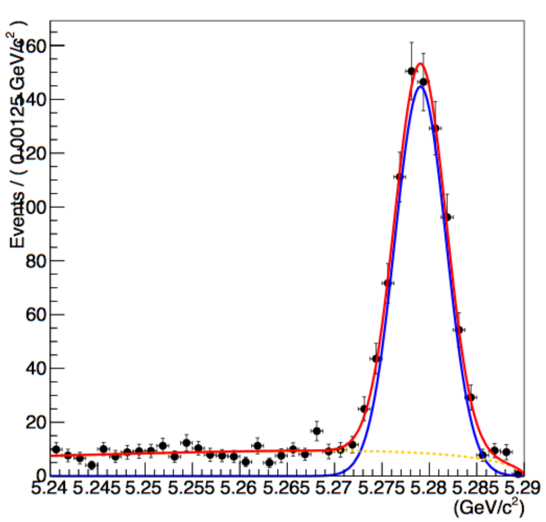
1.9-2.0



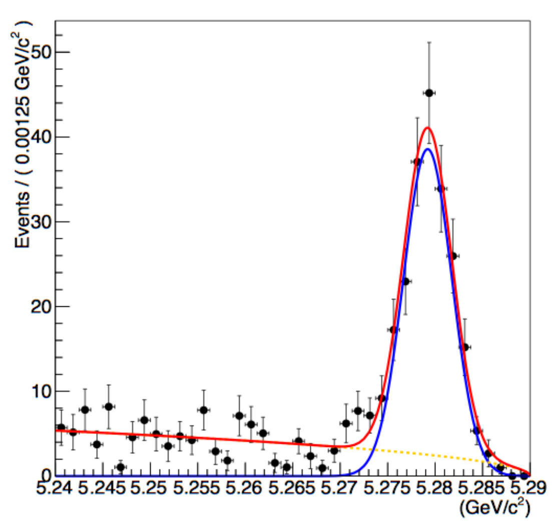
2.0-2.1



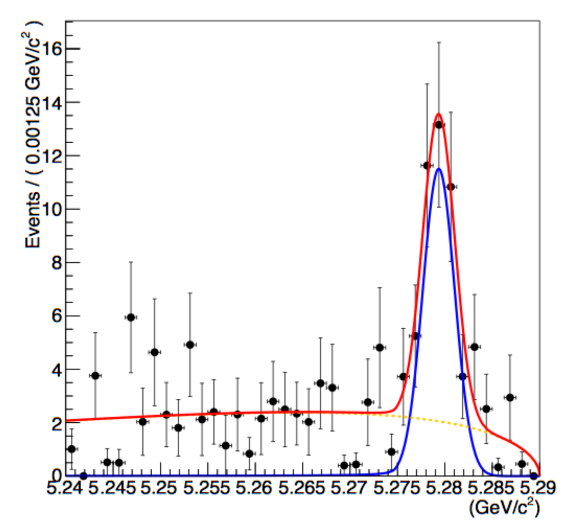
2.1-2.2



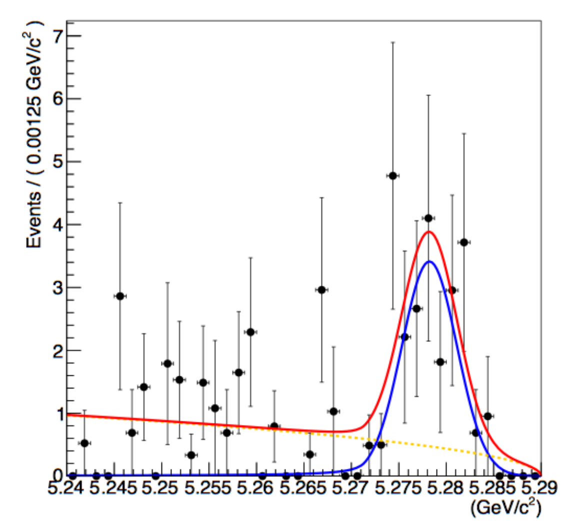
2.2-2.3



2.3-2.4

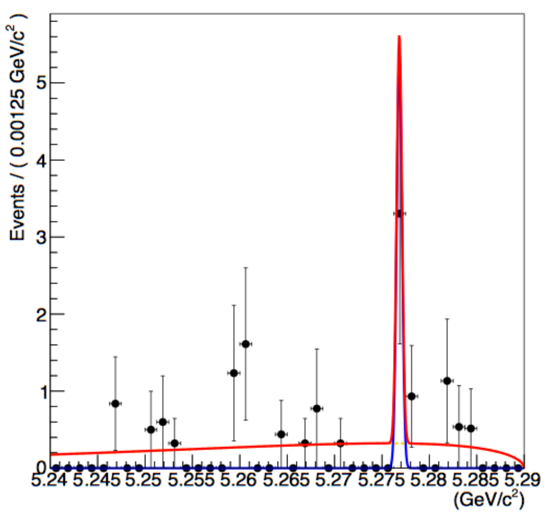


2.4-2.5

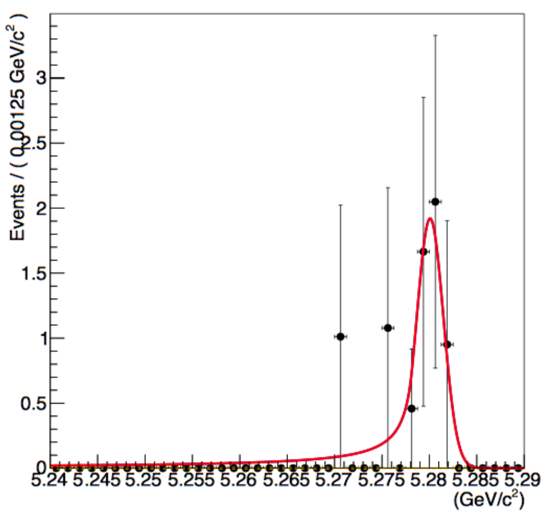


M_{bc} Fit results - BB/qq

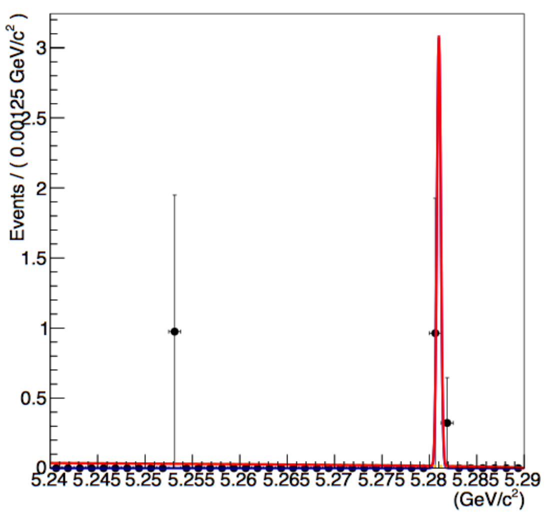
2.5-2.6



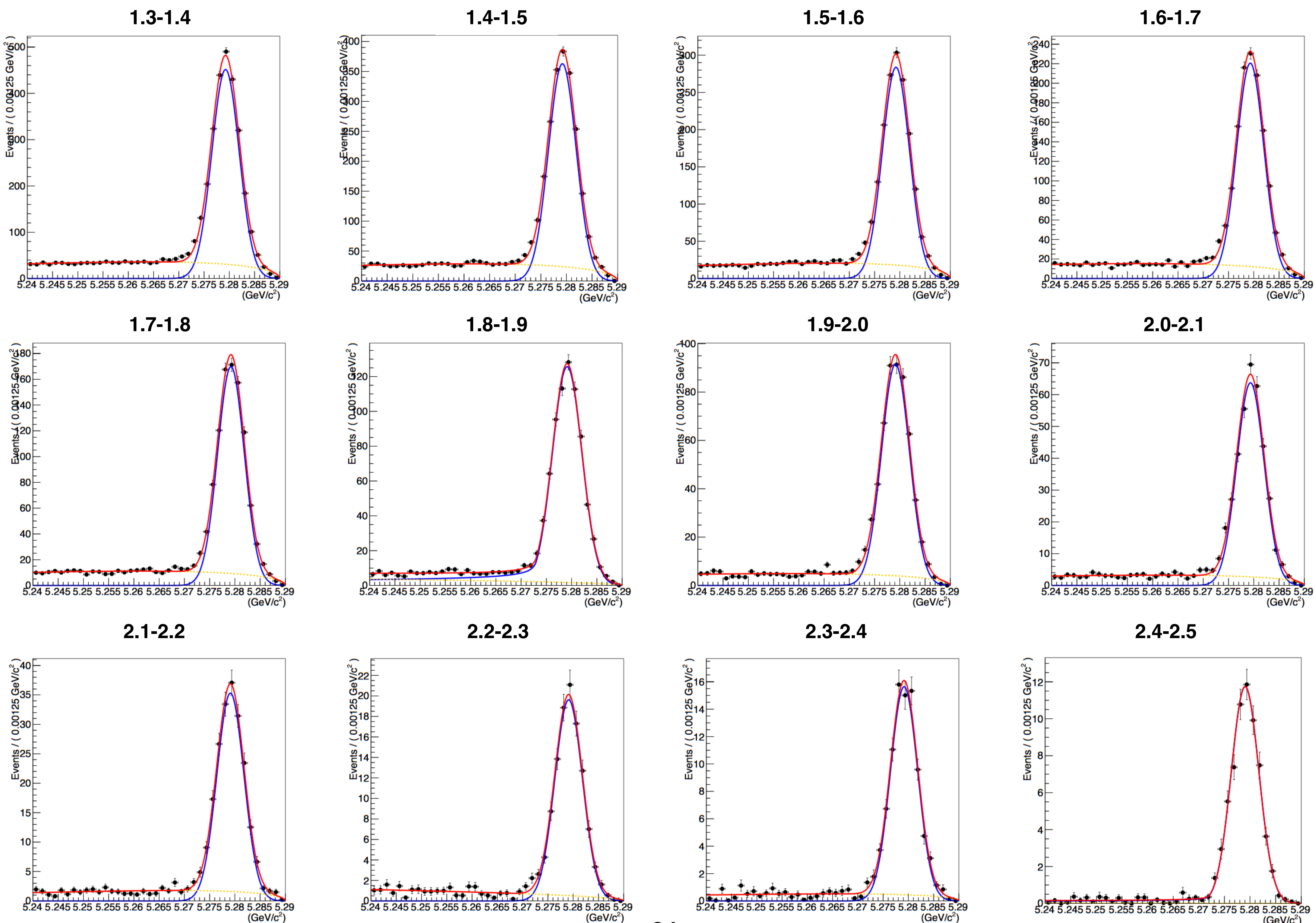
2.6-2.7



2.7-2.8

