On uncertainties of the determination of reheating temperature

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▪ inflation

▪ Inflation

… The rapid expansion that occurred immediately after the birth of the universe.

▪ Inflaton field (Inflaton)

…A scalar field believed to cause rapid accelerated expansion in the early universe

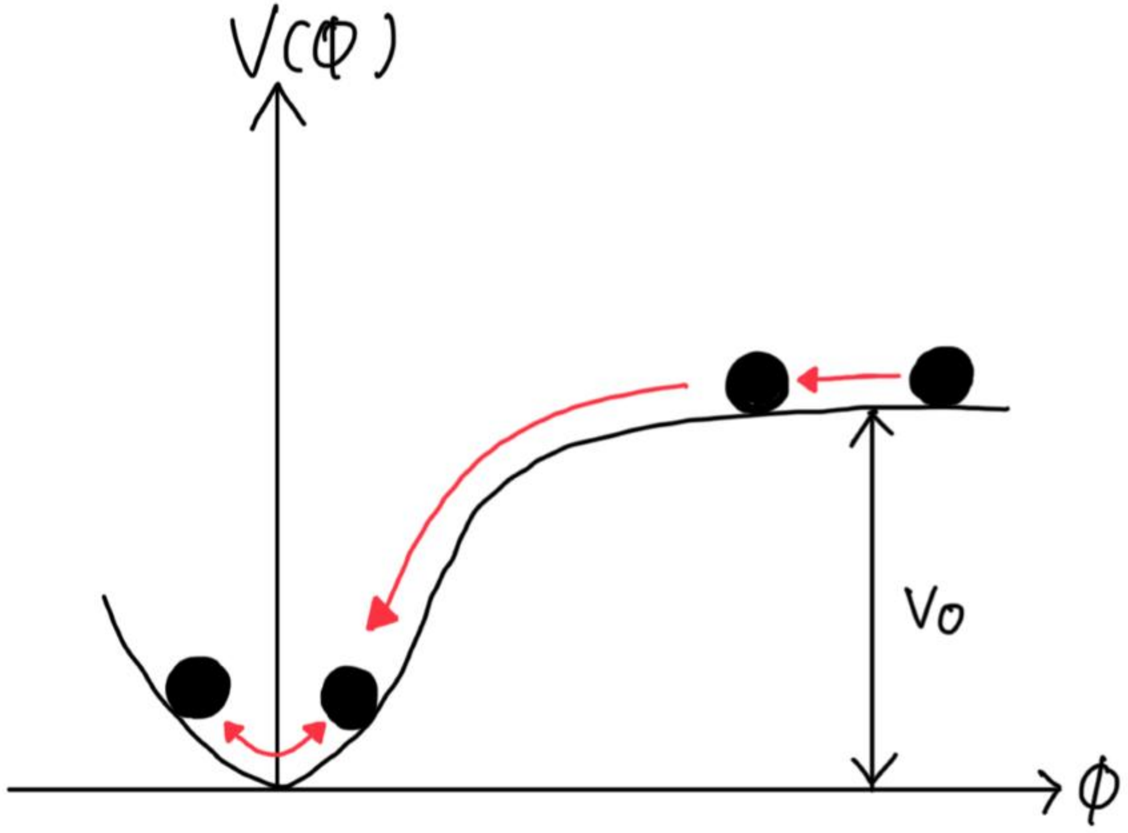


Figure1 Typical potentials of the Inflaton Field.

