Inverse Power-law Potential with Inverse Non-minimal Coupling to Unify Inflation and Late-time Acceleration

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Introduction

Introduction

- **Cosmic Inflation** is one of popular mechanism to cure following issues on the Standard Big Bang Cosmology.
    - Fine-tuning on Parameters (e.g. Flatness problem, Horizon problem, ...)
    - Origin of Density Perturbations
- Various observations tell us that our current Universe is in favor of acceleration. (**late-time acceleration**).

Supernova Search Team collaboration (1998).

Supernova Cosmology Project collaboration (1999).

S.F. Daniel et al. (2008).

Introduction

- Both mechanisms introduces a scalar field (named **inflaton**, and **quintessence**) beyond the Standard Model such that the slow-roll assumption is satisfied. In general, those two fields are not necessarily equal.

- What about a case where the early time inflation and late time acceleration is governed by a "same" scalar field φ?

  $\longrightarrow$ **Quintessence Inflation** (**QI**)

Introduction

- In order to successfully realize cosmic inflation and late time acceleration, potential $V(\varphi)$ at two certain regimes (e.g. small field limit and large field limit) must be flat.
  - To ensure fine-tuning problem (e.g. horizon, flatness problem) during early-time inflation.
  - To ensure late-time acceleration : $\ddot{a}_0 > 0$.

$$\frac{\ddot{a}_0}{a_0} = -\frac{1}{2}\left(\frac{1}{3} + w_{eff,0}\right) \Longrightarrow w_{eff,0} < -\frac{1}{3}$$

where $w_{eff,0} \simeq w_{s,0}\Omega_{s,0}$.

Setup & Motivations

Our Setup

- In our work, we considered following setup :

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\underbrace{\left(1+\frac{\xi}{\varphi^{n/2}}\right)}_{\Omega^2(\varphi)\,:\,\text{non-minimal coupling}}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial^\nu\varphi - \frac{V_0}{\varphi^n}\right]$$

i. Flat potential in a large limit in a minimal way

ii. Attractor Behavior

iii. Origin from dynamical symmetry breaking

I. Affleck et al. (1985).

P. Binetruy (1999).

For simplicity, we will replace the exponential factor : $n \to 2n$.

Weyl Transformation

- Applying Weyl Transformation

$$g_{\mu\nu} \Longrightarrow g_{E,\mu\nu} = \Omega^2(\varphi) g_{\mu\nu}$$

results in an Einstein-frame action with a *canonical* field s

$$S_E = \int d^4x \sqrt{-g_E} \left[\frac{1}{2} R_E - \frac{1}{2} (\partial s_E)^2 - V_E \right], \qquad V_E \equiv \frac{V_J(s(\varphi))}{\Omega^4(s(\varphi))}$$

where canonical field s and Jordan frame field φ are related by following equation.

$$\frac{ds}{d\varphi} = \sqrt{\frac{1}{\Omega^2} + \frac{3(\Omega^2_{,\varphi})^2}{2\Omega^4}} \equiv F(\varphi)$$

Why NM Coupling with Inverse Power-Law?

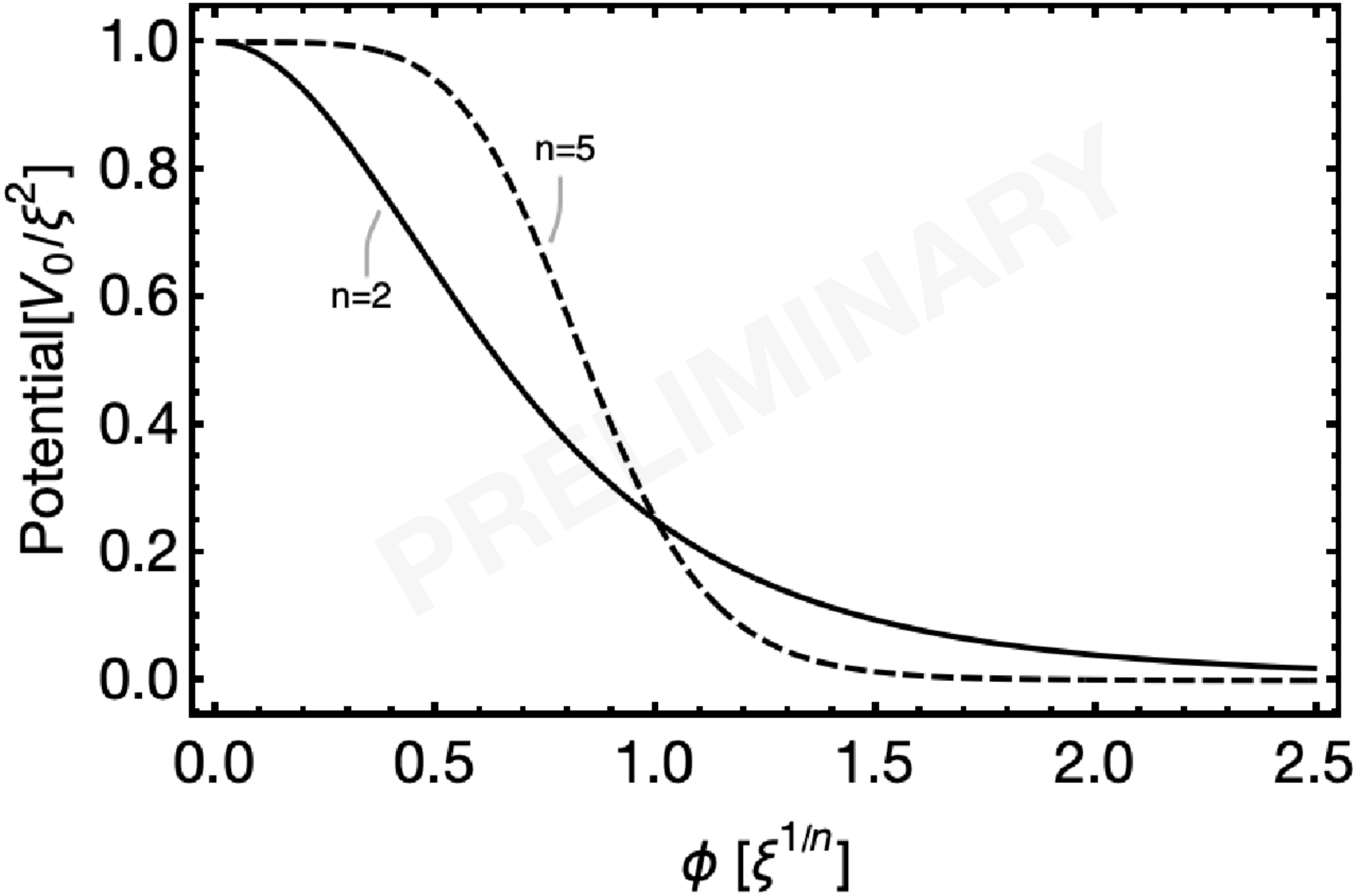
- In a small field limit $(\varphi^n \ll \xi)$, the Einstein-frame potential is approximated by

$$V_E = \frac{V_0}{(\varphi^n + \xi)^2} \simeq \frac{V_0}{\xi^2}\left(1 - 2\frac{\varphi^n}{\xi}\right)$$

