Classification of inflation models using k-means method

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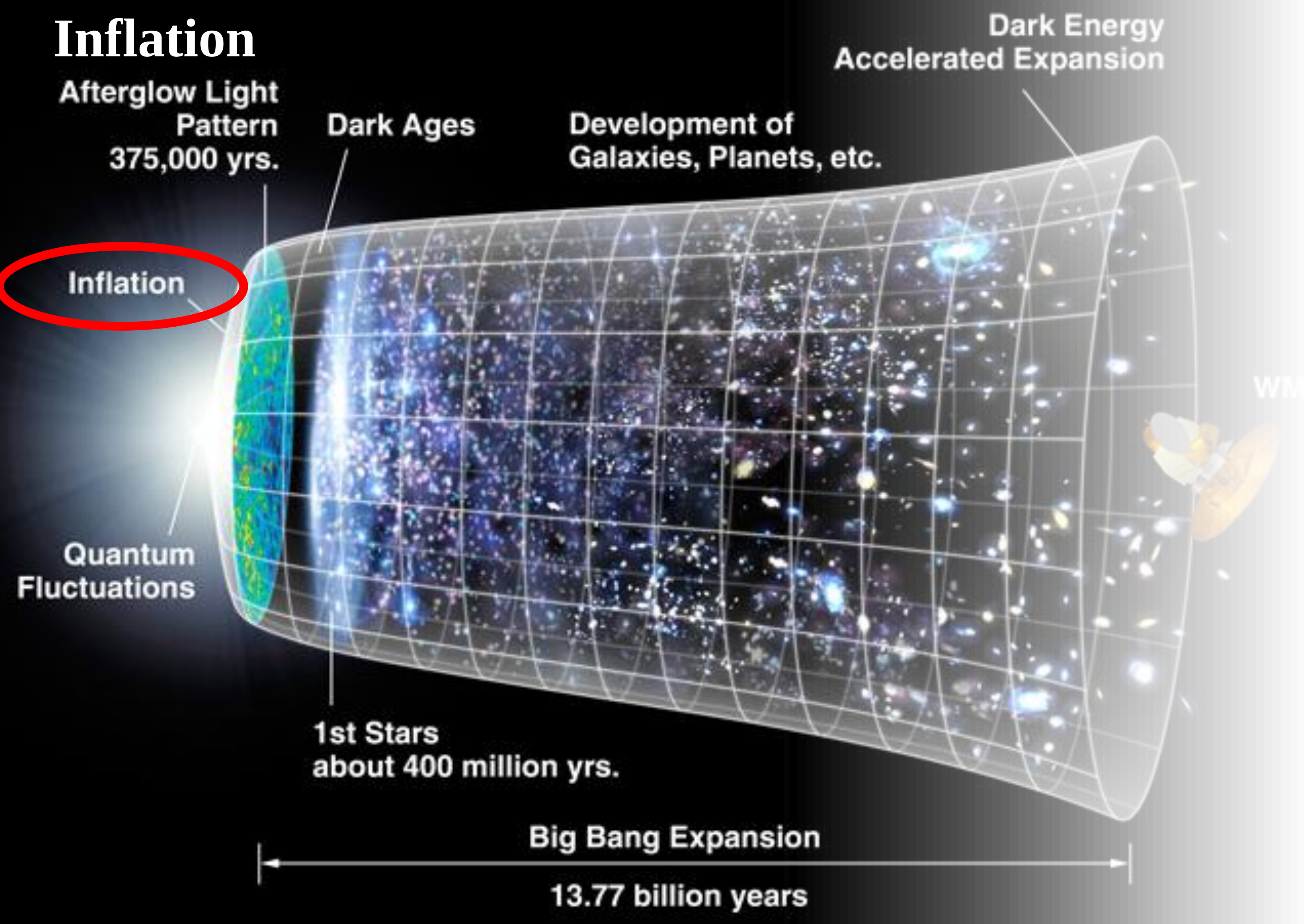
Inflation



In the early Universe, a rapid exponential expansion, called inflation, is considered to occur.

Inflation is believed to solve the problems of the Big bang theory.

/map.gsfc.nasa.gov/media/060915/index.html

Inflation

Inflation is driven by a slow-rolling scalar field

A scalar field depends on position and time.

Inflaton field

$$\phi(\vec{r}, t)$$

Inflation

V(ϕ) : the potential for the inflaton.

The energy density ρ_ϕ and pressure p_ϕ of the scalar field ϕ

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Condition：$\dot{\phi}^2 \ll |V(\phi)|$

$$\rho_\phi \simeq -p_\phi \simeq V(\phi)$$

When the potential $V(\phi)$ is approximately constant, inflation occurs.

Dynamics of inflation

Friedmann equation : Representing the expansion of the universe

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

H:Hubble parameter
$\dot{\phi}$:Time derivative of the Inflaton field
$V(\phi)$:Potential energy

Equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

Dynamics of inflation

To apply the slow-roll approximation, we make two assumptions.

The kinetic energy is much smaller than the potential energy. $\dot{\phi}^2 \ll \mathrm{V}(\phi)$

The acceleration of the scalar field is small. $\ddot{\phi} \approx 0$

Slow-roll approximation

Friedmann Equation $H^2 \approx \frac{8\pi G}{3} V(\phi)$

Equation of Motion $3H\dot{\phi} \approx -V'(\phi)$

Dynamics of inflation

Definition of slow-roll parameters

$$\epsilon \equiv \frac{1}{2} M_{pl}^2 \left(\frac{V'}{V}\right)^2$$

$$\eta \equiv M_{pl}^2 \frac{V''}{V}$$

V': $First - order\ derivative\ of\ potential\ \phi$

V'': $Second - order\ derivative\ of\ potential \phi$

M_{pl} : $Planck\ mass$

$\epsilon \ll 1$

$\eta \ll 1$

Inflation continues as long as this condition holds.

