Amplification of Attosecond High-Harmonic X-Ray Pulses by Plasma-Based X-Ray Lasers

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The metric system

Attoseconds??

We'll need to really know the metric system because the pulses are incredibly short and the powers and intensities can be incredibly high.

Prefixes:

Small		Big	
Milli (m)	10^{-3}	Kilo (k)	10^{+3}
Micro (μ)	10^{-6}	Mega (M)	10^{+6}
Nano (n)	10^{-9}	Giga (G)	10^{+9}
Pico (p)	10^{-12}	Tera (T)	10^{+12}
Femto (f)	10^{-15}	Peta (P)	10^{+15}
Atto (a)	10^{-18}	Exa (E)	10^{+18}
Pulse Duration		Pulse Peak Power	

What could happen in attoseconds (1as = 10^{-18} s)?

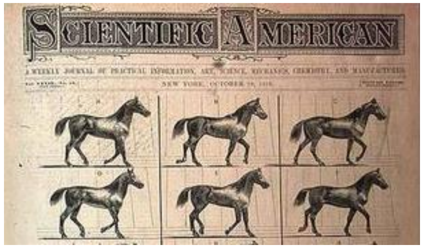
1 s

1 ms

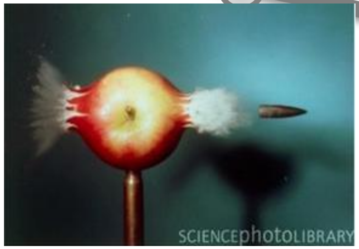
1 μs

Camera

1 ns

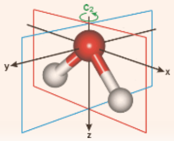


In 1878, E. Muybridge



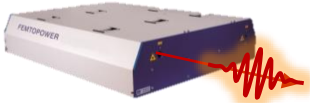
In 1964, H. Edgerton

Molecules rotate



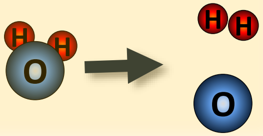
1 ps

Femtosecond Laser



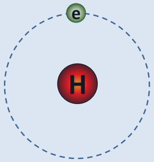
1 fs

Chemical reactions

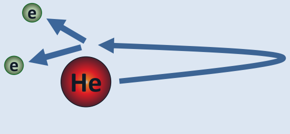


Electron dynamics

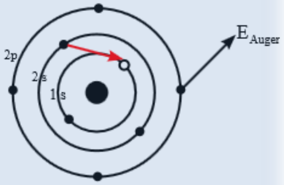
Revolution cycle
~160 as



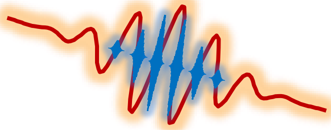
NSDI
~fs



Auger decay
~fs



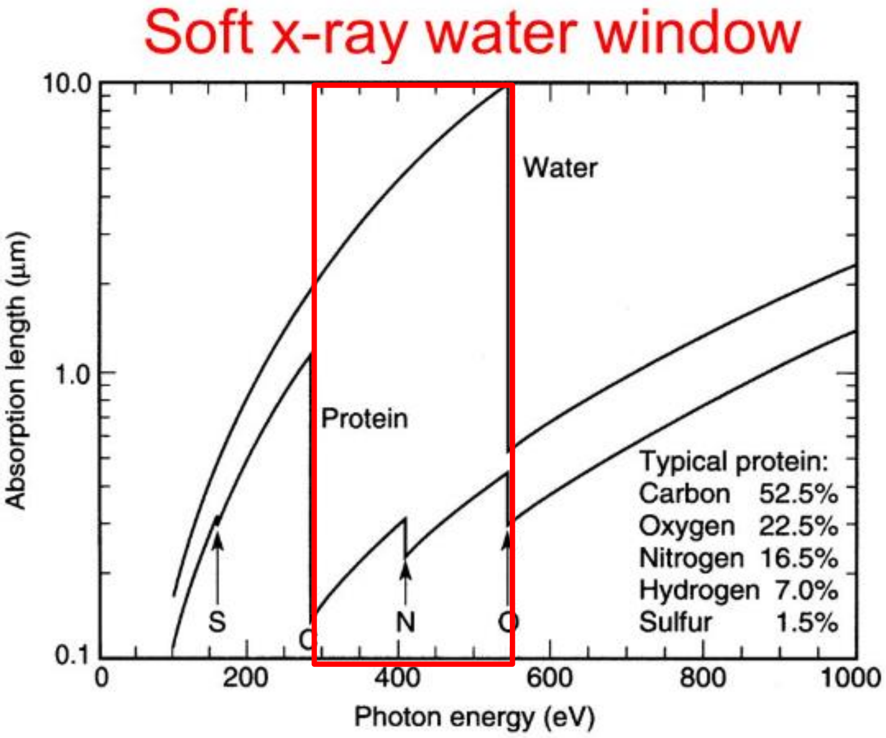
Attosecond Pulse



1 as = 10^{-18} s

Motivation

- Coherent attosecond X-ray pulses provide a **unique combination of extremely high temporal and spatial resolution.** Applications for dynamical, element-specific imaging in biochemistry and material science require **sufficiently large number of photons per pulse.**
- **2.3-4.4 nm wavelength band ("Water Window") giving to low absorption of O and high absorption of C** is especially important for **protein imaging in a cell.**



"Water window": 2.3 nm - 4.4 nm
Photon energy: 530 eV - 280 eV

What is 'water window'?

- The **water window** is a region of the electromagnetic spectrum in which water is transparent to soft x-rays. The window extends from the K-absorption edge of carbon at 282 eV (68 PHz, 4.40 nm wavelength) to the K-edge of oxygen at 533 eV (129 PHz, 2.33 nm wavelength). Water is transparent to these X-rays, but carbon and its organic compounds are absorbing. These wavelengths could be used in an X-ray microscope for viewing living specimens. This is technically challenging because few if any viable lens materials are available above extreme ultraviolet.

From Wikipedia.

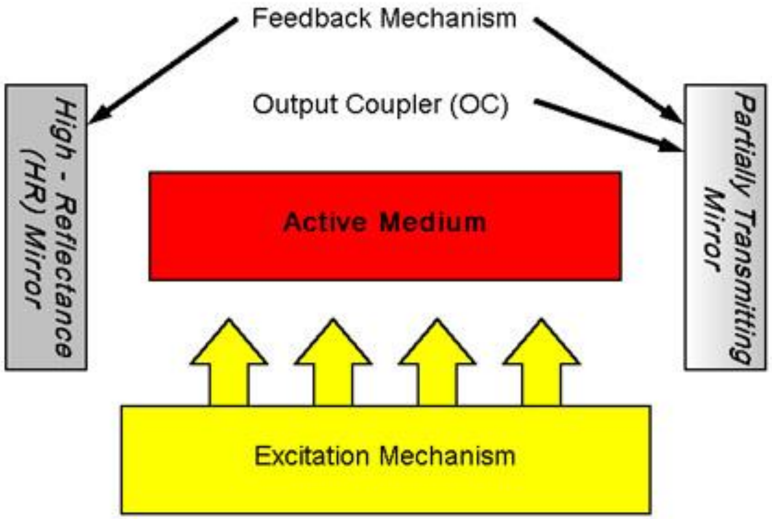
What are problems?

Maser/Laser is a major source of coherent radiation in microwave IR, optical, UV and VUV ranges.

Can the same concept be extended to X-ray and gamma-ray ranges?

Quiz: 누가 처음으로 레이저를 보였는가?

Theodore Maiman on May 16, 1960


Feedback Mechanism

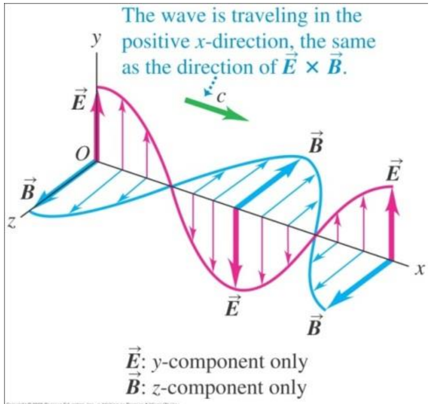
Output Coupler (OC)

High - Reflectance (HR) Mirror

Active Medium

Partially Transmitting Mirror

Excitation Mechanism

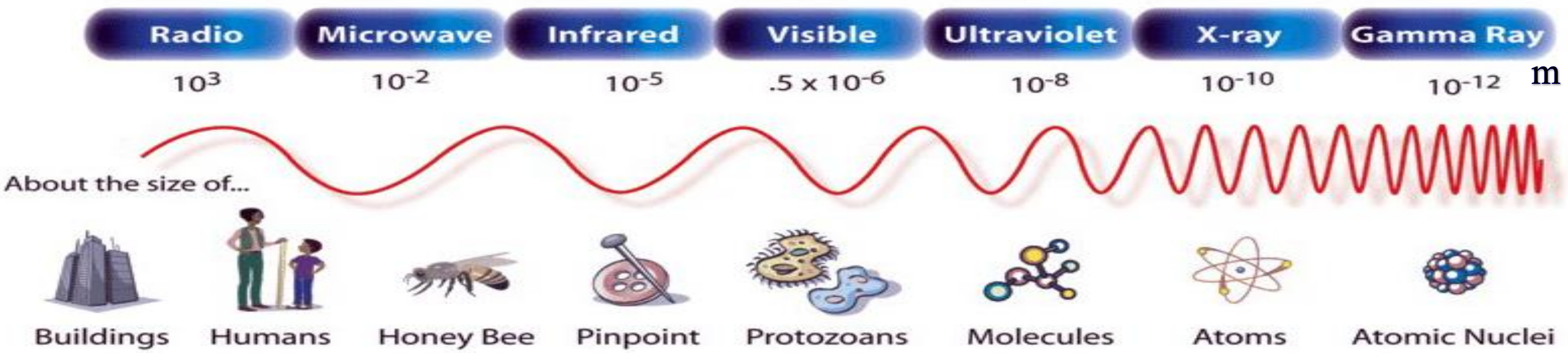


The Nobel in Physics 1964 for invention of maser/ laser


Charles Townes
(1915-2014)


Alexander Prokhorov
(1916-2002))


Nikolay Basov
(1922-2001)



(Z. Huang, SLAC)

A.L. Schawlow and C.H. Townes, “Infrared and Optical Masers”, Phys. Rev. **112**, 1940 (1958): “unless some radically new approach is found, they (*maser systems*) cannot be pushed to wavelengths much shorter than those in the ultraviolet region.”

$$P\sigma_{res} > \frac{1}{T_1}$$

$$\sigma_{res} = \lambda^2 / 2\pi$$

$$\frac{1}{T_1} = \frac{4(2\pi)^3 \mu^2}{3\lambda^3 \hbar}$$

Pumping flux: P ~ $1/\lambda^5$!