・Infrationary dynamics

(i) Slow-roll phase

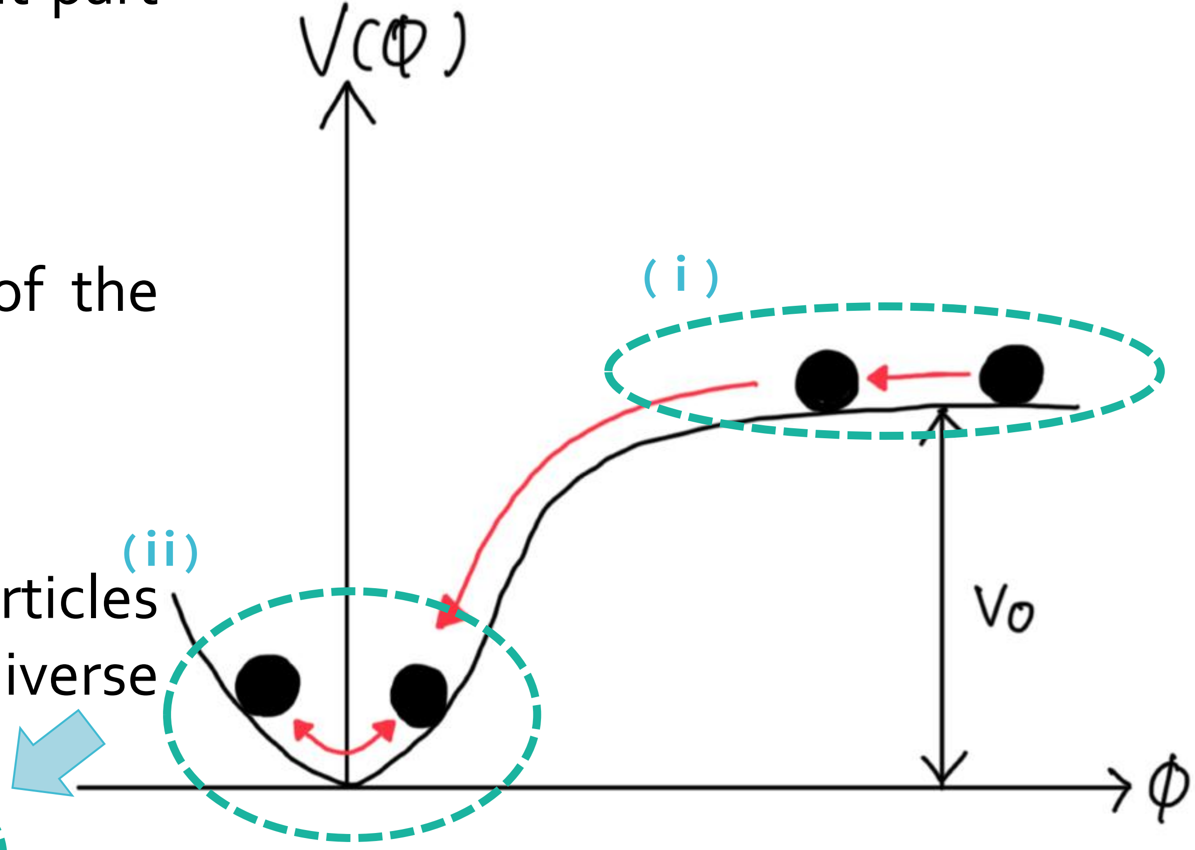
The inflaton field slowly rolls down a flat part of the potential.

(ii) Oscillation phase

The inflaton field falls to the bottom of the potential and oscillates around it.

(iii) Decay into radiation

The inflaton field decays into other particles (mainly radiation), transitioning the universe into a radiation-dominated era.



The dynamics of inflaton field can be considered in three stages

▪ reheating

After inflation ends, the oscillation phase follows, but at that time, the universe is still not radiation-dominated; it is dominated by the oscillating inflaton field.After some time, the inflaton field decays into radiation.

The energy density of the oscillating Φ decreases as $\rho_{\phi} \propto a^{-3}$

↓

H also decreases with time.

↓

When $H \simeq \Gamma_{\phi}$, the rapid decay of ϕ occurs, leading to a radiation-dominated universe."

$\Gamma\phi$:Decay rate of the inflaton field
H :Hubble parameter
$H \simeq \Gamma\phi$: Gamov's criteria

The inflaton field evolves according to the following Klein-Gordon equation, depending on the form of the potential V(ϕ).