Dynamics of inflation

Definition of spectral index n_s

$$n_s - 1 = \frac{dInP_s(k)}{dInk}$$

$$n_s = 1 - 6\epsilon + 2\eta$$

$P_s: Power\ spectrum$

$$P_s = \frac{H^2}{\dot{\phi}^2}\left(\frac{H}{2\pi}\right)^2$$

Definition of tensor-to-scalar ratio

$$\gamma = \frac{P_t(k)}{P_s(k)} = 16\epsilon$$

$P_t(k): Tensor\ power\ spectrum$

$$P_t = \frac{8}{M_{pl}}\left(\frac{H}{2\pi}\right)^2$$

Dynamics of inflation

Running spectrum index

$$\alpha_s = \frac{dn_s}{dInk} \qquad \beta_s = \frac{d\alpha}{dInk}$$

$$\alpha_s = -24\epsilon^2 + 16\epsilon\eta - 2\xi^{(2)}$$

$$\beta_s = -192\epsilon^3 + 192\epsilon^2\eta - 32\epsilon\eta^2 + (-24\epsilon + 2\eta)\xi^{(2)} + 2\sigma^{(3)}$$

High-order slow-roll parameters

$$\xi^{(2)} = M_{pl}^4 \frac{V'V'''}{V^2}$$

$$\sigma^{(3)} = M_{pl}^6 \frac{(V')^2 V''''}{V^3}$$

Constraints on the running spectral index have also been obtained through observations of the Planck satellite.

Various Inflation Models

Chaotic inflation

$$V(\phi) = \lambda M_{pl}^4 \left(\frac{\phi}{M_{pl}}\right)^n$$

Hilltop inflation

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{\mu}\right)^p + \cdot\ \cdot\ \cdot\right]$$

Natural inflation

$$V(\phi) = \Lambda^4 \left[1 - cos\left(\frac{\phi}{f_\phi}\right)\right]$$

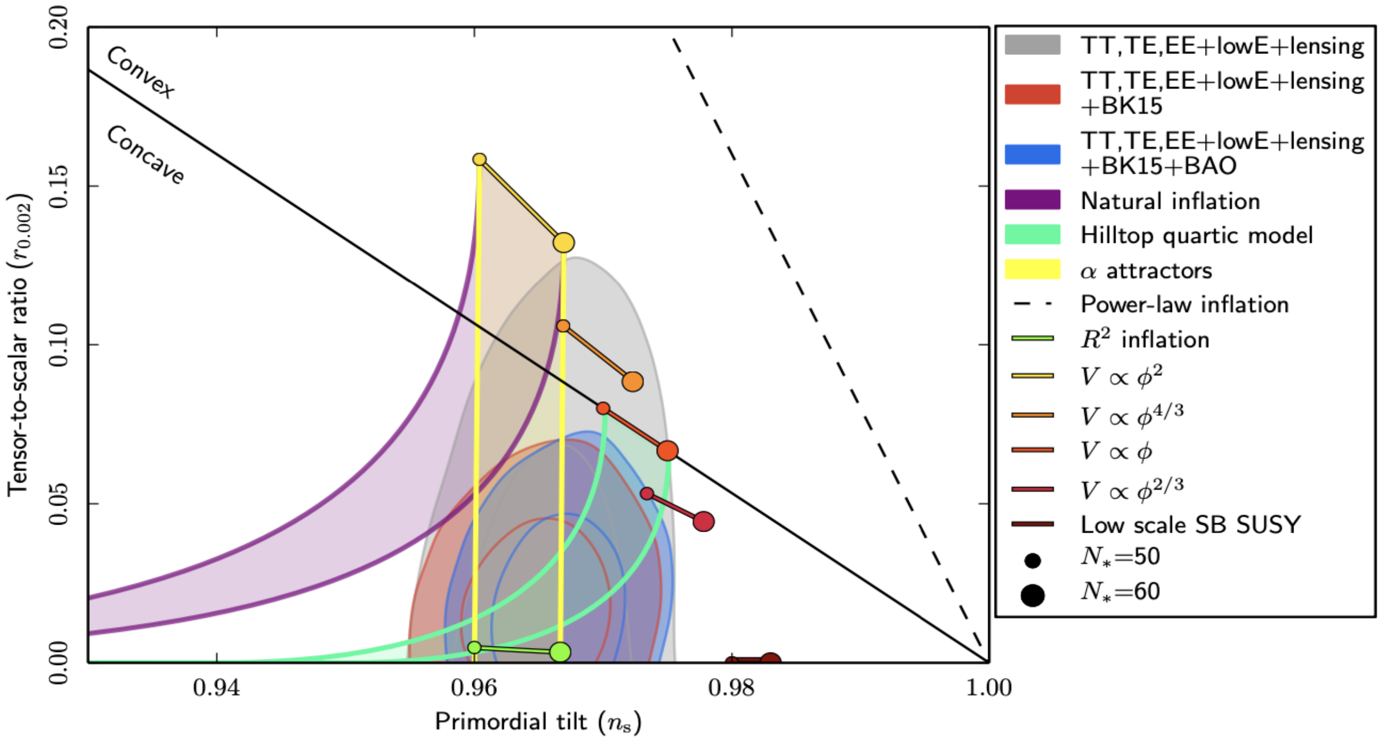
Power-law inflation

$$V(\phi) = V_0 e^{\frac{\alpha\phi}{M_{pl}}}$$

Current observational constraints

This figure compares observational data with theoretical predictions. Shaded regions are constraints from observations. Various potentials have been excluded.

Planck 2018 results



Plank 2018 results X. Constraints on inflation

k-means clustering

Reason for Using the k-means Method for Clustering

Merit

・Low computational cost and easily accommodates large datasets.

・Each data point is clearly classified into a cluster, facilitating subsequent analysis and classification.

