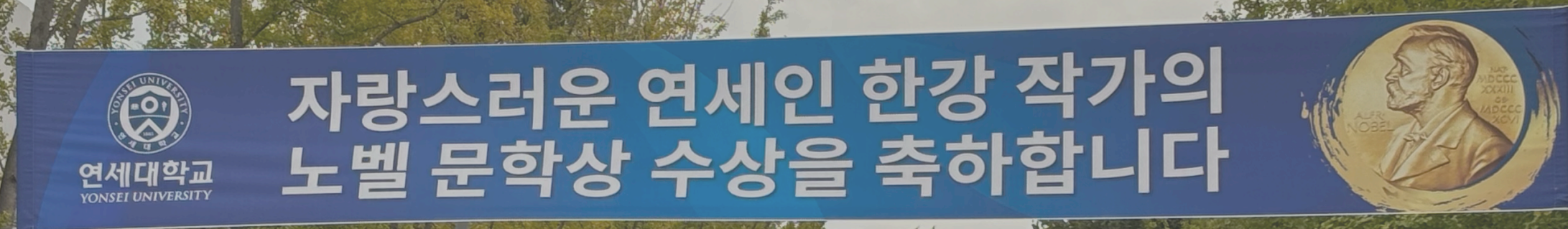


# Updates of $V_{cb}$ , $V_{ub}$ Tensions from Belle (II)



Youngjoon Kwon (Yonsei U.)

Nov. 5, 2024 @ Saga-Yonsei XXI



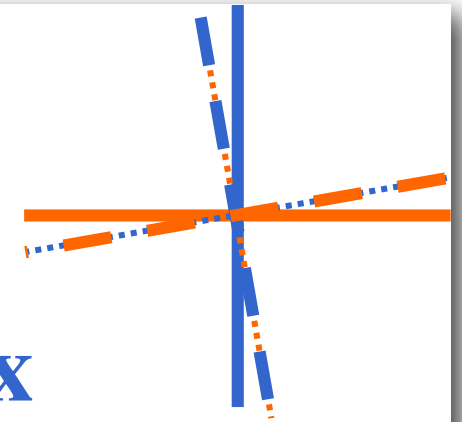
# Fermions of SM

$$\begin{bmatrix} \nu_e \\ e^- \end{bmatrix}, \quad \begin{bmatrix} \nu_\mu \\ \mu^- \end{bmatrix}, \quad \begin{bmatrix} \nu_\tau \\ \tau^- \end{bmatrix}$$

$$\begin{bmatrix} u \\ d' \end{bmatrix}, \quad \begin{bmatrix} c \\ s' \end{bmatrix}, \quad \begin{bmatrix} t \\ b' \end{bmatrix}$$

# Quark flavor mixing and CKM matrix

- For quarks,
  - weak interaction eigenstates  $\neq$  mass eigenstates
  - mixing of quark flavors through a **unitary matrix**



$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \left( V_{\text{CKM}} \right) \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

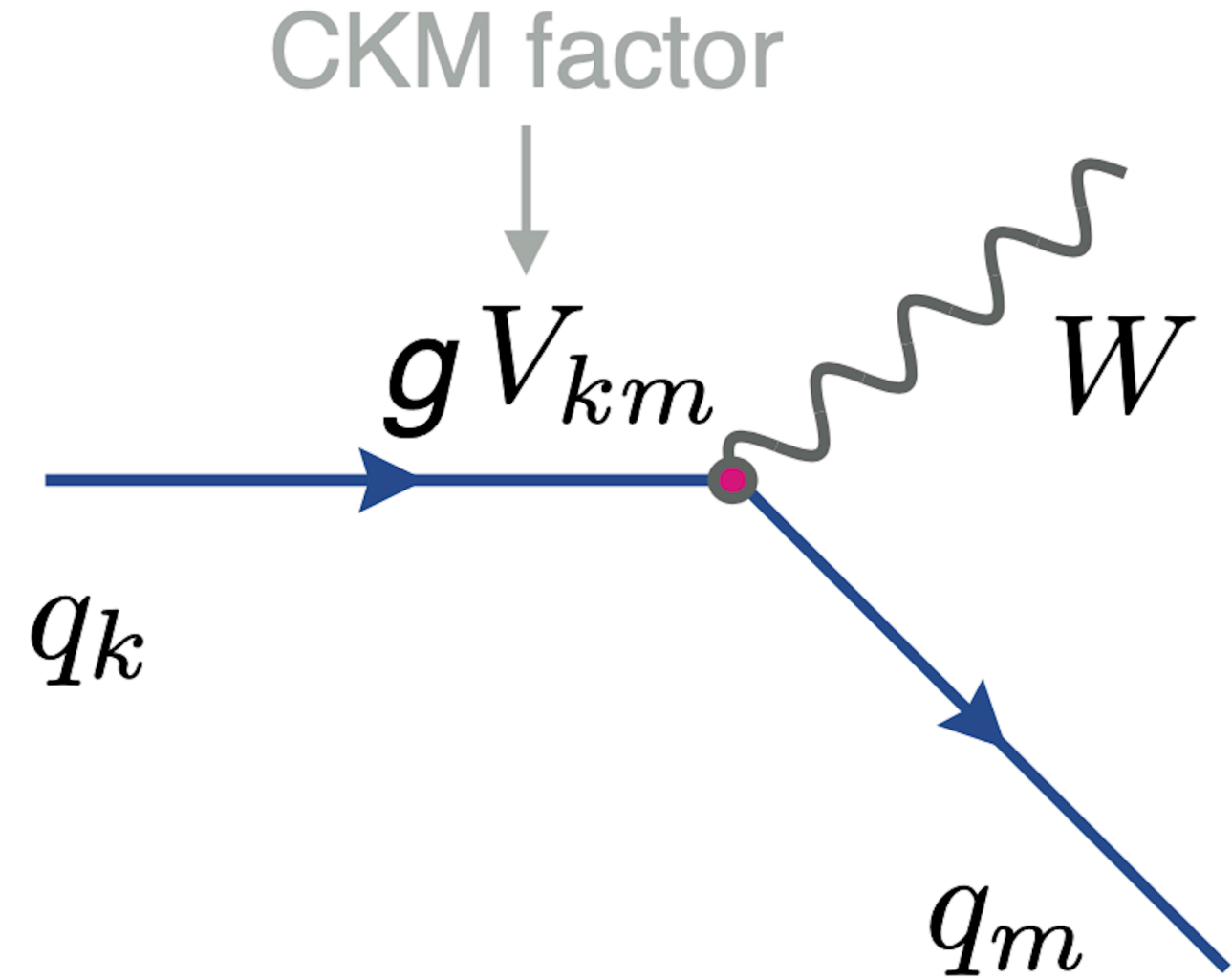
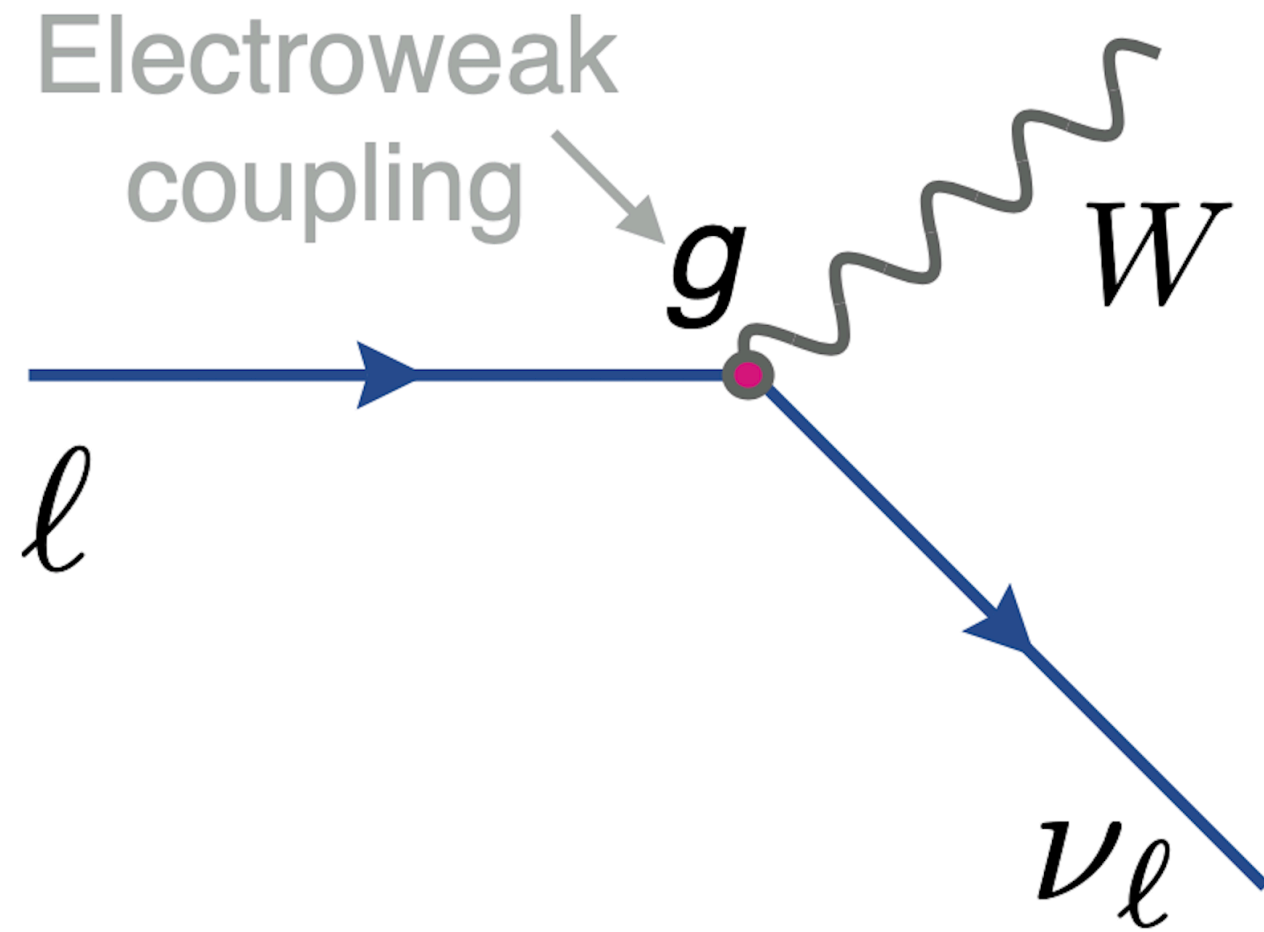
**Wolfenstein parametrization**

$$V_{\text{CKM}} \approx \begin{pmatrix} 1 - \lambda^2/2 & \lambda & \frac{A\lambda^3(\rho - i\eta)}{A\lambda^2} \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ \frac{A\lambda^3(1 - \rho - i\eta)}{A\lambda^2} & -A\lambda^2 & 1 \end{pmatrix}$$

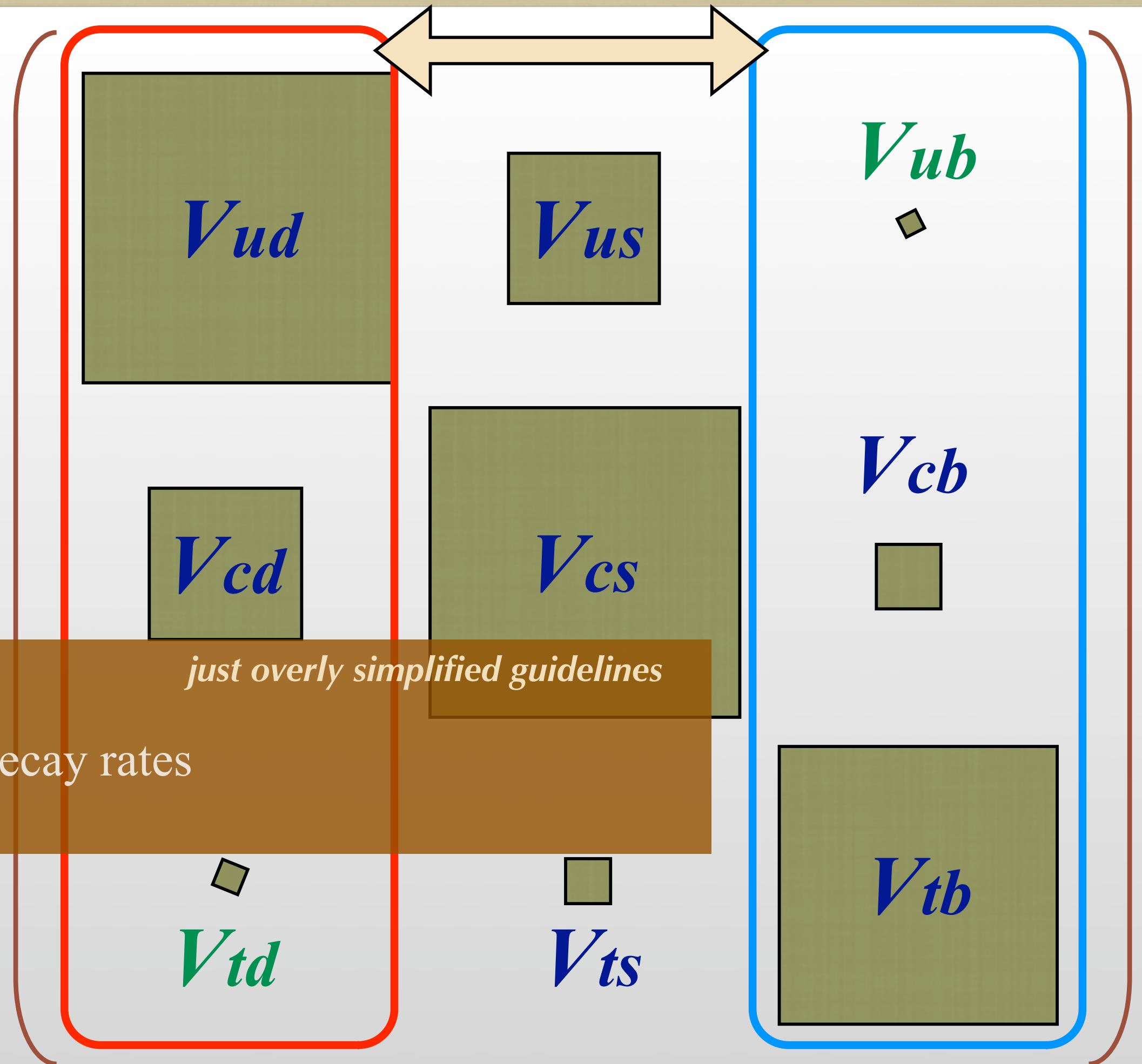
$$|\lambda| \approx O(0.1)$$

3 real parameters ( $\lambda, A, \rho$ ) and 1 phase ( $\eta$ )