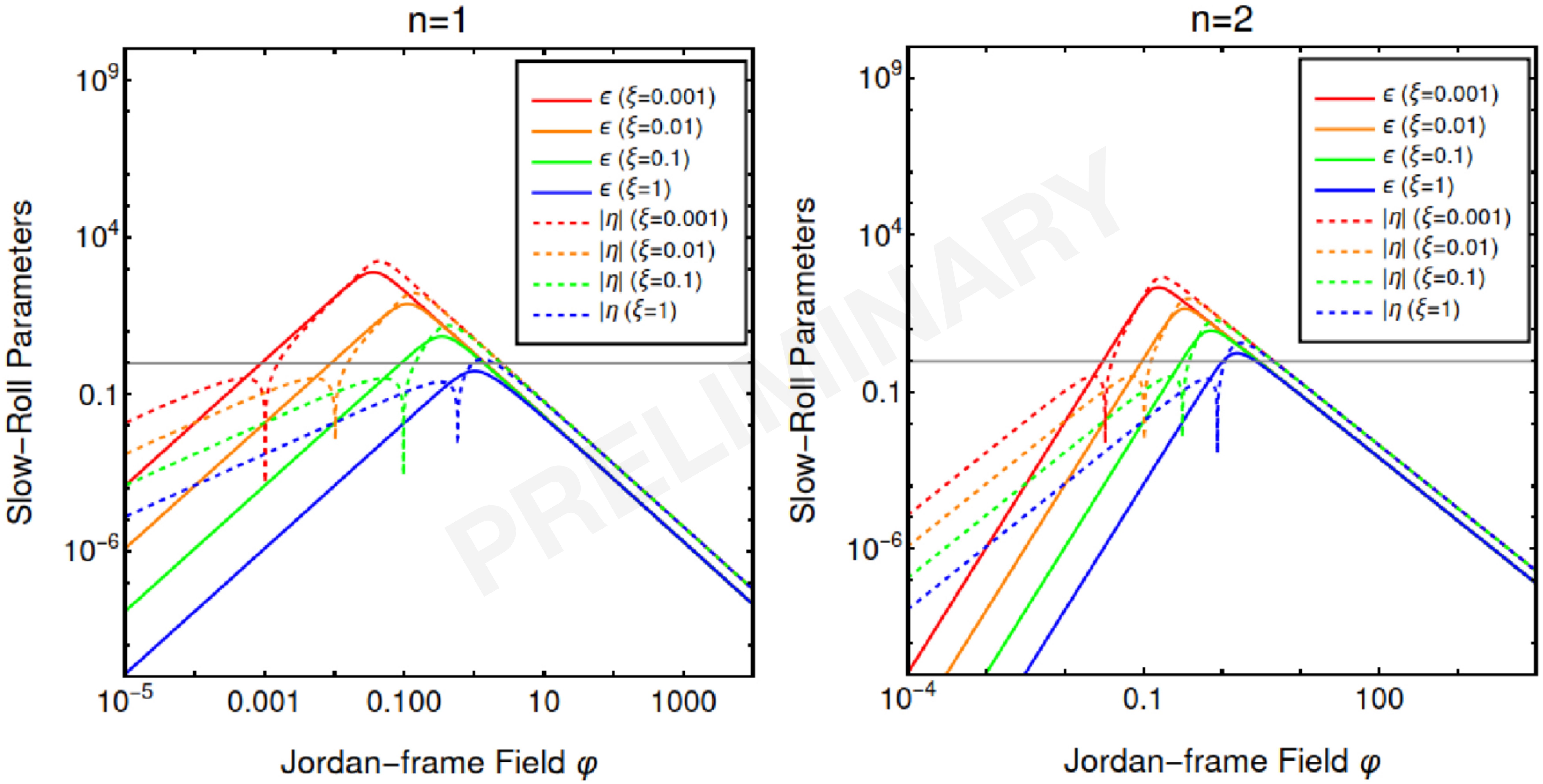
- In a large field limit $(\varphi^n \gg \xi)$, the Einstein-frame potential is approximated by

$$V_E = \frac{V_0}{(\varphi^n + \xi)^2} \simeq \frac{V_0}{\varphi^{2n}}$$

Why NM Coupling with Inverse Power-Law?



Slow-Roll Parameters



Slow-Roll parameters

1. $$\epsilon \equiv \frac{1}{2V_E^2}\left(\frac{dV_E}{ds}\right)^2 = \frac{1}{2V_E^2F^2}\left(\frac{dV_E}{d\varphi}\right)^2$$

2. $$\eta \equiv \frac{1}{V_E}\frac{d^2V_E}{ds^2} = \frac{1}{FV_E}\frac{d}{d\varphi}\left(\frac{1}{F}\frac{dV_E}{d\varphi}\right)$$

For general n and ξ, there always exists ξ_{crit} such that there is no solution of equation $\max\{\epsilon, |\eta|\} = 1$ for $\xi \geq \xi_{crit}$. We will only focus on a case where $\xi < \xi_{crit}$.