・The number of clusters can be specified, allowing for easy adjustment.

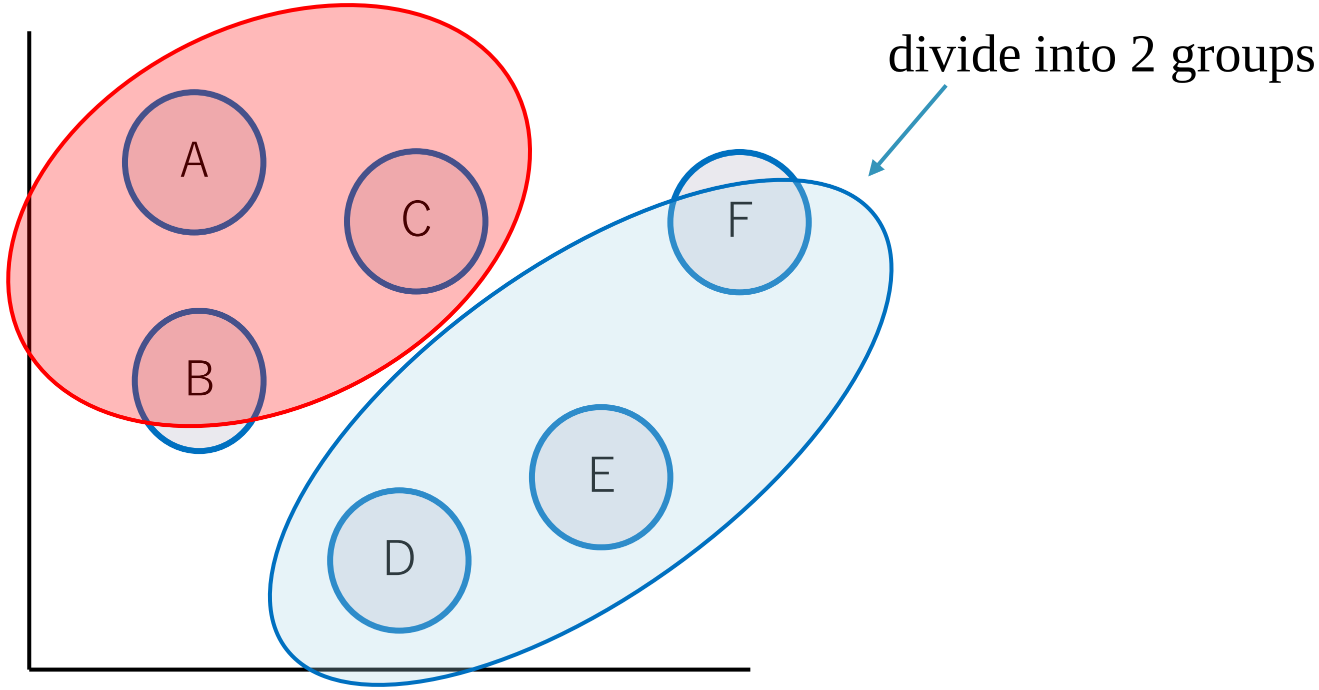
Reason for Clustering Inflation Models

Difficult to analyze from a single perspective due to multiple parameters.

・Clustering allows grouping of models with different parameters and trends.

k-means clustering

non-hierarchical clustering



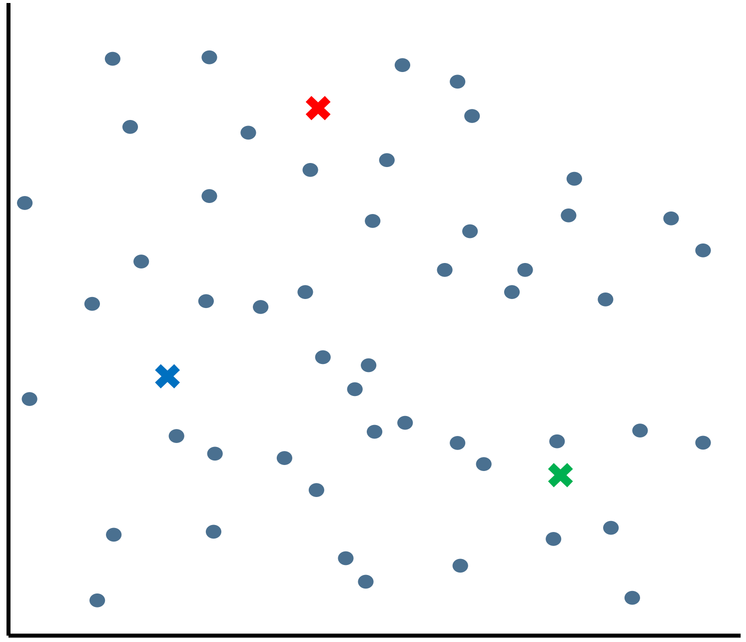
Clustering
A method of unsupervised learning in which given data is grouped together by similarity.

k-means clustering

Step 1 Determine the number of clusters

In this work, we divide them to divide them into three clusters.