# How fermions interact with $W^\pm$

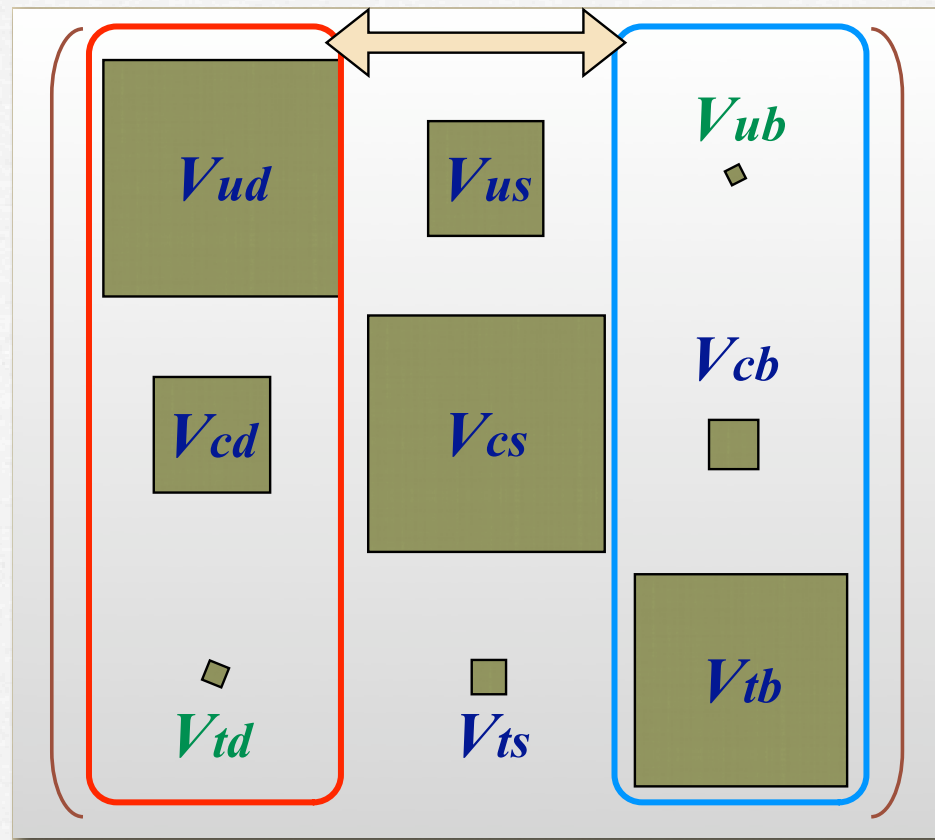


# Test Unitarity?



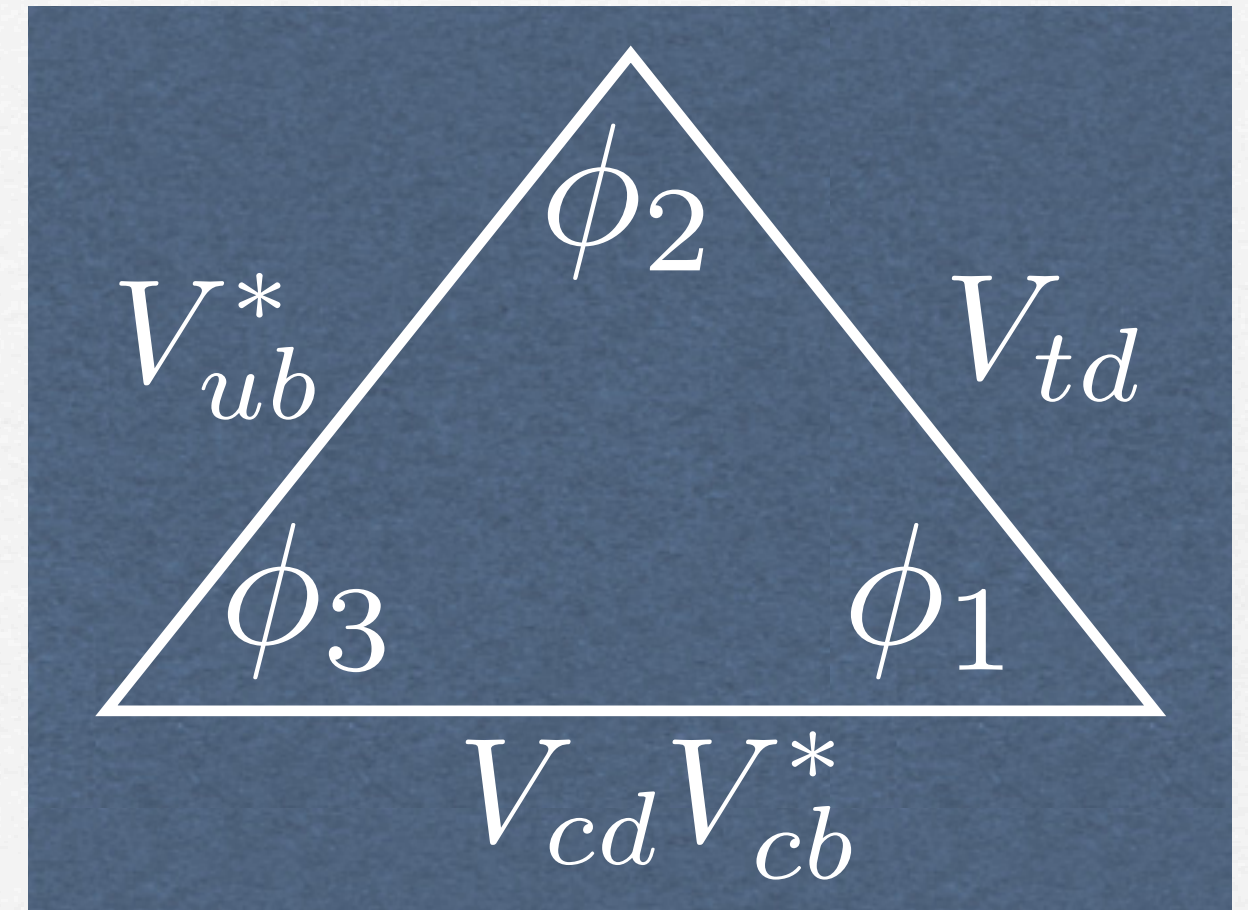
■  $V = |V| \exp(i\phi)$

- $|V|$  from semi-leptonic decay rates
- $\phi$  from  $CP$  asymmetries



$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

$$V_{ud} \cong V_{tb} \cong 1$$



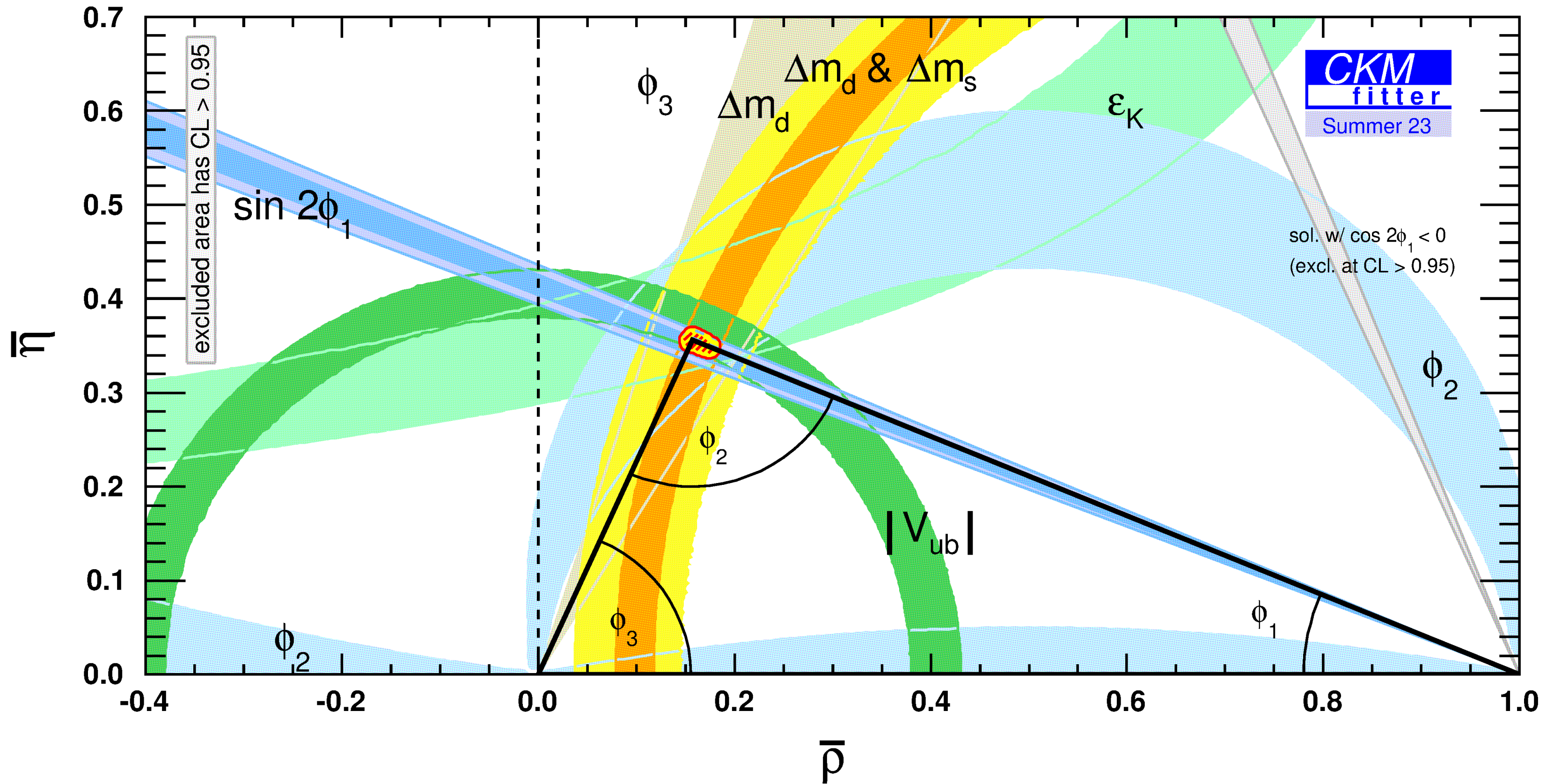
### Unitarity triangle angles

BABAR:  $\beta$     $\alpha$     $\gamma$

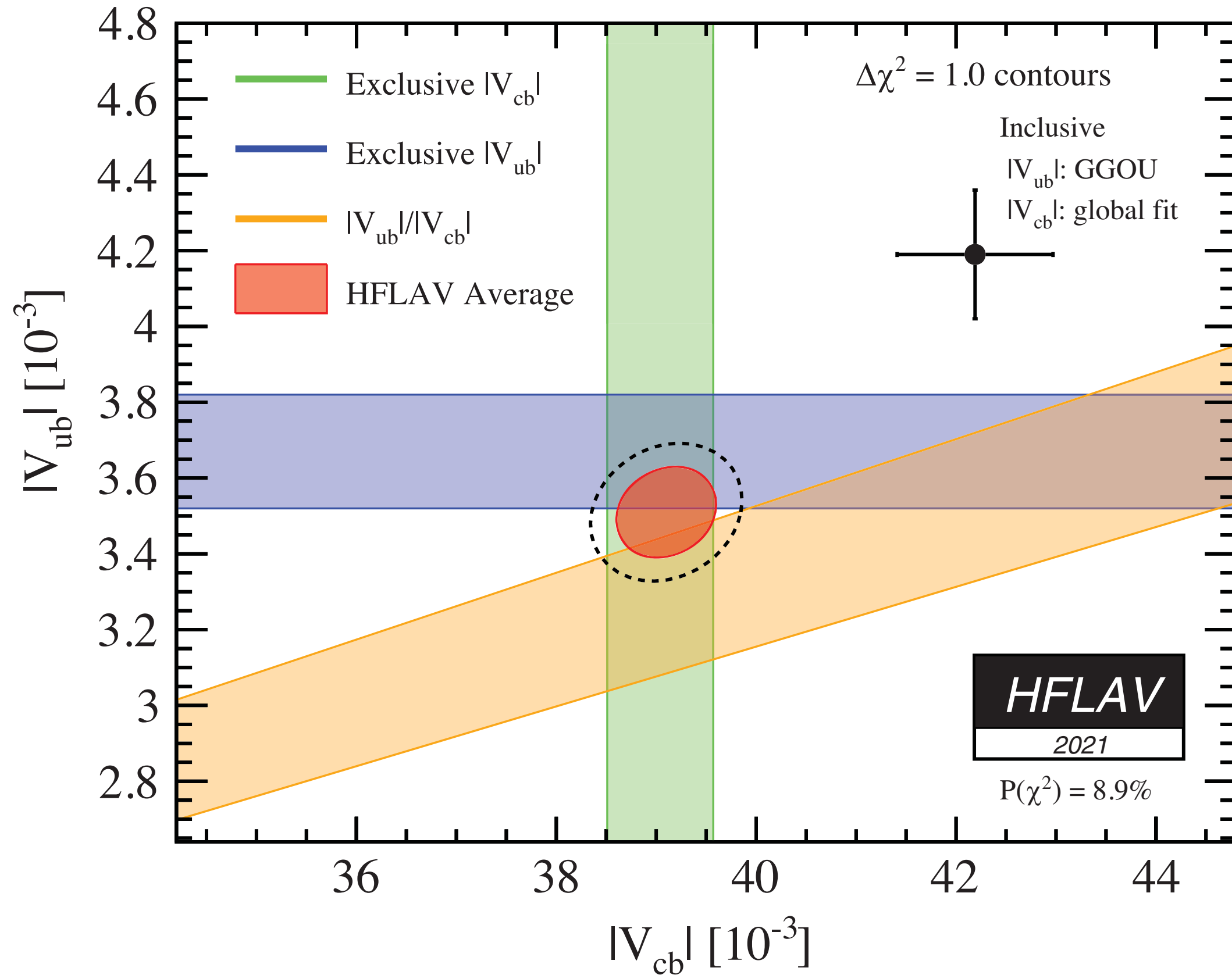
BELLE:  $\phi_1$     $\phi_2$     $\phi_3$

This talk: 易   難   魔

# Current status of CKM unitarity



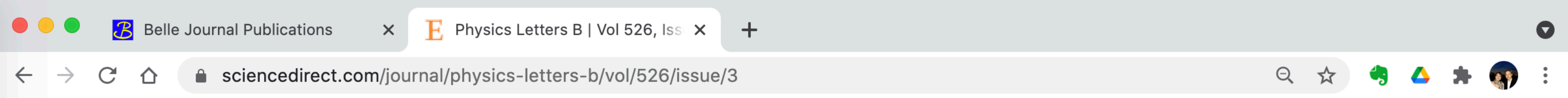
# The $V_{cb}$ , $V_{ub}$ saga





# The “CKM Brothers”

- Dr. Hyunki Jang (SNU)
- Dr. Jiwoo Nam (SKKU)



## Physics Letters B

9.4

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## Volume 526, Issues 3–4

Pages 173-450 (7 February 2002)

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Short communication ○ Abstract only

### Determination of $|V_{cb}|$ using the semileptonic decay $B^0 \rightarrow D^{*+} e^- \bar{\nu}$

Belle Collaboration, K. Abe, K. Abe, R. Abe, ... D. Žontar

Pages 247-257

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Short communication ○ Abstract only

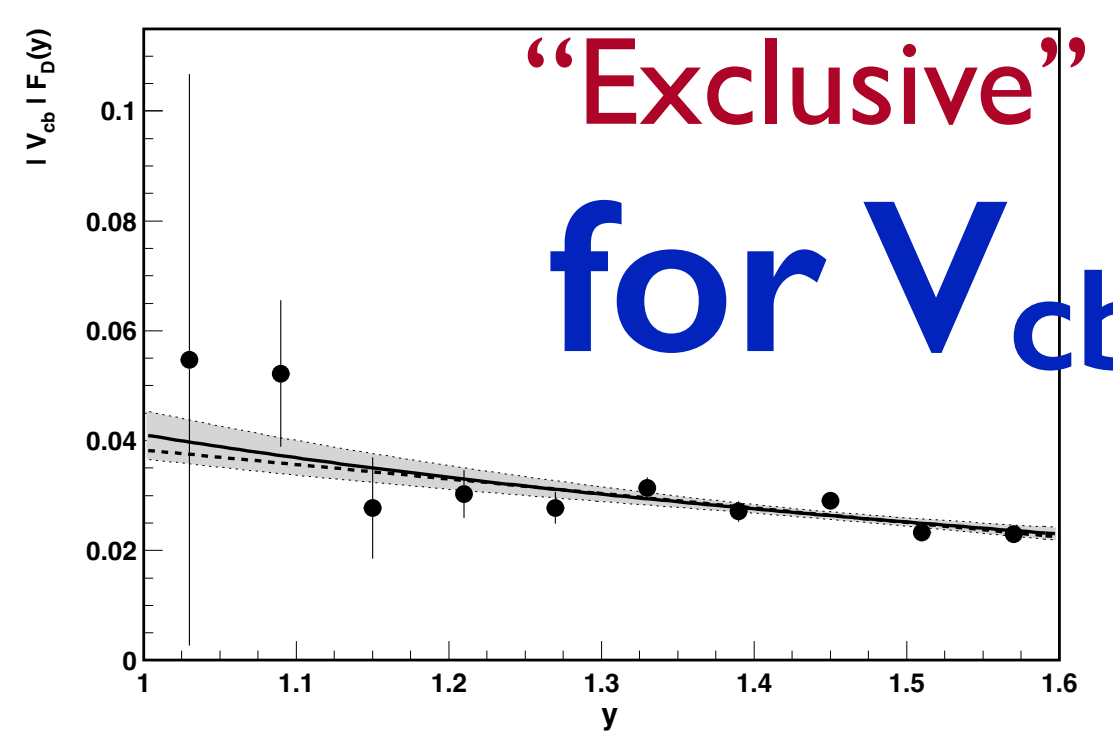
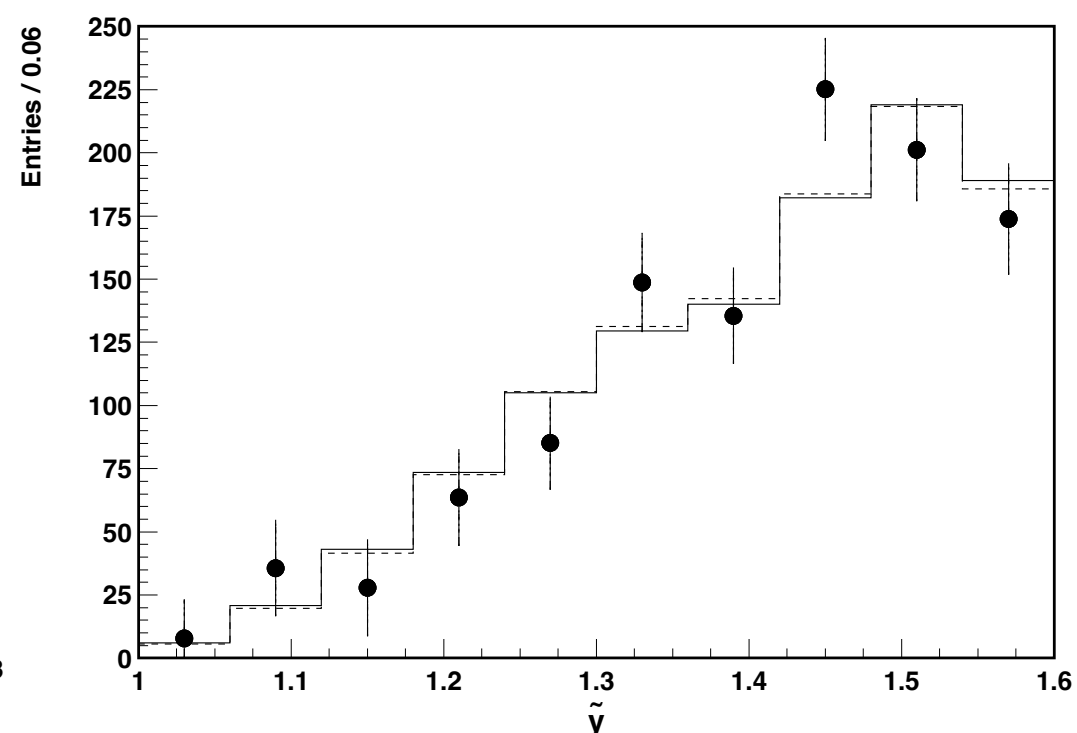
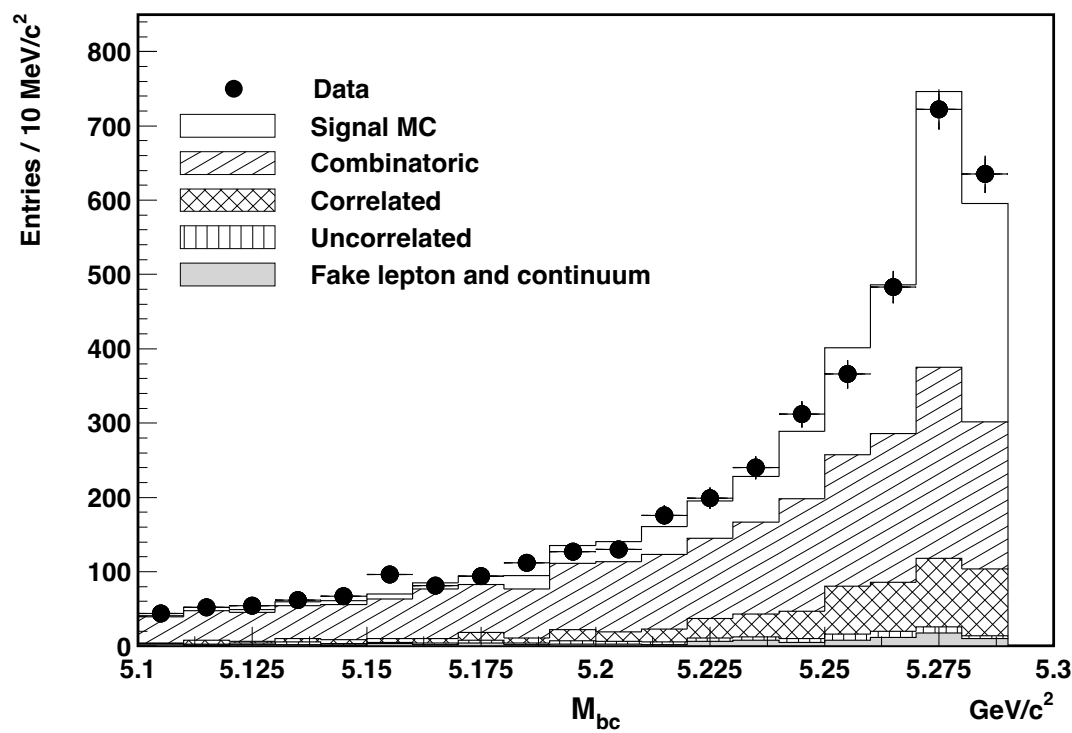
### Measurement of $B(B^0 \rightarrow D^+ \ell^- \bar{\nu})$ and determination of $|V_{cb}|$

Belle Collaboration, K. Abe, K. Abe, R. Abe, ... D. Žontar

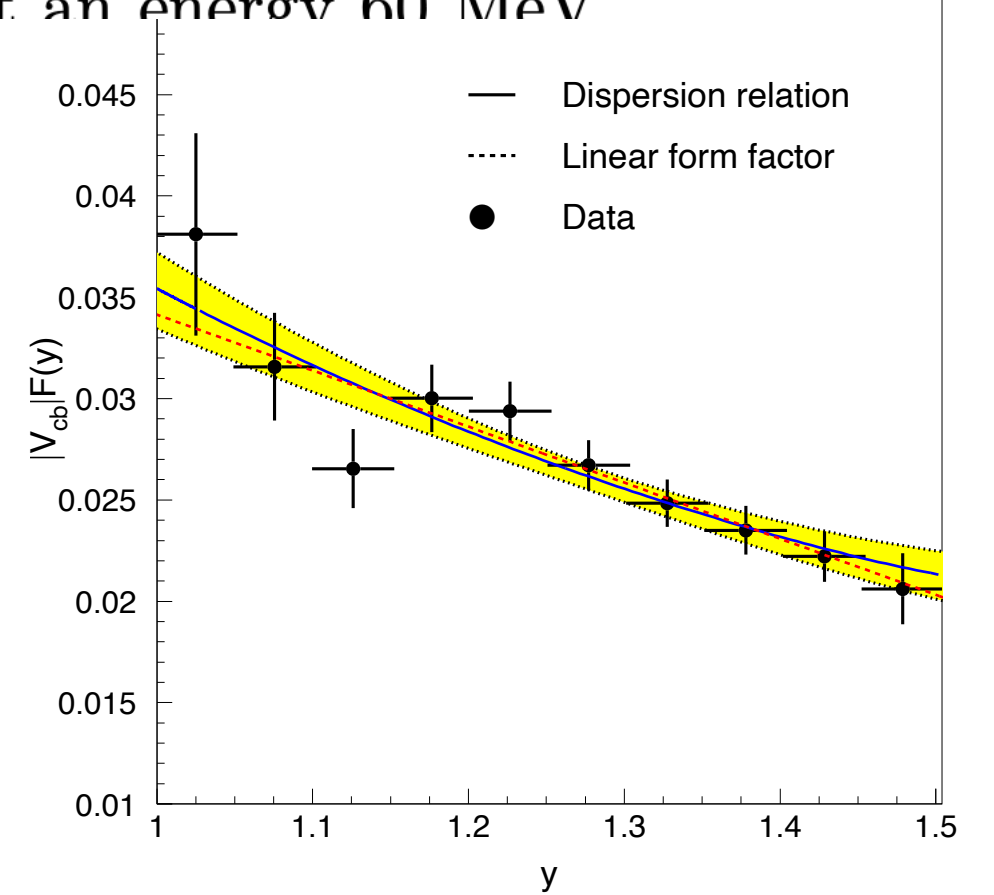
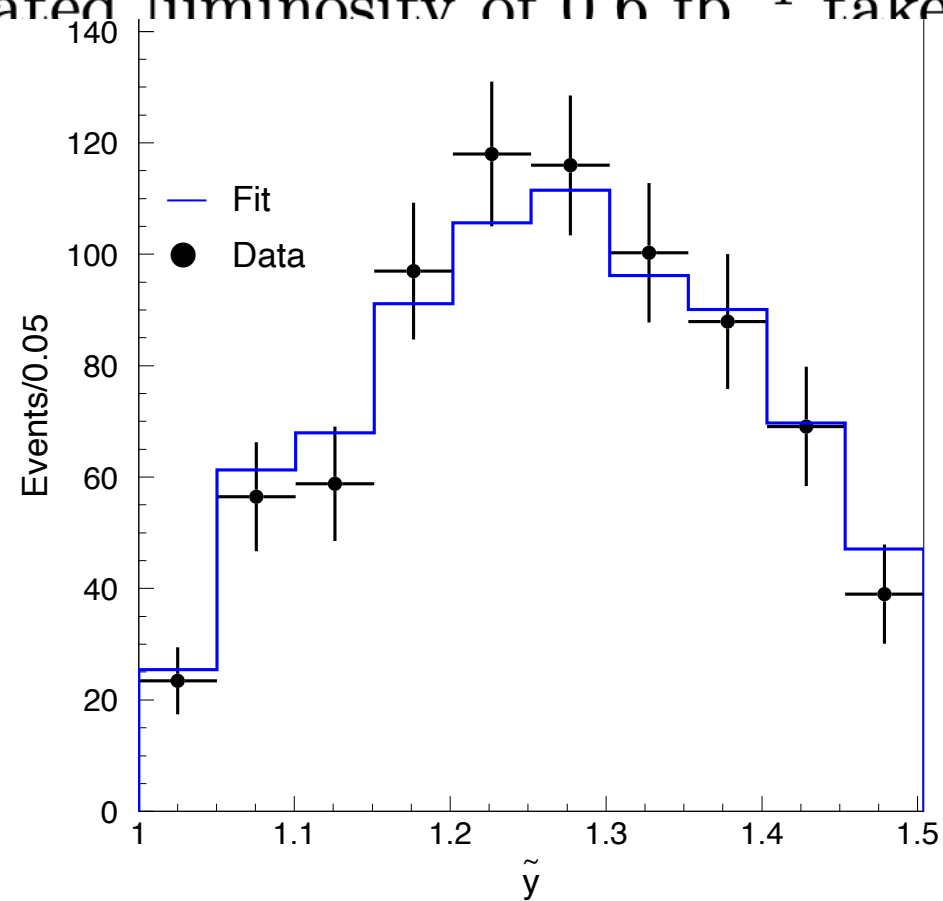
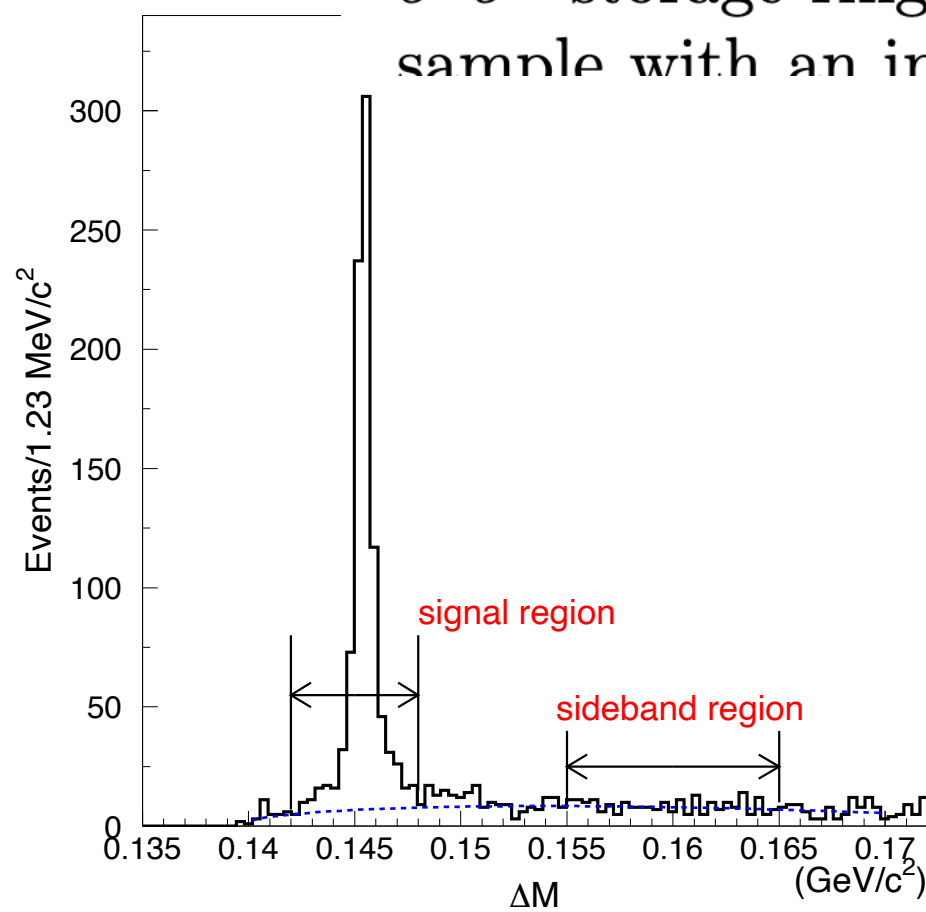
Pages 258-268