Klein-Gordon equation：
$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\partial V}{\partial \varphi} = 0\,. \qquad (H \equiv \dot{a}/a)$$

In the following,we assume that the potential minimum during reheating can be approximated as a monomial functionV∝ϕ^m, specifically,

$$V(\varphi) = \begin{cases} \frac{m_\varphi^2}{2}\varphi^2 \ (m=2) \\ \frac{\lambda}{4}\varphi^4 \quad (m=4) \\ \frac{k}{6}\varphi^6 \quad (m=6) \end{cases}$$

The energy density of the inflaton field is described by the following the continuity equation.

$$\dot{\rho}_\varphi + 3H\left(1 + \omega_\varphi\right)\rho_\varphi \cong -\Gamma_\varphi \rho_\varphi \, .$$

Where, ω_φ is equation of state parameter

$$\omega_\varphi \equiv \frac{\langle p_\varphi \rangle}{\langle \rho_\varphi \rangle} = \frac{m-2}{m+2} = \left\{ \begin{array}{l} 0 \quad (m = 2) \\ 1/3 \ (m = 4) \\ 1/2 (m = 6) \end{array} \right.$$

Pressure : p
Energy density: ρ

▪ Evolution of energy density

Radiation =photon + neutrino

continuity equation

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0\,.$$

$$p_r = \frac{\rho_r}{3}$$

$$\rightarrow \quad \dot{\rho}_r + 4\frac{\dot{a}}{a}\rho_r = 0$$

$$\rightarrow \quad \rho_r \propto a^{-4}$$

Evolution of the inflaton field energy density

continuity equation

$$\rightarrow \quad \rho \propto a^{-3(1+\omega_\varphi)}$$

$$\rho \propto a^{-3} \quad (\omega_\varphi = 0)$$

$$\rightarrow \quad \rho \propto a^{-4} \quad (\omega_\varphi = 1/3)$$

$$\rho \propto a^{-\frac{9}{2}} \quad (\omega_\varphi = 1/2)$$

The ontinuity equation

$$\dot{\rho}_\varphi + 3H(1+\omega_\varphi)\rho_\varphi \cong -\Gamma_\varphi \rho_\varphi$$

Assuming all decayed particles turn into radiation components, this equation includes the effect of φ decay which generates radiation