Step 2 Set the initial value of the center of gravity

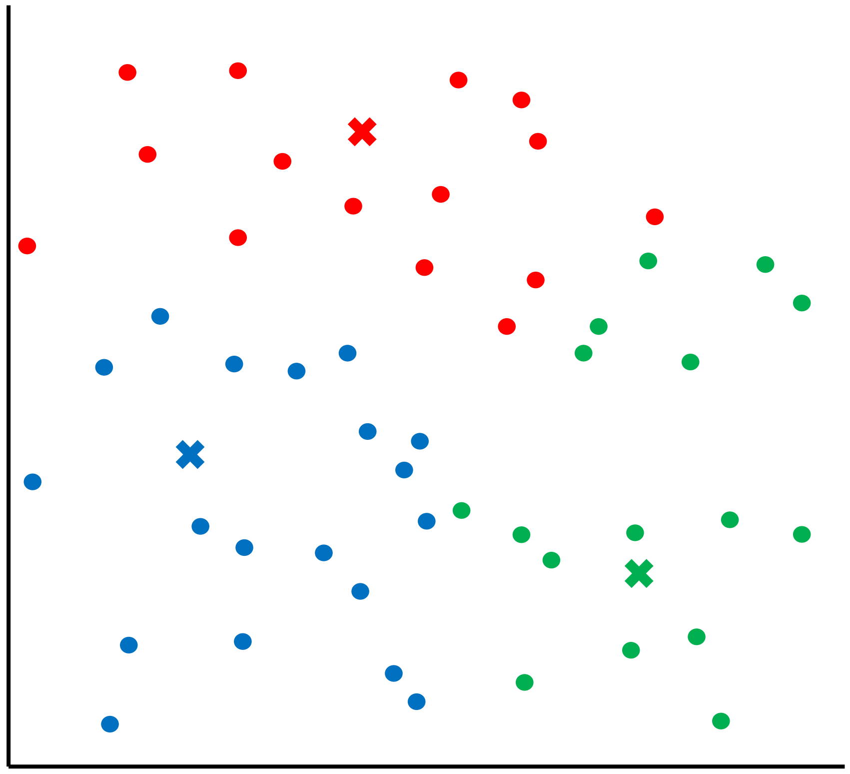


<u>k-means clustering</u>
A method of unsupervised learning in which given data is grouped together by similarity.

k-means clustering

Step 3

Cluster the data based on the distance to the center of gravity.