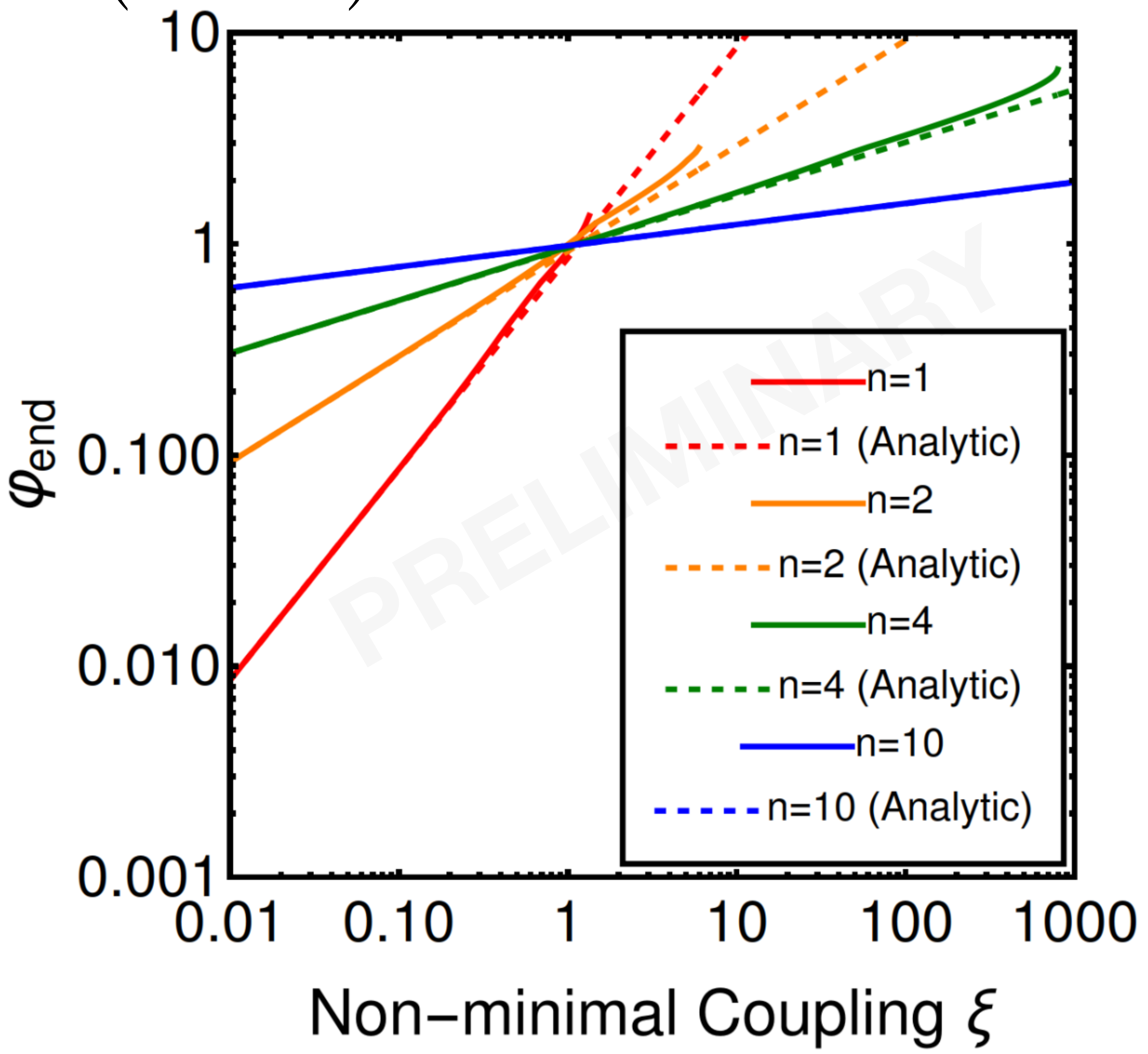
Early Time Inflation

$\left(\sqrt{3}/2\right)^{1/n} \simeq 1$ for every natural number n.

