Orbifold Gauge Breaking in 5D

Application to asymptotic GUTs

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Based on 2409.16137 [hep-ph]

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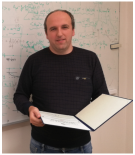

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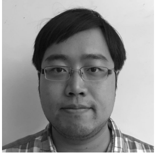

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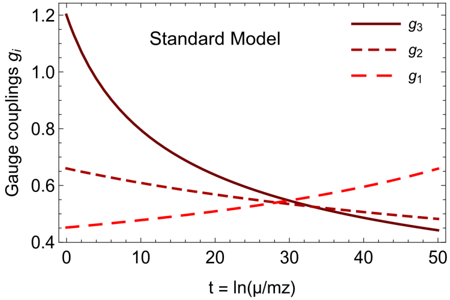
I. Motivation for extra-dimensions: aGUTs

Grand Unified Theories

- The Standard Model relies on local gauge symmetries: $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
- The gauge couplings depends on the energy $\longrightarrow$ running
- At high energy, the gauge couplings seem to merge: $\Lambda_{\mathrm{GUT}} \sim 10^{16}$ GeV



[Cot 2022]

Unique gauge group at high energy?

- Example: $\mathrm{SU}(5)$

$$\psi_{\bar{\mathbf{5}}} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix} \quad \phi_{\mathbf{5}} = \begin{pmatrix} H_1 \\ H_2 \\ H_3 \\ \varphi_+ \\ -\varphi_0 \end{pmatrix} \tag{1}$$

$$\psi_{\mathbf{10}} = \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^c \\ d_1 & d_2 & d_3 & e^c & 0 \end{pmatrix} \tag{2}$$

I. Motivation for extra-dimensions: aGUTs

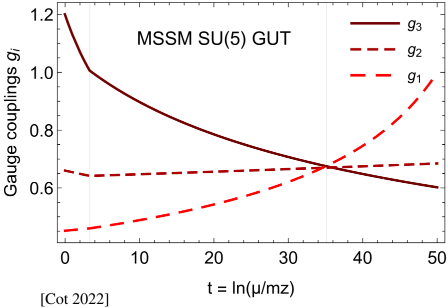
Limits of traditional GUTs

- Assume more gauge bosons → **proton decay**
  Example: SU(5), $\mathbf{24} = G(\mathbf{8},\mathbf{1})_0 \oplus W(\mathbf{1},\mathbf{3})_0 \oplus B(\mathbf{1},\mathbf{1})_0 \oplus X(\mathbf{3},\mathbf{2})_{-5/6} \oplus Y(\mathbf{3},\mathbf{2})_{5/6}$, X and Y allow $p \to e^+ + \pi^0$

- Doublet-triplet splitting: mechanisms to make H heavier than φ

- Large matter representations to break the GUT gauge group → Landau pole

- Need extra scalar/SUSY to have exact unification:



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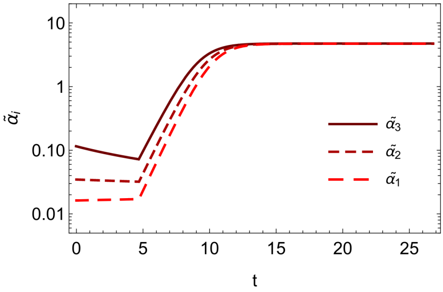
I. Motivation for extra-dimensions: aGUTs

Asymptotic Unification

- New paradigm: No exact unification, gauge couplings tend to the same UV fixed point
- Realized in asymptotic safe theories or **extra dimension** (power law running)



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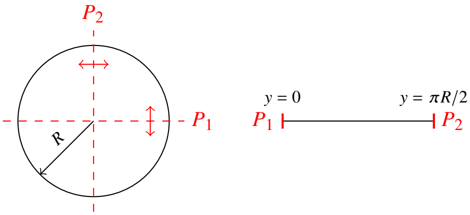
II. Introduction to Orbifolds and GHU

Adding 1 extra dimension

- Higher dimensional QFT:
  - Gauge group $\mathcal{G}$ (GUT)
  - $\mathcal{M} = \mathbb{R}^4 \times C$, coordinates $\{x^\mu, y\}$
- Extra dimension compactified on a circle/ interval, radius R



