Monkey bias with velocity consistency relation

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Work in progress

21th Saga-Yonsei partnership

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Large-scale structure (*LSS*) and Gravity theory

Study gravity theories by using galaxy distribution

The galaxy distribution includes

⋆many-body

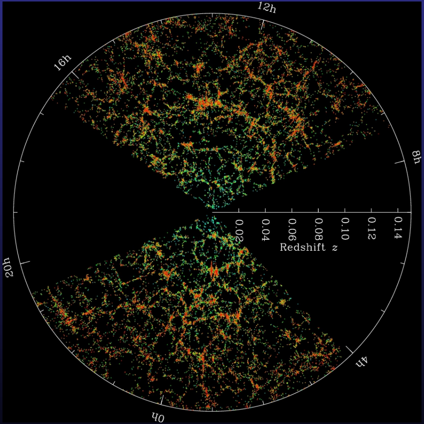
⋆Baryonic interaction and ...

very complicated

introduce a bias model to evade complicated issues

i.e. $\delta_g = \mathcal{F}[\delta_{\rm Gravity}, \delta_{\rm Linear}]$

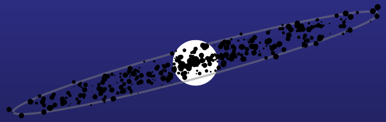
$\longrightarrow$ What is the correct bias model?



Credit: M. Blanton and the Sloan Digital Sky Survey.

Bias model

DARK MATTER ONLY



relying on the fluid equation:

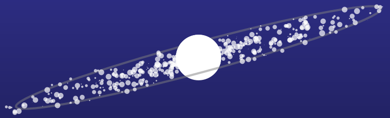
$$\frac{\partial}{\partial\eta}\delta + \nabla\cdot\left[(\delta+1)\boldsymbol{V}\right] = 0,$$

$$\frac{\partial}{\partial\eta}\boldsymbol{V} + \frac{1}{2} + (\boldsymbol{V}\cdot\nabla)\boldsymbol{V} + \frac{3}{2}\nabla\varphi = 0,$$

$$\Delta\varphi = \delta$$

$\Rightarrow$ Solution: δ

GALAXY



relying on the modified fluid equation:

$$\frac{\partial}{\partial\eta}\delta_g + \nabla\cdot\left[(\delta_g+1)\boldsymbol{V}_g\right] = \text{source term (1)},$$

$$\frac{\partial}{\partial\eta}\boldsymbol{V}_g + \frac{1}{2} + (\boldsymbol{V}_g\cdot\nabla)\boldsymbol{V}_g + \frac{3}{2}\nabla\varphi = \text{source term (2)},$$

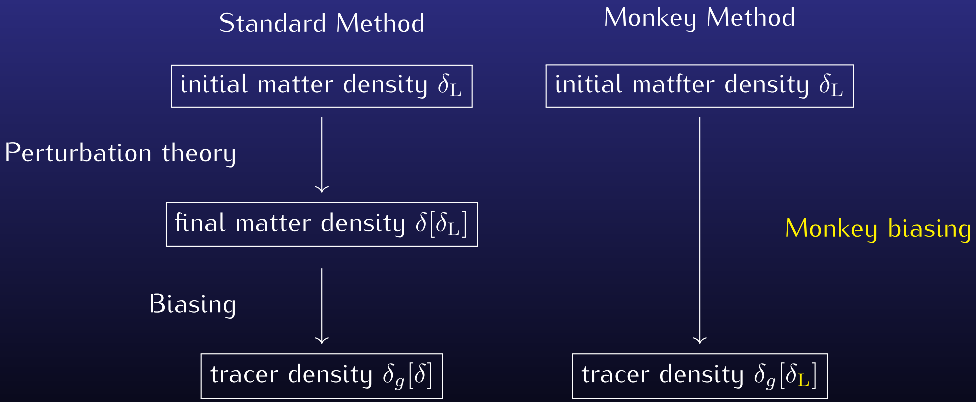
$$\Delta\varphi = \delta_g$$

$\Rightarrow$ Solution: $\delta_{g(\text{tracer})}$ (UNKNOWN)

One of the simplest bias model: $\delta_g = b\cdot\delta$ (b is a bias parameter).