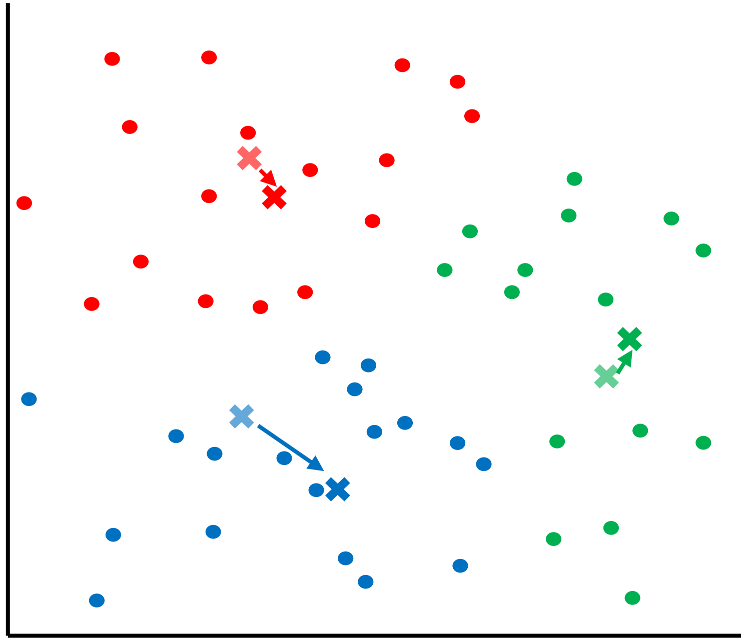


k-means clustering

Step 4 Recalculate the center of gravity for each class

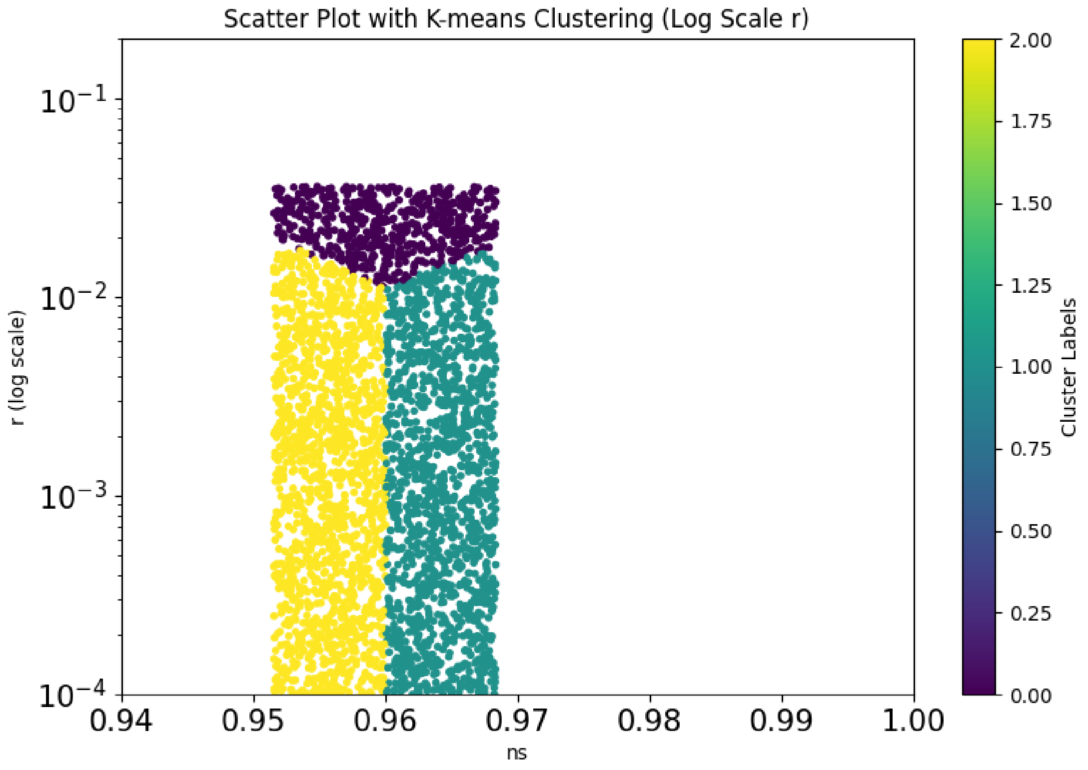
k-means clustering

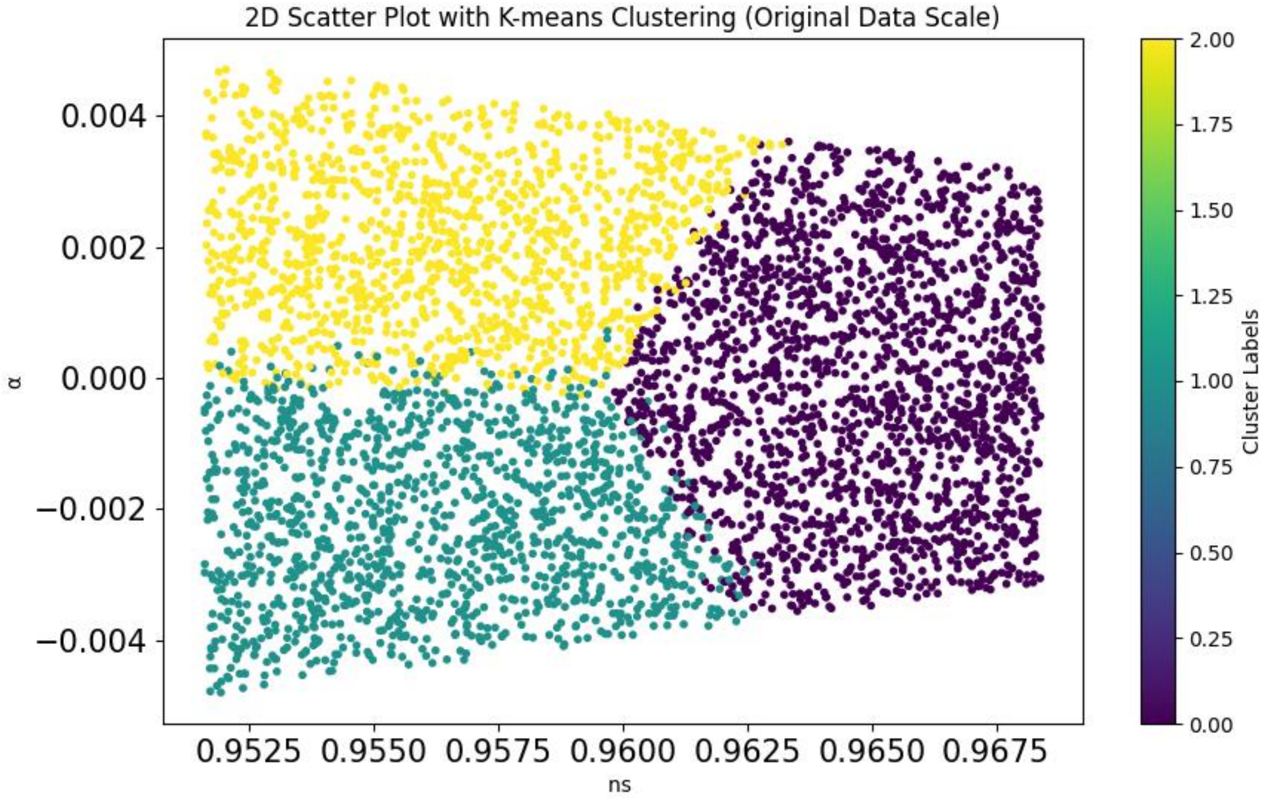
Step 5 Repeat Steps 3 and 4 until no change occurs



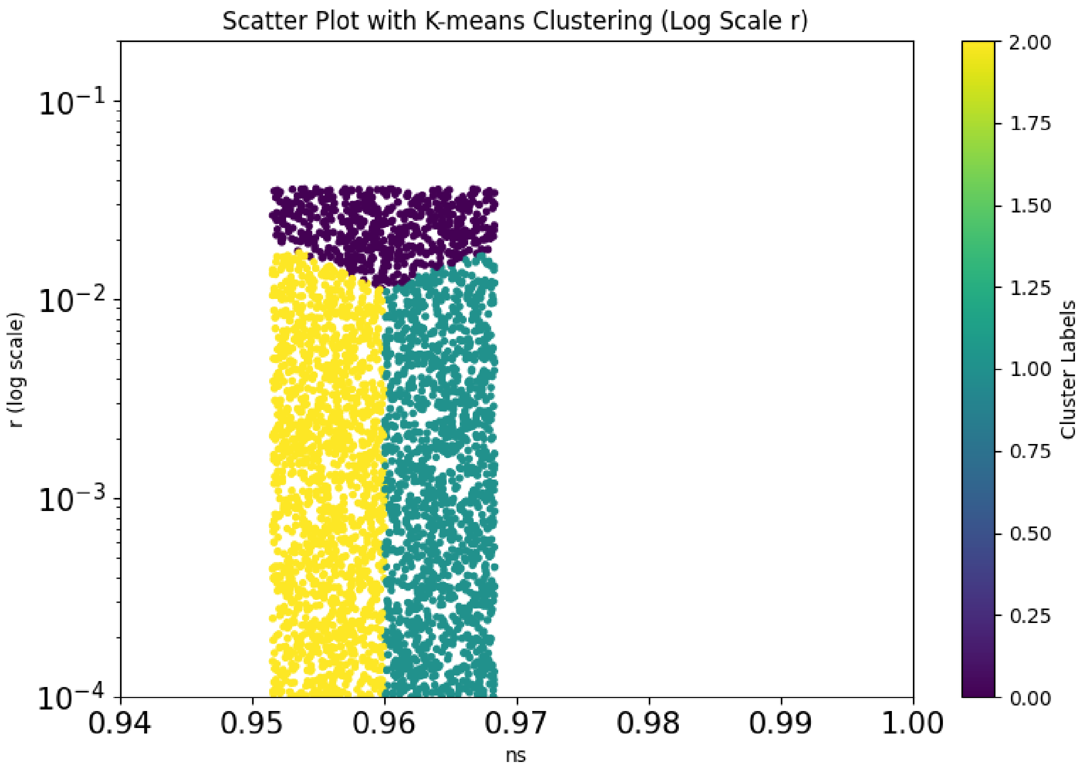
Steps 3 and 4 are repeated, and the state at the point when there are no more changes is the output of the k-means method.