- Mod the theory by $\mathbb{Z}_2$ on each boundary:
  $\mathbb{Z}_2 : \quad y \sim -y$
  $\mathbb{Z}_2' : \quad y' \sim -y'$ with $y' = y + \pi R/2$
- Orbifold $\rightarrow$ Manifold with boundaries (fixed points), in 5d we get $\quad C = S^1/\left(\mathbb{Z}_2 \times \mathbb{Z}_2'\right)$
- Action of the parity on the fields Φ:

$$P : \; \Phi(x^\mu, y) \sim P\,\Phi(x^\mu, -y), \qquad P' : \; \Phi(x^\mu, y') \sim P'\,\Phi(x^\mu, -y') \tag{3}$$

- $P^2 = \mathbb{I}$ and $P'^2 = \mathbb{I} \rightarrow \quad$ diagonal P and P', eigenvalues $\{-1, 1\}$
- Classify fields according to $(P, P') = (\pm, \pm)$

II. Introduction to Orbifolds and GHU

Kaluza-Klein (KK) decomposition

- Kaluza-Klein: each field can be decomposed along the extra dimension:

$$\phi(x,y) = \sum_{n=0}^{+\infty} \phi^{(n)}(x)\, f_n(y)$$

- Use EOM and parities to compute $f_n(y)$:

$$f_n(y) = \begin{cases} (++) & \frac{1}{\sqrt{2\pi R}} + \frac{1}{\sqrt{\pi R}} \cos\left(\frac{ny}{R}\right) & \to m_n = \frac{n}{R} & \to \text{ zero mode} \\ (+-) & \frac{1}{\sqrt{\pi R}} \cos\left(\frac{(n+1/2)y}{R}\right) & \to m_n = \frac{n+1/2}{R} & \\ (-+) & \frac{1}{\sqrt{\pi R}} \sin\left(\frac{(n+1/2)y}{R}\right) & \to m_n = \frac{n+1/2}{R} & \\ (--) & \frac{1}{\sqrt{\pi R}} \sin\left(\frac{(n+1)y}{R}\right) & \to m_n = \frac{n+1}{R} & \end{cases} \tag{4}$$

- $E \ll 1/R \to$ 4D effective field theory, KK towers integrated out, only zero modes remain

- Action of the parities on gauge bosons:

$$A_\mu \to P.\,A_\mu.\,P \qquad A_5 \to -P.\,A_5.\,P \tag{5}$$

- Action of the parities on fermions $\psi_L = \frac{1}{2}(1-\gamma_5)\psi$ and $\psi_R = \frac{1}{2}(1+\gamma_5)\psi$:

$$\psi_L \to \eta_\psi P.\,\psi_L.\,P \qquad \psi_R \to -\eta_\psi P.\,\psi_R.\,P \tag{6}$$

- η_ψ: field specific overall sign

- Each parity breaks the GUT gauge group $\mathcal{G}$ at low energies:

$$\left.\begin{array}{l} P_1 : \mathcal{G} \to \mathcal{H}_1 \\ P_2 : \mathcal{G} \to \mathcal{H}_2 \end{array}\right\} (P_1, P_2) : \mathcal{G} \to \mathcal{H}_1 \cap \mathcal{H}_2$$

- Most general parities ($p + q + r + s = N$):

$$\begin{aligned} P_1 &= \mathrm{diag}(+1, \cdots, +1, +1, \cdots, +1, -1, \cdots, -1, -1, \cdots, -1), \\ P_2 &= \mathrm{diag}(\underbrace{+1, \cdots, +1}_{p}, \underbrace{-1, \cdots, -1}_{q}, \underbrace{+1, \cdots, +1}_{r}, \underbrace{-1, \cdots, -1}_{s}), \end{aligned} \quad (7)$$

- Action of the parities on A_μ:

$$(P_1, P_2)(A_\mu) = \begin{array}{c} \begin{matrix} \quad p \quad & \quad q \quad & \quad r \quad & \quad s \quad \end{matrix} \\ \begin{pmatrix} (+,+) & (+,-) & (-,+) & (-,-) \\ (+,-) & (+,+) & (-,-) & (-,+) \\ (-,+) & (-,-) & (+,+) & (+,-) \\ (-,-) & (-,+) & (+,-) & (+,+) \end{pmatrix} \begin{matrix} p \\ q \\ r \\ s \end{matrix} \end{array} \quad (8)$$