Monkey bias

Fujita and Vlah., *2003.10114*



We need galaxy distribution for testing gravity!!

How can we predict galaxy distribution?

1. We cannot predict the galaxy distribution δ_g.
2. We can predict the dark matter distribution δ.

We need a bias model to obtain δ_g

Why we introduce Monkey bias?

1. Monkey bias doesn't require a detaild gravity theory to be specified.
2. Monkey bias can obtain very general condition by using consistency relation.

Monkey bias method (1)

Fujita and Vlah., *2003.10114*

What is the equations that determine the biased Tracer?

Consider fluid equations ($\theta := \nabla \cdot \boldsymbol{V}$).

$$\partial_\eta \delta + \sim \delta + \sim \theta = \sim \delta\theta + \sim \partial_i \delta \frac{\partial_i}{\partial^2}\theta, \qquad \partial_\eta \theta + \sim \delta + \sim \theta = \sim \frac{\partial_i \partial_j}{\partial^2}\theta \frac{\partial_i \partial_j}{\partial^2}\theta$$

some number

More generally,

$$\longrightarrow \partial_\eta \Delta + \sim \Delta = \sim \Delta\Delta \sim \partial_i \Delta \frac{\partial_i}{\partial^2}\Delta \sim \frac{\partial_i \partial_j}{\partial^2}\Delta \frac{\partial_i \partial_j}{\partial^2}\Delta \qquad (\Delta = \delta \text{ or } \theta)$$

1st order : $\delta^{(1)} = \delta_{\rm L} = D(\eta)\delta_{\rm L0}, \quad \theta^{(1)} \propto \delta_{\rm L} \qquad (\partial_\eta \delta_{\rm L} \propto \delta_{\rm L})$

2nd order : $\delta^{(2)} = \sim \delta_{\rm L}\delta_{\rm L} + \sim \partial_i \delta_{\rm L} \frac{\partial_i}{\partial^2}\delta_{\rm L} + \sim \frac{\partial_i \partial_j}{\partial^2}\delta_{\rm L} \frac{\partial_i \partial_j}{\partial^2}\delta_{\rm L}$

Monkey bias method (2) Fujita and Vlah., *2003.10114*

$$\delta_g = a\delta_{\rm L} + b_1{\delta_{\rm L}}^2 + b_2\partial_i\delta_{\rm L}\frac{\partial_i}{\partial^2}\delta_{\rm L} + b_3\frac{\partial_i\partial_j}{\partial^2}\delta_{\rm L}\frac{\partial_i\partial_j}{\partial^2}\delta_{\rm L} + \dots$$

⬇ Fourier space

$$\delta_g(\boldsymbol{k}) = a\delta_{\rm L}(\boldsymbol{k}) \quad + \int_{\boldsymbol{k}=\boldsymbol{p}_1+\boldsymbol{p}_2}\left[b_1 + b_2\frac{\boldsymbol{p}_1\cdot\boldsymbol{p}_2}{p_1^2} + b_3\frac{(\boldsymbol{p}_1\cdot\boldsymbol{p}_2)^2}{p_1^2p_2^2}\right]\delta_{\rm L}(\boldsymbol{p}_1)\delta_{\rm L}(\boldsymbol{p}_2) \quad + \dots$$

$a, b_1, b_2, b_3, \dots$ include bias and gravity's information (In general, depend on scale).

Connections between the individual bias parameters, a, b_1, b_2, b_3?

Can analytically determine the bias parameters?