Soft X-rays:
λ: 10 nm – 1 nm
ħω: 100eV-1keV

Hard X-rays:
λ: 1nm-0.1Å
ħω: 1keV-100keV

Gamma-rays:
λ<0.1Å
ħω:>100keV

Kocharovskaya. ”What are the ultimate limits for laser photon energies?” pp.1277-1281, in “Light, the universe, and everything…” by G. Agarwal, et al., J. Mod. Phys. <u>65</u>, 1261-1308 (2018)

Three Types of Soft X-ray Lasers (10 nm – 1 nm)

1. **Table-top plasma lasers** based on the population inversion between energy levels of ions in plasma:

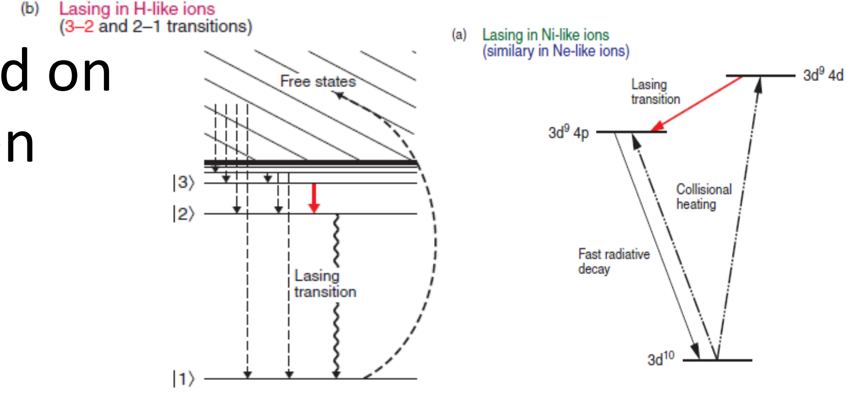
   $U_{\max} \sim 10mJ, \tau_{\min} \sim 1ps$

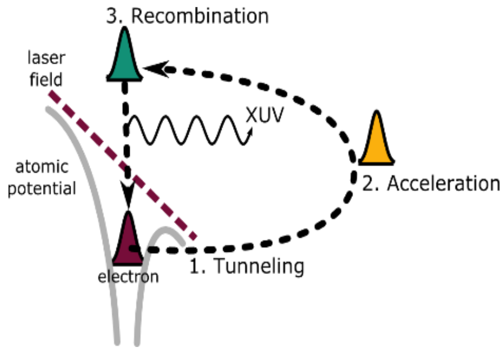
2. **Table- top High Harmonic: Generation Sources:** (high harmonics of an ionizing IR field)

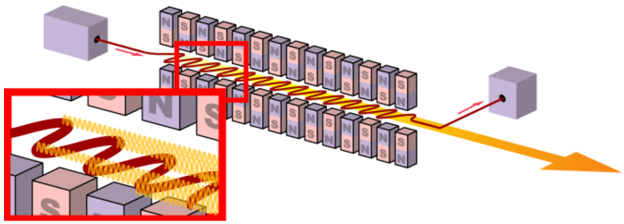
   $U_{\max} \sim 10nJ, \tau_{\min} (\exp ected) \sim 100as$

3. Large-scale free-electron lasers based on the collective synchrotron emission of the high quality electron-bunch trains accelerated to the relativistic energies (~1 GeV) and wiggling in the periodic magnetic field of a long undulator:

$U_{\max} \sim 1mJ, \tau_{\min} \sim 10\,fs$



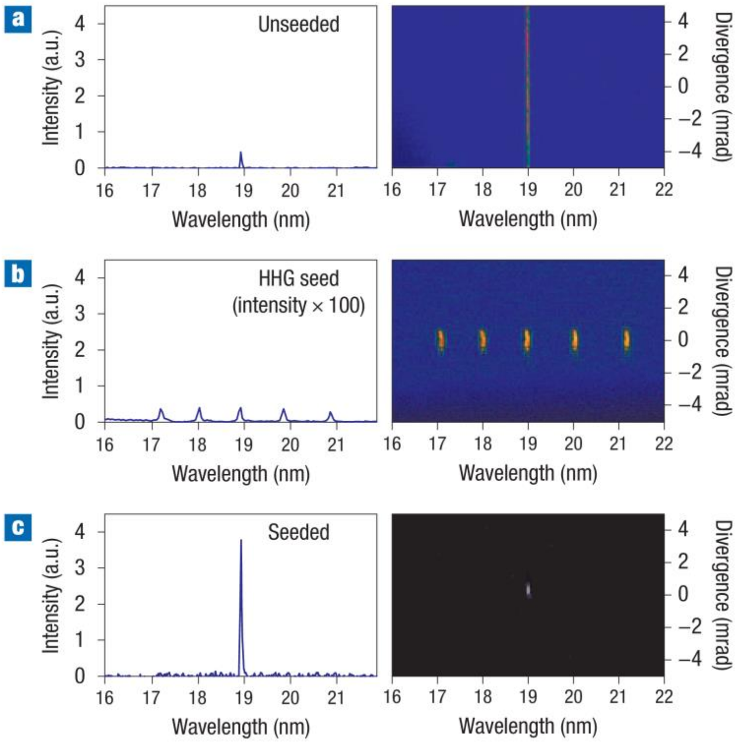




Our goal is;

- **To amplify a set of phase-matched high-order harmonics (attosecond pulses)** to combine element sensitivity and nanometer spatial resolution with attosecond temporal resolution!!
- To build table-top X-ray lasers!!

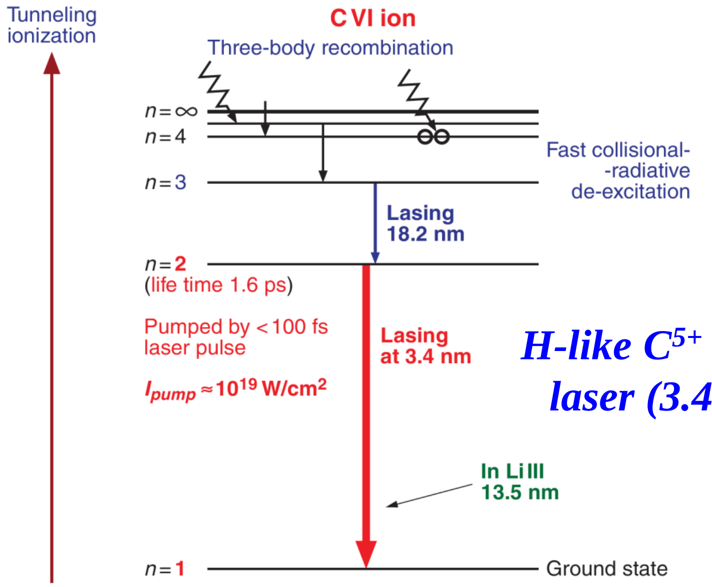
Amplification of a SINGLE high-order harmonic in a plasma-based soft-x-ray laser



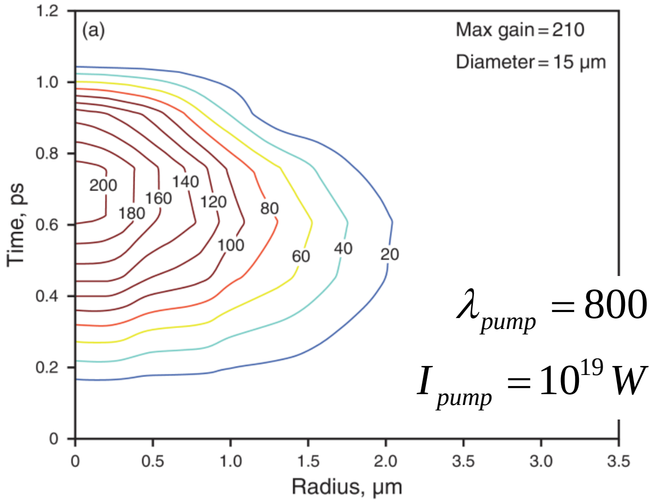
Ni-like Mo^{14+} ; X-ray laser (18.9nm) seeded with harmonics of Ti:Sa laser field

From J. J. Rocca et al., *Nature Phot.* **2**, 94 (2008)

Plasma-based soft-X-ray laser in water window



H-like C^{5+} X-ray laser (3.4nm)



$$\lambda_{pump} = 800nm$$

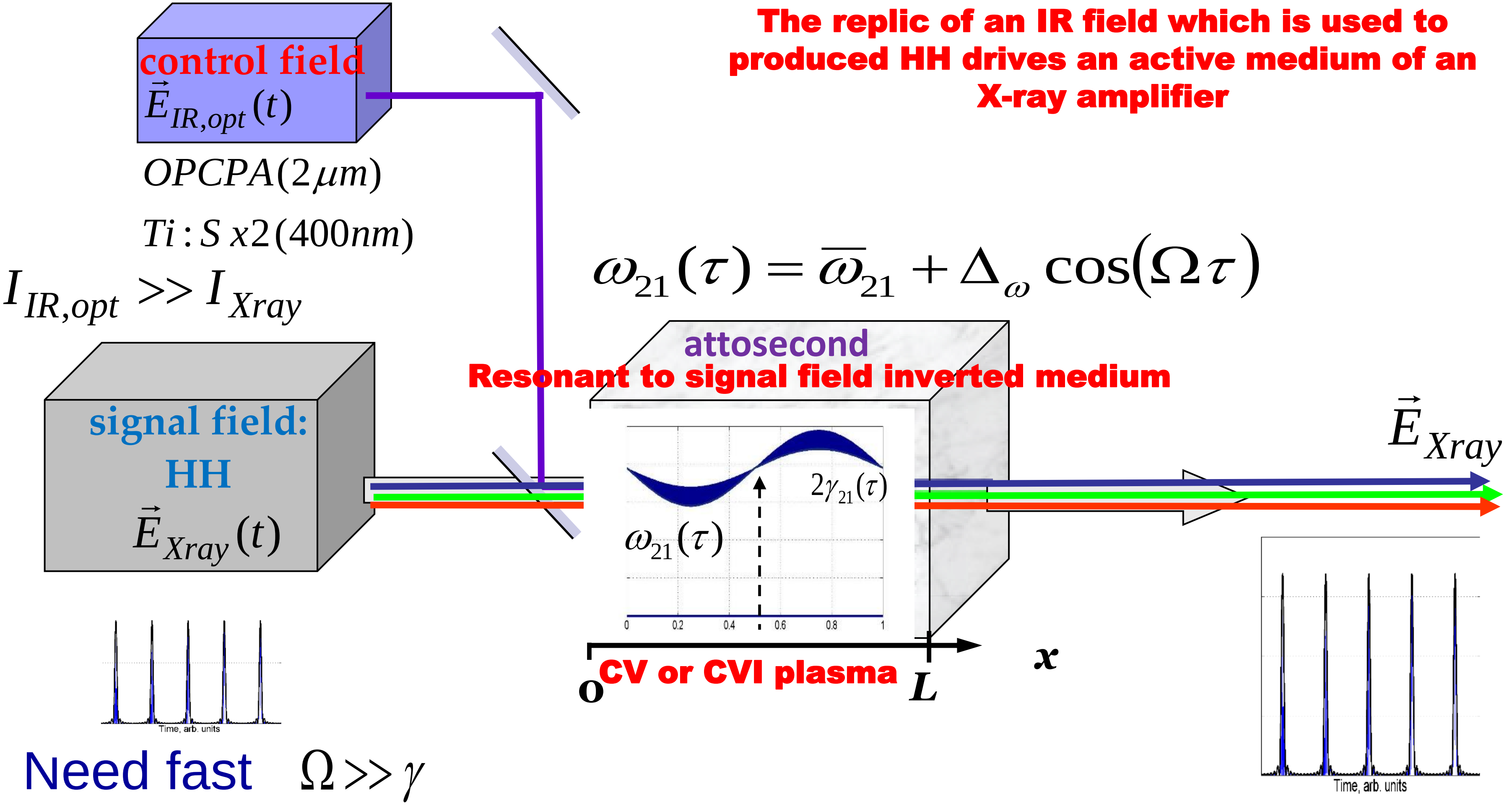
$$I_{pump} = 10^{19} W/cm^2$$

From S. Suckewer, P. Jaeglé, X-Ray laser: past, present, and future, *Laser Phys. Lett.* **6**, 411 (2009)

