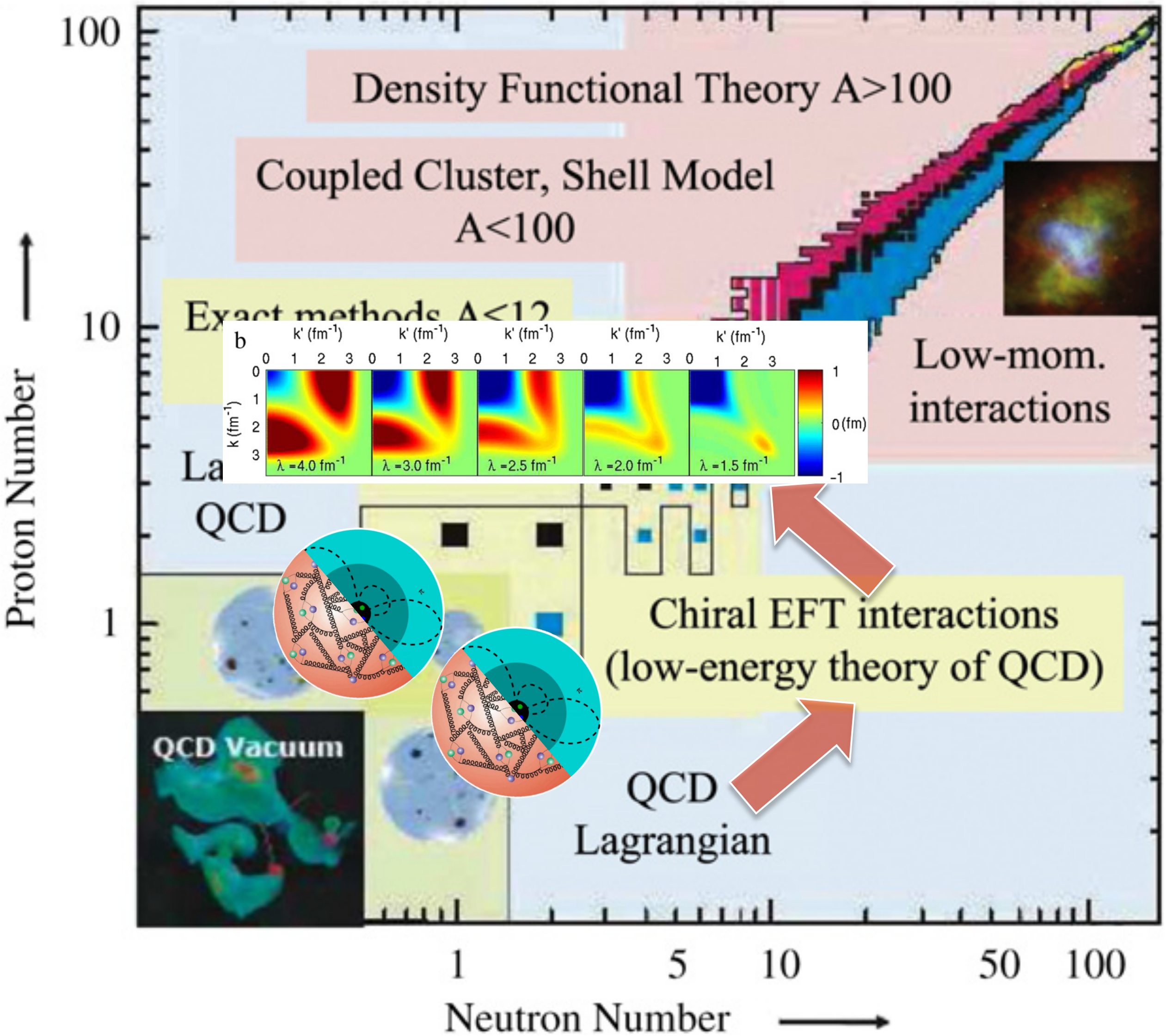
(S)RG and Low-Momentum Interactions

To understand the properties of complex nuclei from first principles



Renormalizing NN Interactions

Basic ideas of RG

Low-momentum interactions

Similarity RG interactions

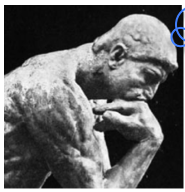
Benefits of low cutoffs

How will we approach this problem:

QCD → NN (3N) forces → Renormalize → "Solve" many-body problem → Predictions

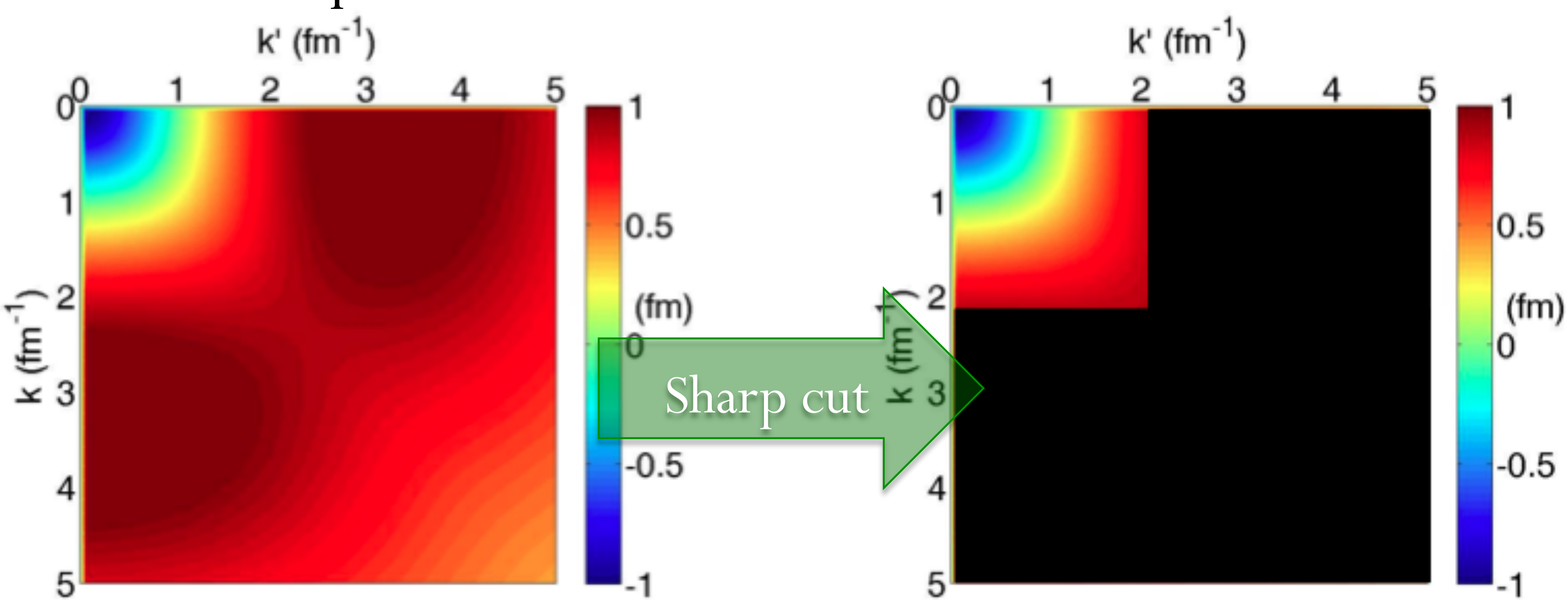
Renormalization of Meson-Exchange Potentials

Ok, high momentum is a pain. I wonder what would happen to low-energy observables…



Low-to-high momentum makes life difficult for low-energy nuclear theorists, so let's get rid of it

Can we just make a sharp cut and see if it works?