Monkey bias method (2)

Fujita and Vlah., *2003.10114*

$$\delta_g = a\delta_{\rm L} + b_1{\delta_{\rm L}}^2 + b_2\partial_i\delta_{\rm L}\frac{\partial_i}{\partial^2}\delta_{\rm L} + b_3\frac{\partial_i\partial_j}{\partial^2}\delta_{\rm L}\frac{\partial_i\partial_j}{\partial^2}\delta_{\rm L} + \dots$$

⬇ Fourier space

$$\delta_g(\boldsymbol{k}) = a\delta_{\rm L}(\boldsymbol{k}) \quad + \int_{\boldsymbol{k}=\boldsymbol{p}_1+\boldsymbol{p}_2}\left[b_1 + b_2\frac{\boldsymbol{p}_1\cdot\boldsymbol{p}_2}{p_1^2} + b_3\frac{(\boldsymbol{p}_1\cdot\boldsymbol{p}_2)^2}{p_1^2p_2^2}\right]\delta_{\rm L}(\boldsymbol{p}_1)\delta_{\rm L}(\boldsymbol{p}_2) \quad + \dots$$

$a, b_1, b_2, b_3, \dots$ include bias and gravity's information (In general, depend on scale).

Connections between the individual bias parameters, a, b_1, b_2, b_3?

Can analytically determine the bias parameters?

➡ Try with consistency relations!!

Consistency relation with *Functional method* (1) Valageas., *1211.12364*

Considering $\delta_{\rm L0}$ as Gaussian,

$$\left\langle \delta_{\rm L0}(x) \cdot \underbrace{\rho}_{\rho[\delta_{\rm L0}]} \right\rangle = \int \mathcal{D}\delta_{\rm L0}(x')\ \delta_{\rm L0}(x) \left[\rho \cdot \exp\left(-{\delta_{\rm L0}}^2/2\sigma\right)\right](x')$$

$$= -\sigma(x) \int \mathcal{D}\delta_{\rm L0}(x')\ \rho(x') \frac{\mathcal{D}[\exp\left(-{\delta_{\rm L0}}^2/2\sigma\right)](x')}{\mathcal{D}\delta_{\rm L0}(x)}$$

$$= \sigma(x) \int \mathcal{D}\delta_{\rm L0}(x')\ \frac{\mathcal{D}\rho(x')}{\mathcal{D}\delta_{\rm L0}(x)} \left[\exp\left(-{\delta_{\rm L0}}^2/2\sigma\right)\right](x')$$

$$= \sigma(x) \left\langle \frac{\mathcal{D}\rho}{\mathcal{D}\delta_{\rm L0}(x)} \right\rangle$$

In Fourier space

$$\langle \delta_{\rm L0}(\boldsymbol{k}) \cdot \rho \rangle = P_{L0}(k) \left\langle \frac{\mathcal{D}\rho}{\mathcal{D}\delta_{\rm L0}(-\boldsymbol{k})} \right\rangle, \qquad \langle \delta_{\rm L0}(\boldsymbol{k}_1) \delta_{\rm L0}(\boldsymbol{k}_2) \rangle = (2\pi)^3 \delta_{\rm D}(\boldsymbol{k}_1 + \boldsymbol{k}_2) P_{L0}(k_1)$$

Consistency relation (2)

$\rho = \delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)$ case

$$\frac{\mathcal{D}\delta(\boldsymbol{k}')}{\mathcal{D}\delta_{L0}(\boldsymbol{k})} = \,?$$

Consider long mode's change, $\Delta\delta_{L0}(\boldsymbol{k})$, ($\boldsymbol{k} \to 0$), the effect is:

$$\delta(t, \boldsymbol{x}) \quad \xrightarrow{\mathcal{D}\delta_{L0}} \quad \delta(t, \boldsymbol{x} + \Delta\boldsymbol{x})$$

$$\delta(t, \boldsymbol{p}) \quad \xrightarrow{\mathcal{D}\delta_{L0}} \quad e^{i\boldsymbol{p}\cdot\Delta\boldsymbol{x}}\delta(t, \boldsymbol{p})$$

What is $\Delta\boldsymbol{x}$?