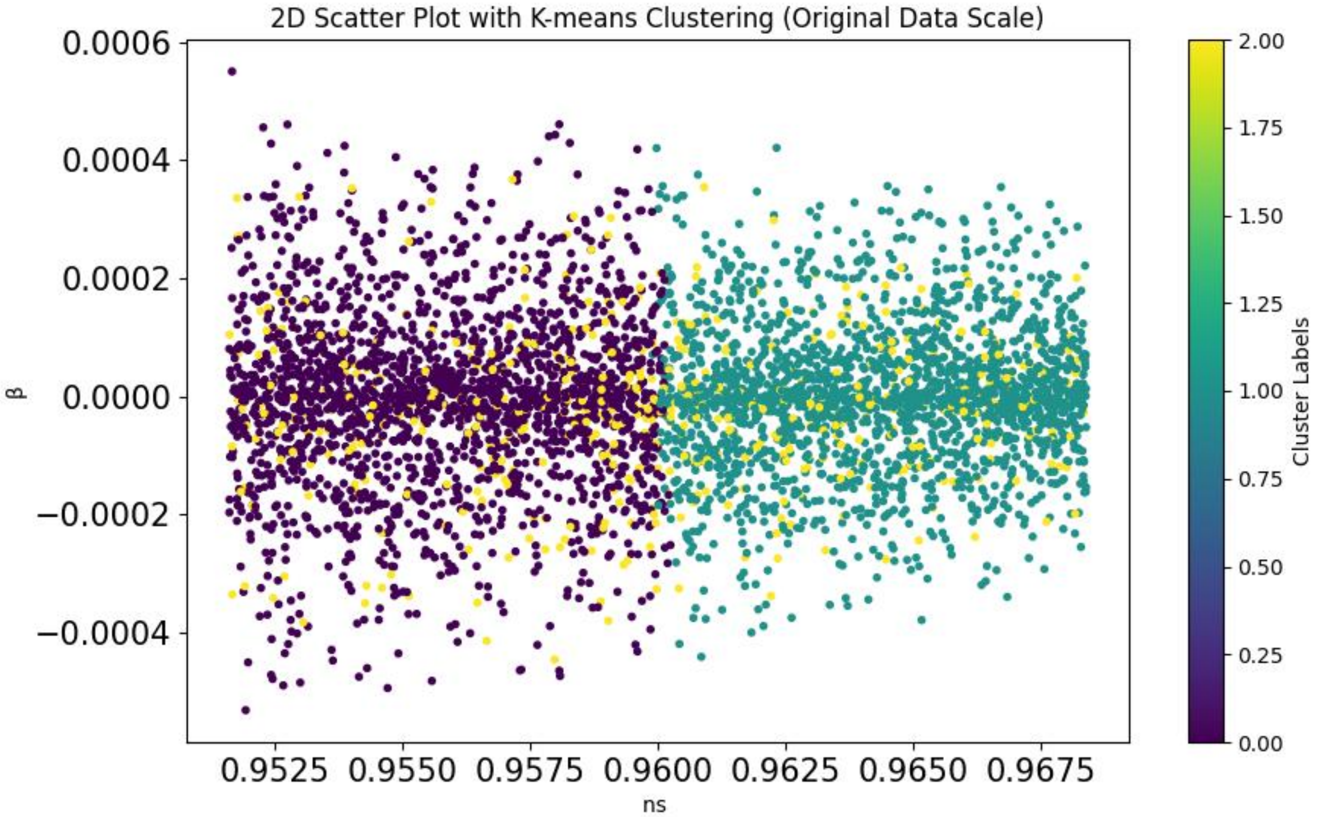
Result


Scatter Plot with K-means Clustering (Log Scale r)
r (log scale)
ns
0.94
0.95
0.96
0.97
0.98
0.99
1.00
Cluster Labels
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