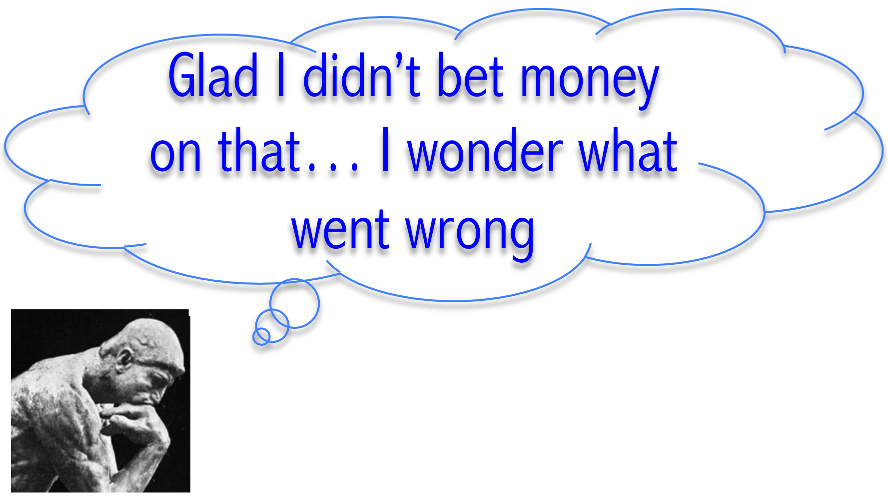


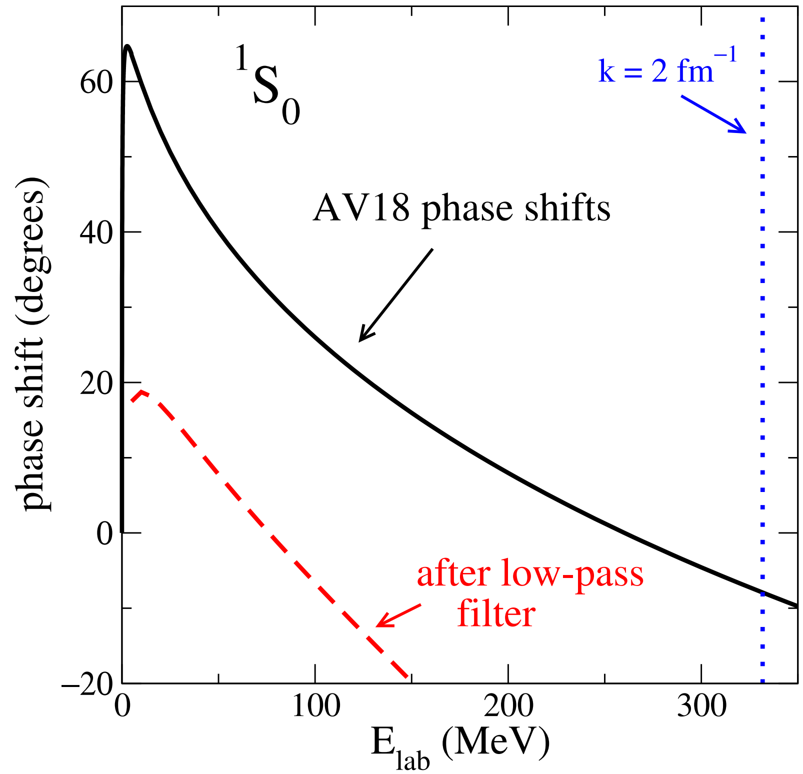
$$V_{\rm filter}(k',k) \equiv 0;\;\; k,k' > 2.2\,\mathrm{MeV}$$

Renormalization of Meson-Exchange Potentials

Can we just make a sharp cut?