Consistency relation (3)

By using velocity linear solution: $\boldsymbol{v}(\boldsymbol{k}) = -i(\boldsymbol{k}/k^2)\partial_\eta \delta_{\rm L}(\boldsymbol{k})$

$$\Delta \boldsymbol{x}(\eta) = \int d\eta \; \Delta \boldsymbol{v}(\boldsymbol{k}) = -i(\boldsymbol{k}/k^2) D(\eta) \Delta \delta_{\rm L}(\boldsymbol{k}),$$

requiring

1. equivalence principle
2. continuity equation $\left(\partial_\eta \delta + \nabla \cdot \left[(\delta + 1)\boldsymbol{V}\right] = 0\right)$

Thus long mode changes as

$$e^{i\boldsymbol{p}\cdot\Delta\boldsymbol{x}}\delta(\eta, \boldsymbol{p}) \simeq [1 + i\boldsymbol{p}\cdot\Delta\boldsymbol{x}]\delta(\eta, \boldsymbol{p}) = \left[1 + D(\eta)\frac{\boldsymbol{k}\cdot\boldsymbol{p}}{k^2}\Delta\delta_{L0}(\boldsymbol{k})\right]\delta(\eta, \boldsymbol{p})$$

$$\therefore \qquad \frac{\mathcal{D}\delta(\eta, \boldsymbol{p})}{\mathcal{D}\delta_{L0}(\boldsymbol{k})} = D(\eta)\frac{\boldsymbol{k}\cdot\boldsymbol{p}}{k^2}\delta(\eta, \boldsymbol{p})$$

Consistency relation (4)

$$\langle \delta_{\mathrm{L0}}(\eta, \boldsymbol{k}) \cdot \delta(\eta_1, \boldsymbol{k}_1)\delta(\eta_2, \boldsymbol{k}_2)\rangle = P_L(k) \left\langle \frac{\mathcal{D}\delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)}{\mathcal{D}\delta_{\mathrm{L0}}(-\boldsymbol{k})} \right\rangle$$

$$\Rightarrow \qquad \langle \delta_{\mathrm{L0}}(\eta, \boldsymbol{k}) \cdot \delta(\eta_1, \boldsymbol{k}_1)\delta(\eta_2, \boldsymbol{k}_2)\rangle_{k\to 0} = -P_L(k) \times \boldsymbol{k} \cdot \left[\frac{D(\eta_1)\boldsymbol{k}_1}{k^2} + \frac{D(\eta_2)\boldsymbol{k}_2}{k^2} \right] \langle \delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\rangle$$

$$\xrightarrow{\eta=\eta_1=\eta_2} \qquad \langle \delta_{\mathrm{L0}}(\boldsymbol{k}) \cdot \delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\rangle_{k\to 0} = -P_L(k)D(\eta) \times \boldsymbol{k} \cdot \left(\frac{\boldsymbol{k}_1 + \boldsymbol{k}_2}{k^2} \right) \langle \delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\rangle = 0$$

For the general density contrast δ: consistency relation

$$\lim_{k_j \to 0} \left\langle \prod_j \delta^{(\alpha_j)}(\boldsymbol{k}_j) \cdot \prod_i \delta^{(\alpha'_i)}(\boldsymbol{k}_i) \right\rangle$$

$$= \prod_j \left[-P_L^{(\alpha'_j)}(\eta_j, \boldsymbol{k}_j) \times \sum_i \frac{\boldsymbol{k}_j \cdot \boldsymbol{k}_i}{k_j^2} \frac{D(\eta_i)}{D(\eta_j)} \right] \times \left\langle \prod_i \delta^{(\alpha_i)}(\eta_i, \boldsymbol{k}_i) \right\rangle$$

Bias constraining

Fujita and Vlah., *2003.10114*

Let's try to constrain bias parameters!