Basic idea: Modulation of atomic states by an IR/optical field

T. Akhmedzhanov, V.Antonov, O. Kocharovskaya, Phys. Rev. A 94, 023821 (2016).

T. Akhmedzanov, et al., Phys. Rev. A 95, 023845 (2017).



The replic of an IR field which is used to produced HH drives an active medium of an X-ray amplifier

$$\omega_{21}(\tau) = \overline{\omega}_{21} + \Delta_{\omega}\cos(\Omega\tau)$$

Resonant to signal field inverted medium

CV or CVI plasma

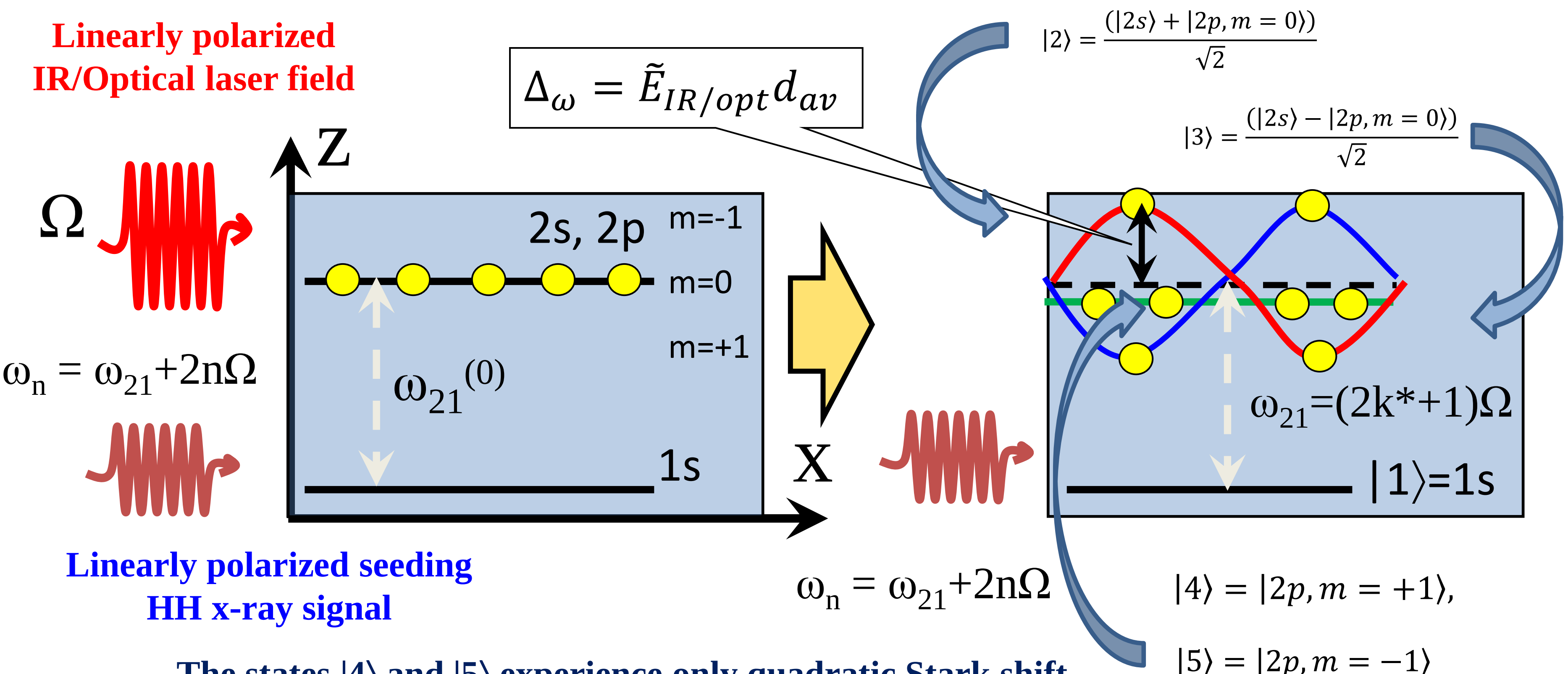
Need fast $\Omega >> \gamma$

and deep $\Delta_{\omega} >> \Omega$ modulation

Amplified attosecond pulses of X-ray radiation

Active medium of a hydrogen-like soft X-ray laser under the action of an optical laser field

The states |2⟩ and |3⟩ experience linear Stark effect:



Linearly polarized IR/Optical laser field

$$\Delta_\omega = \tilde{E}_{IR/opt} d_{av}$$

$$|2\rangle = \frac{(|2s\rangle + |2p, m=0\rangle)}{\sqrt{2}}$$

$$|3\rangle = \frac{(|2s\rangle - |2p, m=0\rangle)}{\sqrt{2}}$$

$\omega_n = \omega_{21} + 2n\Omega$

Linearly polarized seeding HH x-ray signal

$\omega_n = \omega_{21} + 2n\Omega$

$|4\rangle = |2p, m=+1\rangle,$

$|5\rangle = |2p, m=-1\rangle$

The states |4⟩ and |5⟩ experience only quadratic Stark shift

Transitions |2⟩↔|1⟩ and |3⟩↔|1⟩ amplify z-polarized seeding X-ray field,

Transitions |4⟩↔|1⟩, and |5⟩↔|1⟩ are the source of ASE in orthogonal y-polarization and increase population of the ground state |1⟩ reducing amplification of HH signal

T. R. Akhmedzhanov, et. al, PRA 95, 023845 (2017)

Mathematical model:

$$\frac{\partial^2 \vec{E}}{\partial x^2} - \frac{\varepsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 \vec{P}}{\partial t^2}$$ **- wave equation for the X-ray field,**

$$\vec{E}(x,t) = \vec{z}_0 E_z + \vec{y}_0 E_y$$

Amplified spontaneous emission (y-polarization), $E_y(x=0,t)=0,$

Amplified seeding x-ray signal (z-polarization), $E_z(x=0,t) = E_{inc}^{X-ray}(t),$

X-ray polarization of the medium: $\vec{P}(\vec{r},t) = \vec{z}_0 N d_{tr}(\rho_{21}-\rho_{31}) + i\vec{y}_0 N d_{tr}(\rho_{41}+\rho_{51}) + \text{c.c.},$

Equations for the density-matrix elements:

$$\dot{\rho}_{11} = \sum_{i=2}^{5} A_{i1}\rho_{ii} - i[\mathrm{H},\rho]_{11}$$

$$\dot{\rho}_{ij} = -\gamma_{ij}\rho_{ij} - i[\mathrm{H},\rho]_{ij}, ij \neq 11$$

$$\mathrm{H} = \begin{pmatrix} \omega_1 & -E_z d_{tr} & E_z d_{tr} & iE_y d_{tr} & iE_y d_{tr} \\ -E_z d_{tr} & \omega_2 - \tilde{E}_{IR/opt} d_{av}\cos\left\{-i\Omega\left(t - \frac{x n_{pl}}{c}\right)\right\} & 0 & 0 & 0 \\ E_z d_{tr} & 0 & \omega_3 + \tilde{E}_{IR/opt} d_{av}\cos\left\{-i\Omega\left(t - \frac{x n_{pl}}{c}\right)\right\} & 0 & 0 \\ -iE_y d_{tr} & 0 & 0 & \omega_4 & 0 \\ -iE_y d_{tr} & 0 & 0 & 0 & \omega_5 \end{pmatrix}$$

Initially, all the excited states |2⟩, |3⟩, |4⟩, and |5⟩ are populated with equal probability,

$\rho_{22} = \rho_{33} = \rho_{44} = \rho_{55}$ at $t = 0$ **and there are no coherencies,** $\rho_{ij}(x, t=0) = 0$ if $i \neq j$

Analytical solution:

The frequency of transitions |2⟩↔|1⟩ and |3⟩↔|1⟩ is a multiple of Ω:

$(2k^*+1)\Omega = \bar{\omega}_{tr}$, **where** k^* **is an integer number,**

Incident high-harmonic (HH) field:

$$E_{inc}^{X-ray}(x=0,t) = \frac{1}{2}\vec{z}_0 \sum_{l=-L_{\min}}^{L_{\max}} E_{inc}^{[2(k^*+l)+1]} \exp\left\{-i\left(\bar{\omega}_{tr} + 2l\Omega\right)t\right\} + \text{c.c.},$$

Approximations:

1) **Three-level model:** *the states |4⟩ and |5⟩ are neglected*

2) **Linear regime:** *the population differences are constant*

3) **Fixed X-ray field:** *rescattering of the sidebands is unimportant*

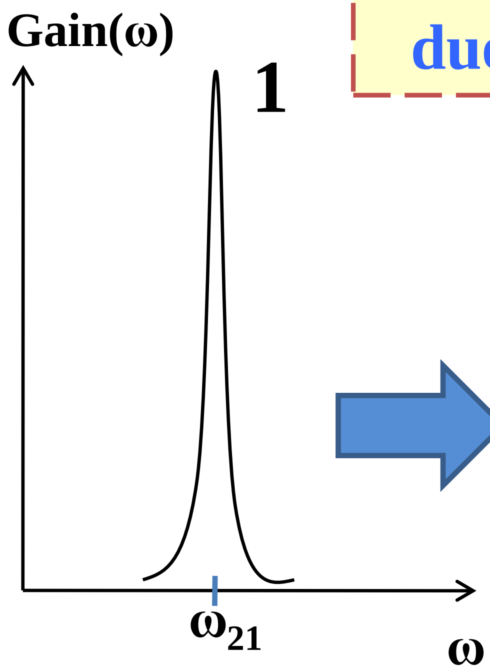
4) **Dense plasma:** $L_{amp}^{(l)} >> L_{coh}, \quad \frac{\omega_{pl}^2}{2\Omega c} >> g_{total}$