Nope! Low-energy physics is not correct





Renormalization of Meson-Exchange Potentials

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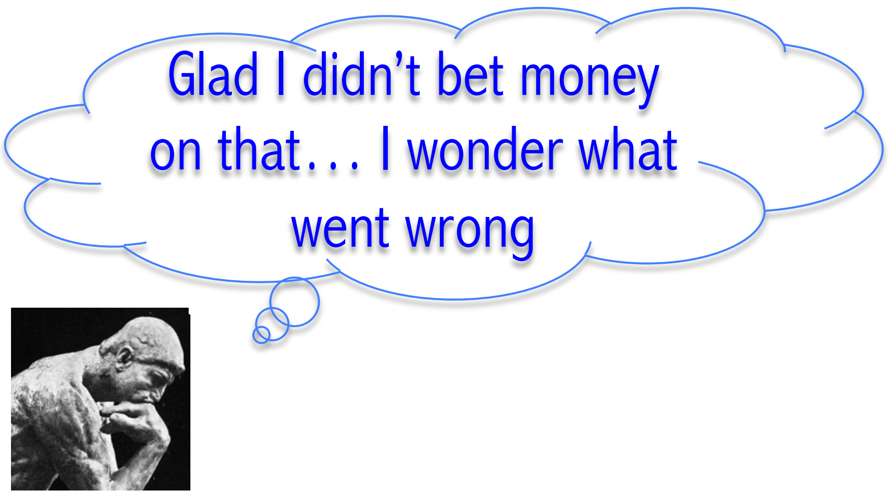
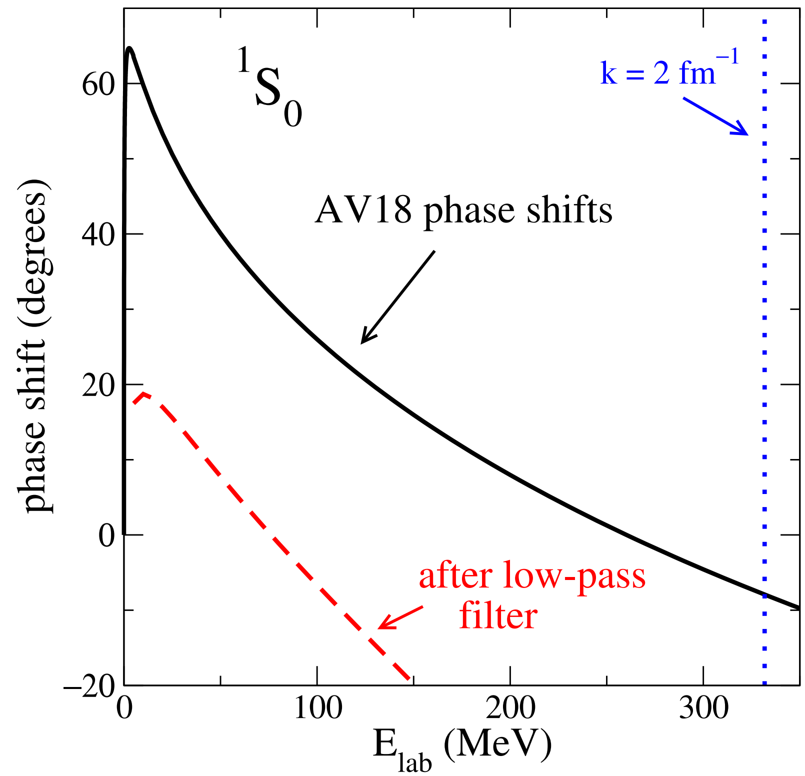




Phase shifts involve couplings of low-to-high momenta

$$\langle k|V|k'\rangle + \sum_{q=0}^{\Lambda} \frac{\langle k|V|q\rangle\langle q|V|k'\rangle}{\epsilon_{k'} - \epsilon_q} + \sum_{q=\Lambda}^{\infty} \frac{\langle k|V|q\rangle\langle q|V|k'\rangle}{\epsilon_{k'} - \epsilon_q}$$

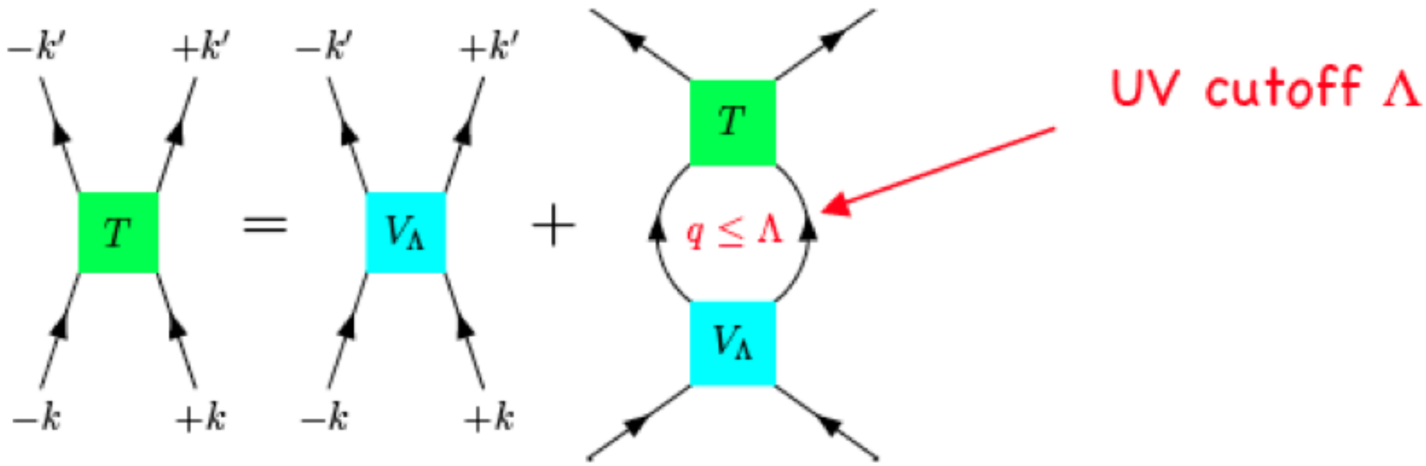
Lesson: Must ensure low-energy physics is preserved!