- At low energy, only the $(++)$ degrees of freedom remain:

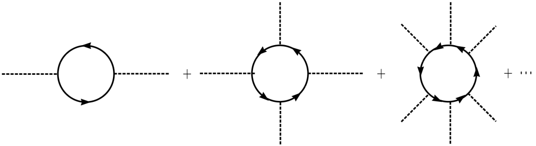
$$\mathrm{SU}(N) \to \mathrm{SU}(p) \times \mathrm{SU}(q) \times \mathrm{SU}(r) \times \mathrm{SU}(s) \times \mathrm{U}(1)^3 \quad (9)$$

II. Introduction to Orbifolds and GHU

Gauge-Higgs Unification

- The 5D gauge field A_M decomposes as a 4D vector field A_μ and a scalar A_5
- Gauge invariance forbids tree level potential for A_5 but consider quantum corrections $\rightarrow$ Higgs mechanism, Gauge-Higgs unification
- 1-loop effective potential (Coleman Weinberg) for I fields:

$$V_{\text{eff}}(A_5) = \frac{1}{2}\sum_I (-1)^{F_I} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \log\left(p^2 + m_I^2\right) \quad \text{with} \quad F_I = \{0, 1\}$$



[Quiros 1999]

- KK expansion:

$$m_{I,n}^2 = \frac{(n+a_I)^2}{R^2} \quad \text{for} \quad (++)/(--)$$
$$m_{I,n}^2 = \frac{\left(n+\frac{1}{2}+a_I\right)^2}{R^2} \quad \text{for} \quad (+-)/(-+)$$
with $a_I \propto \langle A_5 \rangle$

III. Orbifold stability

Gauge Transformation and VEV

- Consider a VEV for A_5: $A_5 = \langle A_5 \rangle T$
- Perform gauge transformation along the VEV: $U = \exp\left(i \langle A_5 \rangle T\, y\right)$

$$A_M^U = U A_M U^{-1} + U^{-1} \partial_5 U, \qquad \phi^U = U\phi \tag{10}$$

- This gauge transformation cancels A_5 VEV:

$$-\left(\partial_y - i \langle A_5 \rangle T\right)^2 \phi = m_n^2 \phi \quad \longrightarrow \quad -\partial_y^2 \phi = m_n^2 \phi \tag{11}$$

- As a counterpart, the boundary conditions of the fields are modified:

$$\phi(2\pi R) = \exp\left(-i 2\pi R \langle A_5 \rangle T\right) \phi(0) \tag{12}$$

- In practice, with CW potential, $\langle A_5 \rangle = 1/2R$, $\phi(2\pi R) = -\phi(0)$
  $\rightarrow$ Modify the signs of the parity matrices
- Use gauge transformation to build equivalence class of parities:

$$(p, q, r, s) \sim (p-1, q+1, r+1, s-1) \sim (p+1, q-1, r-1, s+1) \tag{13}$$

III. Orbifold stability

- Contributions to the effective potential coming from all kind of fields:

$$V_{\text{eff}}(A_5) = V_{\text{eff}}^{\text{gauge}}(A_5) + V_{\text{eff}}^{\text{fermion}}(A_5) + V_{\text{eff}}^{\text{scalar}}(A_5) \tag{14}$$

- $V_{\text{eff}}^{\text{gauge}}(A_5)$ can't lead to the breaking of the gauge group ($\langle A_5 \rangle \neq 0$) as it would make the theory inconsistent

- If it does, can use gauge transformation to remove the VEV $\rightarrow$ modification on the parities, different breaking patterns

Orbifold Stability

$V_{\text{eff}}^{\text{gauge}}(A_5)$ must have his minimum $\langle A_5 \rangle = 0$

Goals

- Classify all the allowed gauge configurations for every GUT group in the bulk ($\text{SU}(N)$, $\text{Sp}(2N)$, $\text{SO}(N)$, Exceptionnal groups...)