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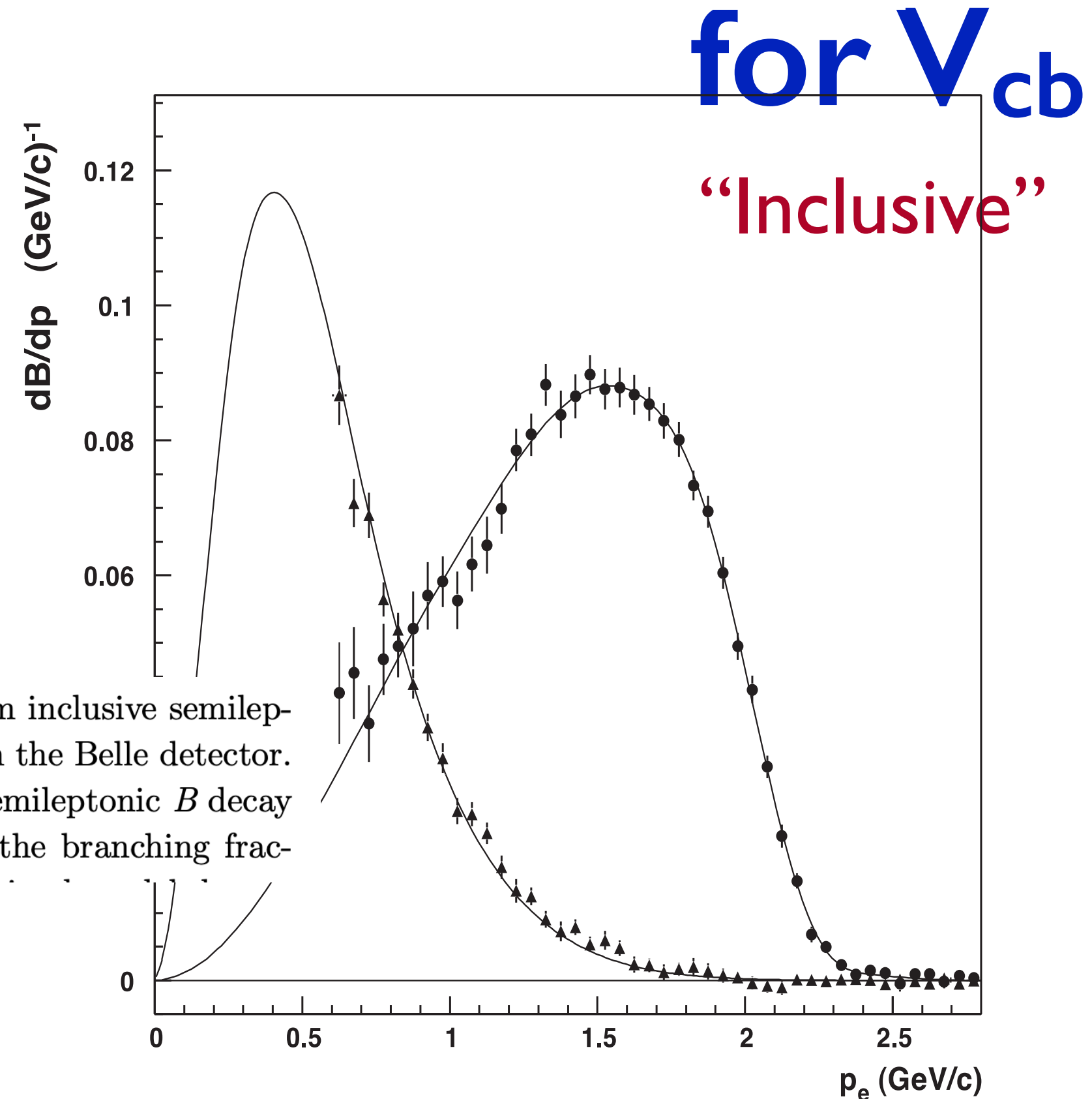
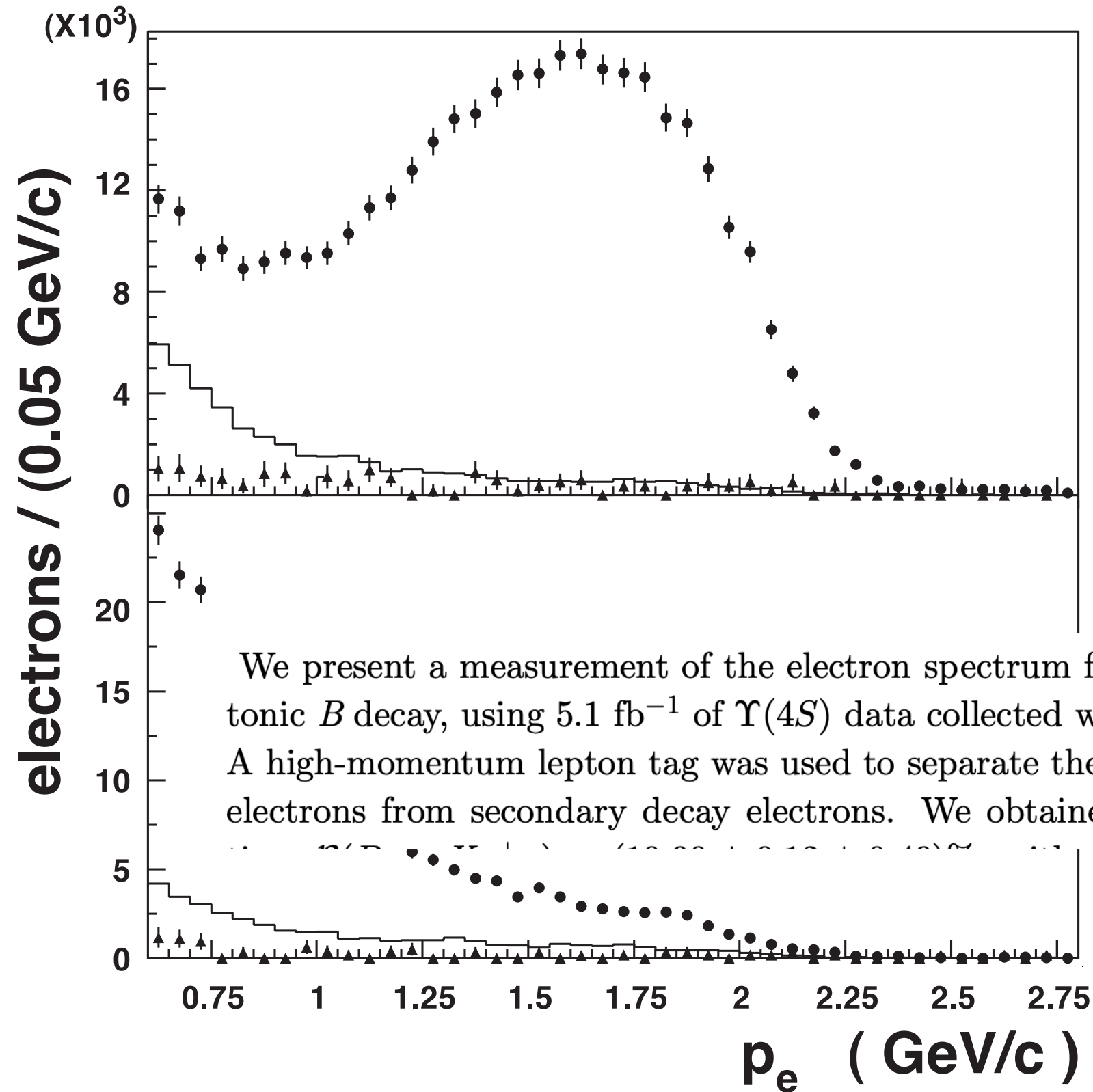


The data correspond to an integrated luminosity of  $10.2 \text{ fb}^{-1}$  accumulated at the  $\Upsilon(4S)$  resonance with the Belle detector [15] at KEKB [16], an asymmetric  $e^+e^-$  storage ring. This corresponds to  $10.8 \times 10^6 B\bar{B}$  events. An additional sample with an integrated luminosity of  $0.6 \text{ fb}^{-1}$  taken at an energy  $60 \text{ MeV}$



# Measurement of the inclusive semileptonic branching fraction of $B$ mesons and $|V_{cb}|$

Belle Collaboration



18506

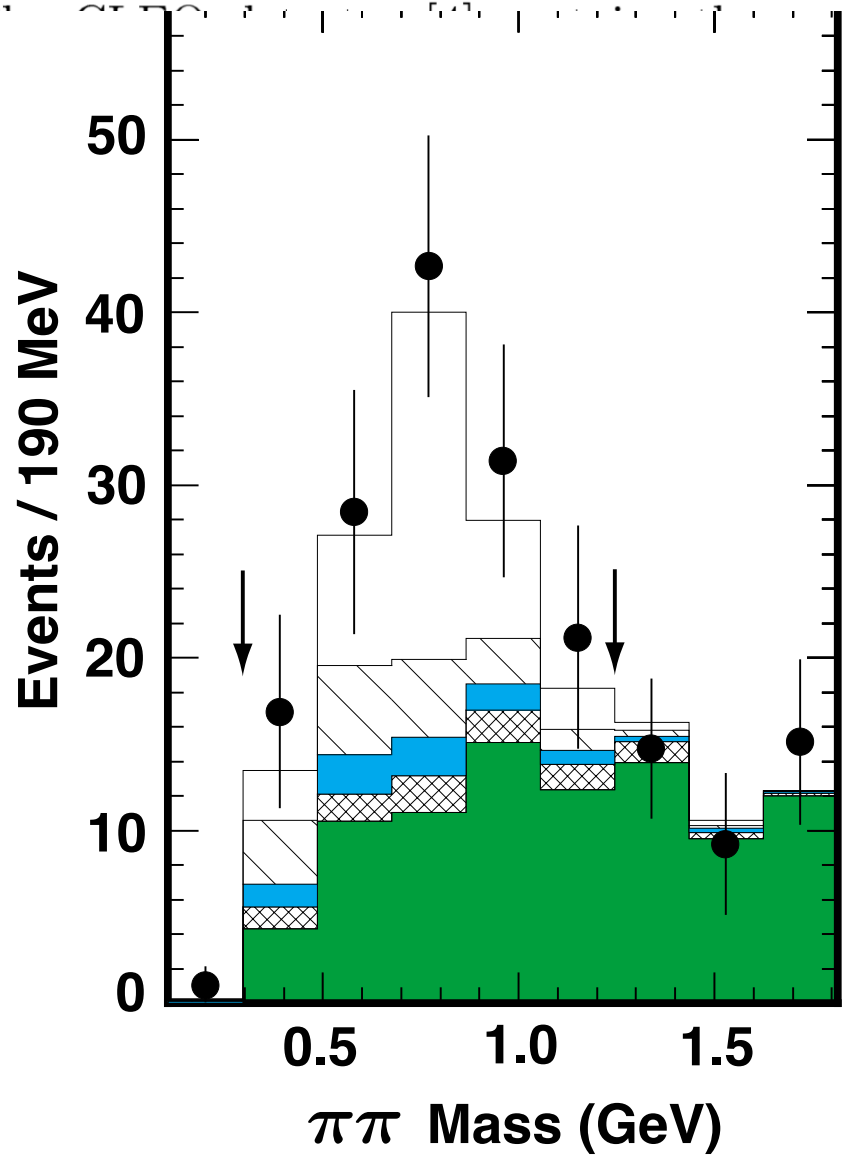
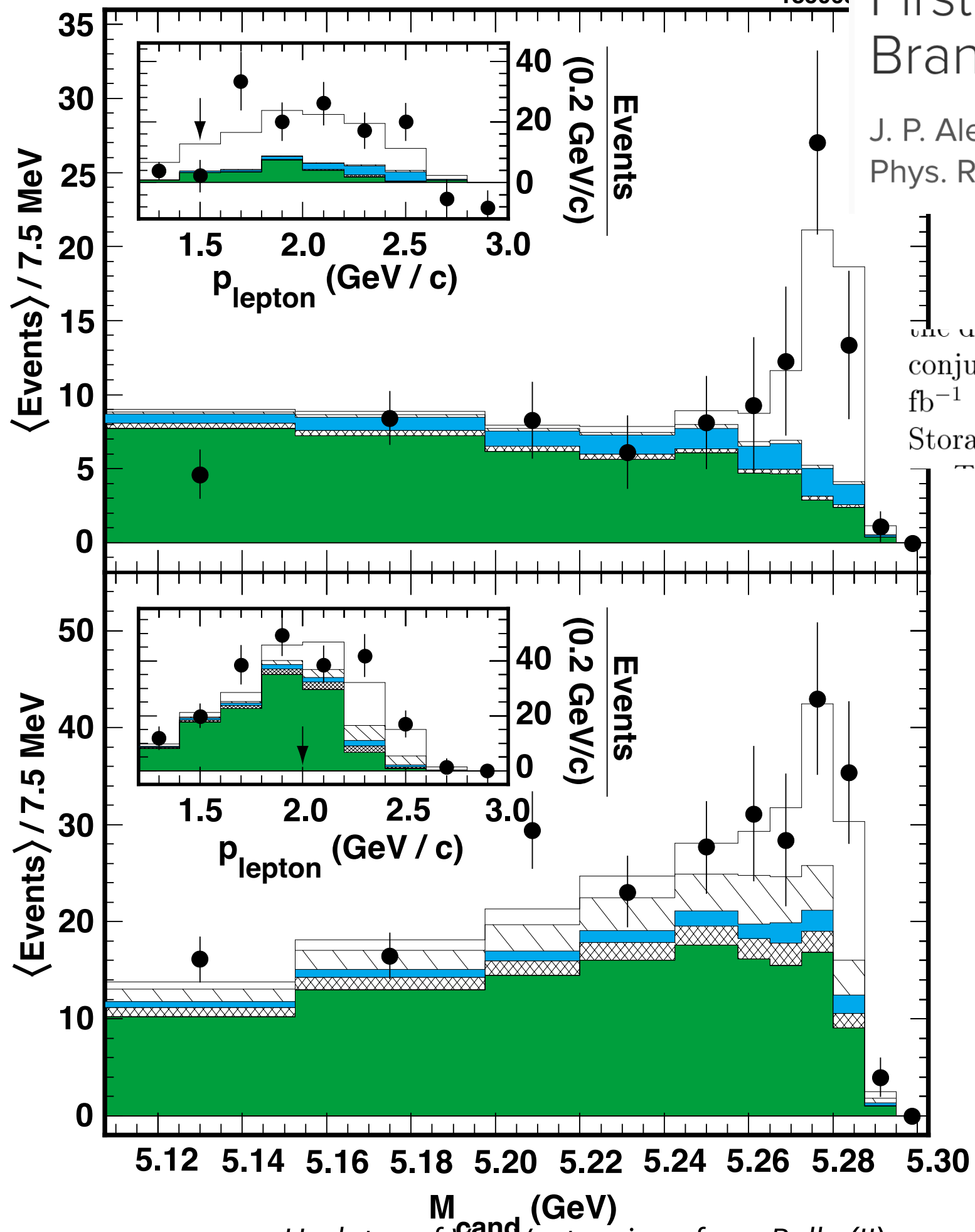
# First Measurement of the $B \rightarrow \pi \ell \nu$ and $B \rightarrow \rho(\omega) \ell \nu$ Branching Fractions

J. P. Alexander *et al.* (CLEO Collaboration)  
Phys. Rev. Lett. **77**, 5000 – Published 16 December 1996

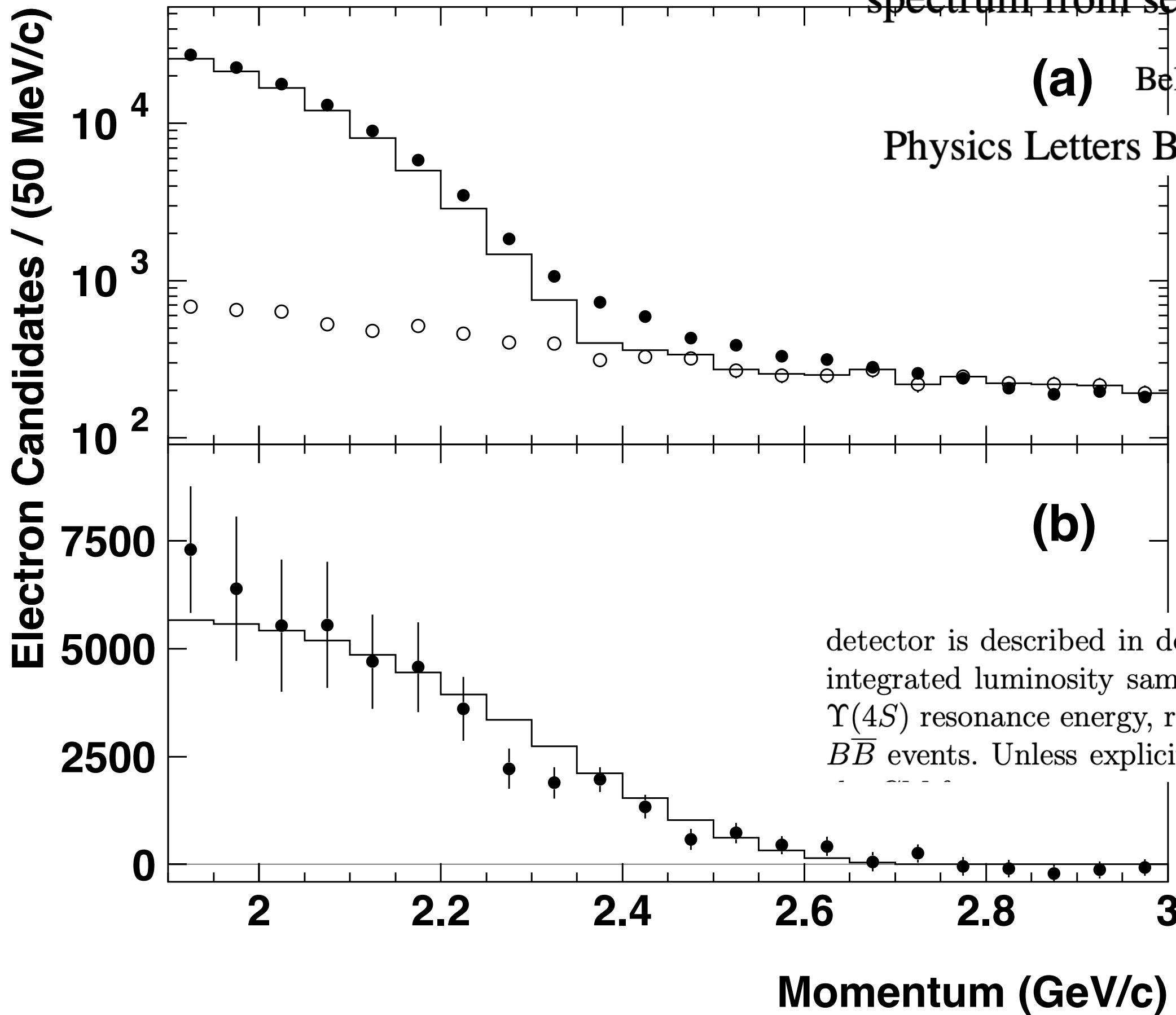
# for $V_{ub}$

## “Exclusive”

the decays  $B \rightarrow \pi \ell \nu$ ,  $B \rightarrow \rho \ell \nu$ ,  $B \rightarrow \omega \ell \nu$ , and charge conjugate modes, where  $\ell = e$  or  $\mu$ . The study is based on an  $\Upsilon(4S)$  data sample of  $2.66 \text{ fb}^{-1}$  ( $2.84 \times 10^6 B\bar{B}$  pairs) accumulated by the CLEO experiment at the Cornell Electron Storage Ring (CESR).