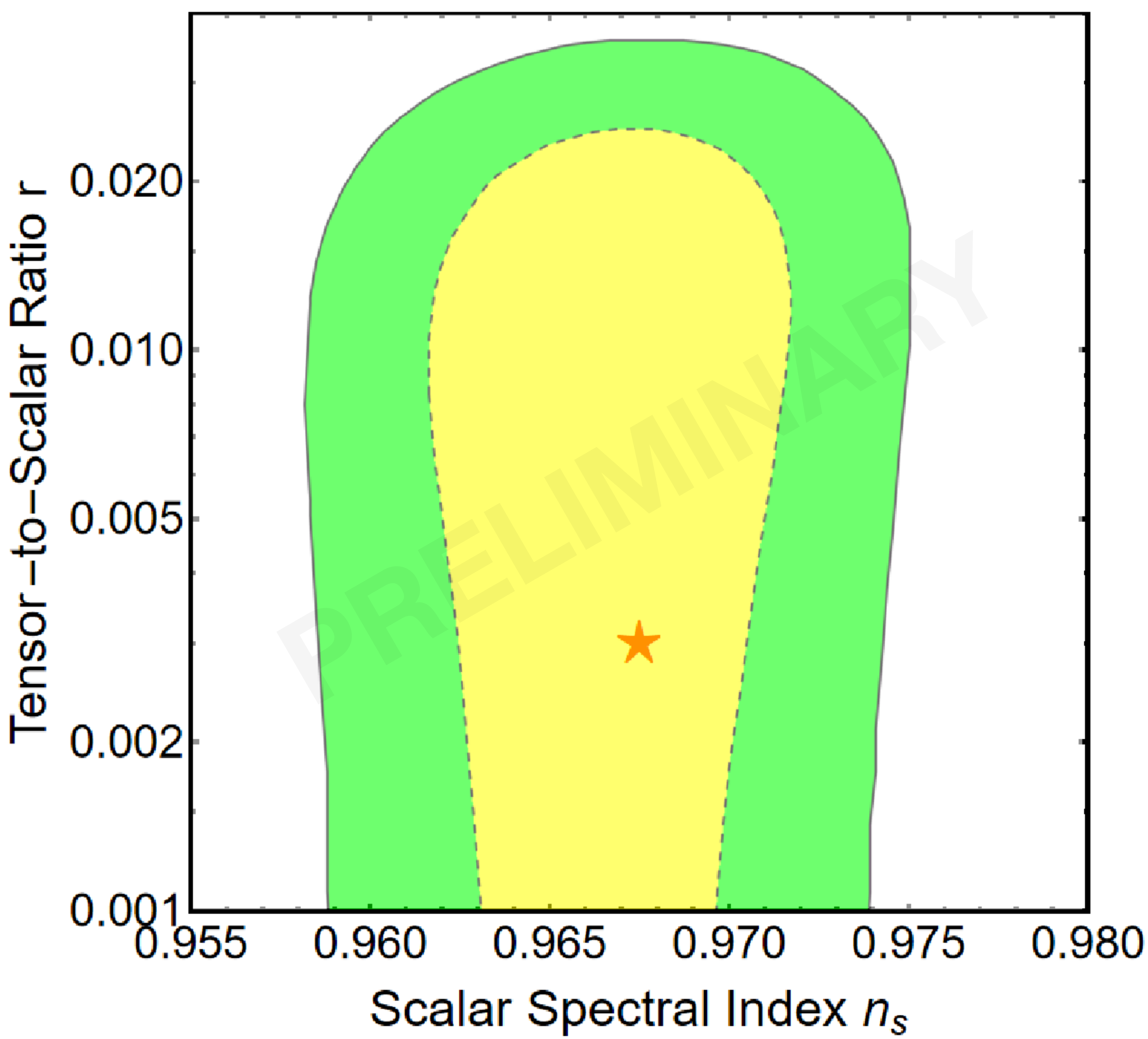
End of Inflation

- Numerical calculation of end-of-inflation by solving $\max\{\epsilon, |\eta|\} = 1$ results in following scaling behavior : $\varphi_{end} \simeq \left(\sqrt{3}\xi/2\right)^{1/n}$.

- From this fact, we can roughly divide whole evolution of the scalar field into two regimes :
  - Small field limit ($\varphi \ll \xi^{1/n}$) during *inflation*.
  - Large field limit ($\varphi \gg \xi^{1/n}$) during *late-time acceleration*.