1. Now, we have unknown bias parameters: a, b_1, b_2, b_3.
2. If bias parameters are constrained, we can obtain galaxy distribution: δ_g.

$$\delta_g(\boldsymbol{k}) = a\delta_{\rm L}(\boldsymbol{k}) + \int_{\boldsymbol{k}=\boldsymbol{p}_1+\boldsymbol{p}_2} \left[b_1 + b_2 \frac{\boldsymbol{p}_1 \cdot \boldsymbol{p}_2}{p_1^2} + b_3 \frac{(\boldsymbol{p}_1 \cdot \boldsymbol{p}_2)^2}{p_1^2 p_2^2} \right] \delta_{\rm L}(\boldsymbol{p}_1)\delta_{\rm L}(\boldsymbol{p}_2) + \ldots$$

Constraints with equal-time consistency relation (m :dark matter, α, β : galaxis' name)

$$\langle \delta_m(\boldsymbol{k}) \cdot \delta_\alpha(\boldsymbol{k}_1)\delta_\beta(\boldsymbol{k}_2)\rangle_{k\to 0} = 0$$

$$\langle \delta_m(\boldsymbol{k}) \cdot \delta_\alpha(\boldsymbol{k}_1)\delta_\beta(\boldsymbol{k}_2)\rangle \simeq \int_{\boldsymbol{k}_1=\boldsymbol{p}_1+\boldsymbol{p}_2} \left[\left\langle \delta_{\rm L}(\boldsymbol{k}) \cdot \delta_\alpha^{(2)}(\boldsymbol{k}_1)\delta_\beta^{(1)}(\boldsymbol{k}_2) \right\rangle + \left\langle \delta_{\rm L}(\boldsymbol{k}) \cdot \delta_\alpha^{(1)}(\boldsymbol{k}_1)\delta_\beta^{(2)}(\boldsymbol{k}_2) \right\rangle \right]$$

$$\left\langle \delta_{\rm L}(\boldsymbol{k}) \cdot \delta_\alpha^{(2)}(\boldsymbol{k}_1)\delta_\beta^{(1)}(\boldsymbol{k}_2) \right\rangle_{\boldsymbol{k}_1=\boldsymbol{p}_1+\boldsymbol{p}_2} = a^{(\beta)} \left[b_1^{(\alpha)} + b_2^{(\alpha)} \frac{\boldsymbol{p}_1 \cdot \boldsymbol{p}_2}{p_1^2} + b_3^{(\alpha)} \frac{(\boldsymbol{p}_1 \cdot \boldsymbol{p}_2)^2}{p_1^2 p_2^2} \right] \langle \delta_{\rm L}(\boldsymbol{k}) \cdot \delta_{\rm L}(\boldsymbol{p}_1)\delta_{\rm L}(\boldsymbol{p}_2)\delta_{\rm L}(\boldsymbol{k}_2) \rangle$$

$$= a^{(\beta)} \left[b_1^{(\alpha)} + b_2^{(\alpha)} \frac{\boldsymbol{p}_1 \cdot \boldsymbol{p}_2}{p_1^2} + b_3^{(\alpha)} \frac{(\boldsymbol{p}_1 \cdot \boldsymbol{p}_2)^2}{p_1^2 p_2^2} \right] \times (2\pi)^6 P_L(\boldsymbol{k}) P_L(\boldsymbol{k}_2)$$

$$\times \left[\delta_{\rm D}(\boldsymbol{k} + \boldsymbol{p}_1)\delta_{\rm D}(\boldsymbol{p}_2 + \boldsymbol{k}_2) + \delta_{\rm D}(\boldsymbol{k} + \boldsymbol{p}_2)\delta_{\rm D}(\boldsymbol{p}_1 + \boldsymbol{k}_2) \right]$$

$$\stackrel{k\to 0}{\simeq} a^{(\beta)} b_2^{(\alpha)} \frac{\boldsymbol{k} \cdot \boldsymbol{k}_2}{k^2} \times (2\pi)^6 P_L(k) P_L(k_2) \delta_{\rm D}(\boldsymbol{k} + \boldsymbol{p}_1)\delta_{\rm D}(\boldsymbol{p}_2 + \boldsymbol{k}_2)$$