- Use those models to study the phenomenology of aGUTs

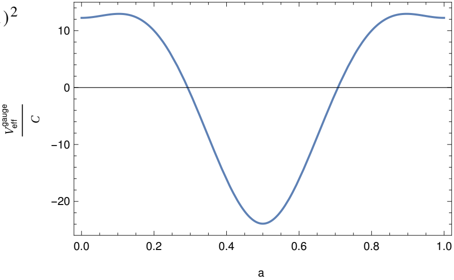
III. Orbifold Stability

Example with SU(6)

$\mathrm{SU}(6) \to \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)^2$

$\phi_{A_5} = (\mathbf{1}, \mathbf{2})_{1/2,3}$

$p = 2$
$q = 3$
$s = 1$

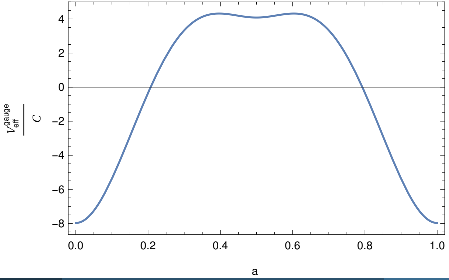


→ **UNSTABLE**

$\mathrm{SU}(6) \to \mathrm{SU}(4) \times \mathrm{U}(1) \times \mathrm{U}(1)$

$\phi_{A_5} = \mathbf{4}_{5,1}$

$p = 1$
$q = 4$
$r = 1$



→ **STABLE**

III. Orbifold Stability

Classification of stable orbifolds

Model	Breaking pattern	Stability criteria
$\mathrm{SU}(N)$	$\mathrm{SU}(N) \to \mathrm{SU}(A) \times \mathrm{SU}(N-A) \times \mathrm{U}(1)$	stable $\forall$ A
	$\mathrm{SU}(N) \to \mathrm{SU}(p) \times \mathrm{SU}(q) \times \mathrm{SU}(s) \times \mathrm{U}(1)^2$	$p \geq N/2$
$\mathrm{Sp}(2N)$	$\mathrm{Sp}(2N) \to \mathrm{Sp}(2A) \times \mathrm{Sp}(2(N-A))$	stable $\forall$ A
	$\mathrm{Sp}(2N) \to \mathrm{Sp}(2p) \times \mathrm{Sp}(2q) \times \mathrm{Sp}(2s)$	$p \geq N/2$
	$\mathrm{Sp}(2N) \to \mathrm{SU}(p) \times \mathrm{SU}(q) \times \mathrm{U}(1)^2$	stable $\forall p, q$
$\mathrm{SO}(2N)$	$\mathrm{SO}(2N) \to \mathrm{SO}(2A) \times \mathrm{SO}(2(N-A))$	stable $\forall$ A
	$\mathrm{SO}(2N) \to \mathrm{SO}(2p) \times \mathrm{SO}(2q) \times \mathrm{SO}(2s)$	$p \geq N/2$
	$\mathrm{SO}(2N) \to \mathrm{SU}(p) \times \mathrm{SU}(q) \times \mathrm{U}(1)^2$	stable $\forall p, q$

IV. Example: SU(6) aGUT

- Choice of parities leading to a stable orbifold:

$$P_1 = \mathrm{diag}(+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1)\,,$$
$$P_2 = \mathrm{diag}(\underbrace{+1, \cdots, +1}_{p=3}, \underbrace{-1, \cdots, -1}_{q=2}, \underbrace{-1, \cdots, -1}_{s=1})\,. \tag{15}$$

- Orbifold breaking: $\mathrm{SU}(6) \to \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)^2$

- Gauge-Higgs: $\phi_{A_5} = (\mathbf{3}, \mathbf{1})_{-1/3,3}$

- Fermion representations leading to SM zero modes:

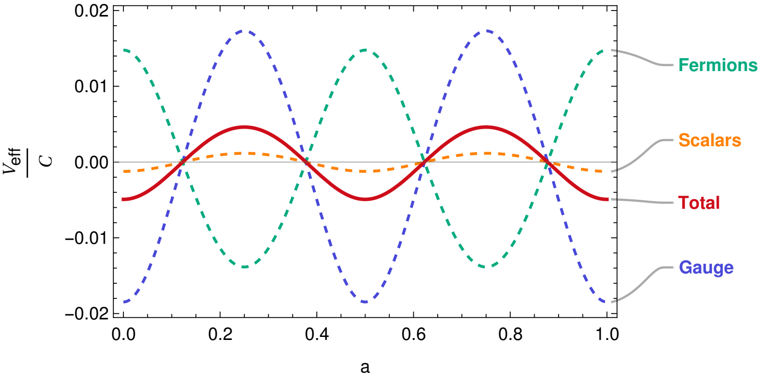
$$\Psi_{\mathbf{15}}^{(+,-)} \supset q_L + l_L^c \quad \text{and} \quad \Psi_{\overline{\mathbf{15}}}^{(-,-)} \supset u_R + e_R + d_R^c \tag{16}$$