Main Results

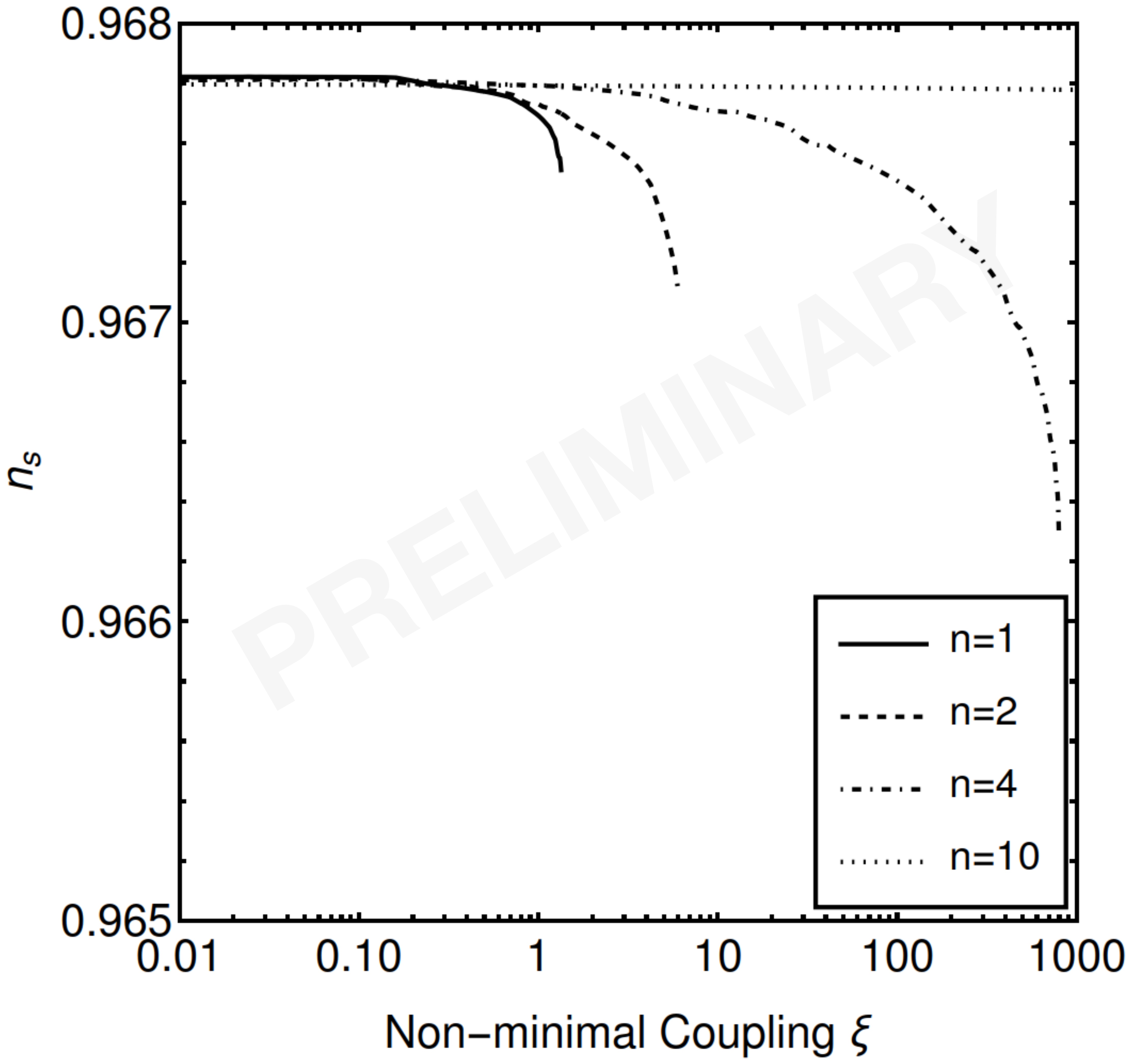


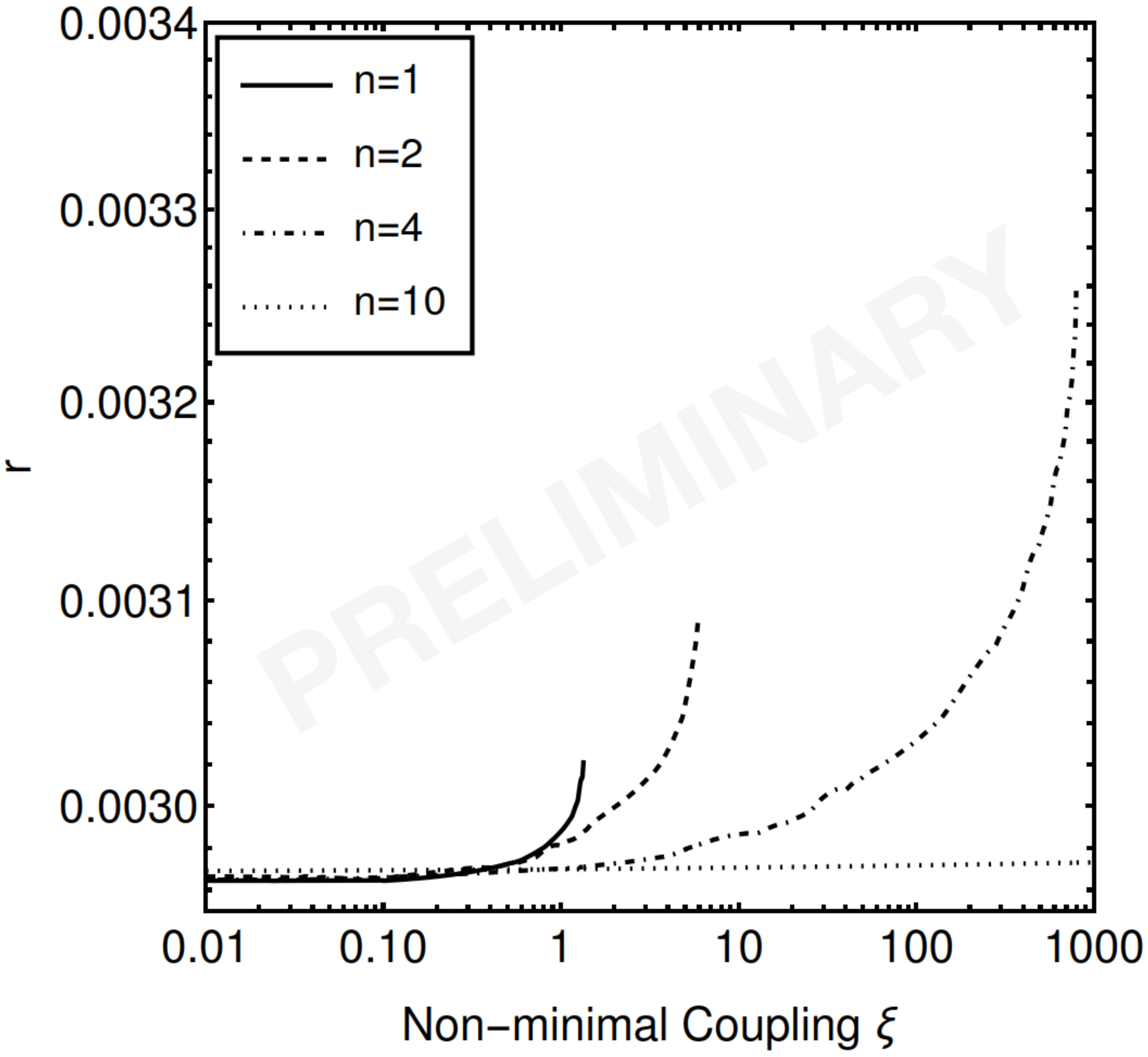
PLANCK Collaboration (2020).
BICEP, Keck collaboration (2021).

- Observational Constraints by Planck results and BICEP/Keck (BK) data are drawn respectively.
  - 1σ-bound colored with **green**.
  - 2σ-bound colored with **yellow**.

- We picked $N_e = 60$, where N_e denotes an e-folds between pivot scale and an event where inflation terminates.

Main Results





Independence of Observables with Free Parameters

- In a small field limit ($\varphi \ll \xi^{1/n}$), CMB observables are approximated by

$$n_s = 1 - 6\epsilon_* + 2\eta_* \simeq 1 - \frac{8}{3}\frac{\varphi_*^n}{\xi} - 8\left(\frac{\varphi_*^n}{\xi}\right)^2 \simeq 1 - \frac{2}{N_e} - \frac{9}{N_e^2}, \qquad r \simeq \frac{64}{3}\left(\frac{\varphi_*^n}{\xi}\right)^2 \simeq \frac{12}{N_e^2},$$

where $\varphi_* \simeq (3\xi/4N_e)^{1/n}$.

PLANCK Collaboration (2020).

- The last degree of freedom V_0 is *fixed* by normalization condition, which constrains the scalar amplitude at the pivot scale (denoted by $*$) : $\ln(10^{10}A_s) = 3.044 \pm 0.014$ (TT,TE,EE+lowE+lensing). Relation between V_0 and non-minimal coupling is given by :

$$\frac{V_0}{\xi^2} \simeq \frac{18\pi^2 A_s}{N_e^2} \simeq (1.01 - 1.05) \times 10^{-9} \times \left(\frac{N_e}{60}\right)^{-2}.$$

Late Time Acceleration

Late Time Acceleration

- In order to check whether our model is compatible with observational results regarding late-time acceleration, we've checked following items.

1. Hubble parameter at current Universe :
   $\longrightarrow H_0 \simeq 67.36 \pm 0.54 \text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1} \simeq 10^{-51} M_{Pl}$ PLANCK Collaboration (2020).
2. The equation-of-state (EoS) parameter of the quintessence
   $\longrightarrow w_{DE,0} \leq -0.9$ PLANCK Collaboration (2015), S. Alam et al. (2017), Y. Wang et al. (2017).
3. After last scattering event, matter-dominated Universe ensues, and finally dark-energy dominated one.

Classical Equation of Motions

- We solved classical equation of motions for quintessence field

$$H^2\left(1-\frac{1}{6}F^2\varphi_{,N}^2\right)=\frac{V_E}{3}+H_0^2\left(\frac{\Omega_{m,0}}{\exp(3N)}+\frac{\Omega_{r,0}}{\exp(4N)}\right)$$

$$H^2\varphi_{,NN}+\left(3H^2+\frac{1}{2}(H^2)_{,N}\right)\varphi_{,N}+H^2\frac{F_{,\varphi}}{F}\varphi_{,N}^2+\frac{V_{E,\varphi}}{F^2}=0$$

E-folds
$N\equiv\ln(a/a_0)$

with suitable two input initial conditions

$$\varphi(N=N_i),\quad \varphi_{,N}(N=N_i)\,.$$

- In our work, we set $N_i=0$ (today) and tuned $\alpha\equiv\varphi(N=0)$ and $\beta\equiv\varphi_{,N}(N=0)\,.$

Setting Adequate (α, β)

- Two initial conditions (α, β) are uniquely fixed by applying slow-roll conditions and a large field limit as follows :