5) **High-frequency modulation:** $\Omega >> \bar{\gamma}_{tr}$

$$\vec{E}_{X-ray}(x,\tau) = \frac{1}{2}\vec{z}_0 \sum_{l=-L_{\min}}^{L_{\max}} E_{inc}^{[2(k^*+l)+1]} \exp\left\{g_{total} J_{2l}^2(p_\omega)x\right\} \exp\left\{-i\left(\bar{\omega}_{tr} + 2l\Omega\right)\tau\right\} + \text{c.c.},$$

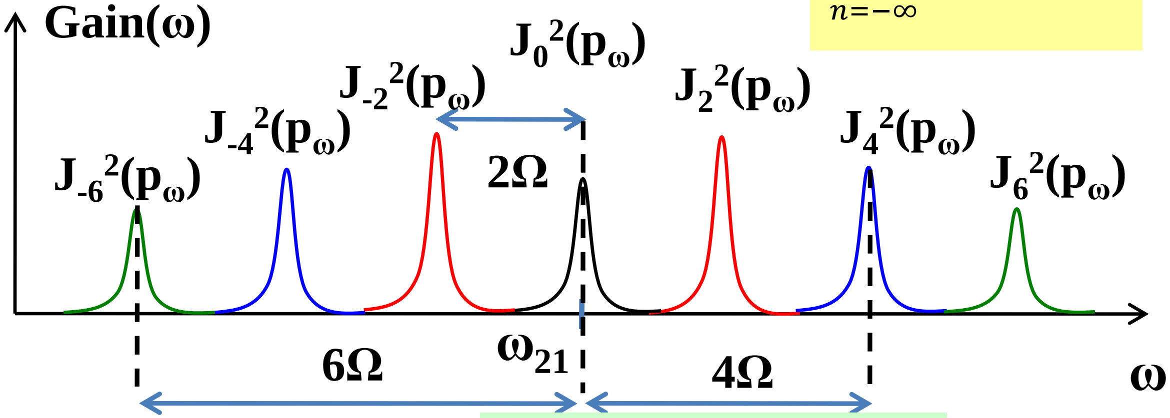
where $\tau = t - x/c$ **is local time, and** $g_{total} = \frac{4\pi n_{tr} N d_{tr}^2 \bar{\omega}_{tr}}{\hbar \bar{\gamma}_{tr} c}$ **is unperturbed amplification coefficient**

Gain redistribution for x-ray field due to modulation of the inverted transitions



$$J_{-2n}(x) = J_{2n}(x)$$

$$\sum_{n=-\infty}^{\infty} J_n^2(x) = 1$$



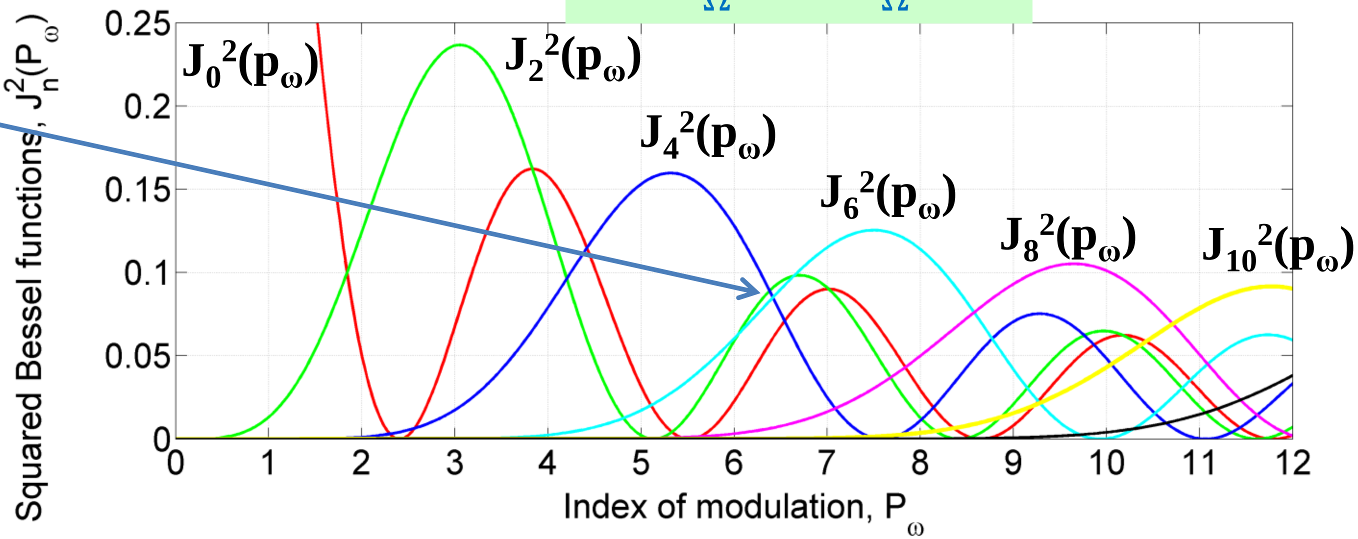
Modulation index: $p_\omega = \frac{\Delta_\omega}{\Omega} = \frac{\tilde{E}_{IR/opt} d_{av}}{\Omega}$

Optimal value of modulation index:

$p_\omega = 6.4$

corresponds to nearly uniform amplification of 7 harmonics

$J_0^2 \approx J_2^2 \approx J_4^2 \approx J_6^2$



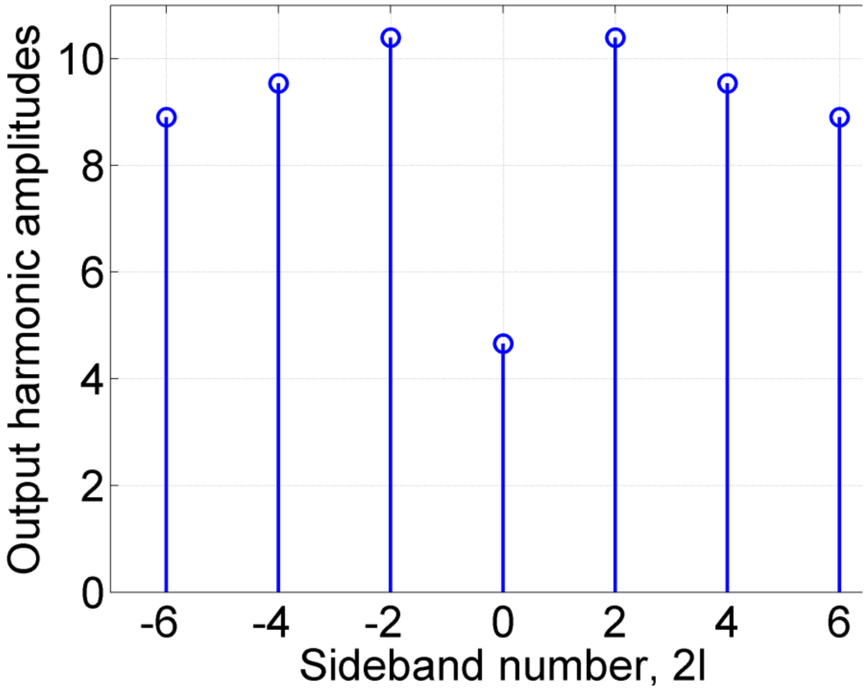
The results of analytical solution:

Incident field:

$$E_{inc}^{X-ray}(x=0,t)=\frac{1}{2}\vec{z}_0 E_0\sum_{l=-3}^{3}\exp\{-i(\bar{\omega}_{tr}+2l\Omega)t\}+\text{c.c.}$$

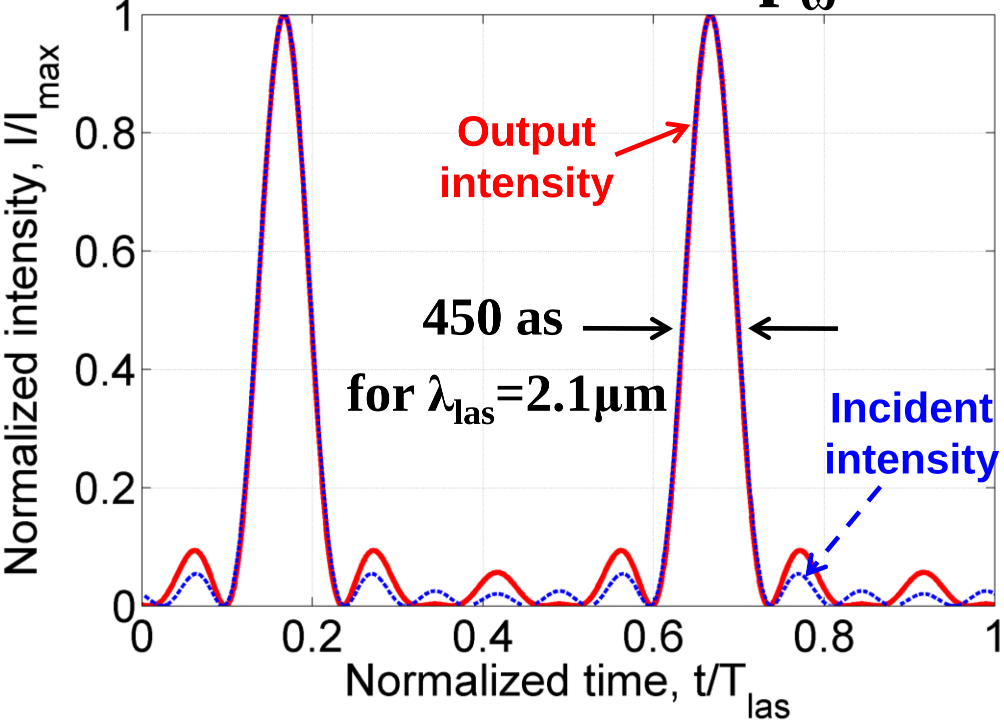
7 harmonics of equal amplitude

Output harmonic spectrum



Normalized intensities

$p_\omega=6.4$



Output intensity

450 as

for $\lambda_{las}=2.1\mu m$

Incident intensity

Output field:

There are almost no distortions!

$$\vec{E}_{X-ray}(x=L,\tau)=\frac{1}{2}\vec{z}_0 E_0\sum_{l=-3}^{3}\exp\{J_{2l}^2(p_\omega)g_{total}L\}\exp\{-i(\bar{\omega}_{tr}+2l\Omega)\tau\}+\text{c.c.}$$

$$T_{las}=\frac{2\pi}{\Omega}$$

Peak output intensity $I_{max}=79.2I_0$

$$g_{total}L=26$$

- For quantitative analysis, take into account variation of the population differences between the states $|1\rangle = |1s\rangle, \quad |2\rangle = (|2s\rangle + |2p, m=0\rangle)/\sqrt{2}$
  and $|3\rangle = (|2s\rangle - |2p, m=0\rangle)/\sqrt{2}$
  as well as influence of the states $|4\rangle = |2p, m=1\rangle, \quad |5\rangle = |2p, m=-1\rangle$
- Neutral plasma consisting of C VI ions, electrons, and some other ions
- Concentrations of C VI ions and electrons: $N_{ion} = 10^{19} cm^{-3}$ and $N_{el} = 15\ N_{ion}$
- $\lambda_{IR} = 623\lambda_{21} \approx 2.1\ \mu\text{m}$
  Modulation index : $\mathrm{p}_{\omega} = 6.4$
  Modulating field intensity: $I_C = 2.7 \times 10^{15} W/cm^2$