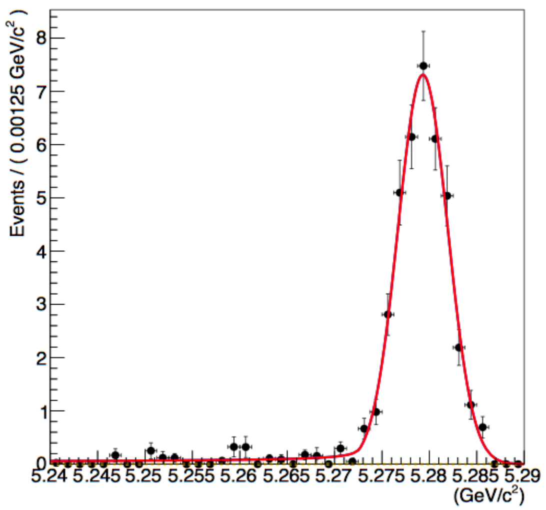


M_{bc} Fit results - Sig + BB/qq

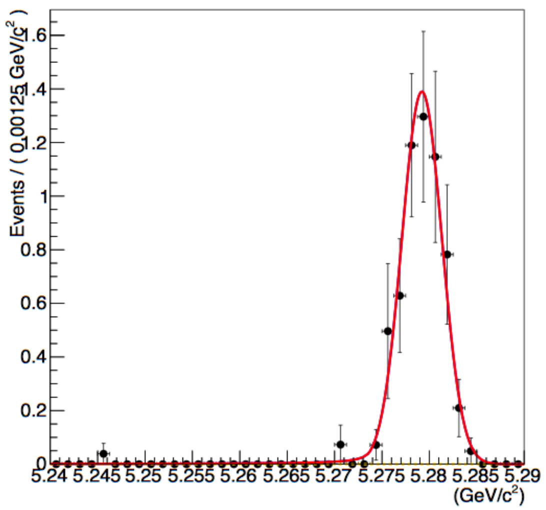


M_{bc} Fit results - Sig + BB/qq

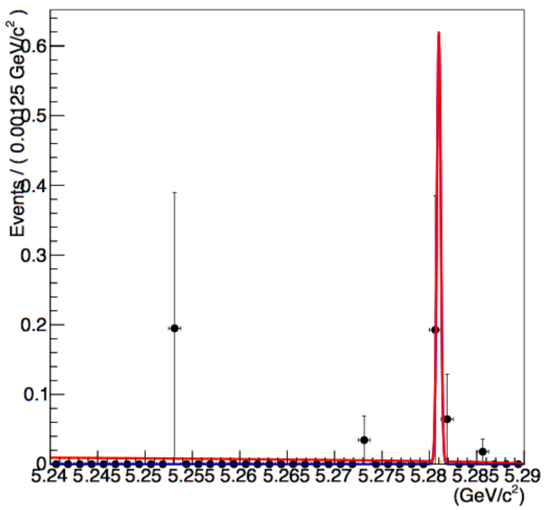
2.5-2.6



2.6-2.7



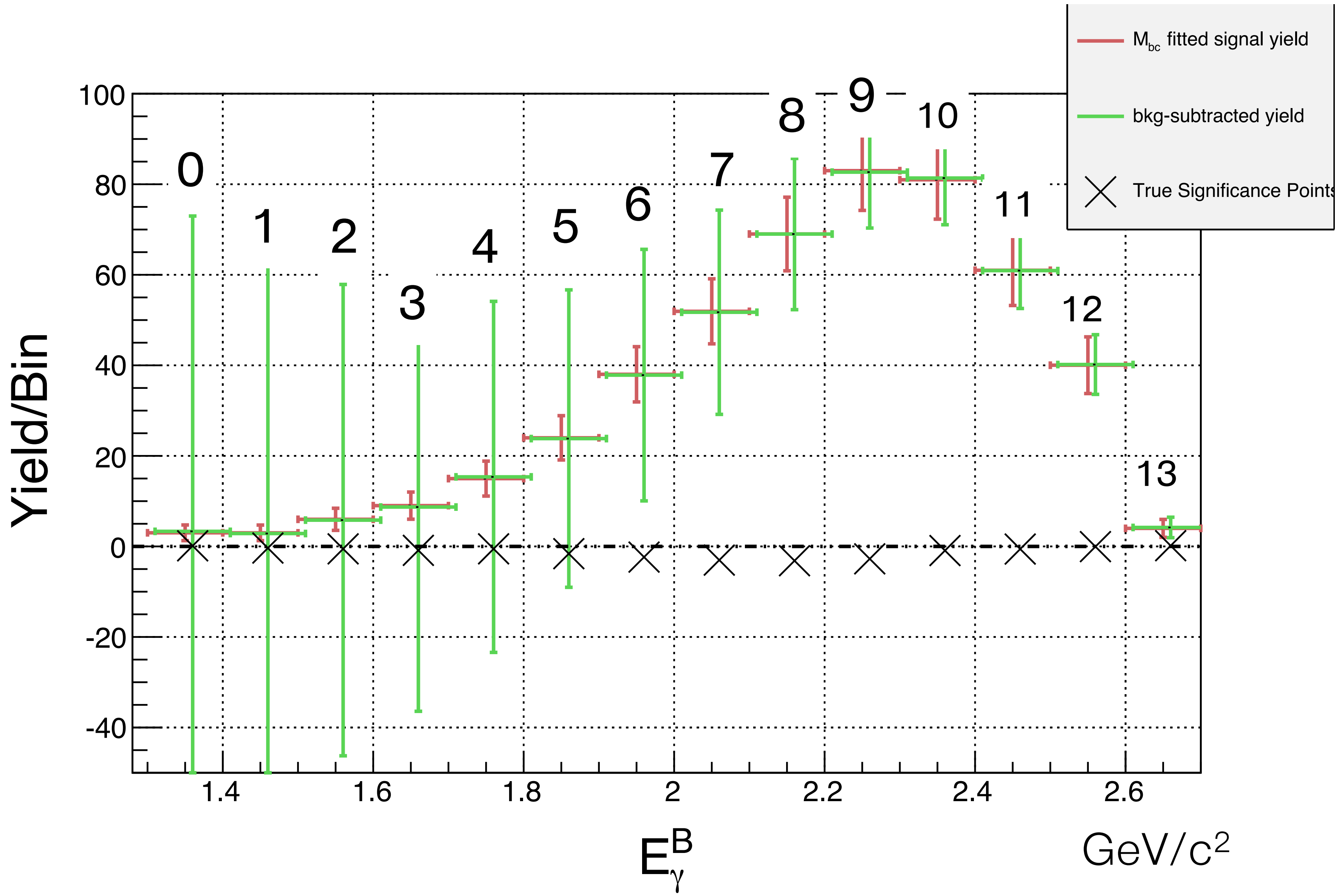
2.7-2.8



Bkg subtracted yield

Significances

Bin#	range	TRUE		Subtract.
0	1.3-1.4	0.046	<	0.048
1	1.4-1.5	0.053	>	0.047
2	1.5-1.6	0.122	>	0.112
3	1.6-1.7	0.212	>	0.193
4	1.7-1.8	0.410	>	0.396
5	1.8-1.9	0.774	>	0.725
6	1.9-2.0	1.447	>	1.362