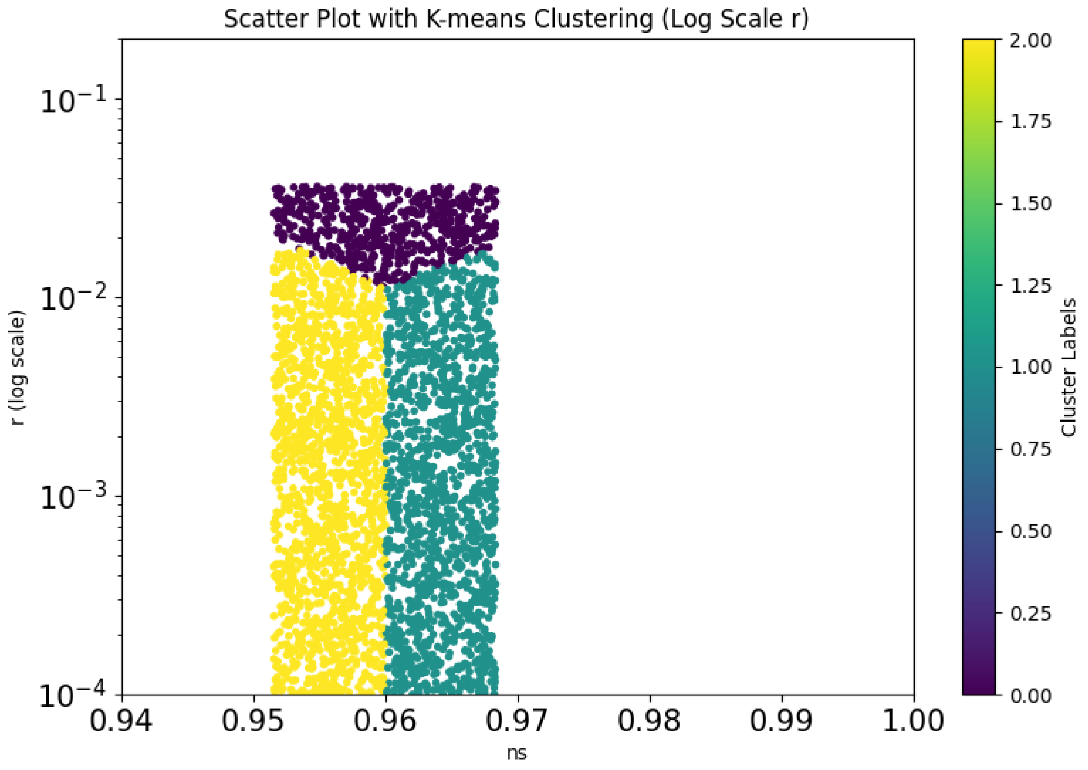

2D Scatter Plot with K-means Clustering (Original Data Scale)
α
0.004
0.002
0.000
−0.002
−0.004
ns
0.9525
0.9550
0.9575
0.9600
0.9625
0.9650
0.9675
Cluster Labels
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

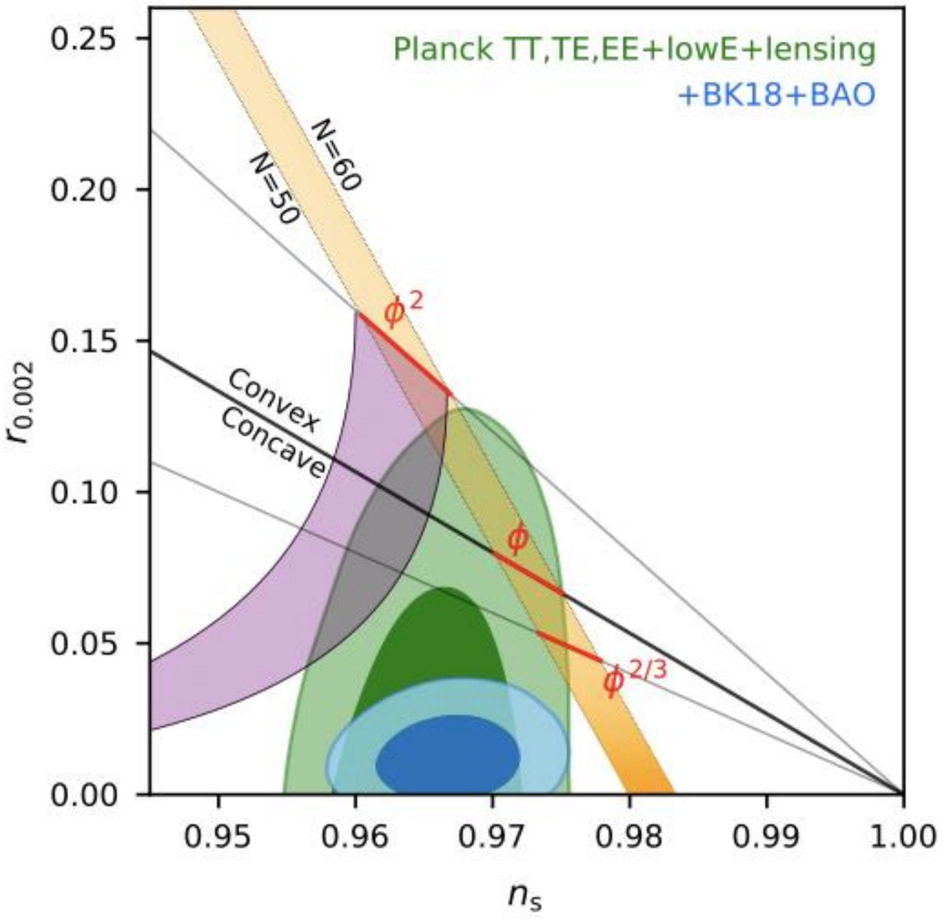
Result


Scatter Plot with K-means Clustering (Log Scale r)
r (log scale)
ns
Cluster Labels
0.94
0.95
0.96
0.97
0.98
0.99
1.00
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00


2D Scatter Plot with K-means Clustering (Original Data Scale)
β
ns
Cluster Labels
0.0006
0.0004
0.0002
0.0000
−0.0002
−0.0004
0.9525
0.9550
0.9575
0.9600
0.9625
0.9650
0.9675
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