Y. Avitzour, S. Suckewer,
***J. Opt. Soc. Am. B* 24, 819 (2007)**

- Transitions $|4\rangle \leftrightarrow |1\rangle$ and $|5\rangle \leftrightarrow |1\rangle$ result in generation of y-polarized ASE considered by Gross-Haroche's approach. Phys. Rept. 93, 301 (1982)
- Assume to be a train of attosecond pulses with Gaussian envelope centered at t_{peak} and the duration (FWHM of intensity) $t_{1/2}$:

$$E_z(t, x=0) = \frac{1}{2}\hat{z}_0 E_{hh} \exp\left[-2\ln 2(t - t_{peak})^2/t_{1/2}^2\right] \sum_{l=-L_{\max}}^{l=L_{\max}} \exp\left[-i(\bar{\omega}_{tr} + 2l\Omega)t\right] + \text{c.c.}$$

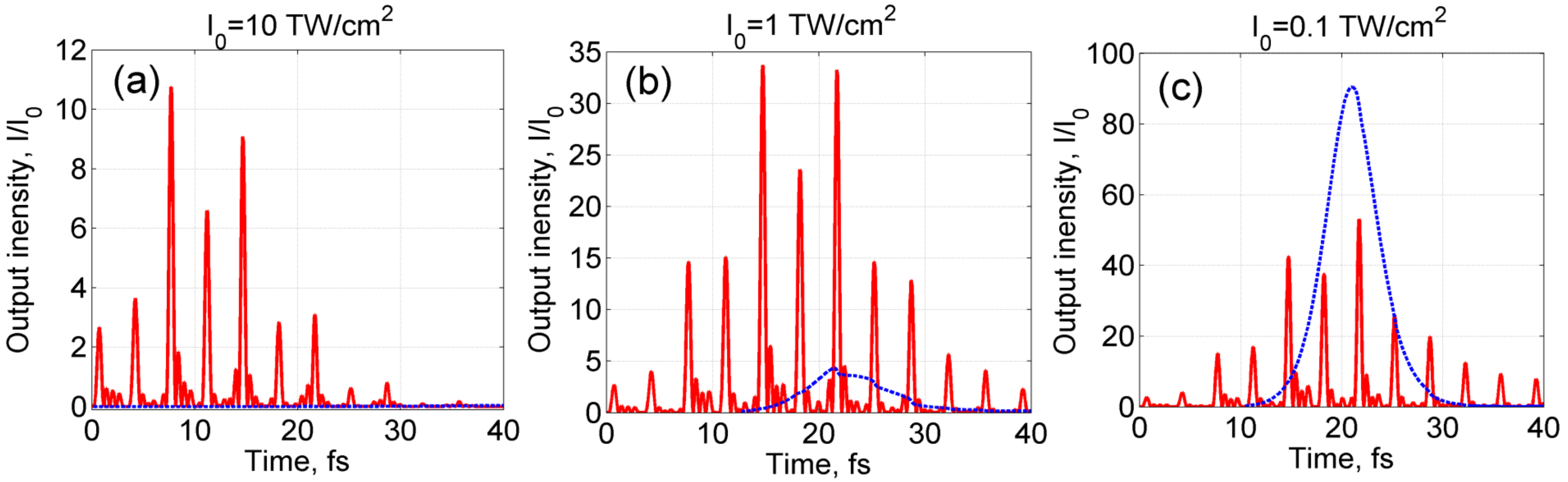
- Length and radius of the active medium: L=1 mm and R=1 µm
- Different peak intensities: $I_0(t=0) = 10, 1, 0.1, \text{and } 0.01\ \text{TW}/cm^2$

$$I_0 = \frac{c}{2\pi}(2L_{max} + 1)^2 \mathrm{E}_{hh}^2$$

- $t_{peak} = 10$ fs, $t_{1/2} = 35$ fs, $L_{max} = 3$

Antonov, KCH, Akhmedzhanov, Scully, and Kocharovskaya, PRL 123, 243903 (2019)

Amplification of a train of ~450 as pulses (in red) at 3.2nm (produced via HHG) by a H-like CVI plasma laser, modulated with an IR laser field for different intensities of the seeding pulses vs an amplified spontaneous emission (in blue) with an orthogonally polarized radiation.



Plasma Parameters:
$N_{C^{5+}} = 10^{19} cm^{-3}, N_e = 1.5 \times 10^{20} cm^{-3}, L = 1mm$

as in Y. Avitzour and S. Suckewer,
J. Opt. Soc. Am. B **24**, 819 (2007).

Modulating IR field:

λ_{las}=2.1 μm, I_{las}=2.7×10^{15} W/cm^2
(modulation index p_ω=6.4)

Possibilities for amplification of the shorter pulses:

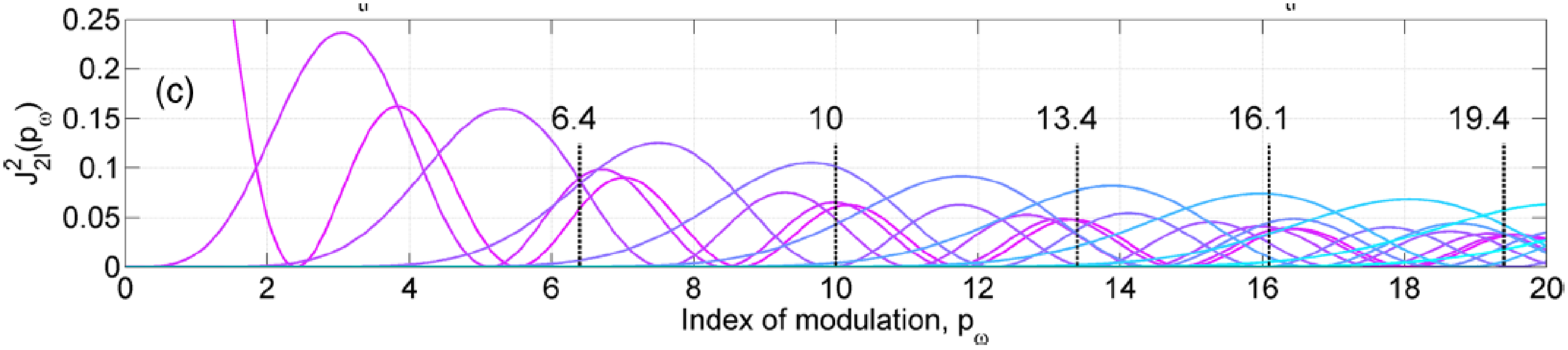
$$\tau_{pulse} \sim \frac{1}{(p_\omega \Omega)} \sim \frac{1}{\tilde{E}_{IR/opt}}$$

(1) **To increase the value of modulation index at fixed frequency of the modulating laser field (λ_{las}=2.1μm)**

(2) **To increase frequency of the modulating laser field at fixed value of modulation index (p_ω=6.4)**

In both cases, $\widetilde{E}_{\mathrm{IR/opt}}$ s are limited by ionization threshold from the upper lasing state of the ions

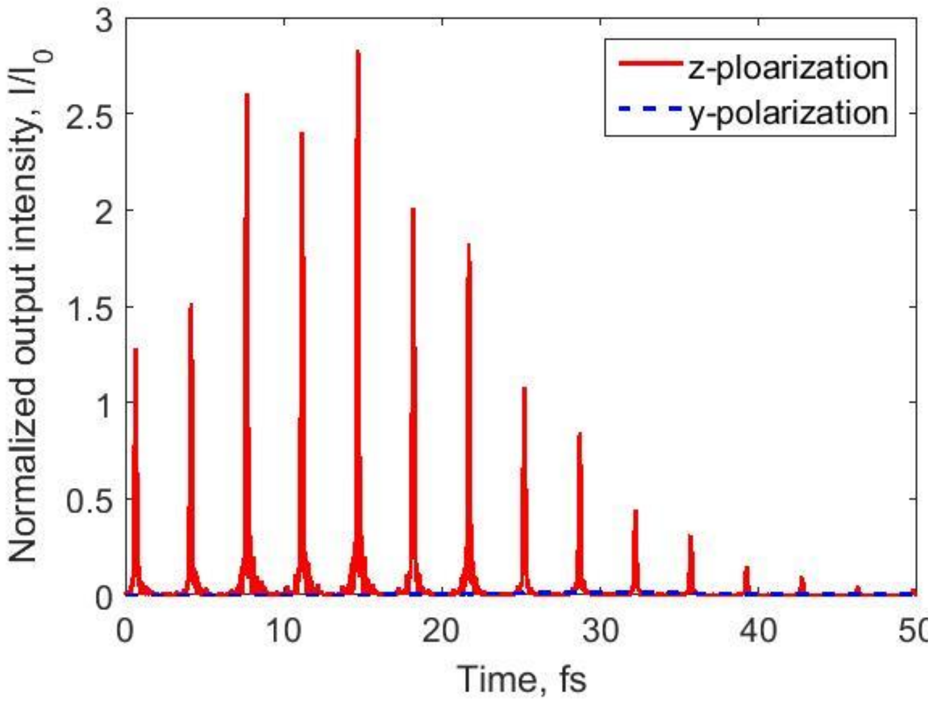
For C^{5+} ions max{I_{las}}=3.5×10^{16} W/cm^2 , corresponding to ionization time ~ 60 fs,