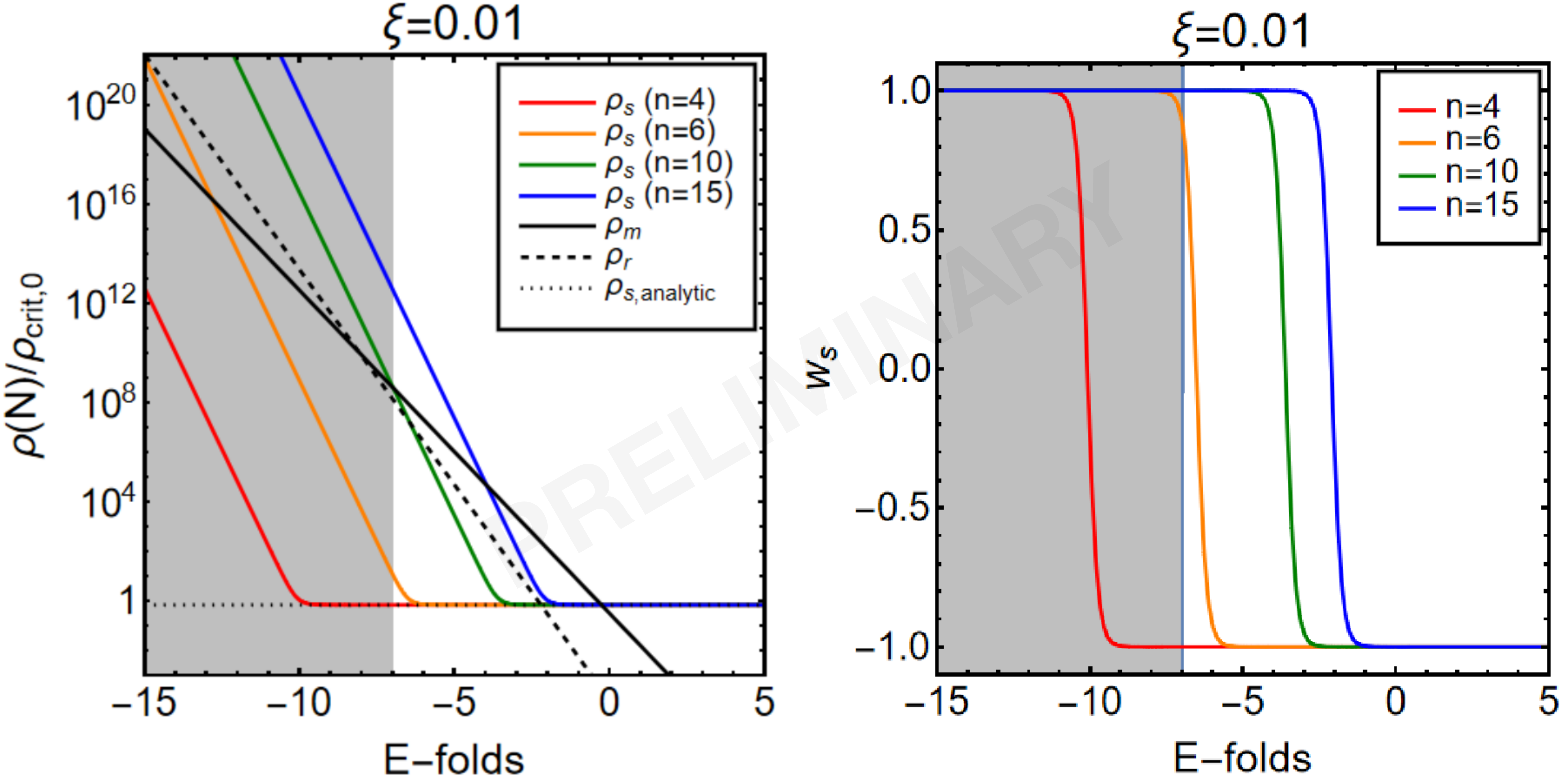
$$\alpha \simeq \left(\frac{V_0}{3H_0^2 \Omega_{s,0}} \right)^{\frac{1}{2n}}, \qquad \beta \simeq 2n\Omega_{s,0} \,.$$

- Inputting above initial conditions, we have tried to numerically calculate the evolution of energy density of the quintessence with respect to an e-fold, as well as an EoS(Equation-of-State) parameter of that field, especially in a regime where $N \in [-15, 5]$.

Main Results

The grey regime corresponds to $N < N_{CMB} \simeq -6.97$, in which our numerical results become unreliable.

(N_{CMB} denotes an event where last-scattering happens.)



Domination of matter components immediately after last scattering is required.

$\Longrightarrow$ **There exists lower bound for n not to harm Big Bang scenario!**

Indistinguishability between s and φ

- Dominance of quintessence field comparing with other components happens in a large-field regime : $\varphi^n \gg \xi$.
- The classical equation of motion with respect to the *Jordan-frame* field is approximated by φ

$$\ddot{\varphi} + 3H\dot{\varphi} + V_{E,\varphi} \simeq 0$$

In other words, the Jordan-frame field φ in our setup behaves like one in a single-field scenario with no non-minimal coupling. In that sense, s and φ are indistinguishable each other. : $s \simeq \varphi$.

$F(\varphi) \equiv \frac{ds}{d\varphi} = \sqrt{\frac{1}{1+\xi\varphi^{-n}} + \frac{3n^2\xi^2\varphi^{-2n}}{2\varphi^2\left(1+\xi\varphi^{-n}\right)^2}} \simeq 1$ on a large field limit.

Explanations of Shift of Dominance of Quintessence

- Suppose we backward quintessence from $N = 0$ to $N = -dN$, with infinitesimal dN. The energy loss due to evolution of quintessence field by Hubble drag is given by

$$|\Delta E_{loss}(N=0)| = -3H_0^2\beta^2 dN$$

- On the other hand, the potential difference by the evolution of quintessence field is given by

$$|\Delta V(N=0)| = (V_0/\alpha^{2n})(2n\beta dN/\alpha) \simeq 3H_0^2(2n\beta/\alpha)dN$$

- One important fact is that for decently small $n \gtrsim \mathcal{O}(1)$,

$$|\Delta E_{loss}(N=0)| \gg |\Delta V(N=0)|$$

e.g., when $(n, \xi) = (4, 0.01)$, $\alpha \sim \mathcal{O}(10^{13})$.