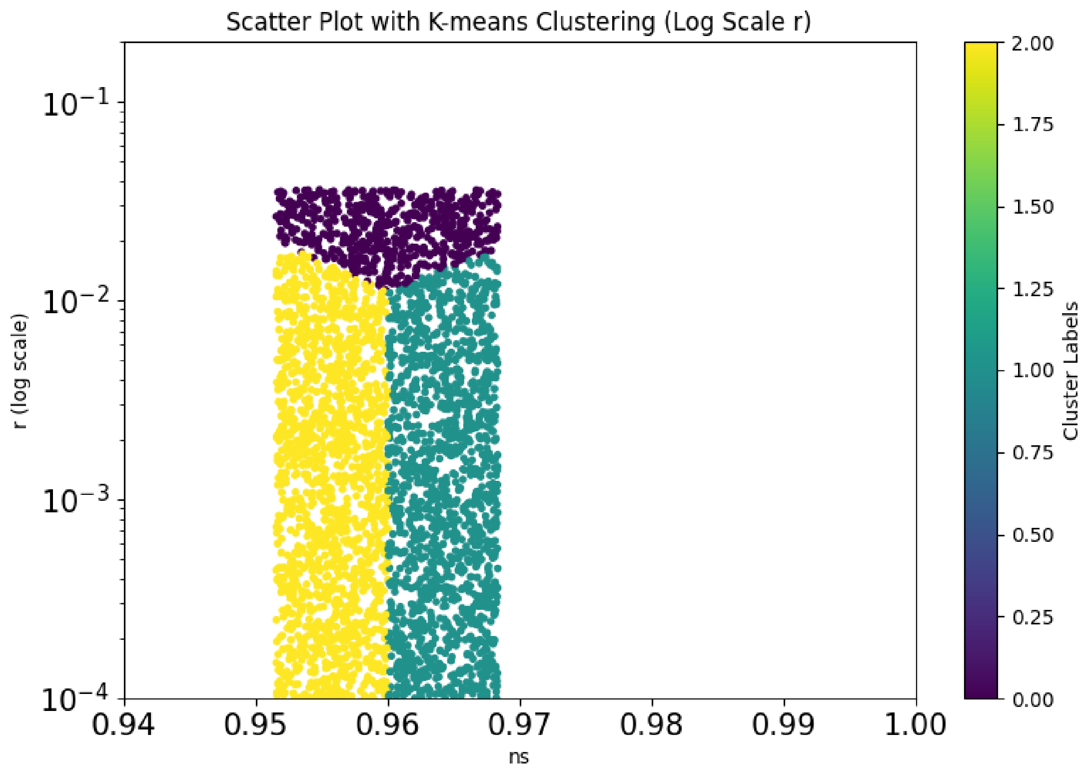
Result



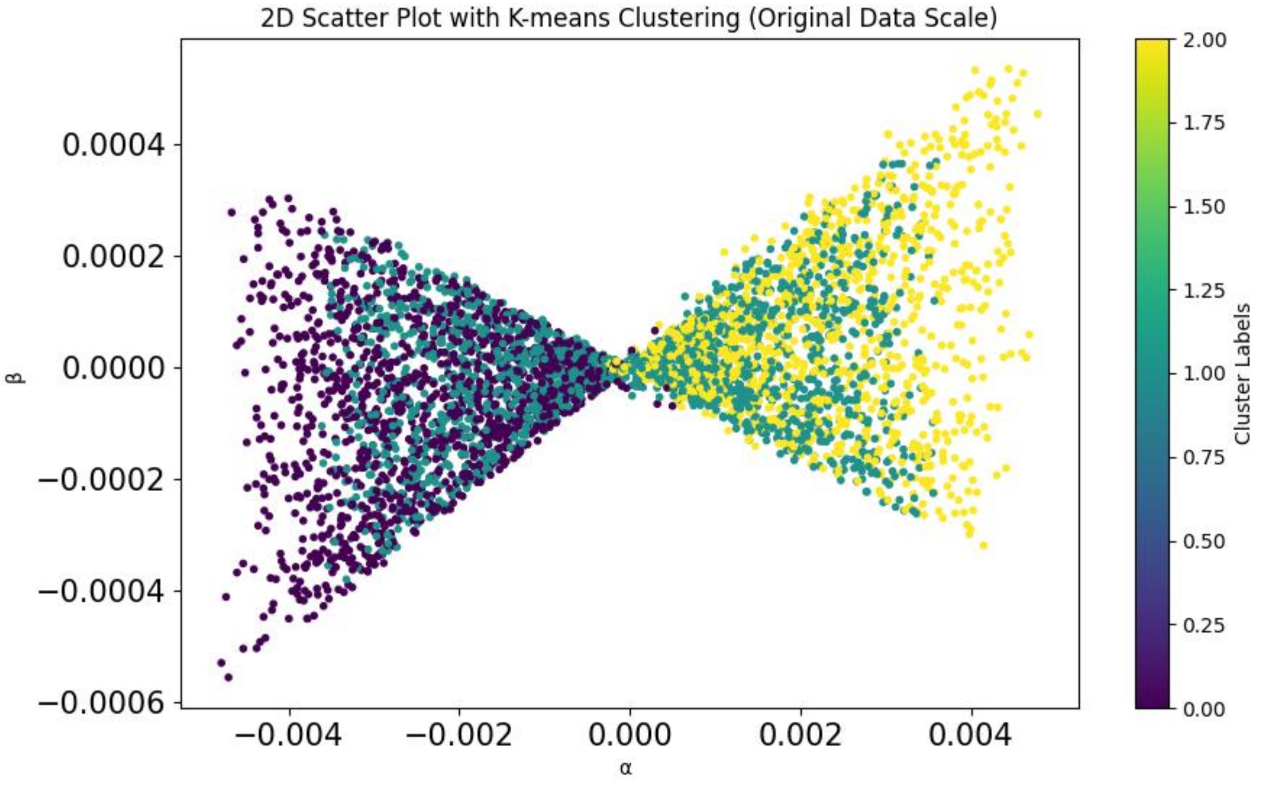


BICEP / Keck XIII: Improved Constraints on Primordial Gravitational Waves using Planck, WMAP, and BICEP/Keck Observations through the 2018 Observing Season

Result



Scatter Plot with K-means Clustering (Log Scale r)
r (log scale)
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0.94
0.95
0.96
0.97
0.98
0.99
1.00
ns
Cluster Labels
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00



2D Scatter Plot with K-means Clustering (Original Data Scale)
β
0.0004
0.0002
0.0000
−0.0002
−0.0004
−0.0006
−0.004
−0.002
0.000
0.002
0.004
α
Cluster Labels
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00

Summary

Discovery of correlations

It may be possible to identify correlations between various parameters.
(For example, ns, r, α, β)

Discovery of new physical phenomena

The patterns and characteristics of the data obtained through clustering may lead to the discovery of new physical phenomena or mechanisms that cannot be explained by existing theories or models.