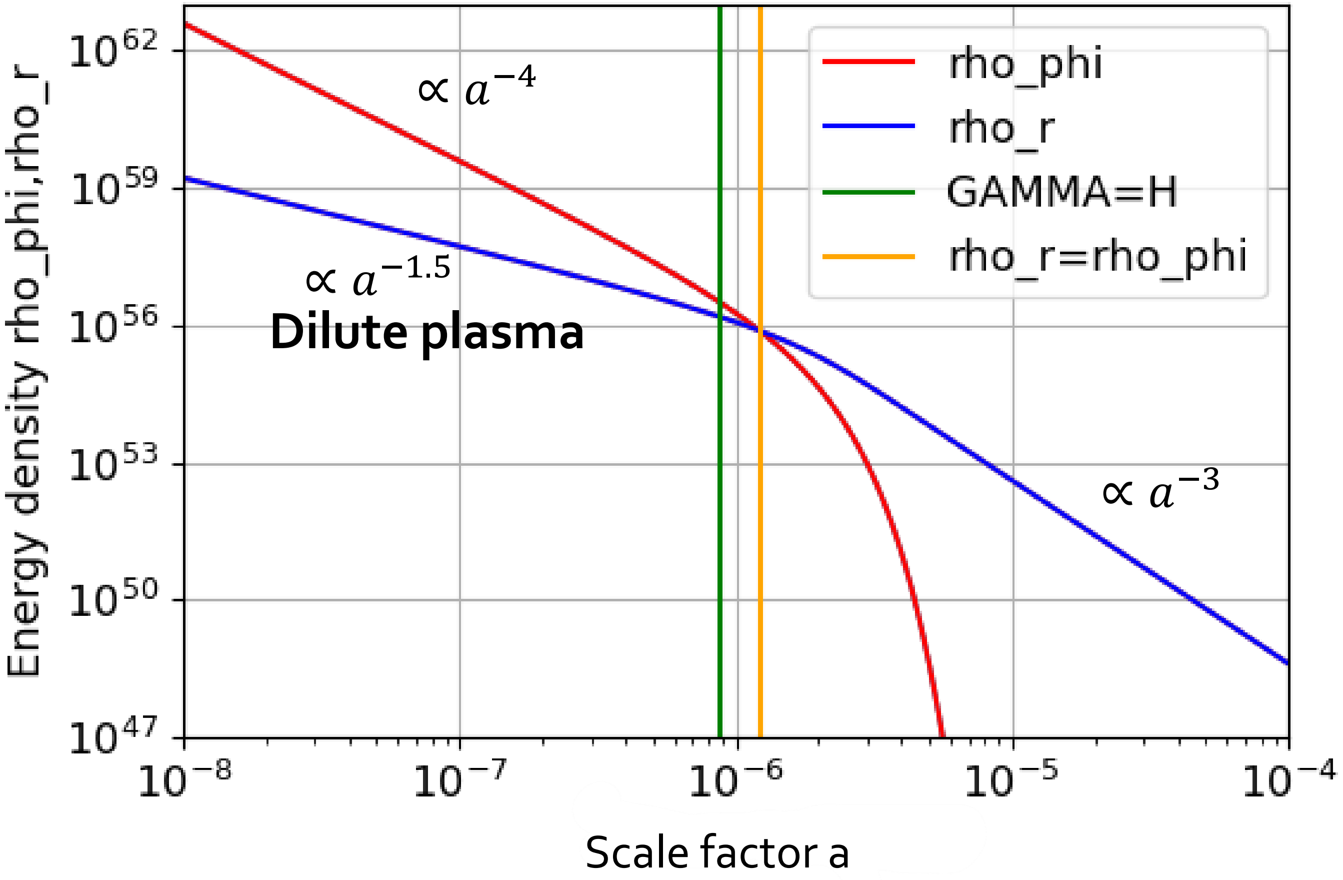
$$\dot{\rho}_r + 4H\rho_r = \Gamma_\varphi \rho_\varphi \,.$$

The friedman equation

$$H^2 = \frac{1}{3M_{pi}^2}(\rho_\varphi + \rho_r)$$

→ Solve simultaneously

· The process of cosmic reheating by the inflaton field



$$\omega_\varphi = 0$$

$a_{reheating}$: scale factor when $\rho_\phi = \rho_r$. When $a < a_{reheating}$, the universe is dominated by the energy of the oscillating inflaton field.

However, when $a > a_{reheating}$, the universe is dominated by the radiation energy density ρ_r.

Dilute plasma: The radiation component present before reaching $a_{reheating}$.

▪ reheating temperature

The expression for the total energy density in the relativistic limit.

$$\rho_r = \frac{\pi^2}{30} g_* T_r^4 .$$

$$\longrightarrow \quad T_r = \left(\frac{30\rho_r}{\pi^2 g_*}\right)^{\frac{1}{4}}$$

Where, g_* : effective degrees of freedom

$$g_* = \sum_{i=bosons} g_i \left(\frac{T_i}{T}\right)^4 + \sum_{i=fermions} g_i \left(\frac{T_i}{T}\right)^4 .$$

▪ Gamow criterion

Friedman equation is

$$H^2 = \frac{1}{3M_{pl}^2} \frac{\pi^2}{30} g_* T^4 .$$

From $H \simeq \Gamma\phi$ (Gamow criterion),

$$H^2 = \frac{1}{3M_{pl}^2} \frac{\pi^2}{30} g_* T^4 = \Gamma_\varphi^2 \quad .$$