Recall that $\beta \simeq 2n\Omega_{s,0}$.

Explanations

- Suppose we backward quintessence from $N = 0$ to $N = -dN$, with infinitesimal dN.

$$|\Delta E_{loss}(N=0)| = -3H_0^2\beta^2 dN$$

- Hereafter, the most of energy increase $|\Delta E_{loss}(N=0)|$ by backwarding quintessence contributes to the increase of kinetic energy, with potential energy unchanged.
- Therefore, when we increase β with other parameters fixed, the kinetic energy of quintessence overtakes potential energy *faster* than same physical system of low β.

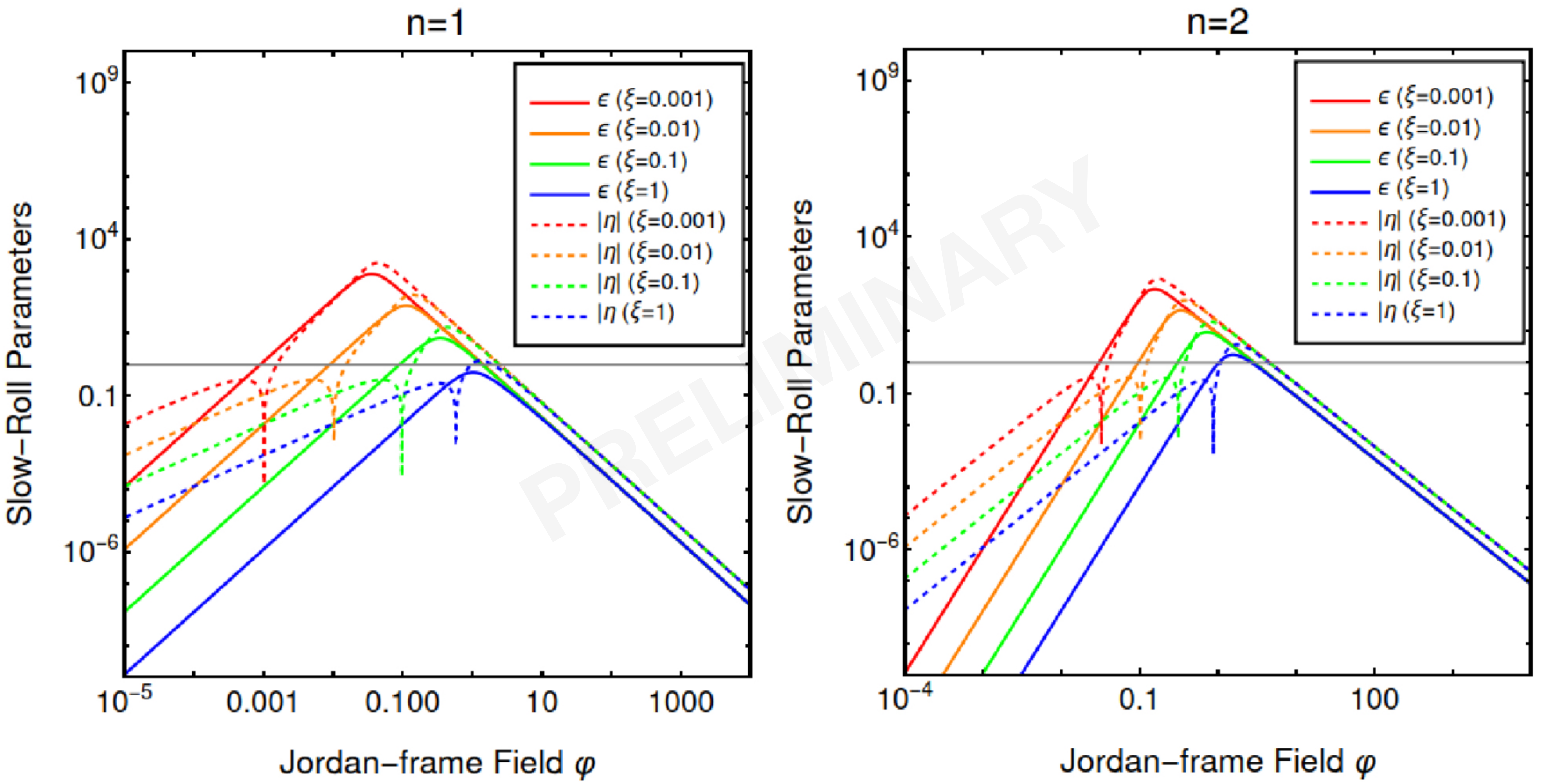
Conclusions

Conclusions

- We have tried a setup which a scalar field is non-minimally coupled with gravity (especially with an inverse non-minimal coupling) with inverse power-law potential.
- CMB observables during inflation are well in accordance with recent bounds, with most of parameter sets : (n, ξ, V_0) .
- This setup also realizes late-time acceleration for some kinds of parameter spaces, and compatible with successful Big Bang scenario, when $n < 10.$

Thank You!