Measurement of  $|V_{ub}|$  near the endpoint of the electron momentum spectrum from semileptonic  $B$ -meson decays

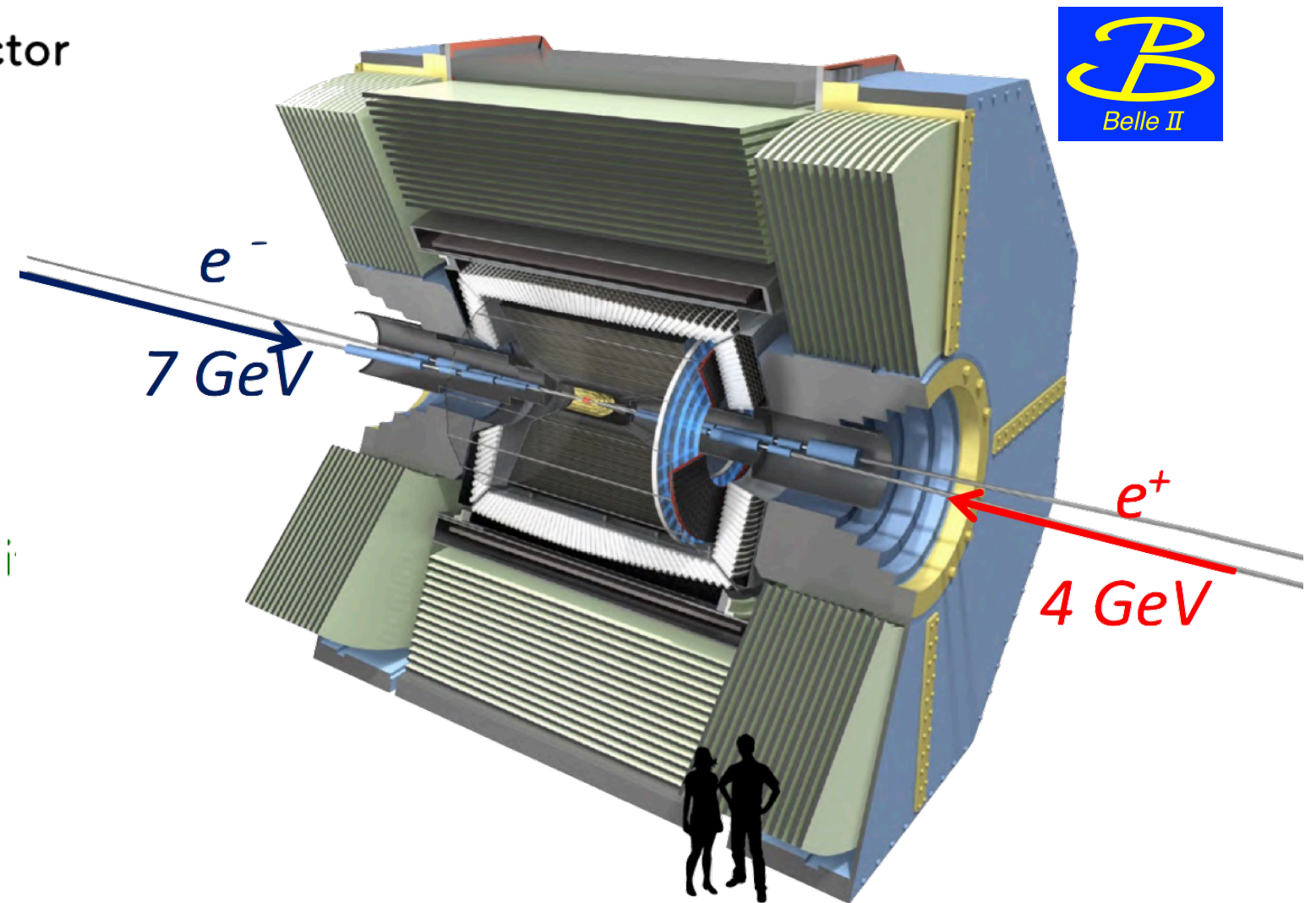
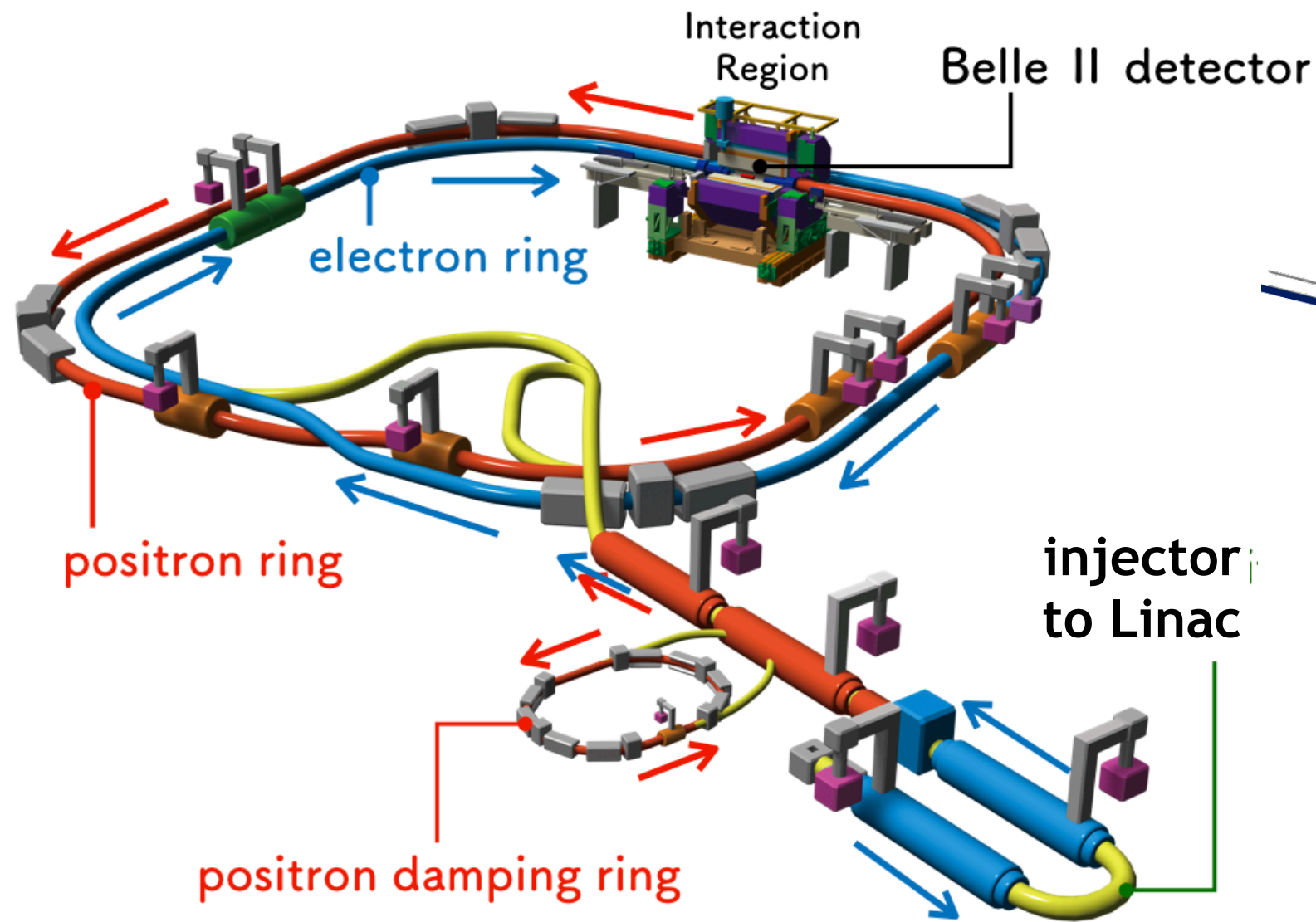


for  $V_{ub}$   
“Inclusive”

detector is described in detail elsewhere [27]. We use  $27.0 \text{ fb}^{-1}$  and  $8.8 \text{ fb}^{-1}$  integrated luminosity samples taken at (ON) and 60 MeV below (OFF) the  $\Upsilon(4S)$  resonance energy, respectively. The ON sample consists of 29.4 million  $B\bar{B}$  events. Unless explicitly stated otherwise, all variables are calculated in

# SuperKEKB

# Belle II



$$e^- \xrightarrow{7 \text{ GeV}} (\star) \xleftarrow{4 \text{ GeV}} e^+$$

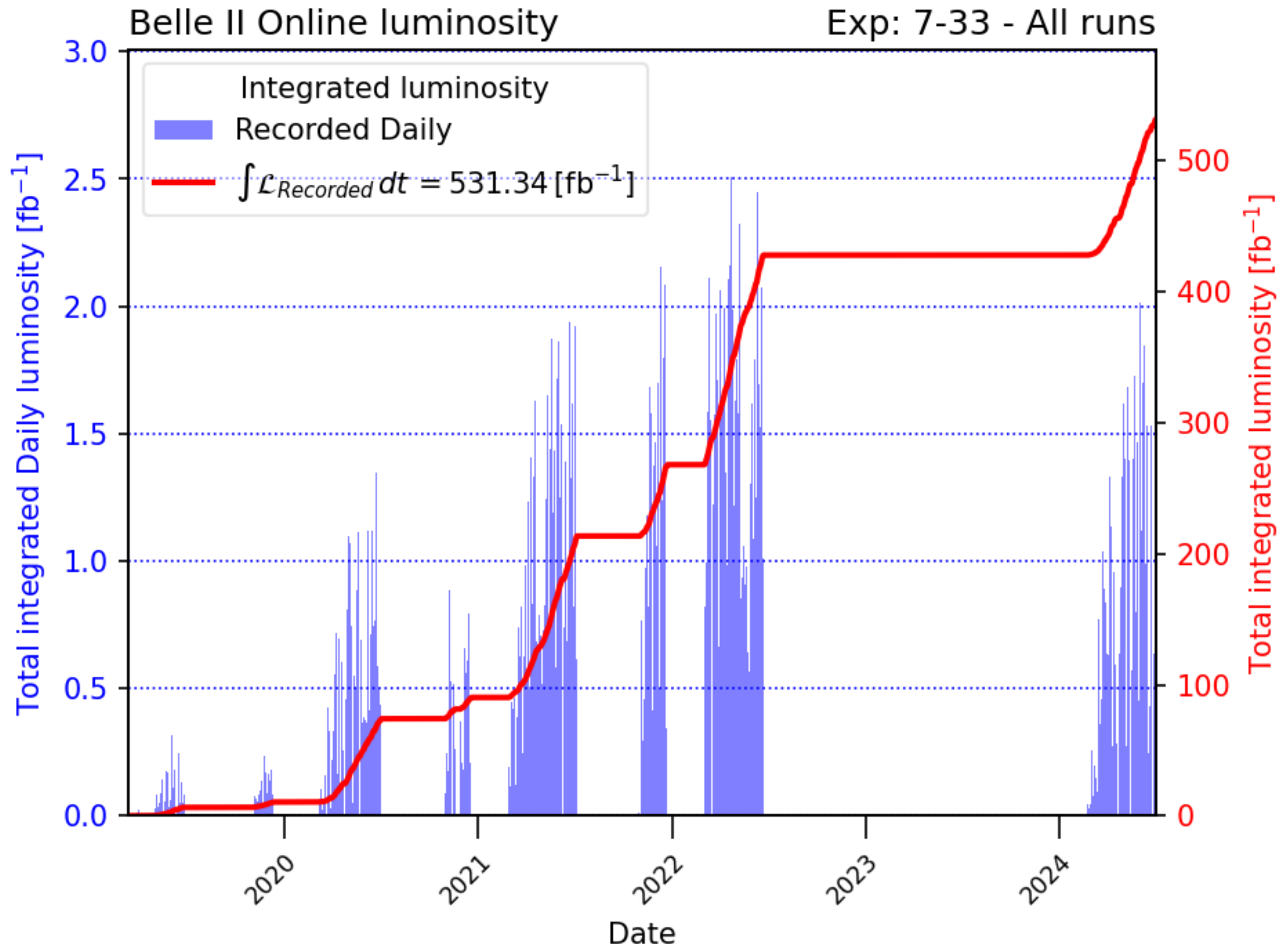
$$\sqrt{s} = 10.58 \text{ GeV} = m_{\Upsilon(4S)} c^2$$

We also have data taken off-resonance as well as energy scan around  $\Upsilon(5S)$

- $\mathcal{B}(\Upsilon(4S) \rightarrow B\bar{B}) > 96\%$ , with  $p_B^{CM} \sim 0.35 \text{ GeV}/c$

- nothing else but  $B\bar{B}$  in the final state

$\therefore$  if we know  $(E, \vec{p})$  of one  $B$ , the other  $B$  is also constrained **“B-tagging”** *unique to  $e^+e^-$  B-factory*



Belle (1999-2010)  
Luminosity

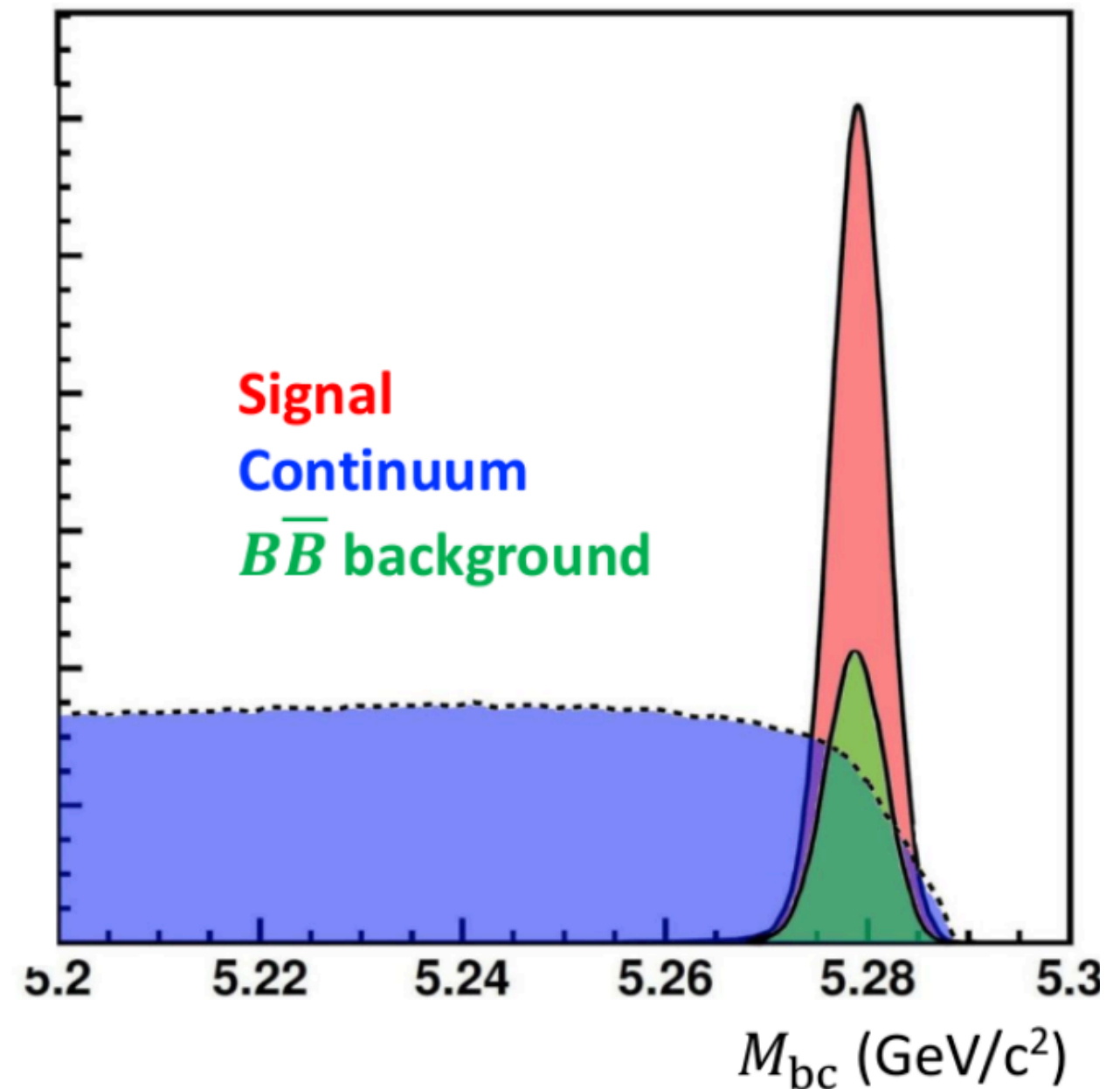
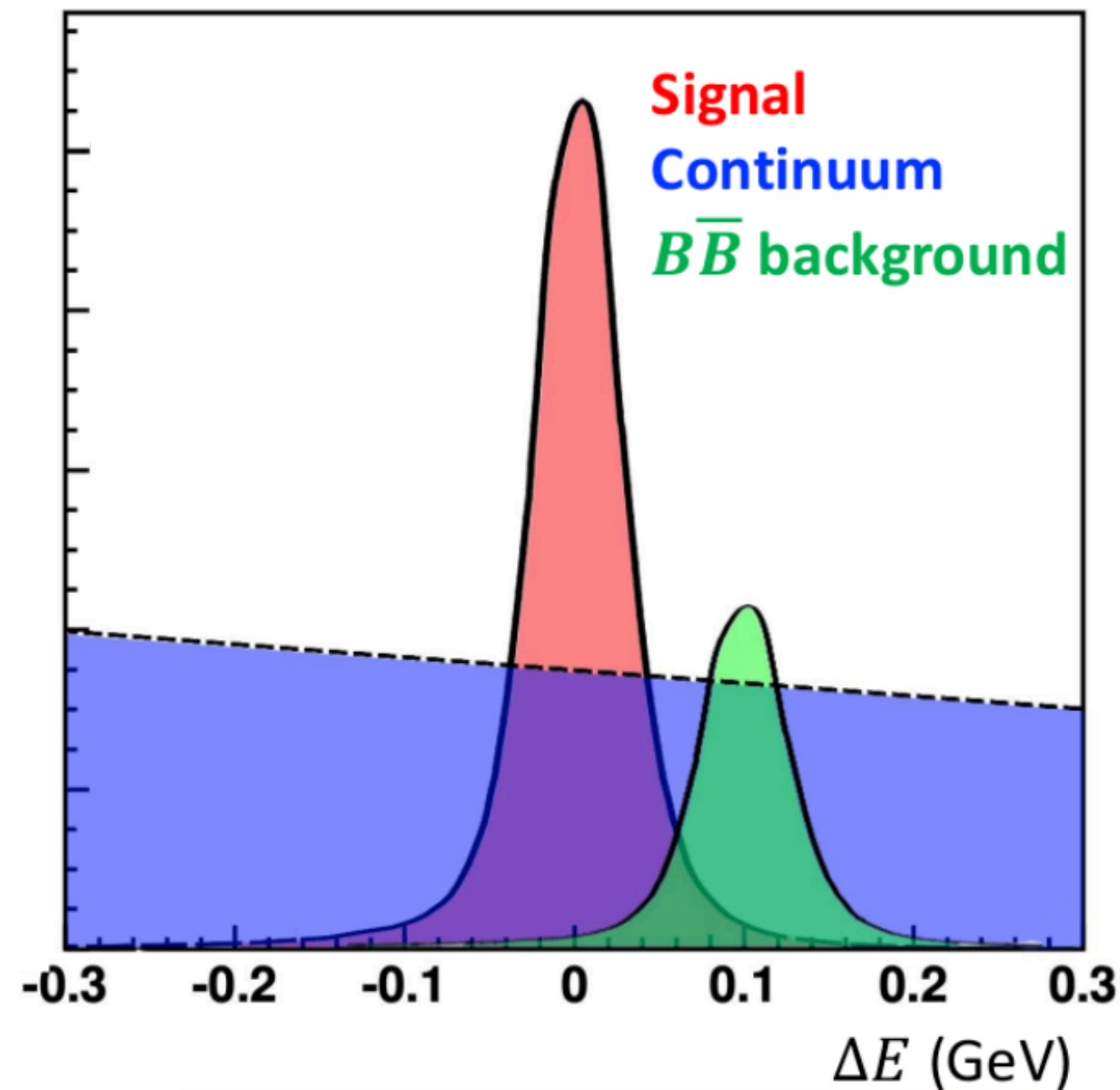
- $\int \mathcal{L}_{total} = 1039 \text{ fb}^{-1}$
- $\int \mathcal{L}_{\Upsilon(4S)} = 711 \text{ fb}^{-1}$

Updated on 2024/07/01 09:43 JST

# Key variables of $B$ decays

$$\Delta E = E_B^* - \sqrt{s}/2$$

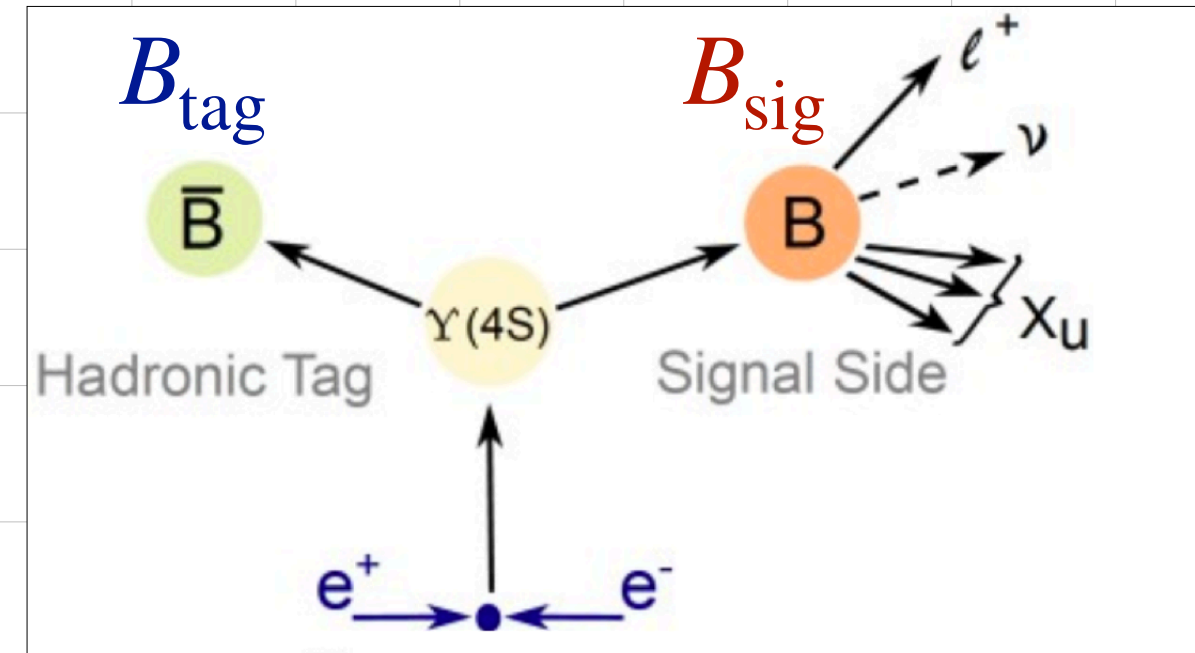
$$M_{bc} = \sqrt{(\sqrt{s}/2)^2 - \vec{p}_B^{*2}}$$