- Scalar that contains the SM Higgs field:

$$\Phi_{\mathbf{15}}^{(-,+)} \supset \phi_h \tag{17}$$

IV. Example: SU(6) aGUT

- Effective potential:



- Gauge Higgs doesn't break the gauge group, $SU(3) \times SU(2) \times U(1)^2$ preserved

Example: SU(6) aGUT

- Compute mass of the gauge scalar ϕ_{A_5}:

$$m^2_{\phi_{A_5}} = \frac{R^2}{2} \frac{\partial^2}{\partial a^2} V_{\text{eff}}(a)\Big|_{a=0} = \frac{3}{16}\zeta(3)\frac{1}{\pi^4 R^2}, \tag{18}$$

- Bulk gauge interactions allow leptoquark coupling with the gauge scalar:

$$\mathcal{L} \supset \overline{\Psi}^{(+,-)}_{\mathbf{15}} iD_M \Gamma^M \Psi^{(+,-)}_{\mathbf{15}} \supset \bar{q}_L \phi_{A_5} l^c_L \tag{19}$$

- Leptoquark searches at LHC: $m_{\phi_{A_5}} \leq 2$ TeV

- Constraints on the compactification scale:

$$m_{KK} = \frac{1}{R} \geq 50\text{TeV} \tag{20}$$

Conclusion

- aGUTs: alternative to traditional GUTs, can be realized in 5D
- Compactification of the 5th dimension on a orbifold $\frac{S^1}{\mathbb{Z}^2 \times \mathbb{Z}_2'}$
- Stable Orbifold: the Gauge-Higgs scalar has to preserve the gauge part of the theory
- Only a few scenario are compatible with the orbifold stability criteria, constrains on aGUTs theories that can be built

Thank you for your attention

Asymptotic Unification: α running

- In 5D, α carries a mass dimension, define effective t'Hooft coupling:

$$\tilde{\alpha} = \mu R \alpha \tag{21}$$

- 1-loop 5D Beta function:

$$2\pi \frac{\mathrm{d}\tilde{\alpha}}{\mathrm{d}\ln\mu} = 2\pi\tilde{\alpha} - b_5\tilde{\alpha}^2 \tag{22}$$

$$b_5 = \frac{7}{3}C(\mathcal{G}) - \frac{4}{3}\sum_f T(R_f) - \frac{1}{3}\sum_s T(R_s) \tag{23}$$

- UV fixed point for $b_5 > 0$:

$$\tilde{\alpha}^* = \frac{2\pi}{b_5} \tag{24}$$

- Similar for Yukawa couplings, RGE given by:

$$2\pi \frac{\mathrm{d}\tilde{\alpha}_y}{\mathrm{d}\ln\mu} = 2\pi\tilde{\alpha}_y + c_y\tilde{\alpha}_y^2 - d_y\tilde{\alpha}\tilde{\alpha}_y \tag{25}$$

- Fixed point when $d_y > 0$, $c_y > 0$ and $d_y\tilde{\alpha}^* > 2\pi$:

$$\tilde{\alpha}_y^* = \frac{d_y\tilde{\alpha}^* - 2\pi}{c_y} \tag{26}$$

Asymptotic Unification: SU(6)

- SU(6) Beta function:

$$b_5 = \frac{61 - 16n_g}{3} \tag{27}$$

- $b_5 > 0$ for $n_g \leq 3$
- Yukawa term: $\mathcal{L} \supset -Y_u \overline{\Psi}_{\overline{\mathbf{15}}} \Phi_{\mathbf{15}} \Psi_{\mathbf{15}}$
- We get the following fixed point:

$$d_y = 28, \qquad c_y = 144 \qquad \tilde{\alpha}_y = \frac{23 + 16n_g}{72(61 - 16n_g)}\pi \tag{28}$$

- We can have at most $n_g \leq 3$