7	2.0-2.1	2.427	>	2.293
8	2.1-2.2	4.321	>	4.131
9	2.2-2.3	6.917	>	6.693
10	2.3-2.4	7.981	>	7.889
11	2.4-2.5	7.349	>	7.276
12	2.5-2.6	6.104	>	6.096
13	2.6-2.7	1.784	<	1.851



True Significance Points = $N_{sub.}$ - ($\boldsymbol{S}_{true}$ x $\sigma_{sub.}$)

(Each point should be located under the zero line)

$\boldsymbol{S}_{true}$: True Significance = $\dfrac{N_{sig,true}}{\sigma_{whole}}$

Fitted Signal Yield

E	1.3-	1.4-	1.5-	1.6-	1.7-	1.8-	1.9-	2.0-	2.1-	2.2-	2.3-	2.4-	2.5-	2.6-
Yield	3.01	3.00	5.99	9.01	14.98	23.99	38.02	51.93	69.01	83.00	80.99	61.00	40.03	3.99
err	1.73	1.72	2.44	3.00	3.86	4.89	6.10	7.18	8.13	8.77	8.70	7.78	6.26	1.98

Fitted Total Yield

E	1.3-	1.4-	1.5-	1.6-	1.7-	1.8-	1.9-	2.0-	2.1-	2.2-	2.3-	2.4-	2.5-	2.6-
Yield	4,302.0	3,254.0	2,406.0	1,808.0	1,336.0	961.0	690.0	458.0	255.0	144.0	103.0	68.9	43.0	5.0
err	65.6	57.0	49.0	42.5	36.5	31.0	26.3	21.4	16.0	12.0	10.1	8.3	6.6	2.2

Fitted bkg MC yield

E	1.3-	1.4-	1.5-	1.6-	1.7-	1.8-	1.9-	2.0-	2.1-	2.2-	2.3-	2.4-	2.5-	2.6-
Yield	4298.7	3251.2	2400.2	1799.3	1320.6	937.2	652.2	406.3	186.0	61.3	21.6	8.0	2.8	0.8
err	23.4	20.4	17.5	15.1	12.9	10.9	9.0	7.1	4.9	2.9	1.8	1.1	0.7	0.3

Subtracted Yield

E	1.3-	1.4-	1.5-	1.6-	1.7-	1.8-	1.9-	2.0-	2.1-	2.2-	2.3-	2.4-	2.5-	2.6-
Yield	3.33	2.85	5.81	8.70	15.36	23.82	37.84	51.73	69.00	82.68	81.37	60.93	40.18	4.19
err	69.65	60.58	52.06	45.12	38.77	32.85	27.78	22.56	16.70	12.35	10.31	8.37	6.59	2.26