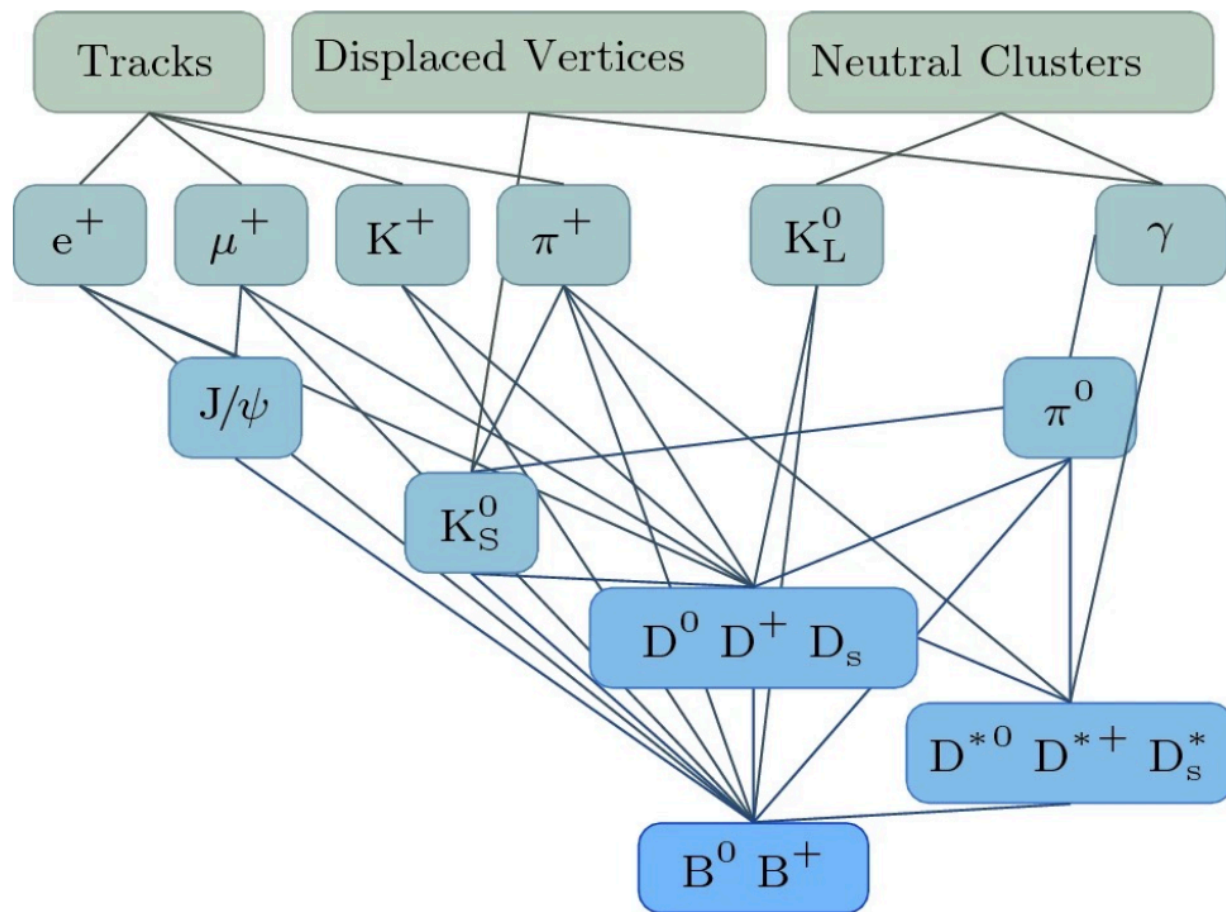


# Full Event Interpretation

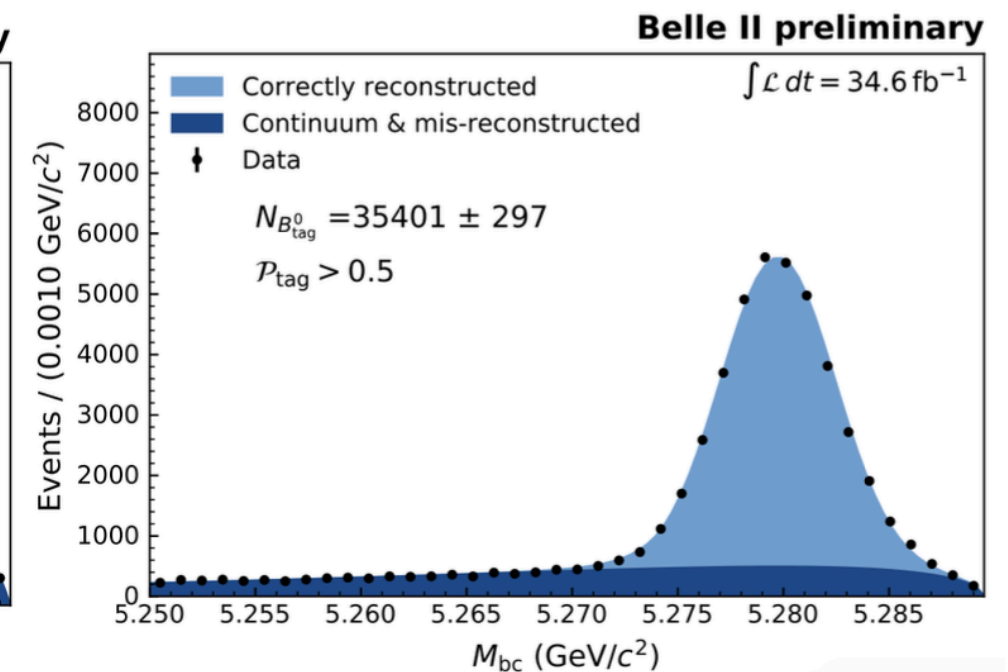
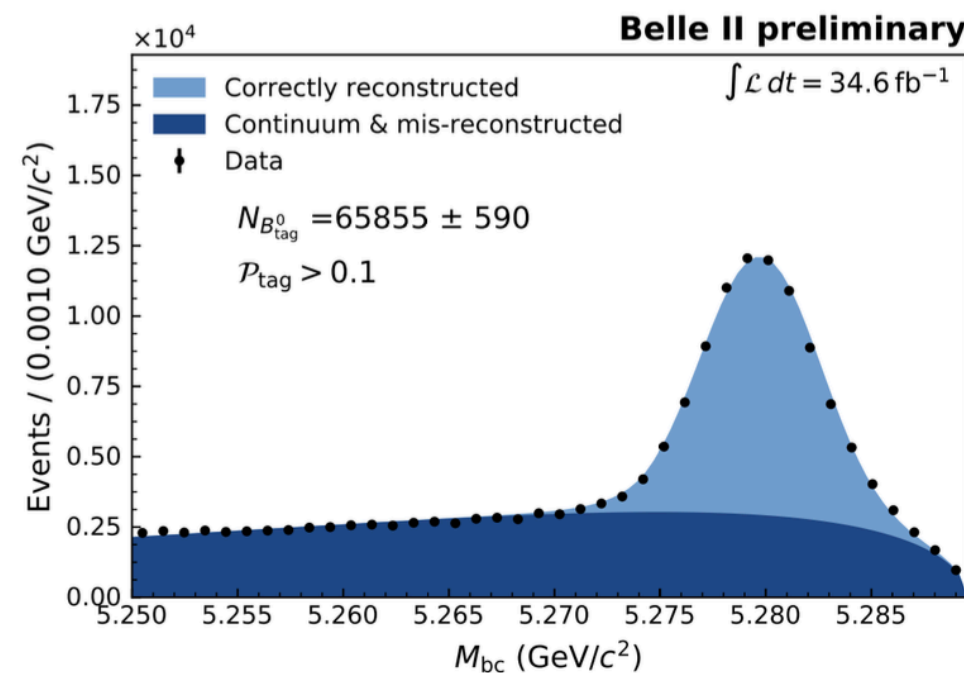
- FEI algorithm to reconstruct  $B_{\text{tag}}$ 
  - uses  $\sim 200$  BDT's to reconstruct  $\mathcal{O}(10^4)$  different  $B$  decay chains
  - assign signal probability of being correct  $B_{\text{tag}}$



Comput Softw Big Sci 3, 6 (2019)



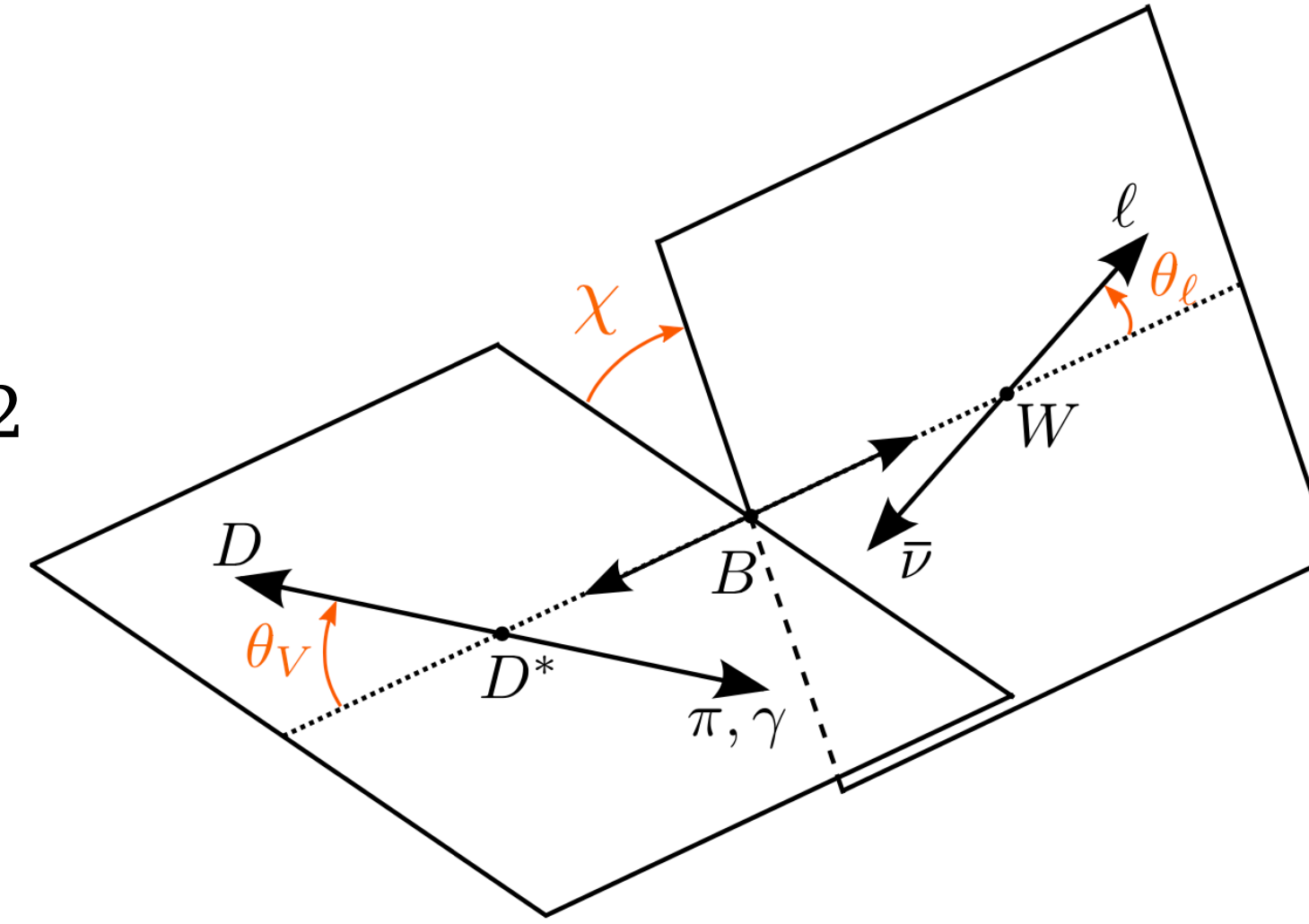
arXiv:2008.060965



# $|V_{cb}|$ from angular coeff's of $B \rightarrow D^* \ell \nu$

- Obtain the differential rates in three angles,  $\theta_\ell$ ,  $\theta_V$ ,  $\chi$ , and a kinematic variable,  $w$ .
- differential rates are expressed in terms of 12 functions  $J_i$  that depend only on  $w$ .
- possible for SM test & LFU test

## Belle data sample of $711 \text{ fb}^{-1}$



$$w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}}$$

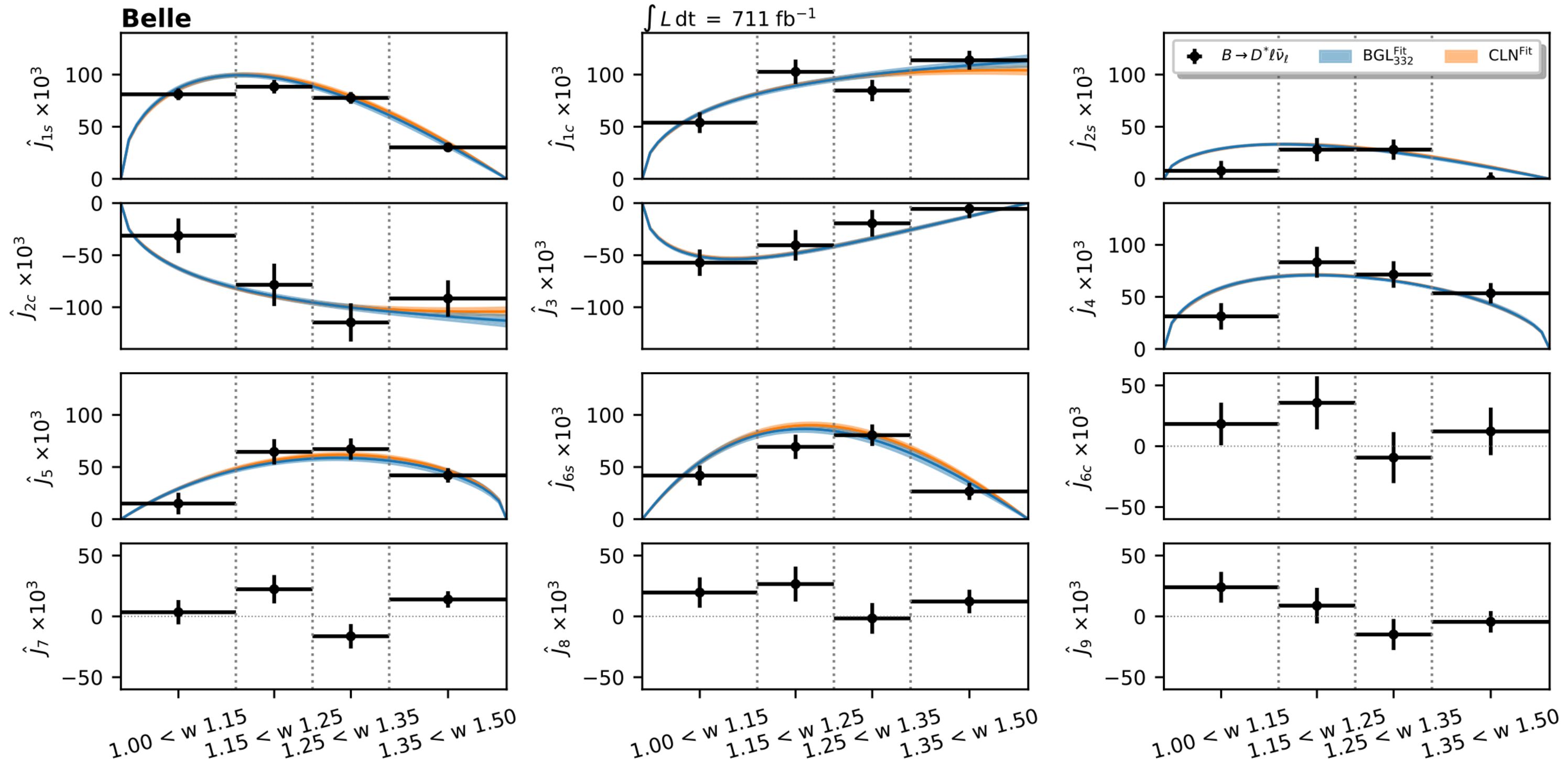
$$\begin{aligned} \frac{d\Gamma(\bar{B} \rightarrow D^* \ell \bar{\nu}_\ell)}{dw \, d\cos\theta_\ell \, d\cos\theta_V \, d\chi} = & \frac{2G_F^2 \eta_{EW}^2 |V_{cb}|^2 m_B^4 m_{D^*}}{2\pi^4} \times \left( J_{1s} \sin^2 \theta_V + J_{1c} \cos^2 \theta_V \right. \\ & + (J_{2s} \sin^2 \theta_V + J_{2c} \cos^2 \theta_V) \cos 2\theta_\ell + J_3 \sin^2 \theta_V \sin^2 \theta_\ell \cos 2\chi \\ & + J_4 \sin 2\theta_V \sin 2\theta_\ell \cos \chi + J_5 \sin 2\theta_V \sin \theta_\ell \cos \chi + (J_{6s} \sin^2 \theta_V + J_{6c} \cos^2 \theta_V) \cos \theta_\ell \\ & \left. + J_7 \sin 2\theta_V \sin \theta_\ell \sin \chi + J_8 \sin 2\theta_V \sin 2\theta_\ell \sin \chi + J_9 \sin^2 \theta_V \sin^2 \theta_\ell \sin 2\chi \right). \end{aligned}$$

# $|V_{cb}|$ from angular coeff's of $B \rightarrow D^* \ell \nu$

$$|V_{cb}| = (40.7 \pm 0.3 \pm 0.4 \pm 0.5) \times 10^{-3} \quad (\text{BGL}_{332}),$$

$$|V_{cb}| = (40.3 \pm 0.3 \pm 0.4 \pm 0.4) \times 10^{-3} \quad (\text{CLN}),$$

- [17] C. G. Boyd, B. Grinstein, and R. F. Lebed, Nucl. Phys. B **461**, 493 (1996), arXiv:hep-ph/9508211.  
 [18] C. G. Boyd, B. Grinstein, and R. F. Lebed, Phys. Rev. D **56**, 6895 (1997), arXiv:hep-ph/9705252.  
 [16] I. Caprini, L. Lellouch, and M. Neubert, Nucl. Phys. B **530**, 153 (1998), arXiv:hep-ph/9712417.