$$\left\langle \delta_{\rm L}(\boldsymbol{k}) \cdot \delta_\alpha^{(1)}(\boldsymbol{k}_1)\delta_\beta^{(2)}(\boldsymbol{k}_2) \right\rangle_{\boldsymbol{k}_2=\boldsymbol{p}_1+\boldsymbol{p}_2} \stackrel{k\to 0}{\simeq} a^{(\alpha)} b_2^{(\beta)} \frac{\boldsymbol{k} \cdot \boldsymbol{k}_1}{k^1} \times (2\pi)^6 P_L(k) P_L(k_1) \delta_{\rm D}(\boldsymbol{k} + \boldsymbol{p}_1)\delta_{\rm D}(\boldsymbol{p}_2 + \boldsymbol{k}_1)$$

$$\xrightarrow{\boldsymbol{k}_1+\boldsymbol{k}_2=0} - a^{(\alpha)} b_2^{(\beta)} \frac{\boldsymbol{k} \cdot \boldsymbol{k}_2}{k^2} \times (2\pi)^6 P_L(k) P_L(k_2) \delta_{\rm D}(\boldsymbol{k} + \boldsymbol{p}_1)\delta_{\rm D}(\boldsymbol{p}_2 + \boldsymbol{k}_1)$$

$$\therefore \quad \langle \delta_m(\boldsymbol{k}) \cdot \delta_\alpha(\boldsymbol{k}_1)\delta_\beta(\boldsymbol{k}_2) \rangle_{k\to 0} \simeq \left[a^{(\beta)} b_2^{(\alpha)} - a^{(\alpha)} b_2^{(\beta)} \right] \frac{\boldsymbol{k} \cdot \boldsymbol{k}_2}{k^2} P_L(k) P_L(k_2)$$

equal-time consistency relation

$$\langle \delta_m(\boldsymbol{k}) \cdot \delta_\alpha(\boldsymbol{k}_1)\delta_\beta(\boldsymbol{k}_2)\rangle_{k\to 0} = 0$$

by using this,

$$a^{(\beta)}b_2^{(\alpha)} - a^{(\alpha)}b_2^{(\beta)} = 0 \quad \Longleftrightarrow \quad \frac{b_2}{a} = \text{Constant} =: \mathcal{C}_b$$

The condition, satisfied for arbitrary galaxy.

$$b_2 = \mathcal{C}_b a$$

Velocity (momentum) consistency relation?

Rizzo et al., *1606.03708*

Consider non-equal time consistency relation

$$\langle \delta_{\mathrm{L0}}(\eta,\boldsymbol{k})\cdot\delta(\eta_1,\boldsymbol{k}_1)\delta(\eta_2,\boldsymbol{k}_2)\rangle_{k\to 0} = -P_L(k)\times\boldsymbol{k}\cdot\left[\frac{D(\eta_1)\boldsymbol{k}_1}{k^2}+\frac{D(\eta_2)\boldsymbol{k}_2}{k^2}\right]\langle\delta(\eta_1,\boldsymbol{k}_1)\delta(\eta_2,\boldsymbol{k}_2)\rangle$$

doing η_2 derivative:

$$\left\langle \delta_{\mathrm{L0}}(\eta,\boldsymbol{k})\cdot\delta(\eta_1,\boldsymbol{k}_1)\frac{\partial\delta(\eta_2,\boldsymbol{k}_2)}{\partial\eta_2}\right\rangle_{k\to 0}$$

$$= -P_L(k)\left\{\frac{d\log D(\eta_2)}{d\eta_2}\frac{\boldsymbol{k}\cdot\boldsymbol{k}_2}{k^2}\langle\delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\rangle+\boldsymbol{k}\cdot\left[\frac{D(\eta_1)\boldsymbol{k}_1}{k^2}+\frac{D(\eta_2)\boldsymbol{k}_2}{k^2}\right]\left\langle\delta(\boldsymbol{k}_1)\frac{\partial\delta(\boldsymbol{k}_2)}{\partial\eta_2}\right\rangle\right\}$$

Associating continuity equation: $\partial_\eta\delta = -\nabla\cdot[(1+\delta)\boldsymbol{v}]$,

$\Rightarrow$ Momentum $\boldsymbol{p} := (1+\delta)\boldsymbol{v}$ consistency relation ?

equal time ($\eta = \eta_1 = \eta_2$) derivative consistency relation (New condition is obtained!)