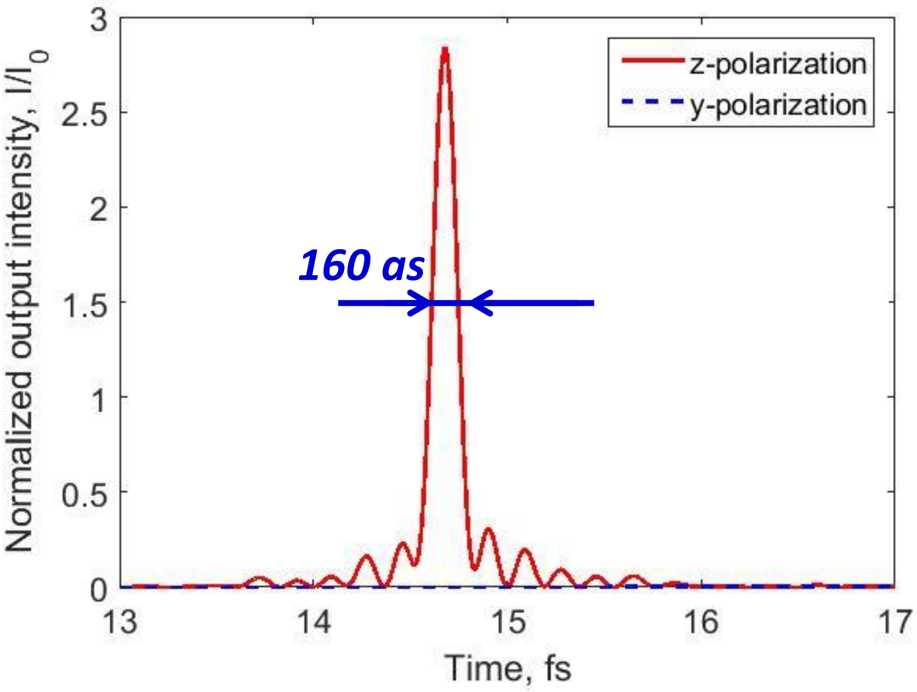


Shorter pulses by increasing p_{ω}=19.4, 21 harmonics

$N_{ion}=10^{19}cm^{-3}$, $N_{el}=1.5\times10^{20}cm^{-3}$, $\lambda_{las}=2.1\mu m$, $I_{las}=2.5\times10^{16}W/cm^{2}$, $L=1mm$

Peak intensity of the incident attosecond pulse train: I_0=10 TW/cm^2

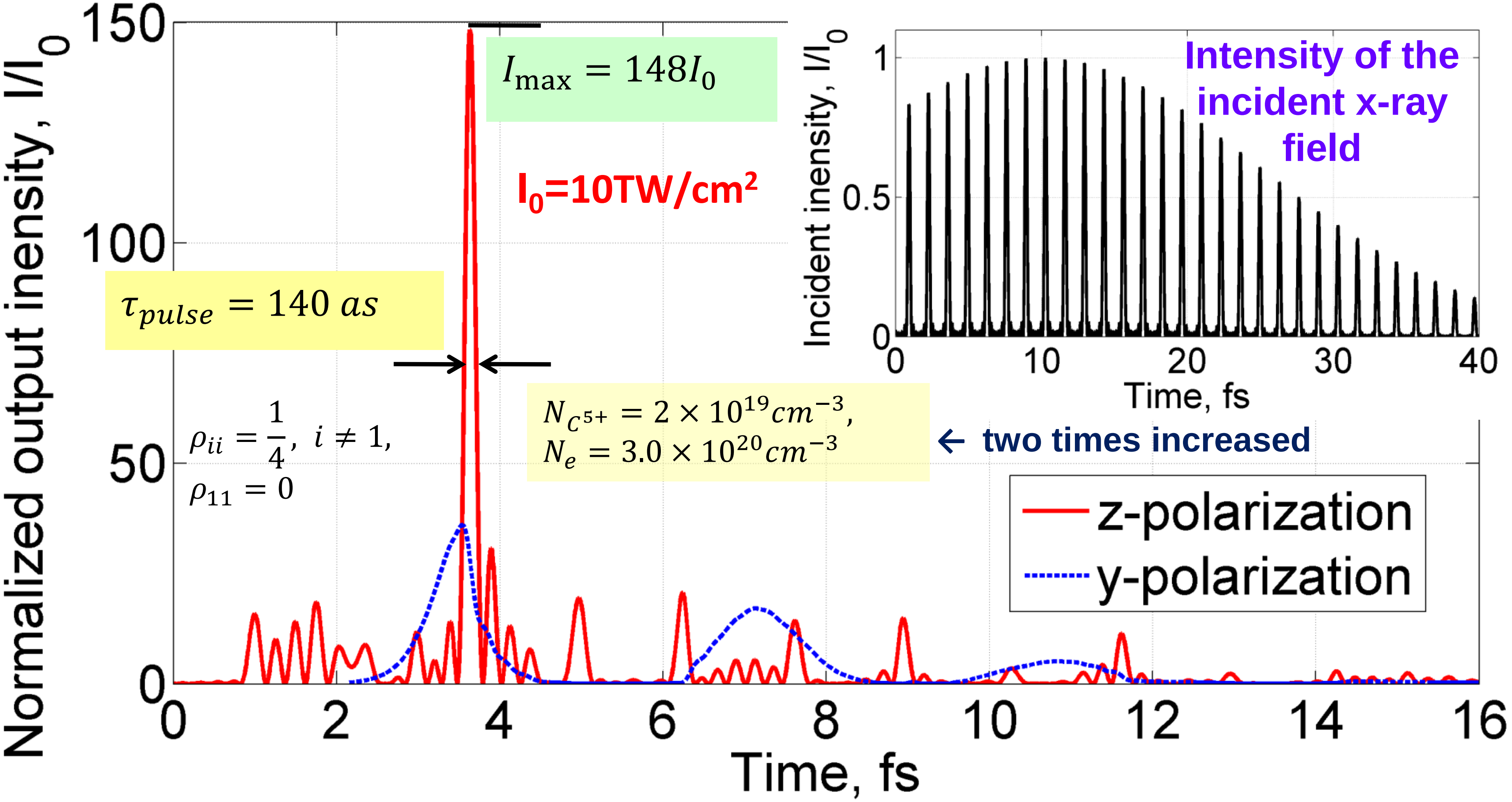




Pulses became shortened (160 as) compared to 450 as but amplification of pulses is reduced due to the higher modulation index

Amplified output x-ray field – numerical solution

Incident x-ray field: 12 harmonics of 0.8 μm laser field
with 35 fs duration of the Gaussian envelope (of HH signal)



$I_{\max} = 148 I_0$

I_0=10TW/cm²

$\tau_{pulse} = 140\ as$

$\rho_{ii} = \frac{1}{4},\ i \neq 1,$
$\rho_{11} = 0$

$N_{C^{5+}} = 2 \times 10^{19} cm^{-3},$
$N_e = 3.0 \times 10^{20} cm^{-3}$

← two times increased

Intensity of the incident x-ray field

Modulating field: λ_{las}=0.8 μm, I_{las}=1.9×10^{16} W/cm² (p_ω= 6.4), **Medium length is L = 3 mm**

Antonov, KCH, Akhmedzhanov, Scully, and Kocharovskaya, PRL 123, 243903 (2019)

Conclusion

- A set of high-order harmonics of an optical laser field can be amplified in an active medium of a hydrogen-like plasma-based X-ray laser, dressed by a replica of the laser field of fundamental frequency.
- In a sufficiently dense plasma, the harmonics are amplified independently from each other, so that their relative phases remain constant.
- We suggest an experimental implementation of this method in active medium of C VI ions and show the possibility to amplify attosecond pulses with duration down to 100 as at the carrier wavelength 3.4 nm in the "water window" range by two orders of magnitude.

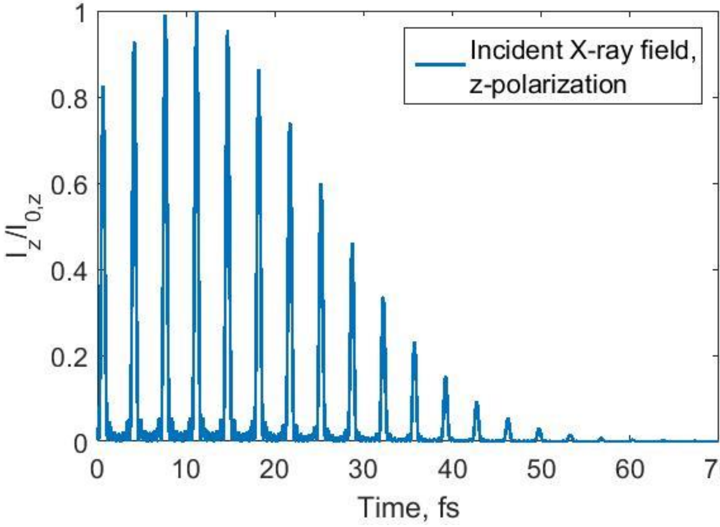
Questions ?

Backup Slides

Amplification of a train of ultrashort pulses in the medium of modulated H-like ions. Effectively 3-level model

Y. Avitzour, S. Suckewer,
***J. Opt. Soc. Am. B* 24, 819 (2007)**

- Neutral plasma consisting of C^{5+} ions, electrons, and some other ions
- Concentrations of C^{5+} ions and electrons: $N_{ion} = 10^{19} cm^{-3}$ and $N_{el} = 15\ N_{ion}$
- Plasmas at low T (few eV)
- $\lambda_{IR} = 623\lambda_{21} \approx 2.1\ \mu\text{m}$
  Modulation index : $\text{p}_\omega = 6.4$
  Modulating field intensity: $I_C = 2.7 \times 10^{15} W/cm^2$
  Incident z-polarized X-ray field is a train of ultrashort pulses resonant to $|1\rangle \leftrightarrow |2\rangle$ and $|1\rangle \leftrightarrow |3\rangle$ transitions with Gaussian train envelope.



Incident z-polarized X-ray pulse train

$$\tilde{E}_z = \tilde{E}_{0,z} e^{-(t-2t_0)^2/(2t_{\text{Gauss}})^2} \sum_{k=-3}^{k=3} e^{-2ik\Omega t - ik^2 a_{\text{chirp}}}$$

$$t_{\text{Gauss}} = t_{\text{env}}/2.35482, \quad t_{\text{env}} = 50\ \text{fs}, \quad t_0 = 5\ \text{fs}$$