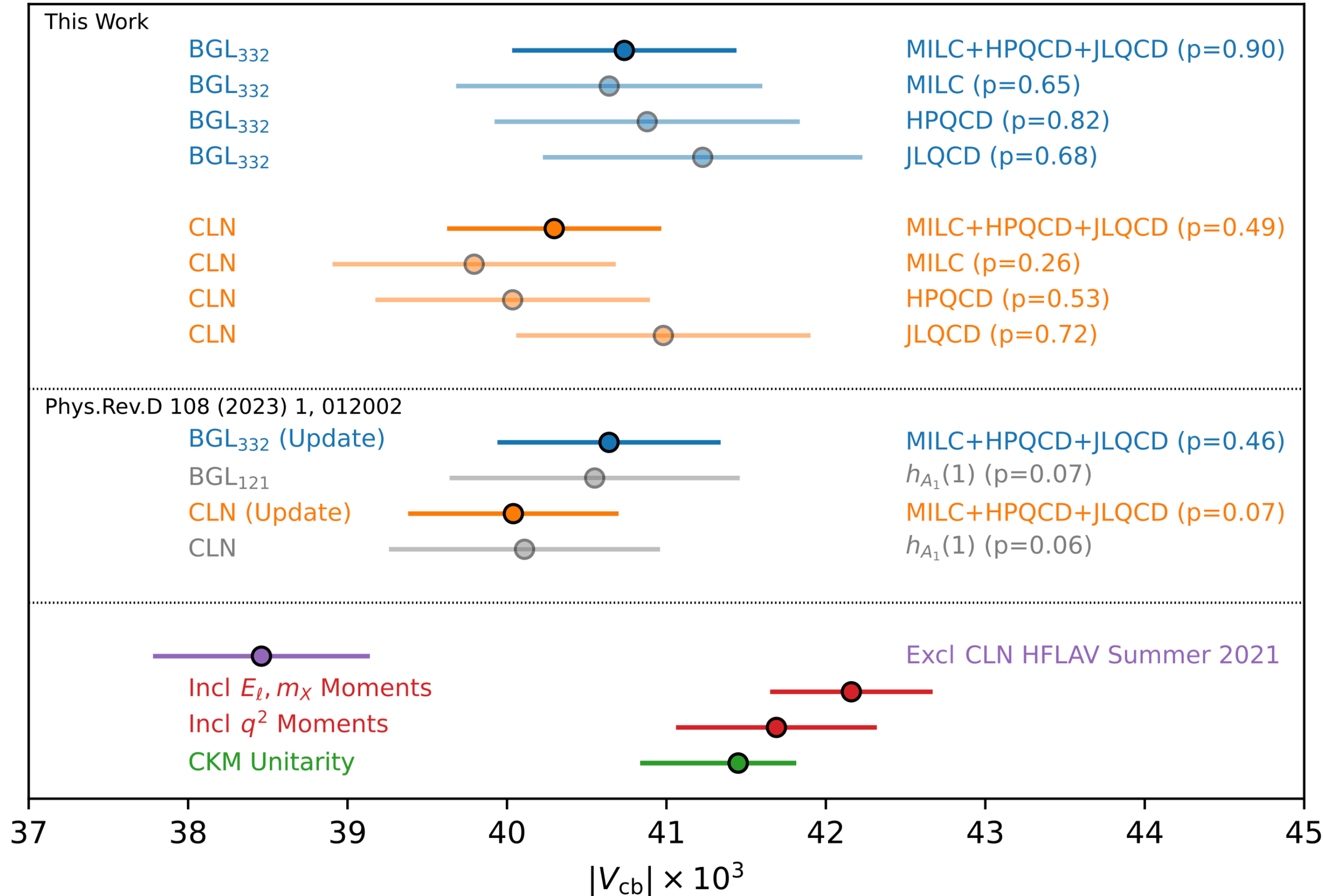




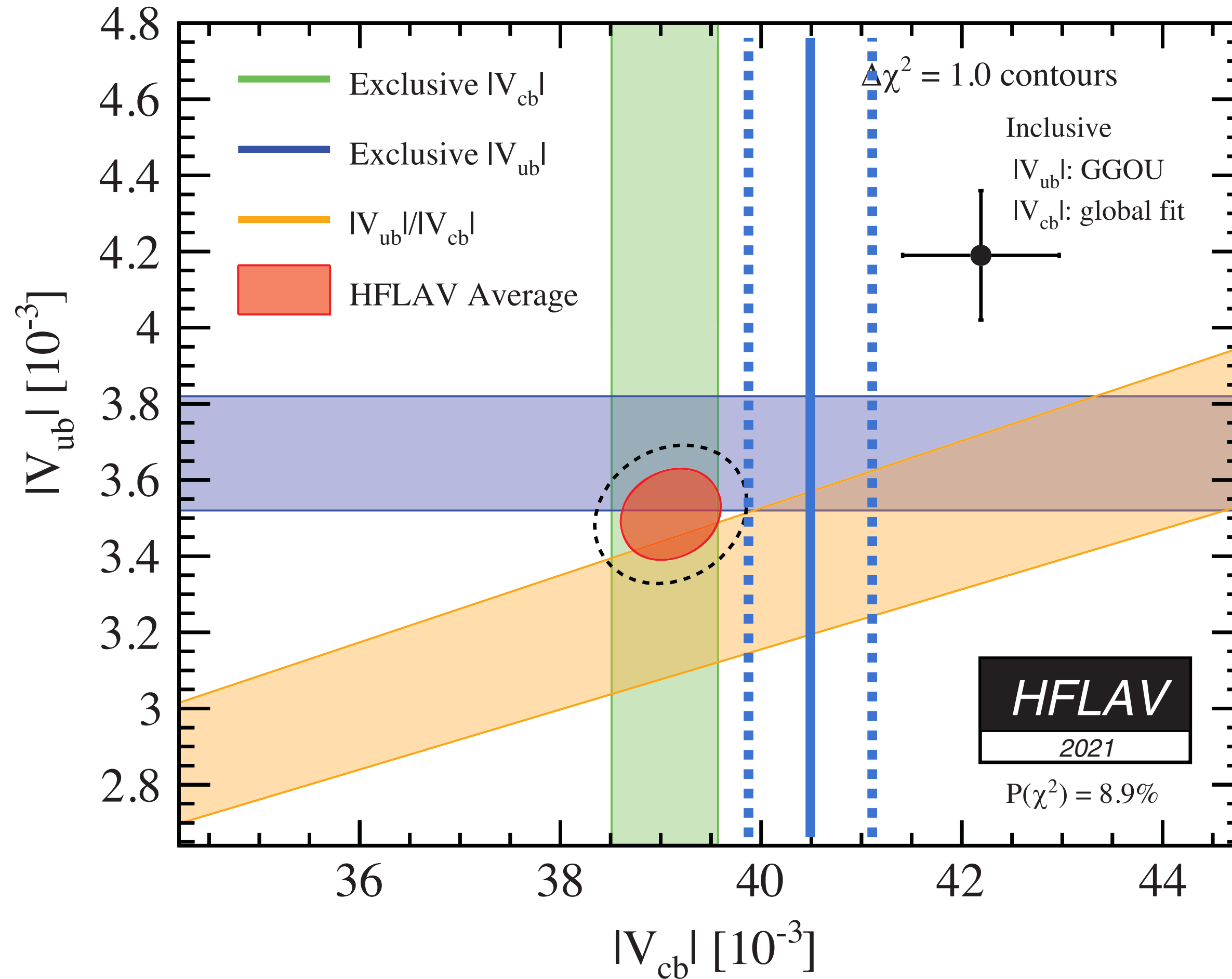
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$$|V_{cb}| = (40.3 \pm 0.3 \pm 0.4 \pm 0.4) \times 10^{-3} \quad (\text{CLN}),$$

$|V_{cb}|$  from angular coeff's  
of  $B \rightarrow D^* \ell \nu$



# $|V_{cb}|$ from angular coeff's of $B \rightarrow D^* \ell \nu$

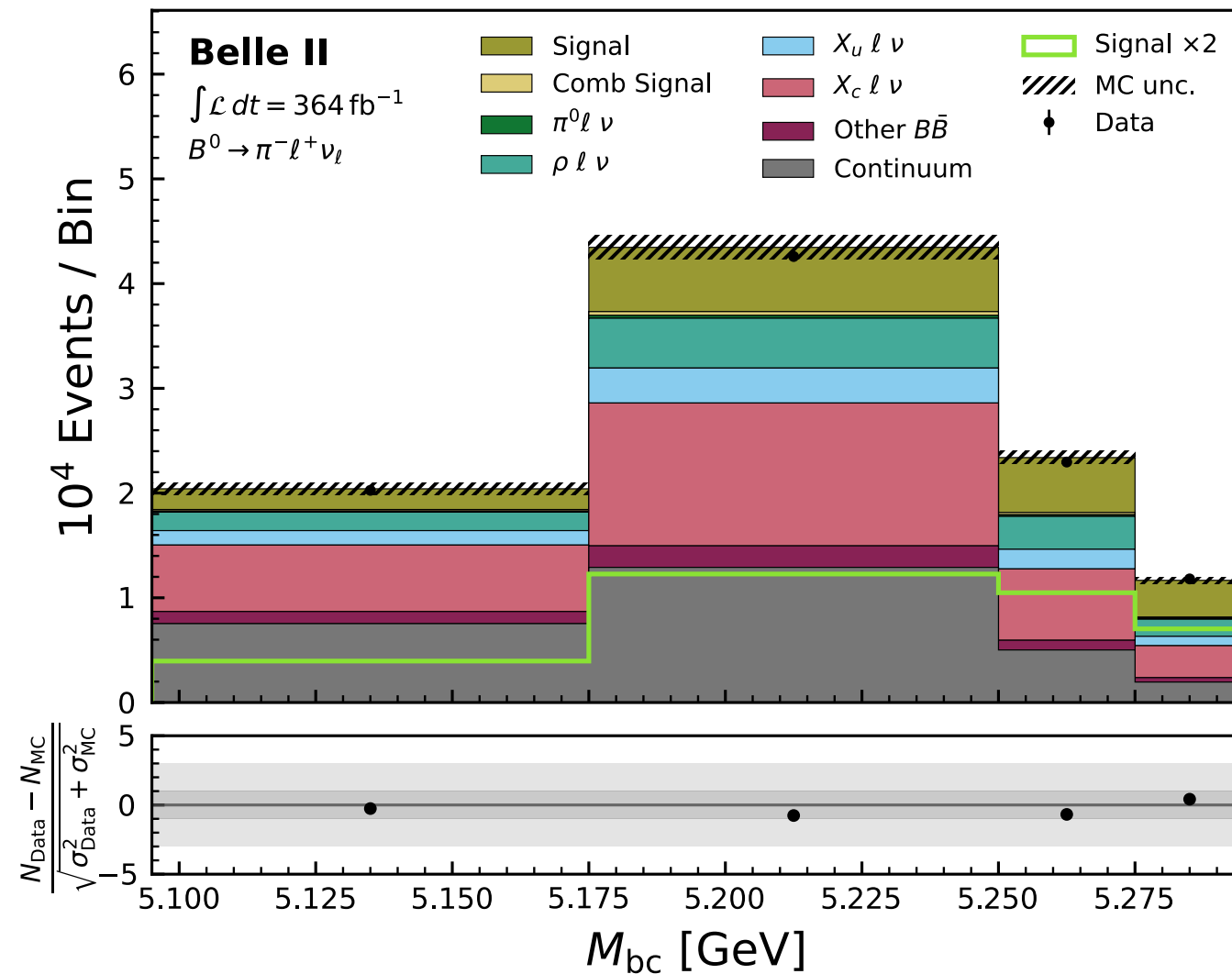
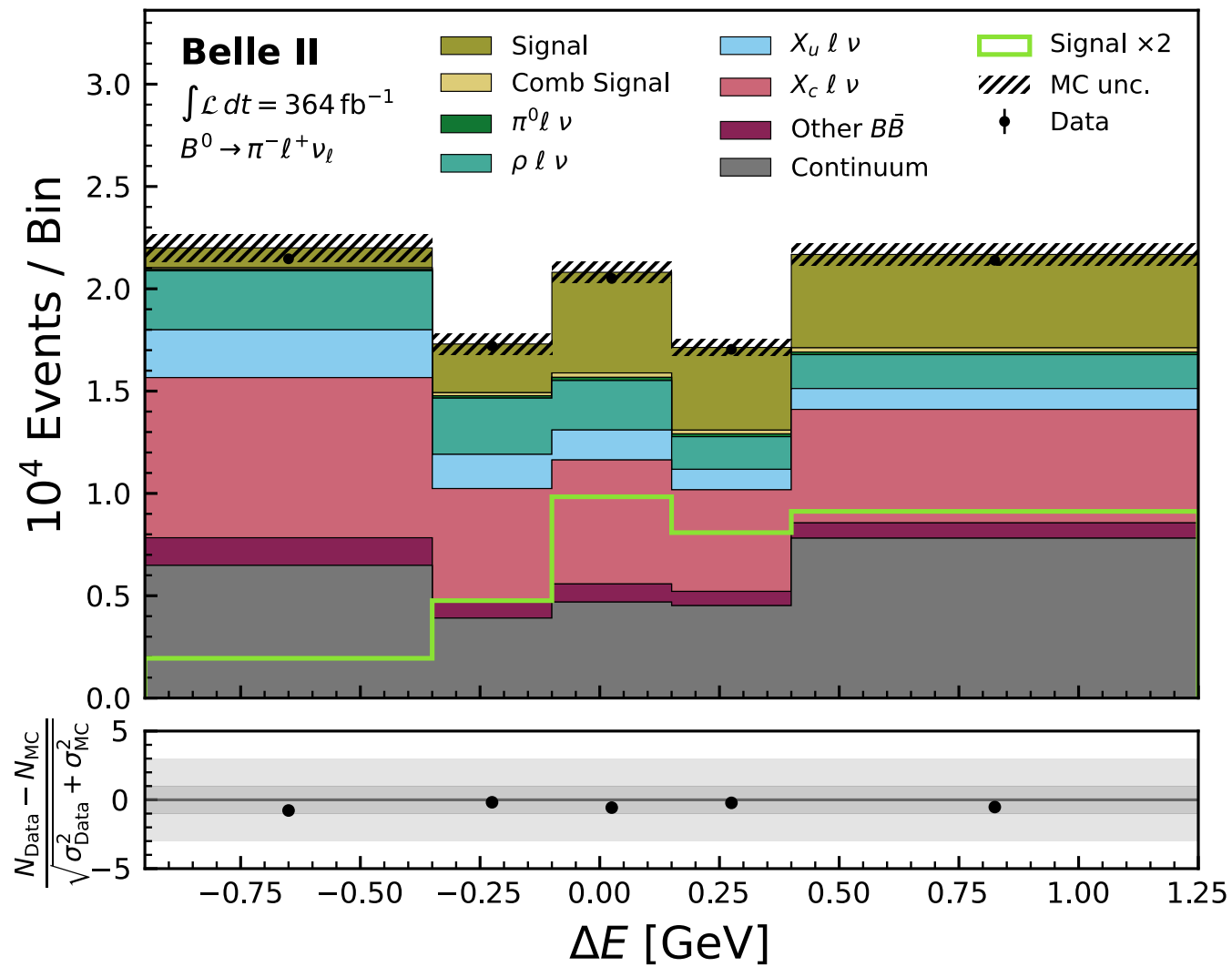
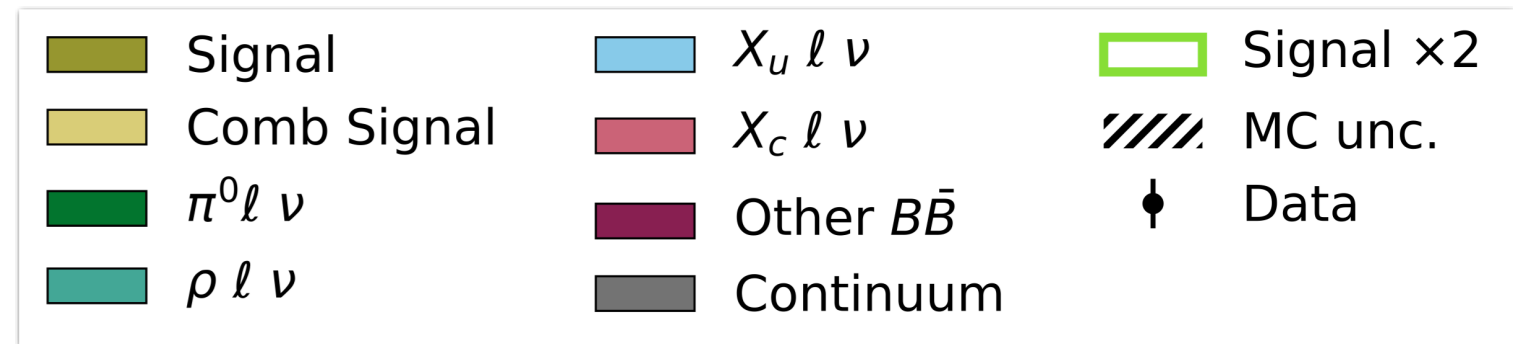


# $|V_{ub}|$ from $B^0 \rightarrow \pi^- \ell^+ \nu$ & $B^+ \rightarrow \rho^0 \ell^+ \nu$

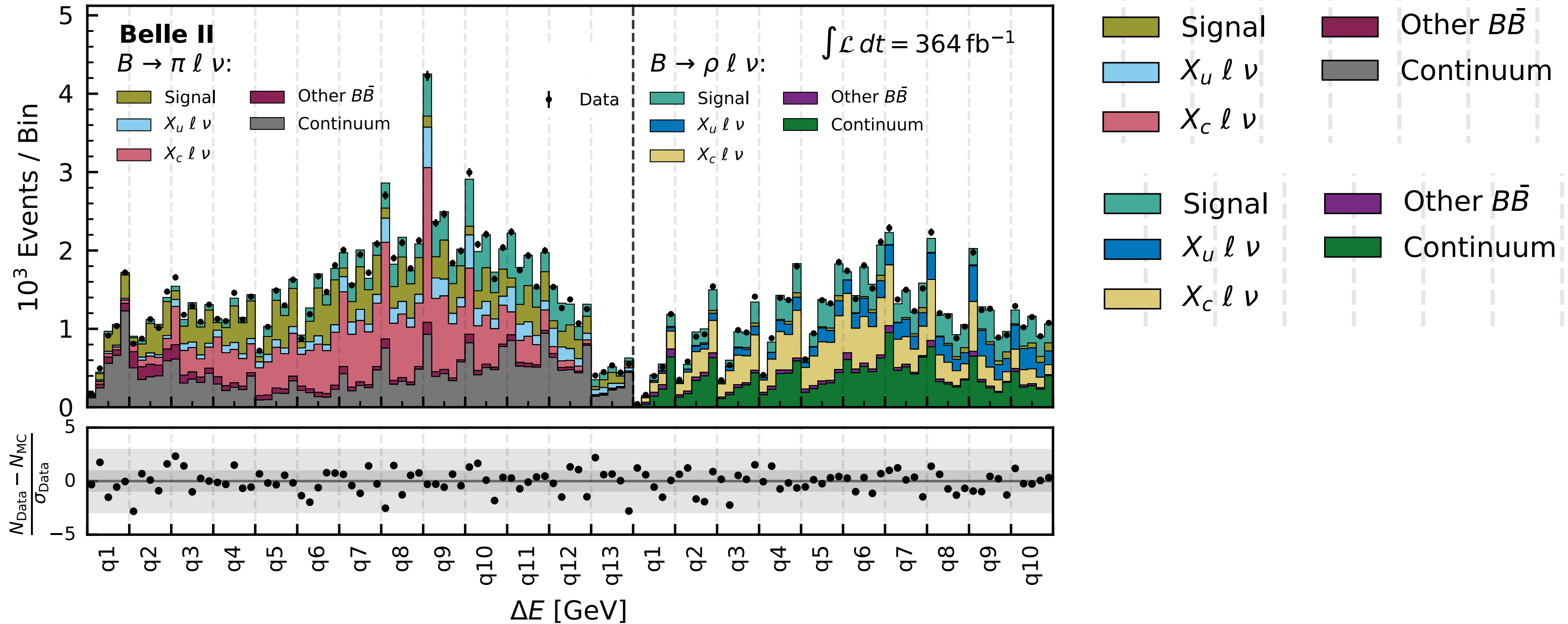
● Extract  $|V_{ub}|$  by simultaneously fitting  $B^0 \rightarrow \pi^- \ell^+ \nu$  &  $B^+ \rightarrow \rho^0 \ell^+ \nu$

● Signal extraction in (13+10)x4x5 bins

- 13 (10) bins in  $q^2$  for  $B^0 \rightarrow \pi^- \ell^+ \nu$  ( $\rho^0 \ell^+ \nu$ )
- 4 bins in  $M_{bc}$ , 5 bins in  $\Delta E$

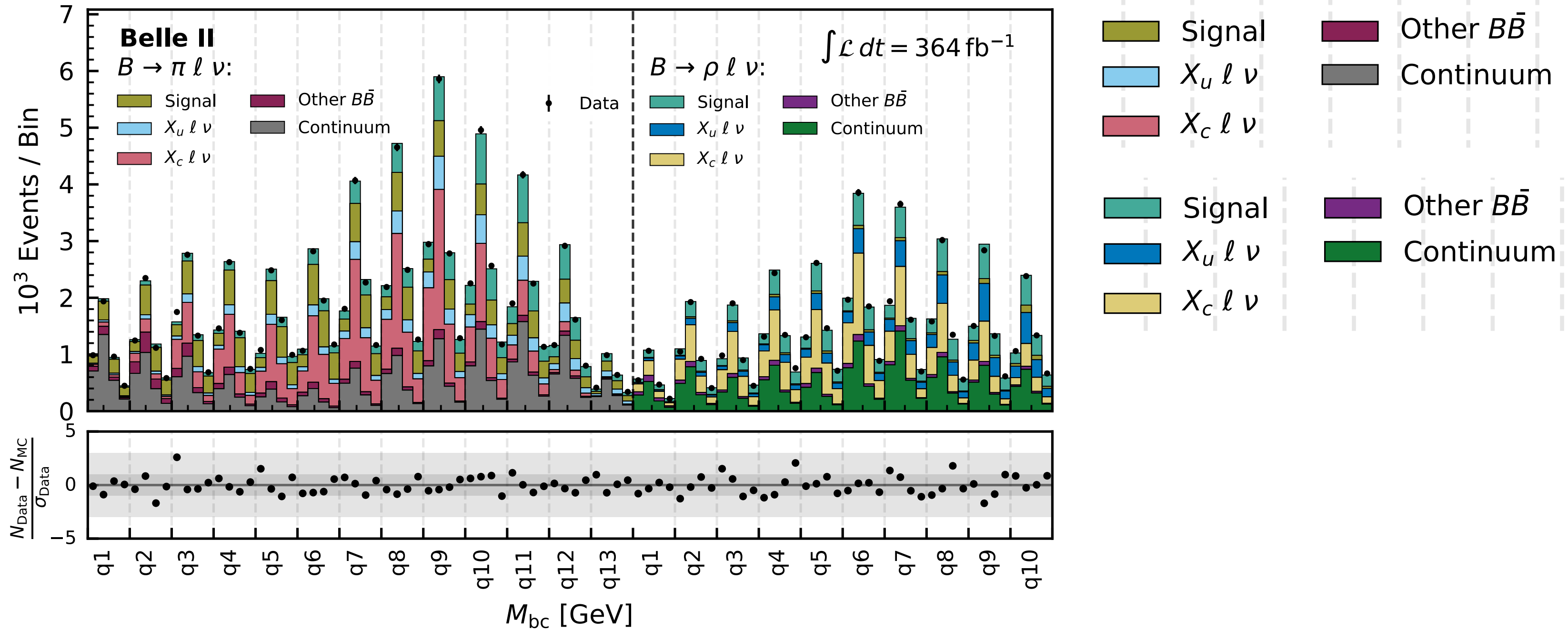


# $|V_{ub}|$ from $B^0 \rightarrow \pi^- \ell^+ \nu$ & $B^+ \rightarrow \rho^0 \ell^+ \nu$



mode. The following are the labels and bin edges for the  $q^2$  bins:  $q1 : q^2 \in [0, 2]$ ,  $q2 : [2, 4]$ ,  $q3 : [4, 6]$ ,  $q4 : [6, 8]$ ,  $q5 : [8, 10]$ ,  $q6 : [10, 12]$ ,  $q7 : [12, 14]$ ,  $q8 : [14, 16]$ ,  $q9 : [16, 18]$ ,  $q10 : [18, 20(20.3)]$ ,  $q11 : [20, 22]$ ,  $q12 : [22, 24]$ ,  $q13 : [24, 26.4] \text{ GeV}^2$ .

# $|V_{ub}|$ from $B^0 \rightarrow \pi^- \ell^+ \nu$ & $B^+ \rightarrow \rho^0 \ell^+ \nu$



mode. The following are the labels and bin edges for the  $q^2$  bins:  $q1 : q^2 \in [0, 2]$ ,  $q2 : [2, 4]$ ,  $q3 : [4, 6]$ ,  $q4 : [6, 8]$ ,  $q5 : [8, 10]$ ,  $q6 : [10, 12]$ ,  $q7 : [12, 14]$ ,  $q8 : [14, 16]$ ,  $q9 : [16, 18]$ ,  $q10 : [18, 20(20.3)]$ ,  $q11 : [20, 22]$ ,  $q12 : [22, 24]$ ,  $q13 : [24, 26.4] \text{ GeV}^2$ .