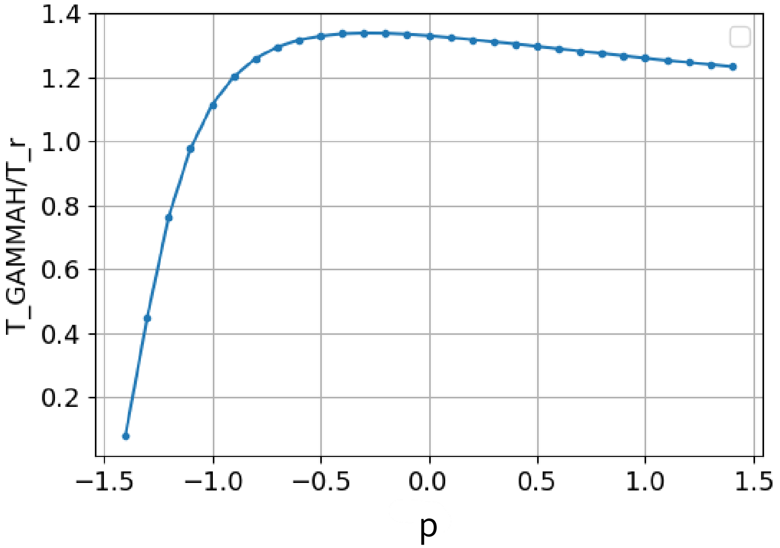
Therefore, the reheating temperature can be evaluated as

$$T_{\Gamma H} = \left(\frac{90}{\pi^2 g_*}\right)^{1/4} \sqrt{M_{pl}\Gamma_\varphi} .$$

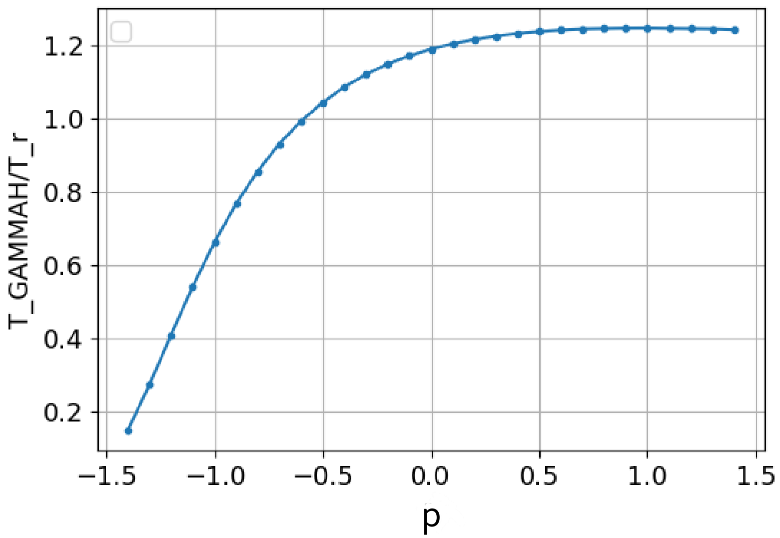
- Time dependence of Γ

define Γ as follows to incorporate its time dependence.

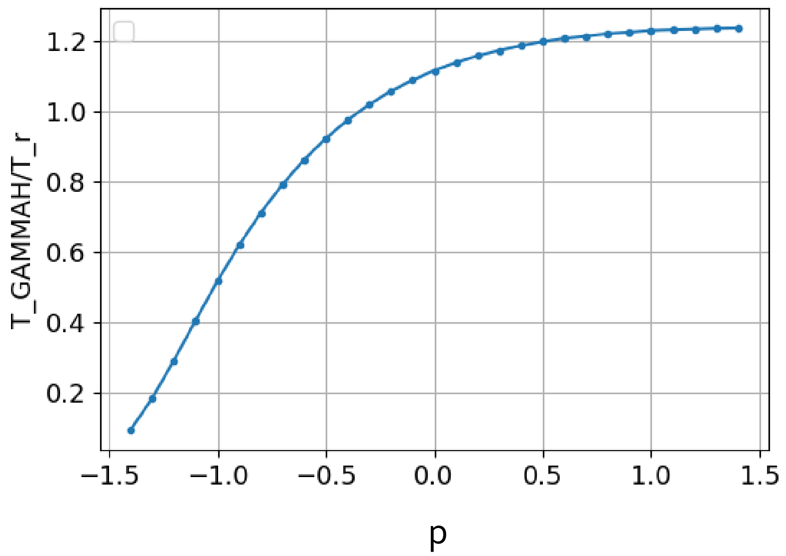
$$\Gamma_\varphi = \Gamma_e \left(\frac{a}{a_e}\right)^p .$$



ω=0



ω=1/3



ω=1/2

→ The reheating temperature is different from Gamow's criterion, depending on the value of p .