$$\left\langle \delta_{\mathrm{L0}}(\eta, \boldsymbol{k}) \cdot \delta(\eta_1, \boldsymbol{k}_1) \frac{\partial \delta(\eta_2, \boldsymbol{k}_2)}{\partial \eta_2} \right\rangle_{k \to 0} \stackrel{\eta=\eta_1=\eta_2}{=} -P_L(k) \frac{\dot{D}}{D} \frac{\boldsymbol{k} \cdot \boldsymbol{k}_2}{k^2} \langle \delta(\boldsymbol{k}_1) \delta(\boldsymbol{k}_2) \rangle$$

Can we derive furthur restriction for the bias?

Derivative of density contrast

By using $\partial_\eta \delta_{\mathrm{L}} = \dot{D}/D\delta_{\mathrm{L}}$,

$$\partial_\eta \delta_g(\eta, \boldsymbol{k}) = A\delta_{\mathrm{L}}(\eta, \boldsymbol{k}) + \int_{\boldsymbol{k}=\boldsymbol{p}_1+\boldsymbol{p}_2} \left[B_1 + B_2 \frac{\boldsymbol{p}_1 \cdot \boldsymbol{p}_2}{p_1^2} + B_3 \frac{(\boldsymbol{p}_1 \cdot \boldsymbol{p}_2)^2}{p_1^2 p_2^2} \right] \delta_{\mathrm{L}}(\eta, \boldsymbol{p}_1)\delta_{\mathrm{L}}(\eta, \boldsymbol{p}_2) + \ldots$$

Derivative of density contrast

By using $\partial_\eta \delta_{\rm L} = \dot{D}/D \delta_{\rm L}$,

$$\partial_\eta \delta_g(\eta, \boldsymbol{k}) = A\delta_{\rm L}(\eta, \boldsymbol{k}) + \int_{\boldsymbol{k}=\boldsymbol{p}_1+\boldsymbol{p}_2} \left[B_1 + B_2 \frac{\boldsymbol{p}_1 \cdot \boldsymbol{p}_2}{p_1^2} + B_3 \frac{(\boldsymbol{p}_1 \cdot \boldsymbol{p}_2)^2}{p_1^2 p_2^2} \right] \delta_{\rm L}(\eta, \boldsymbol{p}_1) \delta_{\rm L}(\eta, \boldsymbol{p}_2) + \ldots$$

remember? $\delta_g(\boldsymbol{k}) = a\delta_{\rm L}(\boldsymbol{k}) + \int_{\boldsymbol{k}=\boldsymbol{p}_1+\boldsymbol{p}_2} \left[b_1 + b_2 \frac{\boldsymbol{p}_1 \cdot \boldsymbol{p}_2}{p_1^2} + b_3 \frac{(\boldsymbol{p}_1 \cdot \boldsymbol{p}_2)^2}{p_1^2 p_2^2} \right] \delta_{\rm L}(\boldsymbol{p}_1) \delta_{\rm L}(\boldsymbol{p}_2) + \ldots$

where $A := \dot{a} + a\dot{D}/D$, $B_i := \dot{b}_i + 2b_i\dot{D}/D$ $(i = 1, 2, 3)$ are the bias parameters of $\partial_\eta \delta_g$.

equal time velocity consistency relaiton

$$\left\langle \delta_{\rm L0}(\eta, \boldsymbol{k}) \cdot \delta(\boldsymbol{k}_1) \frac{\partial \delta(\boldsymbol{k}_2)}{\partial \eta_2} \right\rangle_{k\to 0} \overset{\eta=\eta_1=\eta_2}{=} -P_L(k) \frac{\dot{D}}{D} \frac{\boldsymbol{k}\cdot\boldsymbol{k}_2}{k^2} \langle \delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\rangle$$