# $|V_{ub}|$ from $B^0 \rightarrow \pi^- \ell^+ \nu$ & $B^+ \rightarrow \rho^0 \ell^+ \nu$

$$\mathcal{B}(B^0 \rightarrow \pi^- \ell^+ \nu_\ell) = (1.516 \pm 0.042 \pm 0.059) \times 10^{-4}$$

$$\mathcal{B}(B^+ \rightarrow \rho^0 \ell^+ \nu_\ell) = (1.625 \pm 0.079 \pm 0.180) \times 10^{-4}$$

Total BF by integrating the  $\Delta\mathcal{B}(q^2)$

$|V_{ub}|$  extracted separately from  $\pi\ell\nu$  and  $\rho\ell\nu$  mode using  $\chi^2$  fits to the measured  $q^2$  spectra

$$\chi^2 = \sum_{i,j=1}^N (\Delta B_i - \Delta\Gamma_i\tau) C_{ij}^{-1} (\Delta B_j - \Delta\Gamma_j\tau) + \sum_m \chi_{Theory,m}^2$$

Form-factor coefficients:

BCL for  $B^0 \rightarrow \pi^- l^+ \nu_l$

BSZ for  $B^+ \rightarrow \rho^0 l^+ \nu_l$

C. Bourrely, L. Lellouch and I. Caprini. PRD 79 (2009) 013008

A. Bharucha, D. M. Straub and R. Zwicky, JHEP 08 (2016) 98

$B^0 \rightarrow \pi^- \ell^+ \nu$  (LQCD)

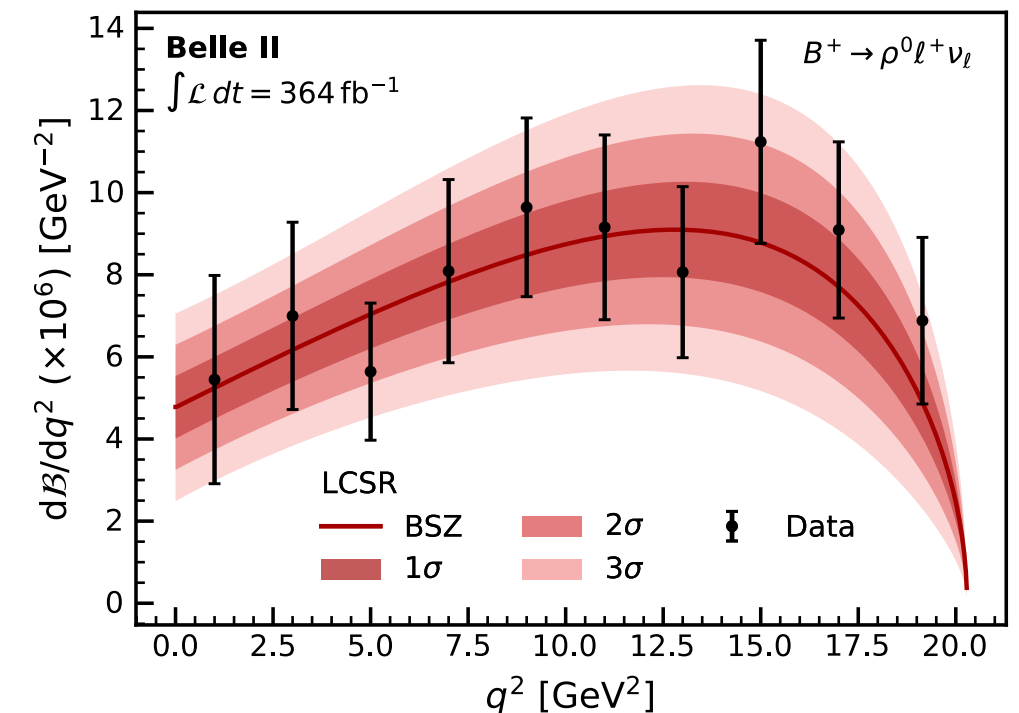
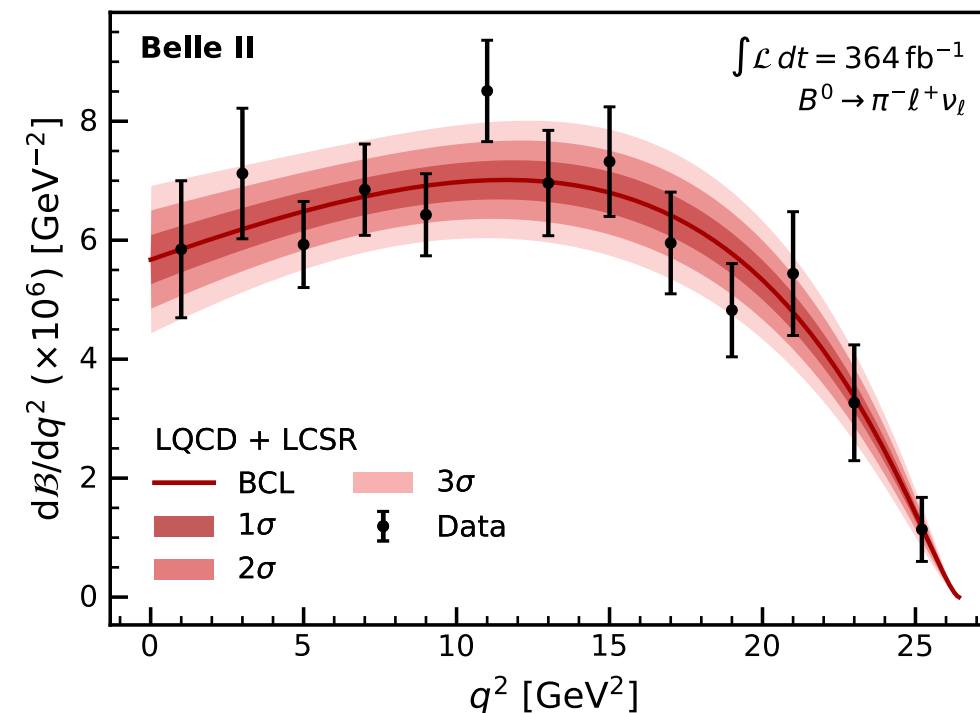
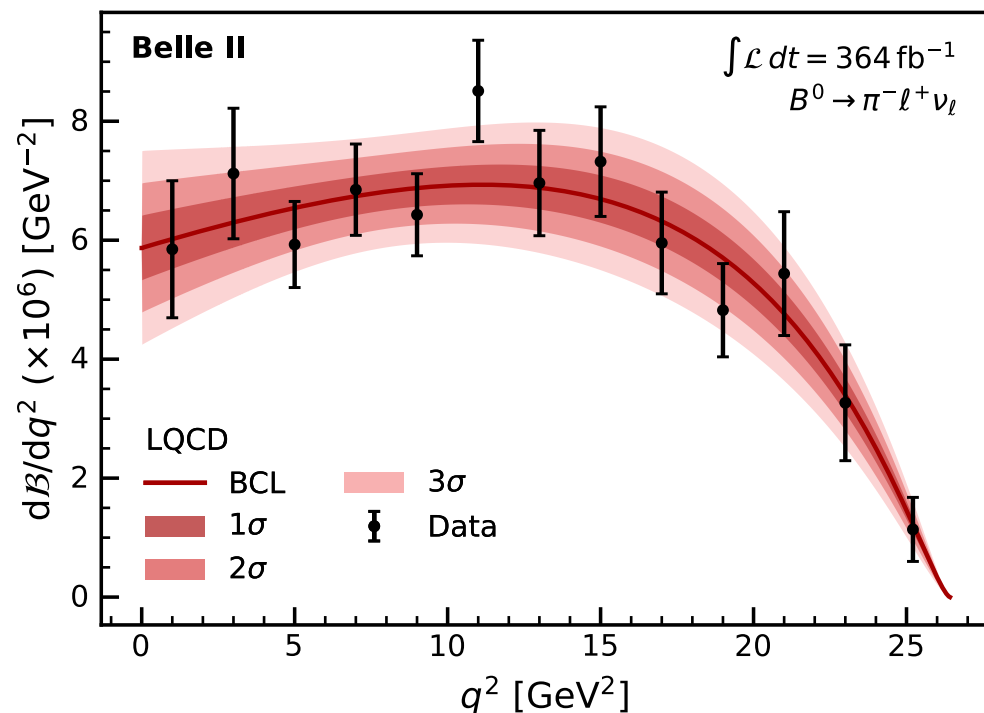
$$|V_{ub}|_{B \rightarrow \pi l \nu_\ell} = (3.93 \pm 0.09 \pm 0.13 \pm 0.19) \times 10^{-3}$$

$B^0 \rightarrow \pi^- \ell^+ \nu$  (LQCD+LCSR)

$$|V_{ub}|_{B \rightarrow \pi l \nu_\ell} = (3.73 \pm 0.07 \pm 0.07 \pm 0.16) \times 10^{-3}$$

$B^+ \rightarrow \rho^0 \ell^+ \nu$  (LCSR)

$$|V_{ub}|_{B \rightarrow \rho l \nu_\ell} = (3.19 \pm 0.12 \pm 0.17 \pm 0.26) \times 10^{-3}$$

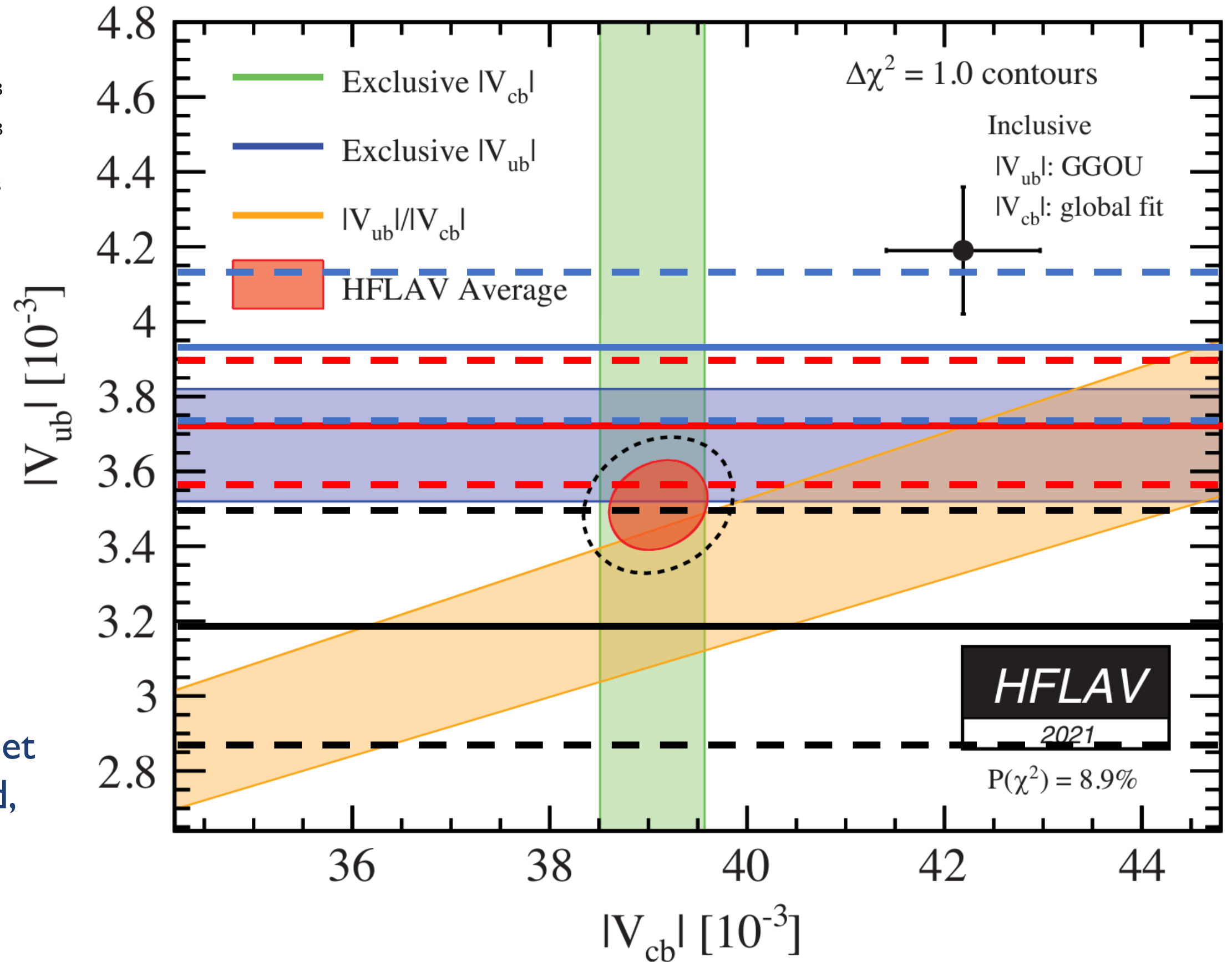


# $|V_{ub}|$ from $B^0 \rightarrow \pi^- \ell^+ \nu$ & $B^+ \rightarrow \rho^0 \ell^+ \nu$

$$|V_{ub}|_{B \rightarrow \pi \ell \nu} = (3.93 \pm 0.09 \pm 0.13 \pm 0.19) \times 10^{-3}$$

$$|V_{ub}|_{B \rightarrow \pi \ell \nu} = (3.73 \pm 0.07 \pm 0.07 \pm 0.16) \times 10^{-3}$$

$$|V_{ub}|_{B \rightarrow \rho \ell \nu} = (3.19 \pm 0.12 \pm 0.17 \pm 0.26) \times 10^{-3}$$



The results are limited by

- size of the off-resonance data set
- non-resonance  $B \rightarrow X_u \ell \nu$  bkgd,

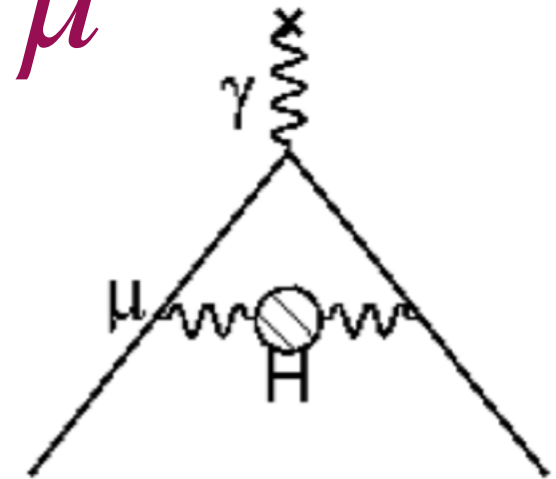
and reduce the tension against

$|V_{ub}|$  inclusive

# For 'light new physics'



$\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^0)$  for  $a_\mu^{\text{HVP}}$



# connections to muon (g-2)

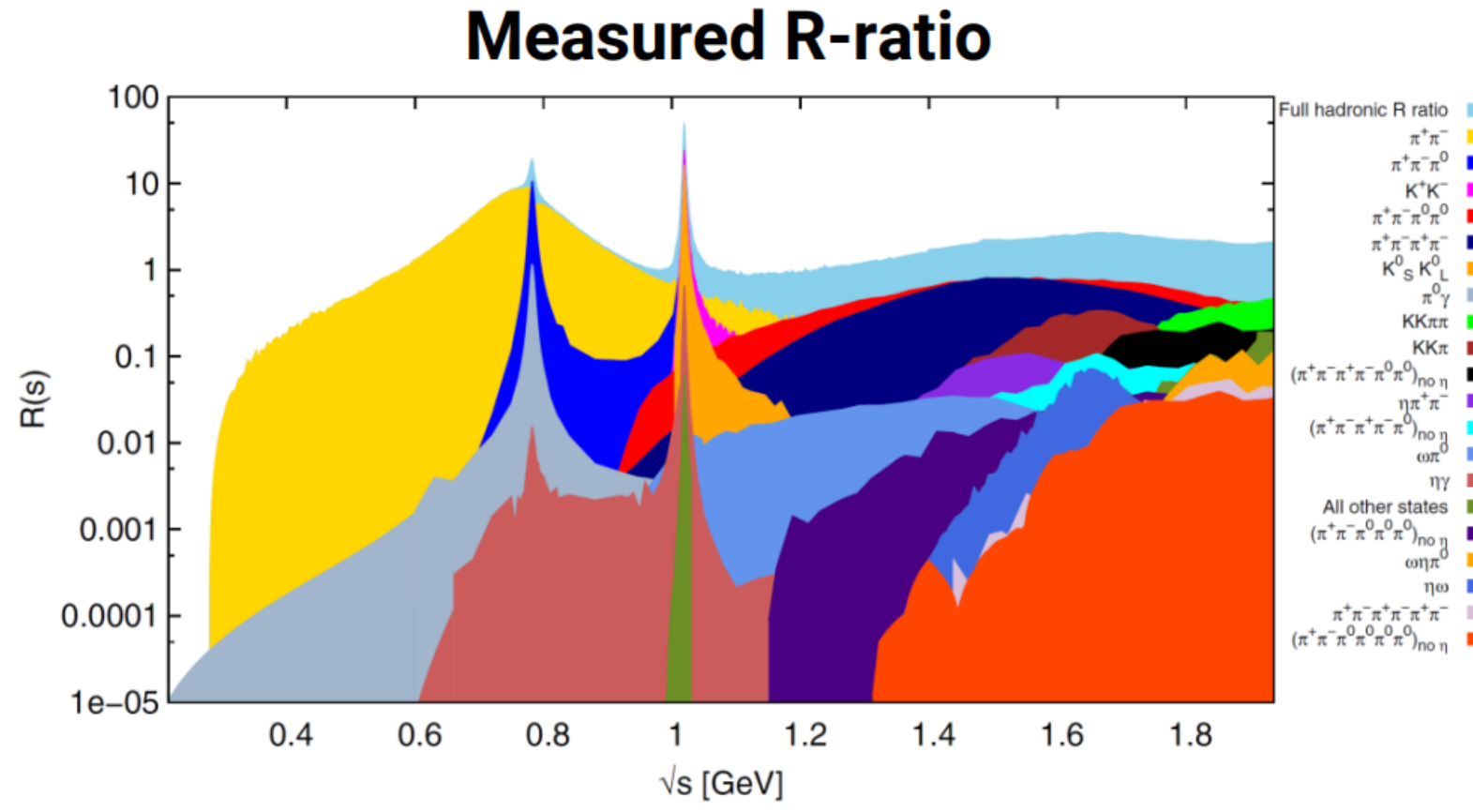
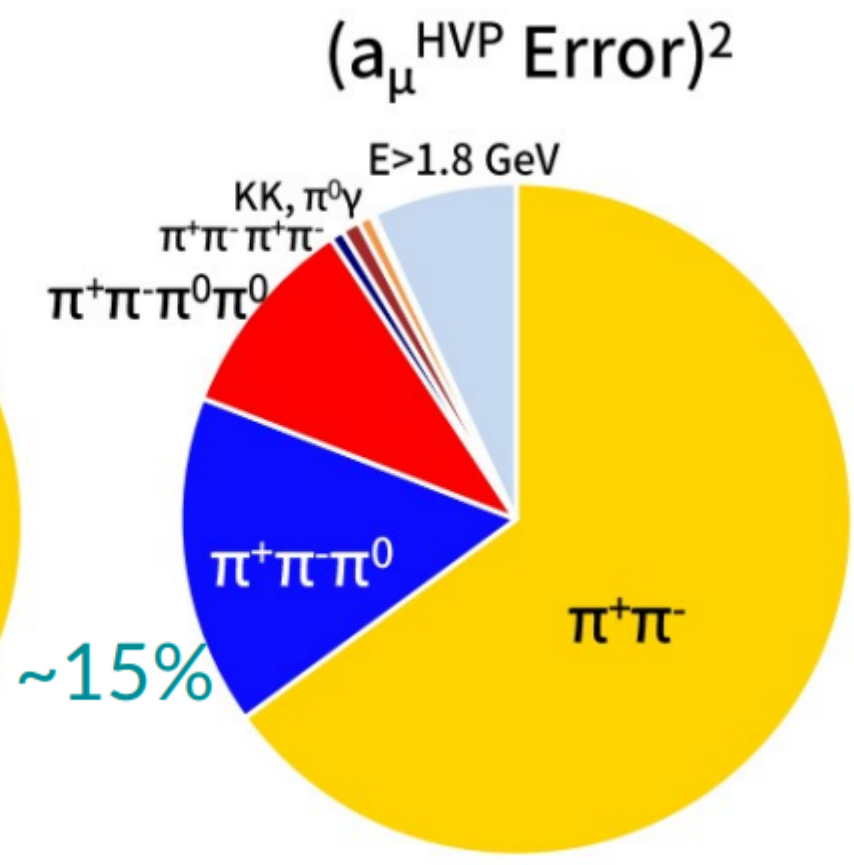
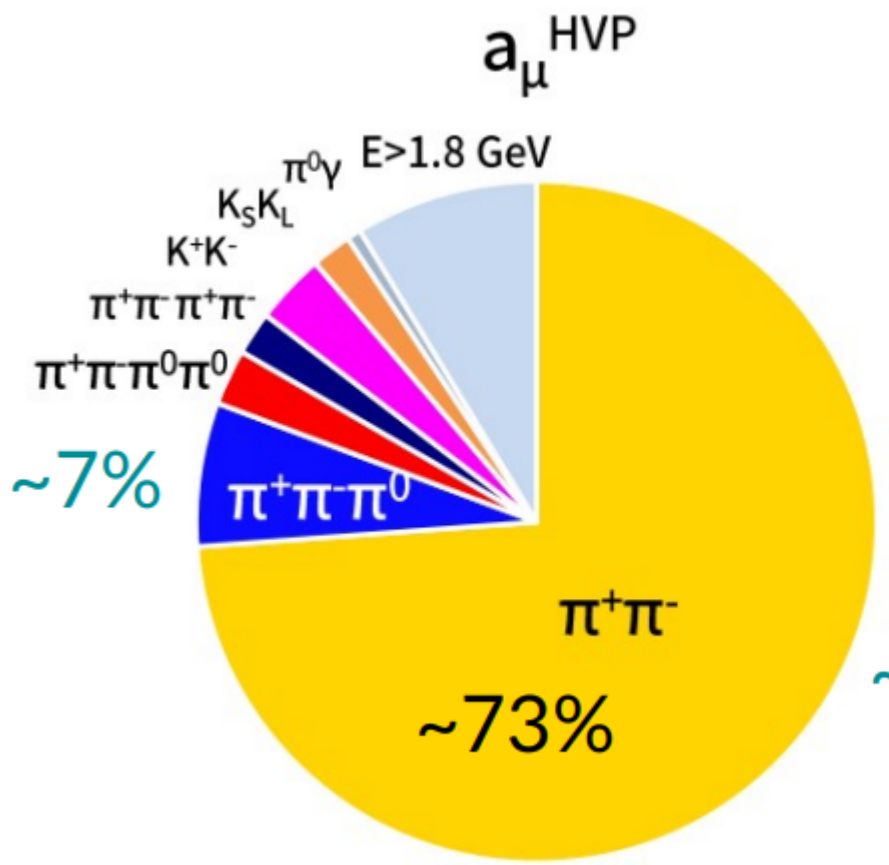
$$a_\mu = \frac{(g-2)_\mu}{2} = a_\mu^{\text{EW}} + a_\mu^{\text{QED}} + a_\mu^{\text{QCD}}$$

$$a_\mu^{\text{QCD}} = a_\mu^{\text{HVP}} + a_\mu^{\text{H,LBL}}$$

(82%)      (18%)

$$a_\mu^{\text{HVP,LO}} = \frac{\alpha}{3\pi^2} \int_{m_\pi^2}^{\infty} \frac{K(s)}{s} R_{\text{had}}(s) ds,$$