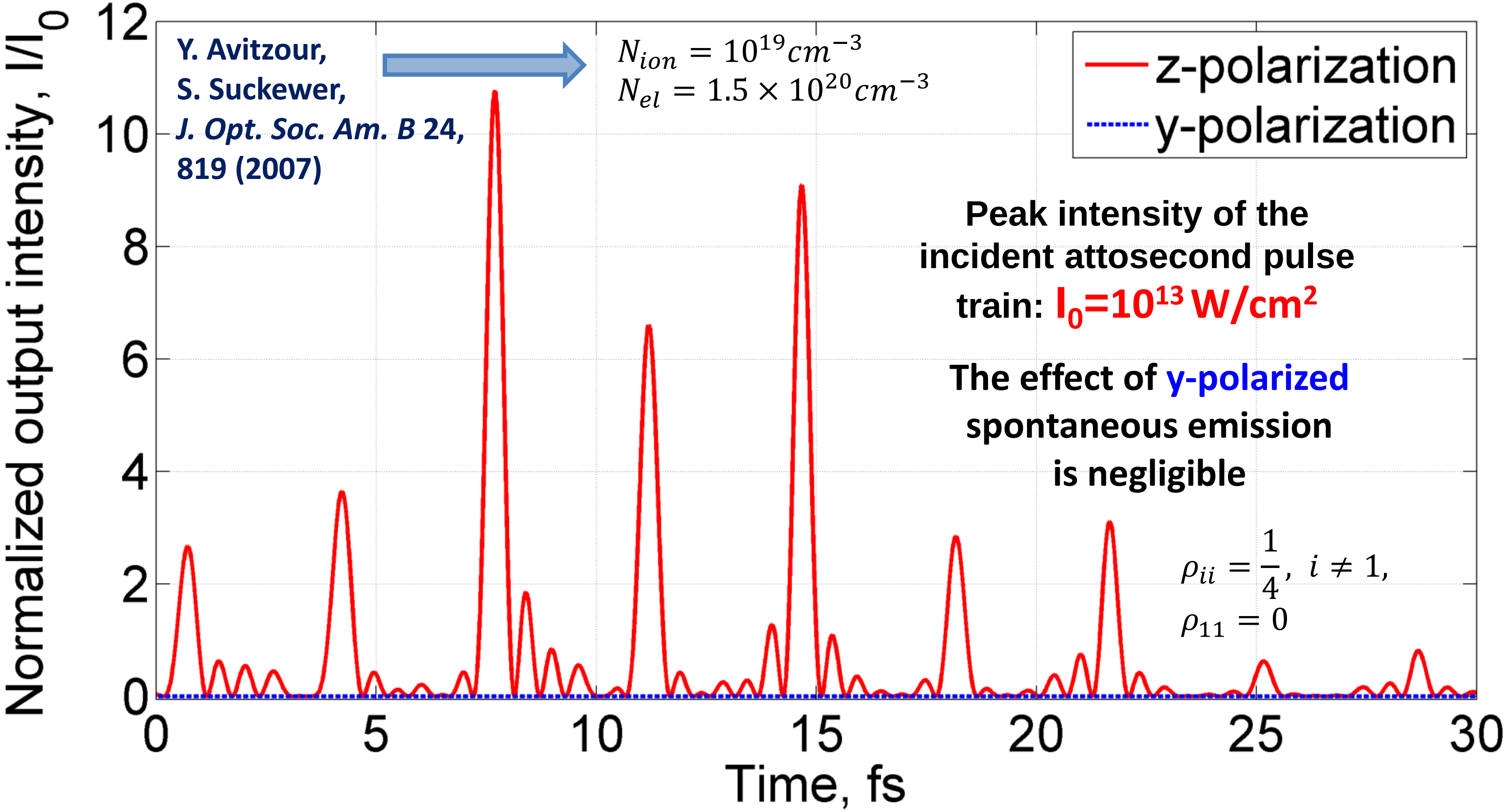
$$a_{\text{chirp}} = \pi/(2n_{\max}), \quad n_{\max} = 763$$

$$I_{z,\max}(t=0) = 10, 1, 0.1, \text{and } 0.01\ TW/cm^2$$
$$I_y(t=0) = 0\ W/cm^2$$

7 harmonics

Amplified output x-ray field – numerical solution

Incident x-ray field: 617-629 harmonics of 2.1μm laser field with 20 fs duration of the Gaussian envelope



Y. Avitzour, S. Suckewer, *J. Opt. Soc. Am. B* 24, 819 (2007)

$N_{ion} = 10^{19} cm^{-3}$
$N_{el} = 1.5 \times 10^{20} cm^{-3}$

Peak intensity of the incident attosecond pulse train: $I_0=10^{13}$ W/cm^2

The effect of y-polarized spontaneous emission is negligible

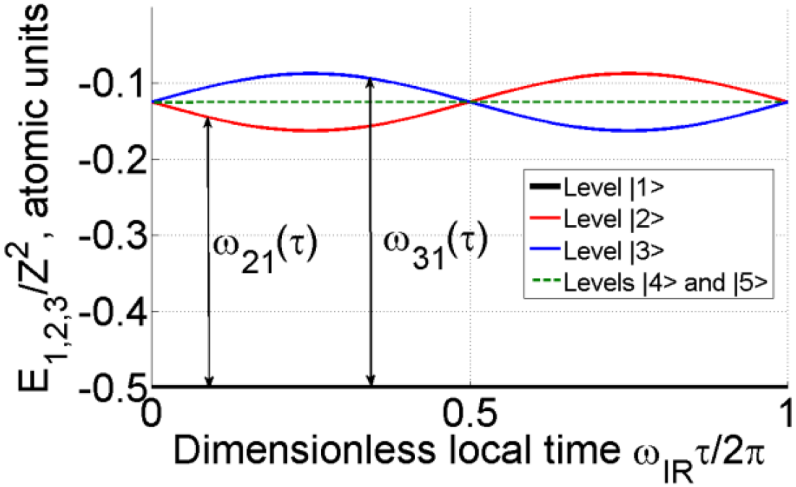
$\rho_{ii} = \frac{1}{4},\ i \neq 1,$
$\rho_{11} = 0$

Modulating field: λ_{las}=2.1 μm, $I_{las}=2.7\times10^{15}$ W/cm^2 (p_ω=6.4), L=1mm

Spontaneous emission is modeled by an incident y-polarized field with $I_y(x=0)=10^4$ W/cm^2

Energy levels of the ground and first excited state of the hydrogenlike-ion dressed by and IR/optical field

- $|1\rangle \leftrightarrow |2\rangle$ and $|1\rangle \leftrightarrow |3\rangle$ are periodically modulated with a period equal to that of the IR/optical field.
- linear AC Stark effect.
- $|1\rangle \leftrightarrow |2\rangle$ and $|1\rangle \leftrightarrow |3\rangle$ have nonvanishing z-components of dipole=> result in modulated polarization of the medium leading to appearance of gain on the sideband frequencies of the resonant frequency for z-polarized field propagating in x-direction.

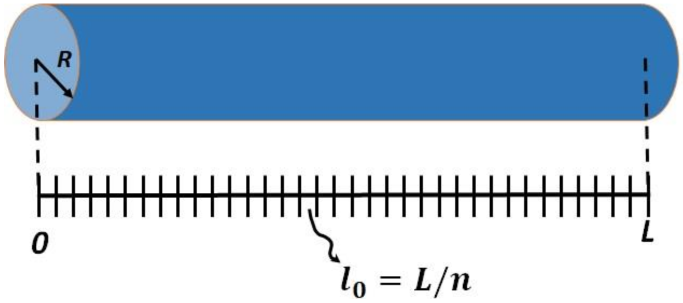


R. J. Damburg and V. V. Kolosov, *In Rydberg states of Atoms and Molecules*, edited by R. F. Stebbings, and F. B. Dunning
(Cambridge University Press, Cambridge, England, 1983)

M. Gross and S. Haroche, *Physics Reports 93*, 301-396 (1982),
Superradiance: An essay on the theory of collective spontaneous emission.

How to take into account spontaneous emission ?

According to the Gross-Haroche's approach, let's split the medium into thin slices of the length $l_0 << L$, in each of which the initial values of atomic coherencies are determined as



$$\tilde{\rho}_{i1}(\tau = 0, x = x_m) = \frac{C_{i1}}{2N_{i,m}d_{tr}} A_{i,m} e^{i\varphi_{i,m}};$$

where $N_{i,m} = N_{ion}\rho_{ii}\pi R^2 l_0$, $C_{21} = 1$, $C_{31} = -1$, $C_{41} = C_{51} = -i$

while $A_{i,m}$ and $\varphi_{i,m}$ are stochastic values:

$$p\left(A_{i,m}^2\right) = \frac{1}{N_{i,m}} \exp\left\{-\frac{A_{i,m}^2}{N_{i,m}}\right\}, 0 \le A_{i,m} < \infty; p\left(\varphi_{i,m}\right) = \frac{1}{2\pi}, 0 \le \varphi_{i,m} \le 2\pi$$

The resonant polarization

$$\vec{P}(\vec{r},t) = N\left(\vec{d}_{12}\rho_{21} + \vec{d}_{13}\rho_{31} + \vec{d}_{14}\rho_{41} + \vec{d}_{15}\rho_{51} + \text{c.c.}\right)$$

Within 5-level model, nonvanishing dipole moments

$$\vec{d}_{12} = \vec{d}_{1s\leftrightarrow 2p,m=0}/\sqrt{2} = \vec{z}_0 d_{\|},\;\; \vec{d}_{13} = -\vec{d}_{1s\leftrightarrow 2p,m=0}/\sqrt{2} = -\hat{z}_0 d_{\|}$$
$$\vec{d}_{22} = \vec{d}_{2s\leftrightarrow 2p,m=0} = \vec{z}_0 d_{\mathrm{av}},\;\; \vec{d}_{33} = -\vec{d}_{2s\leftrightarrow 2p,m=0} = -\hat{z}_0 d_{\mathrm{av}}$$
$$\vec{d}_{14} = \vec{d}_{1s\leftrightarrow 2p,m=1} = i\vec{y}_0 d_{\perp},\;\; \vec{d}_{15} = -\vec{d}_{1s\leftrightarrow 2p,m=-1} = i\hat{y}_0 d_{\perp}$$

$$\vec{d}_{ij} = \vec{d}^{*}_{ji}$$

In atomic units, $d_{\|} = d_{\perp} = \dfrac{2^7}{3^5 Z},\;\; d_{\mathrm{av}} = 3/Z$

Z: ion nucleus charge.

The decay rates

$$\gamma_{12} \approx \gamma_{13} \approx \gamma_{\rm coll}+\Gamma_{\rm ion}/2+\Gamma_{\rm radiative}/2, \quad \gamma_{14} \approx \gamma_{15} \approx \gamma_{\rm coll}+\Gamma_{\rm ion,2}/2+\Gamma_{\rm radiative}/2$$

$$\gamma_{23} \approx \gamma_{\rm coll} + \Gamma_{\rm ion} + \Gamma_{\rm radiative}, \quad \gamma_{24} = \gamma_{25} = \gamma_{34} = \gamma_{35} \approx \gamma_{\rm coll} + \frac{\Gamma_{\rm ion}}{2} + \frac{\Gamma_{\rm ion,2}}{2} + \Gamma_{\rm radiative}$$

$$\gamma_{45} \approx \gamma_{\rm coll} + \Gamma_{\rm ion,2} + \Gamma_{\rm radiative}, \quad \gamma_{22} = \gamma_{33} \approx \Gamma_{\rm ion} + \Gamma_{\rm radiative}$$

$$\gamma_{44} = \gamma_{55} \approx \Gamma_{\rm ion,2} + \Gamma_{\rm radiative}, \quad \gamma_{11} \approx \Gamma_{\rm radiative}$$