Backup Slides



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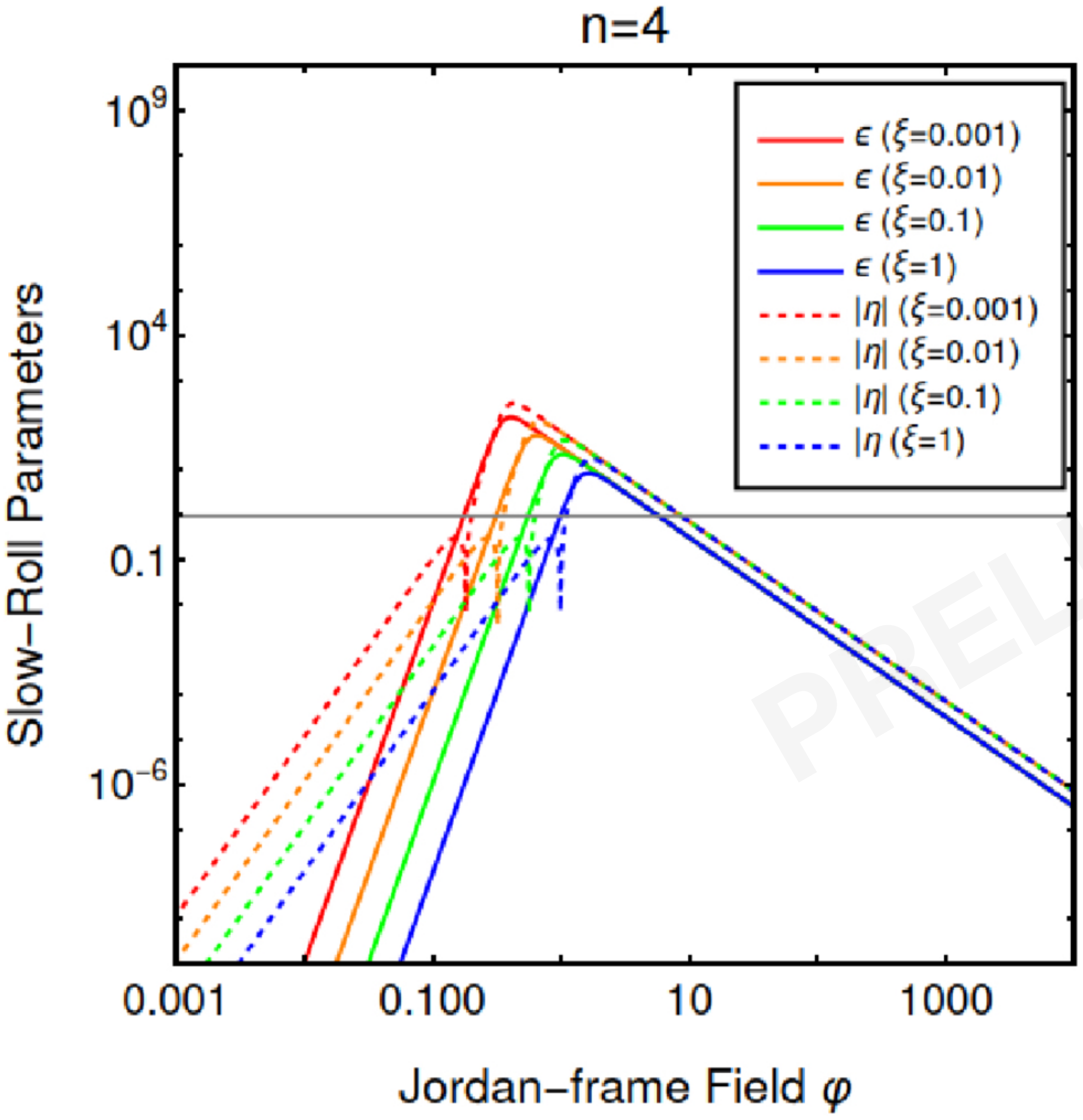

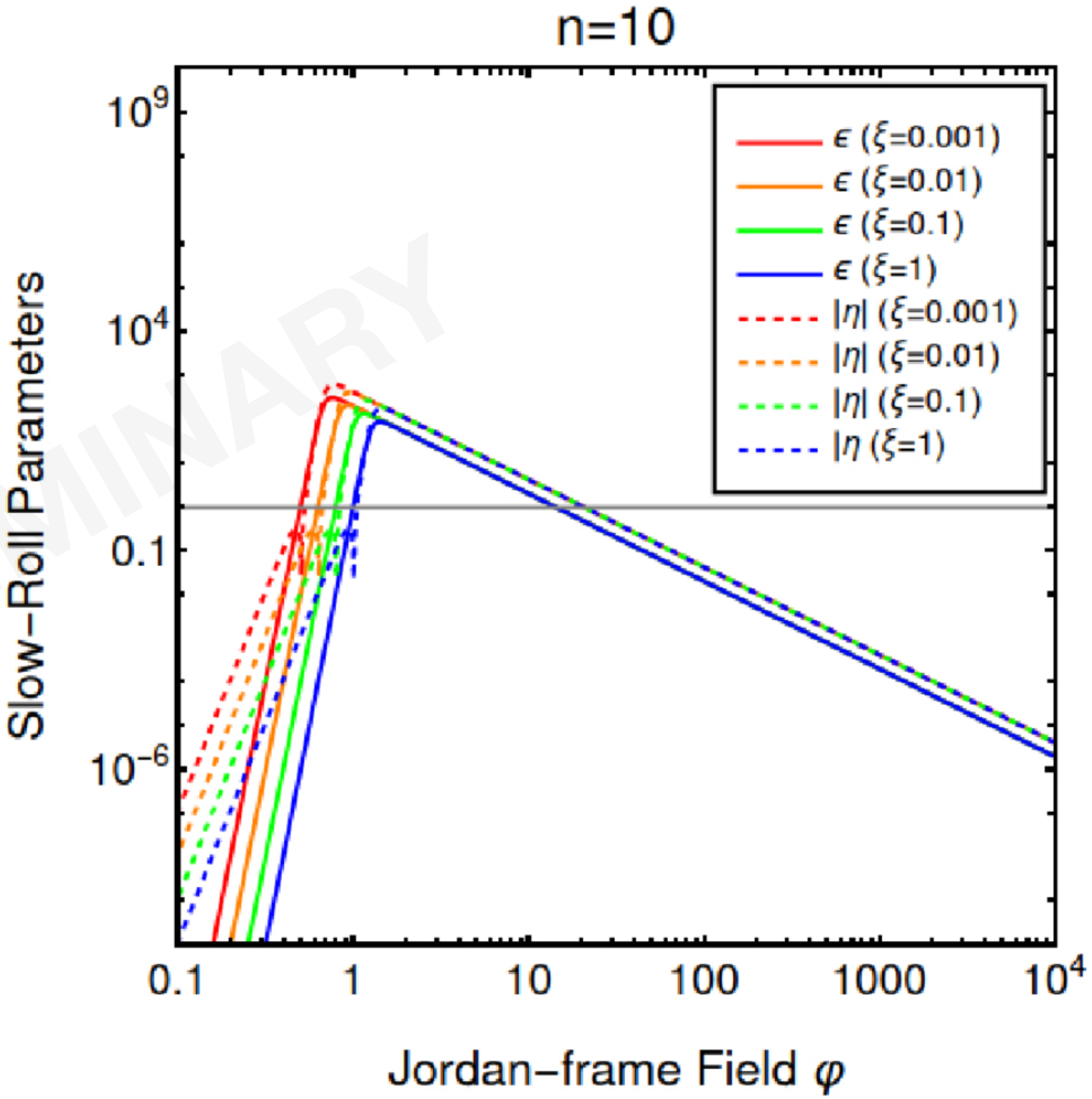

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