・ summary

・ In this presentation, I explored the uncertainties associated with the reheating process and its temperature.

・ The decay of the inflaton field generates radiation components, causing the universe to become radiation-dominated.

・ The reheating temperature (T_R) is the temperature at which the universe becomes radiation-dominated .

・ Generally, the reheating temperature is determined using the Gamow criterion, but it is also necessary to consider the time dependence of the decay rate (Γ).

・ Γ varies depending on the interaction for the decay and the potential, which affects the timing and temperature of reheating.

・ The numerical analysis results indicate that the reheating temperature may deviate significantly from the Gamow criterion.

・ Time dependence of Γ

The decay rate Γ is determined by the interaction and the potential.

Example interaction: $$L_{int} = -y\varphi\bar{\Psi}\Psi,$$

The oscillations of $\varphi(t)$

$$\varphi(t) = \sum_{n=-\infty}^{\infty} \varphi_n e^{-i\omega n t}, \qquad (\omega \equiv 2\pi/T)$$

The transition rate per unit time Γ from the initial state to the final two-particle state:

$$\Gamma = \frac{y^2}{8\pi}\langle\dot{\varphi}^2\rangle$$

Here,$\langle\cdot\cdot\rangle$ denotes the average of ..over one period of the oscillations.

・ Time dependence of Γ

Using the energy conservation,

$$\rho_\varphi \Gamma_\varphi \Delta t = E \Gamma \Delta t \ \ .$$

The left-hand side represents the energy loss of the ϕ field during the infinitesimal time Δt .
The right-hand side the energy gain of two φ –particles . E is the expectation value of the energy of the final two – particles state.

The energy decay rate of φ:

$$\Gamma_\varphi = \frac{y^2}{8\pi} E \frac{\langle \dot{\varphi}^2 \rangle}{\rho_\varphi}, \qquad \left(E = \frac{\sum_{n=1}^{\infty} |\varphi_n|^2 (n\omega)^3}{\sum_{n=1}^{\infty} |\varphi_n|^2 (n\omega)^2} \right).$$

Difine the numerical factor α by

$$\alpha = \frac{\sum_{n=1}^{\infty} |\varphi_n|^2 n^3}{\sum_{n=1}^{\infty} |\varphi_n|^2 n^2}.$$

Then , Γ_φ can be written as

$$\Gamma_\varphi = \frac{y^2}{8\pi} \omega \alpha \frac{\langle \dot{\varphi}^2 \rangle}{\rho_\varphi}.$$

- Time dependence of Γ

Consider the case where the potential $V(\varphi) = \frac{\lambda}{4}\varphi^4$.

$\varphi(t)$ can be written as

$$\varphi(t) = \frac{\sqrt{\pi}\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\varphi_0 \sum_{n=1}^{\infty}\left(e^{i(2n-1)\omega t} + e^{-i(2n-1)\omega t}\right)\frac{e^{-\frac{\pi}{2}(2n-1)}}{1 - e^{-\pi(2n-1)}},$$

Where the frequency ω is given by

$$\omega = \frac{1}{2}\sqrt{\frac{\pi}{6}}\frac{\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}m_\varphi^{eff}, \qquad \left(m_\varphi^{eff} \equiv \sqrt{3\lambda}\varphi_0\right)$$

φ_o: the overall amplitude of the field φ

Using these ,we find $\alpha \approx 1.036$, $\frac{\langle\dot{\varphi}^2\rangle}{\rho_\varphi} = \frac{4}{3}$. Then the decay rate of the inflaton can be written as

$$\Gamma_\varphi = A\,\frac{y^2}{8\pi}m_\varphi^{eff}.$$