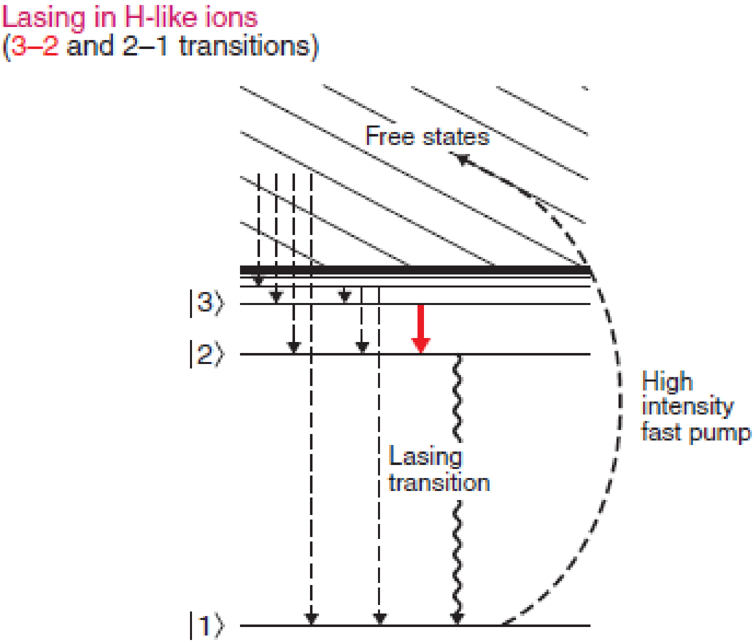
$\gamma_{\rm coll}$: collisional broadening,
$\Gamma_{\rm ion}$ and $\Gamma_{\rm radiative}$: ionize and radiative decay rates

$$\Gamma_{\rm ion} \approx \frac{Z^2}{16}\sqrt{\frac{3F_c}{\pi}}\left[\left(\frac{4}{F_c}\right)e^3 + \left(\frac{4}{F_c}\right)^3 e^{-3}\right]\exp\left(-\frac{2}{3F_c}\right)$$

$\Gamma_{\rm ion,2}$: can be found using Popov-Perelomov-Terentiev equations

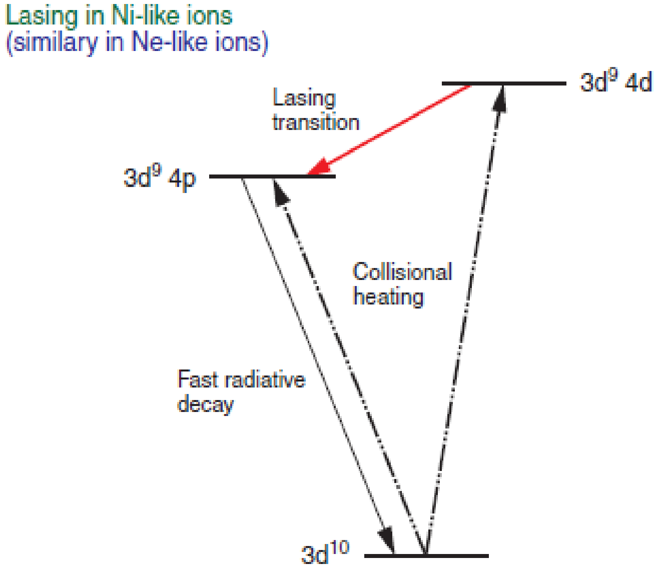
V. S. Popov, Phys.-Usp. 47, 855 (2004)

- A.L. Schawlow and C.H. Townes, “Infrared and Optical Masers”, Phys. Rev. **112**, 1940 (1958): “unless some radically new approach is found, they (*maser systems*) cannot be pushed to wavelengths much shorter than those in the ultraviolet region.”
- V.L. Ginzburg, Nobel Lecture, 2003: Problem № 12 in the list of 30 fundamental problems
- Donna Strickland (2018 Nobel Laureate in Physics): “…I point out that light intensity was greatly increased with the invention of the laser in 1960, then in 1985 the light intensity was again increased by orders of magnitude with the invention of CPA and now it is once again time for a new Nobel prize winning idea to again greatly boost the laser intensity…”



1.The recombination lasers. They imply a fast optical laser ionization via tunneling resulting in a complete striping of all the electrons without their appreciable heating, followed by a a three-body collisional recombination process (**S. Suckewer** and P. Jaegl´e, Laser Phys. Lett. **1**, 26 (2009)).

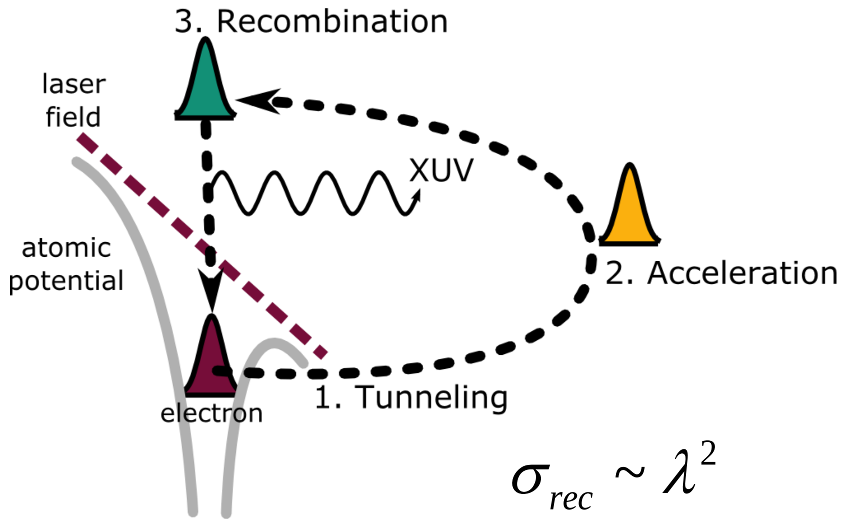
They currently provide the shortest wavelength achieved in plasma lasers: 4.03 nm (S. Suckewer et al., Eds. T. Kawachi, S.V. Bulanov, H. Daido, Y. Kato. Springer Proc. in Phys., to be published)



2. The collisional lasers with pumping by electron collisional excitation (B.A. Reagan, M. Berrill, K.A. Wernsing, C. Baumgarten, M. Woolston, and **J. J. Rocca**, Phys. Rev. **A 89**, 053820 (2014).

The pulse energy and the pulse repetition rate in collisiona plasma x-ray lasers are as high as several mJ and 100 Hz .

Pulse duration in all X-ray plasma lasers >1ps.



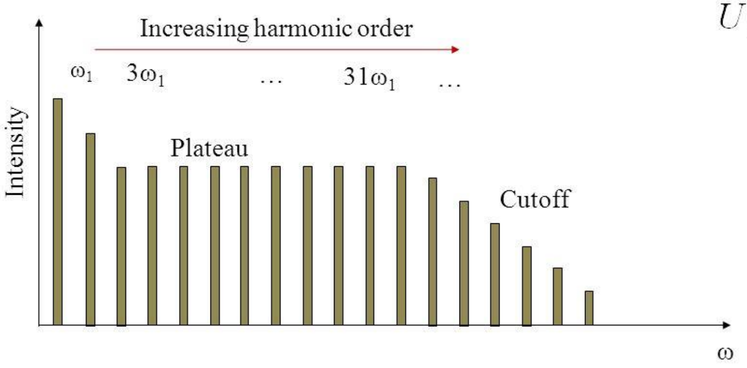
$$\sigma_{rec} \sim \lambda^2$$

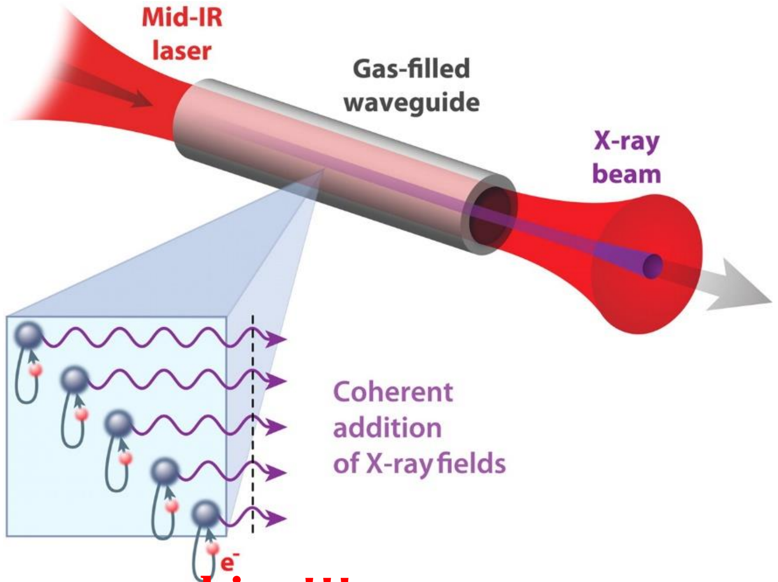
P. B. Corkum, F. Krausz, Nature Physics **3**, 381 (2007).

Typical Shape of the HH Spectrum

$$E_{cutoff} = I_P + 3.17 U_P$$

$$U_P \sim I \times \lambda^2$$





Phase matching!!!

Reasonable transformation efficiency, high coherence and attosecond pulses formation.

In soft X-ray range (up to 4nm):

$$U_{\max} \sim 1 nJ, \tau_{\min} \sim 10\, fs \ (\text{expected})$$

M. Chini, K. Zhao, and Z. Chang, Nat. Photonics **8**, 178 (2014)

T. Popmintchev, et al., Science **336**, 1287 (2012): up to 1,6keV

Soft X-ray (up to 1nm) and hard X-ray (up to 0.5 Å) XFELs:

LCLS/SLAC (1.5 Å) , SACLA/Spring-8 (0.6 Å), PAL in Seoul, Korea (0.5nm), **European XFEL (0.5 Å, 30kHz rep. rate),** SwissXFEL (**1.3 Å**) and SINAP in Shanghai (**1.3 Å**).

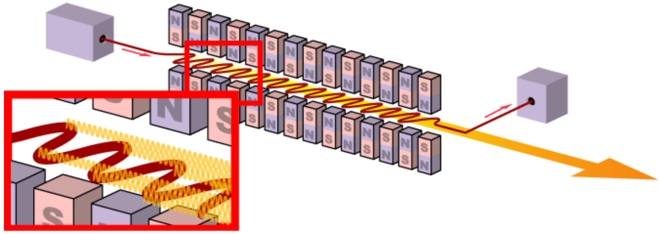
Synchrotron radiation of electron beams accelerated by ~ 1km size RF linear accelerators to the relativistic energies (~ 1 GeV) and wiggled in the periodic magnetic field of a long (~10m) undulators.

J. Madey, J. Appl. Phys., 1971; **F. A. Hopf, P. Meystre, M. O. Scully and W. H. Louisell, Opt. Comm.18: 413; Phys. Rev. Lett. 37, 1342,1976; Bonifacio, Pelligrini,Narducci, Opt.Comm.1984.**

M. Fuchs, et al. Nature Phys. 11, 964 (2015).

$$\tau_p \sim 1\, fs, U \sim 1 mJ, rep.\ rate \sim 10-100 Hz.$$

$$I \sim 10^{20} cm^{-2} \rightarrow 2d\ harmonic\ was\ generated$$



$$\Delta\omega\tau_p \sim 100$$

Seeding to improve coherence: J. Amann et al., Nature Photon., 2012