$$\begin{cases} \text{LHS: } \left\langle \delta_{\rm L0}(\eta, \boldsymbol{k}) \cdot \delta_\alpha(\boldsymbol{k}_1) \frac{\partial \delta_\beta(\boldsymbol{k}_2)}{\partial \eta_2} \right\rangle_{k\to 0} = \left[b_2^{(\alpha)} A^{(\beta)} - a^{(\alpha)} B_2^{(\beta)} \right] \frac{\boldsymbol{k}\cdot\boldsymbol{k}_2}{k^2} P_L(\boldsymbol{k}) P_L(\boldsymbol{k}_2) \\ \text{RHS: } -P_L(k) \frac{\dot{D}}{D} \frac{\boldsymbol{k}\cdot\boldsymbol{k}_2}{k^2} \langle \delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\rangle = -a^{(\alpha)} a^{(\beta)} \frac{\dot{D}}{D} \frac{\boldsymbol{k}\cdot\boldsymbol{k}_2}{k^2} P_L(\boldsymbol{k}) P_L(\boldsymbol{k}_2) \end{cases}$$

Comparing both term (A, B_i are expanded by a, b_i)

$$a^{(\alpha)} \left[\dot{b}_2^{(\beta)} + 2 b_2^{(\beta)} \dot{D}/D \right] - b_2^{(\alpha)} \left[\dot{a}^{(\beta)} + a^{(\beta)} \dot{D}/D \right] = a^{(\alpha)} a^{(\beta)} \dot{D}/D$$

Considering $\alpha = \beta$ and applying $b_2 = \mathcal{C}_b a$,

$$a\left[\dot{b}_2 + 2b_2\dot{D}/D\right] - b_2\left[\dot{a} + a\dot{D}/D\right] = a^2\dot{D}/D$$

$$\Rightarrow a\partial_\eta(\mathcal{C}_b a)D + \mathcal{C}_b a^2\dot{D} - a\dot{a}\mathcal{C}_b D = a^2\dot{D}$$

$$\Rightarrow \partial_\eta\left[(\mathcal{C}_b - 1)D\right] = 0$$

$$\therefore \qquad \mathcal{C}_b - 1 \propto \frac{1}{D} \quad \text{or} \quad \mathcal{C}_b = 1.$$

$\mathcal{C}_b$ condition

$$\mathcal{C}_b - 1 \propto \frac{1}{D} \quad \text{or} \quad \mathcal{C}_b = 1.$$

Considering $\alpha = \beta$ and applying $b_2 = \mathcal{C}_b a$,

$$a\left[\dot{b}_2 + 2b_2\dot{D}/D\right] - b_2\left[\dot{a} + a\dot{D}/D\right] = a^2\dot{D}/D$$

$$\Rightarrow a\partial_\eta(\mathcal{C}_b a)D + \mathcal{C}_b a^2\dot{D} - a\dot{a}\mathcal{C}_b D = a^2\dot{D}$$

$$\Rightarrow \partial_\eta\left[(\mathcal{C}_b - 1)D\right] = 0$$

$$\therefore \qquad \mathcal{C}_b - 1 \propto \frac{1}{D} \quad \text{or} \quad \mathcal{C}_b = 1.$$

$\mathcal{C}_b$ condition

$$\mathcal{C}_b - 1 \propto \frac{1}{D} \quad \text{or} \quad \mathcal{C}_b = 1.$$

$$\delta_g(\boldsymbol{k}) = a\delta_{\rm L}(\boldsymbol{k}) \quad + \int_{\boldsymbol{k}=\boldsymbol{p}_1+\boldsymbol{p}_2}\left[b_1 + \mathcal{C}_b a\frac{\boldsymbol{p}_1\cdot\boldsymbol{p}_2}{p_1^2} + b_3\frac{(\boldsymbol{p}_1\cdot\boldsymbol{p}_2)^2}{p_1^2p_2^2}\right]\delta_{\rm L}(\boldsymbol{p}_1)\delta_{\rm L}(\boldsymbol{p}_2) \quad + \ldots$$

SUMMARY

- Consistency relations give general conditions and can be derived with a few conditions. (But they require continuity equation.)
- By considering generalized fluid equation (for the biased tracer), the perturbative solution can be obtained.
- By using (density,velocity) consistency relations, we derived the conditions for the bias parameters.