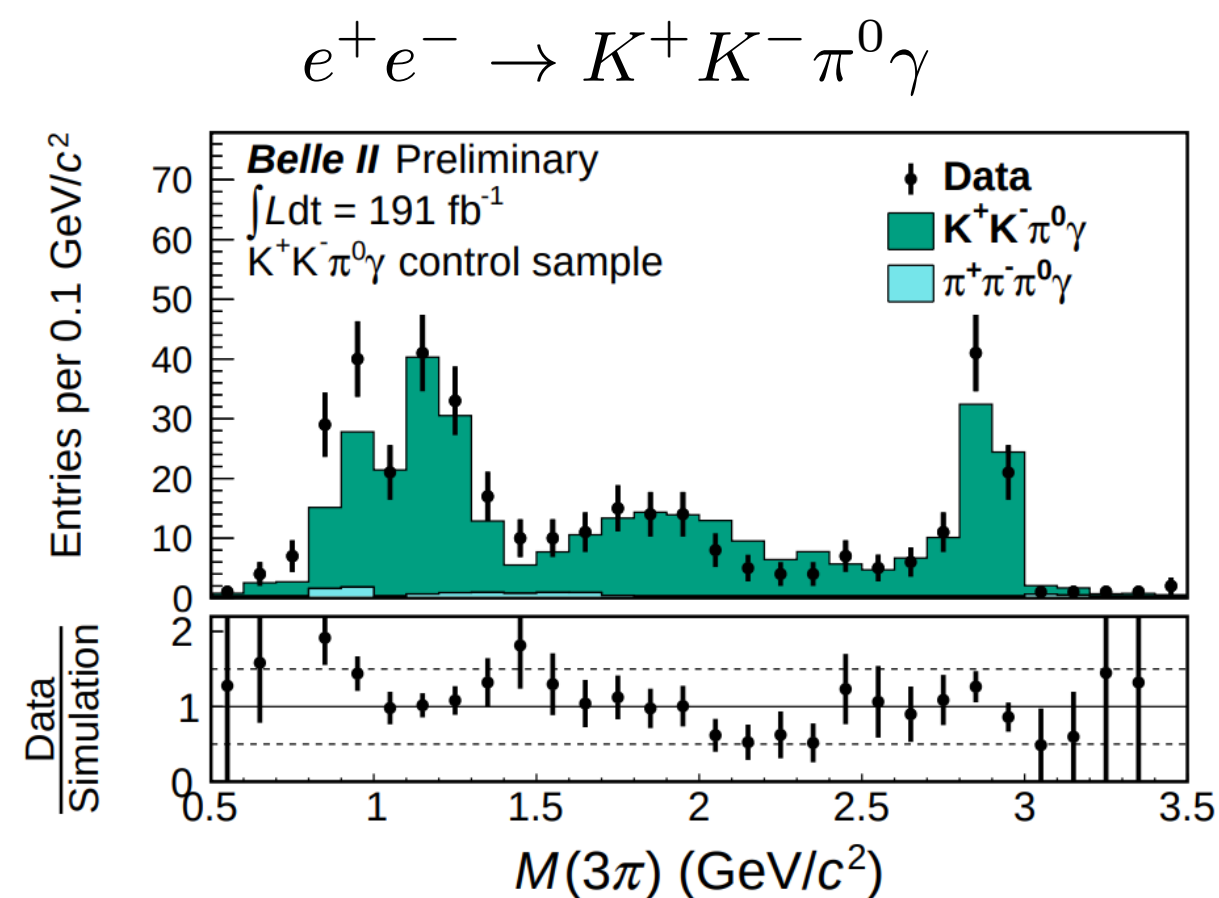
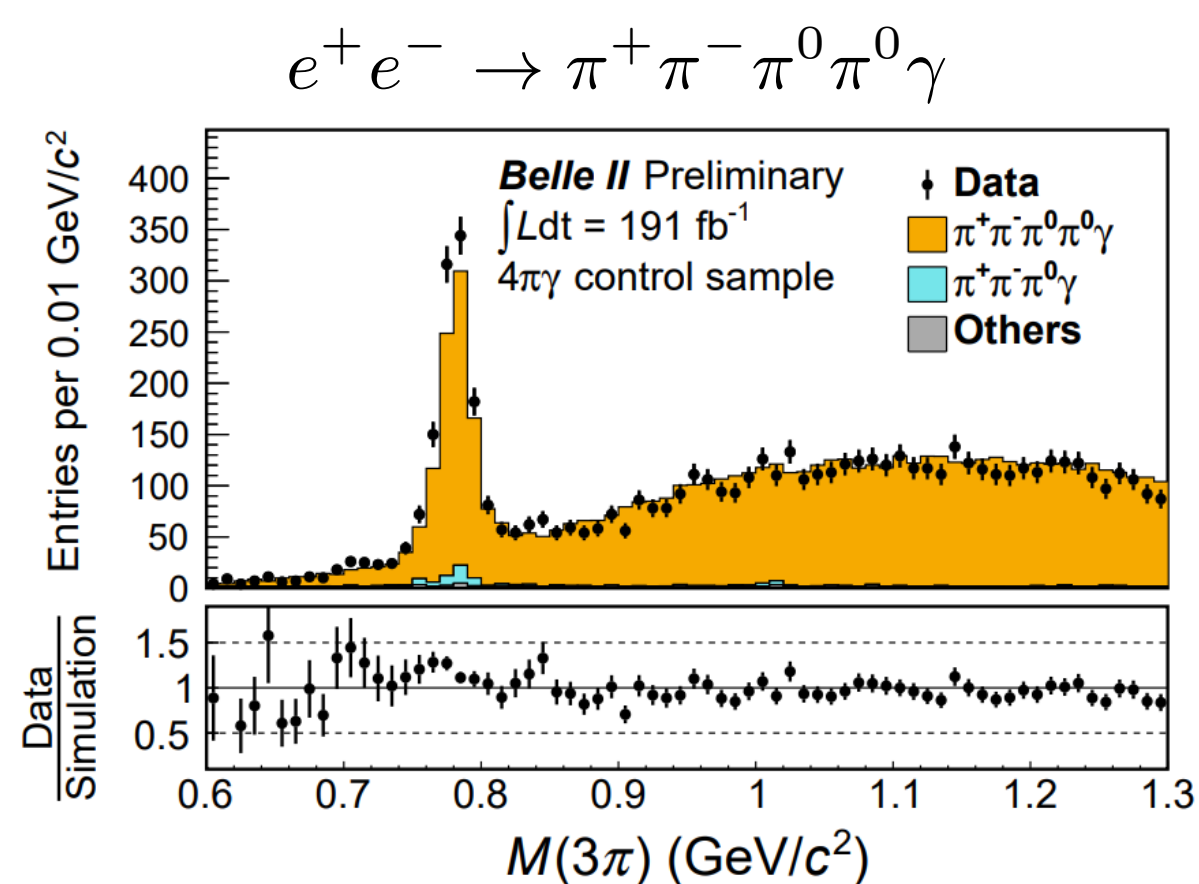
$$R_{\text{had}}(s) = \frac{\sigma_0(e^+e^- \rightarrow \text{hadrons})}{\sigma_{\text{pt}}(e^+e^- \rightarrow \mu^+\mu^-)};$$



(a) The hadronic R-ratio.

$$\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^0)$$

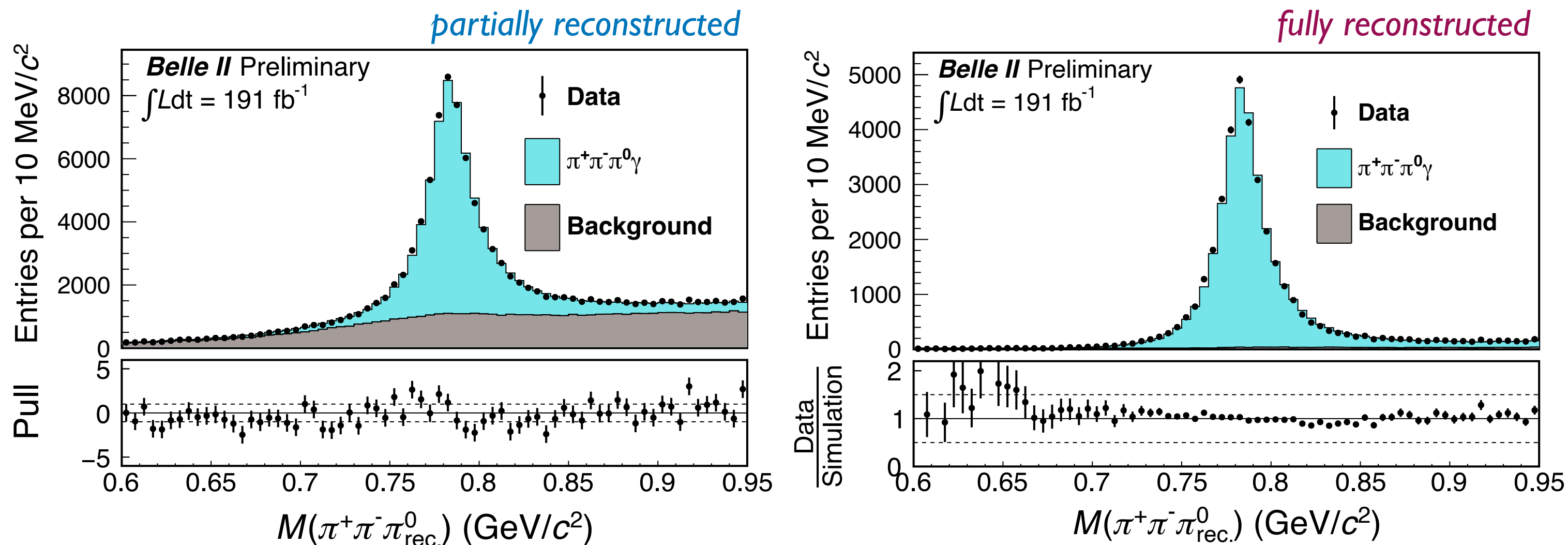
- Study  $e^+e^- \rightarrow \pi^+\pi^-\pi^0$  decays in  $\mathcal{L} = 191 \text{ fb}^{-1}$
- as a function of  $\sqrt{s'}$  by using **ISR** technique
  - reconstruct  $e^+e^- \rightarrow \pi^+\pi^-\pi^0\gamma_{\text{ISR}}$ , for  $0.62 < \sqrt{s'} = M(3\pi) < 3.50 \text{ GeV}$
- **Kinematic fit** for background suppression
  - constrain  $(E, \vec{p})$  of  $\pi^+\pi^-\pi^0\gamma_{\text{ISR}}$  to that of  $e^+e^-$  beams
- Validation (“**scale factor**”) of backgrounds in control samples



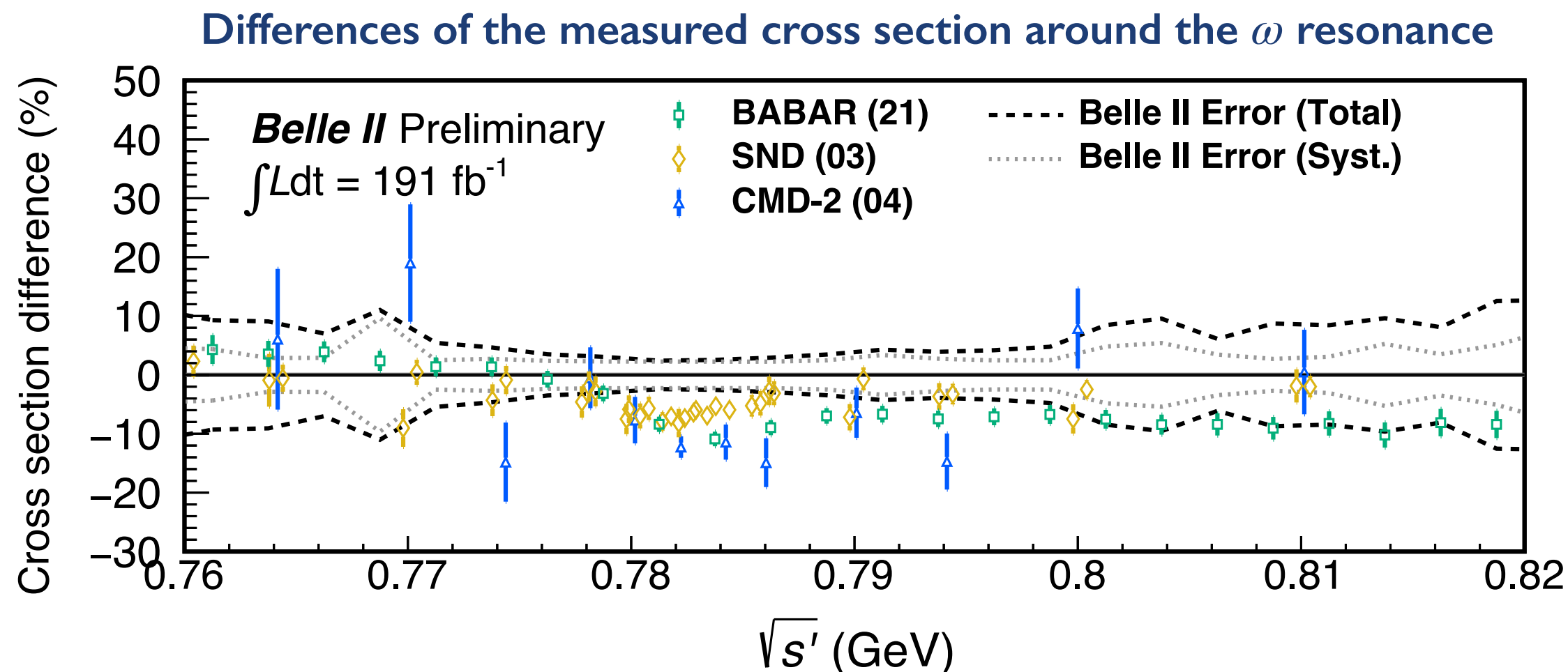
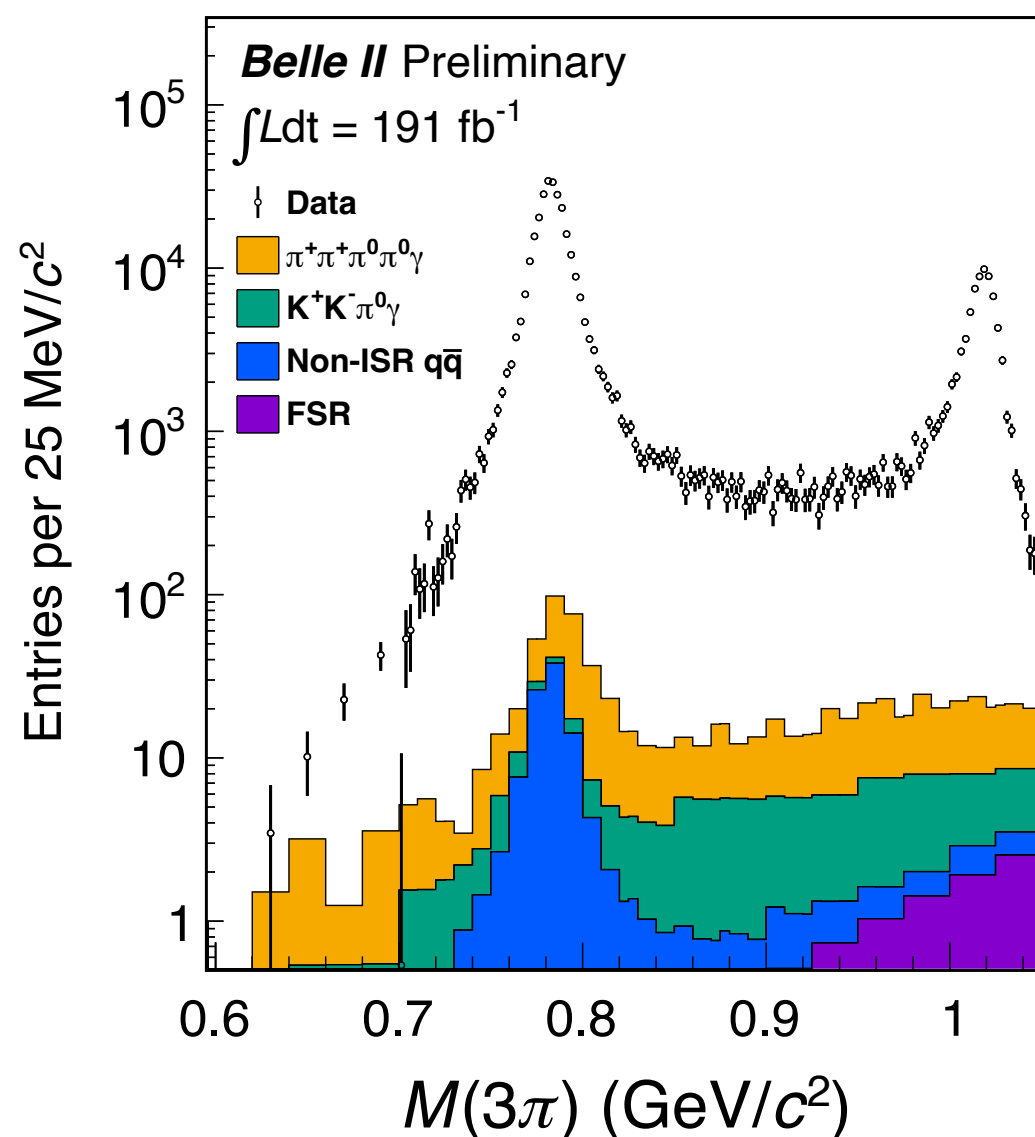
$$\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^0)$$

$$\varepsilon(\pi^0) = \frac{N_{\text{full}}(\gamma_{\text{ISR}}\pi^+\pi^-\pi^0)}{N_{\text{partial}}(\gamma_{\text{ISR}}\pi^+\pi^-)}$$

- $\pi^0$  efficiency as a major analysis challenge
- The  $\varepsilon(\pi^0)$  is determined to an accuracy of  $\sim 1\%$  by comparing full- and partial-reconstruction in the  $\omega \rightarrow \pi^+\pi^-\pi^0$  region



# $\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^0)$ Results



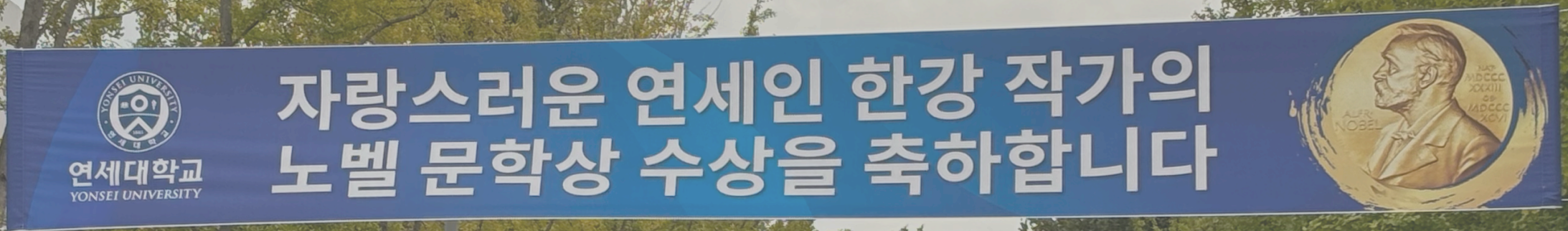
- $a_\mu^{3\pi}(0.62 - 1.8 \text{ GeV}) = (48.91 \pm 0.23 \pm 1.07) \times 10^{-10}$
- main syst. uncertainties from efficiency and absence of NNLO in the MC
- **6.5% higher (2.5 $\sigma$  significant)** than the global fit  $\rightarrow$  **move to smaller 'anomaly'**

$$a_\mu^{3\pi}(0.62-1.8 \text{ GeV}) = (45.91 \pm 0.37 \pm 0.38) \times 10^{-10}.$$

# Closing remarks

- Belle II has collected over  $0.4 \text{ ab}^{-1}$  data sample in its first 3 years of operation before LS1, and started Run 2 data-taking in Feb. this year.
- With the data set of  $\sim 1/2$  the size of Belle, the physics precision of Belle II results are comparable or better in many analyses.
- Recent Belle II physics highlights include first evidence for  $B^+ \rightarrow K^+ \nu \bar{\nu}$ , and inclusive test of LFU with  $B \rightarrow X \tau \nu$ . (PRD 109, 112006 (2024))
- Belle II started her endeavor to understand the ‘Incl.-Excl. tension’ on  $|V_{ub}|$  and  $|V_{cb}|$ .
- After summer shutdown, Run 2 will resume very soon with the goal of collecting a several  $\text{ab}^{-1}$  data in the next few years. Please stay tuned!





*Thank you!*