Renormalization of Meson-Exchange Potentials

To do properly, from T-matrix equation, define **low-momentum** equation:

$$T^{\alpha}(k,k') = V^{\alpha}_{\rm NN}(k,k') + \frac{2}{\pi}\int_0^{\infty} q^2 {\rm d}q \, \frac{V^{\alpha}_{\rm NN}(k,q) T^{\alpha}(q,k')}{k^2 - q^2 + i\varepsilon}$$

$$\rightarrow V^{\Lambda}_{{\rm low}\,k}(k,k') + \frac{2}{\pi}\int_0^{\Lambda} q^2 {\rm d}q \, \frac{V^{\Lambda}_{{\rm low}\,k}(k,q) T(q,k')}{k^2 - q^2 + i\varepsilon}$$



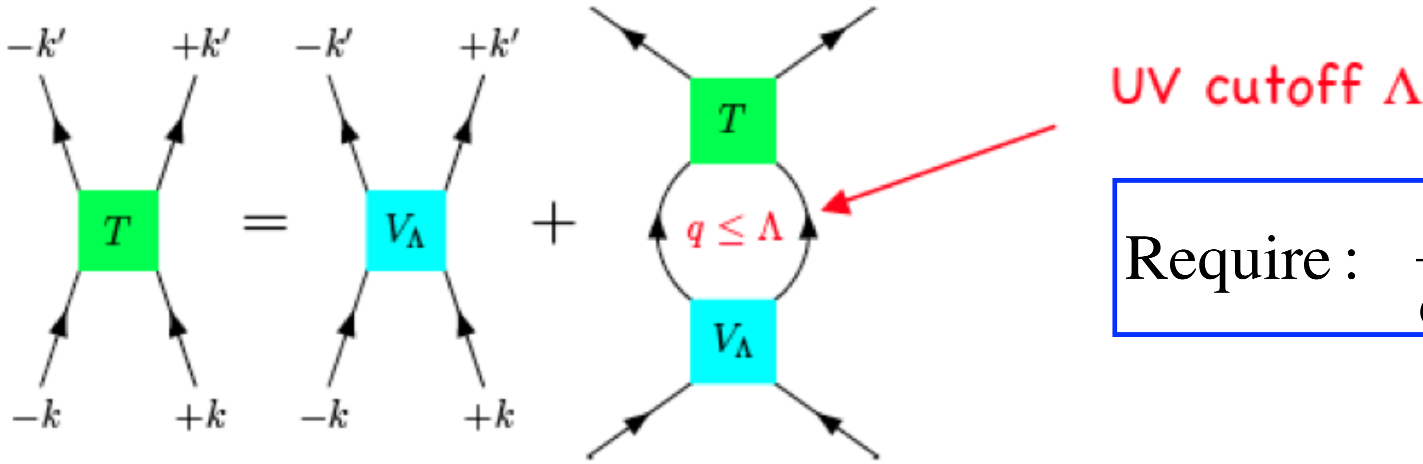
Lower UV cutoff, but preserve low-energy physics!

Renormalization of Meson-Exchange Potentials

To do properly, from T-matrix equation, define **low-momentum** equation:

$$T^{\alpha}(k,k') = V_{\rm NN}^{\alpha}(k,k') + \frac{2}{\pi}\int_0^{\infty} q^2 {\rm d}q \, \frac{V_{\rm NN}^{\alpha}(k,q)T^{\alpha}(q,k')}{k^2-q^2+i\varepsilon}$$

$$\rightarrow V_{{\rm low}\,k}^{\Lambda}(k,k') + \frac{2}{\pi}\int_0^{\Lambda} q^2 {\rm d}q \, \frac{V_{{\rm low}\,k}^{\Lambda}(k,q)T(q,k')}{k^2-q^2+i\varepsilon}$$



Require: $\dfrac{{\rm d}}{{\rm d}\Lambda}T = 0$

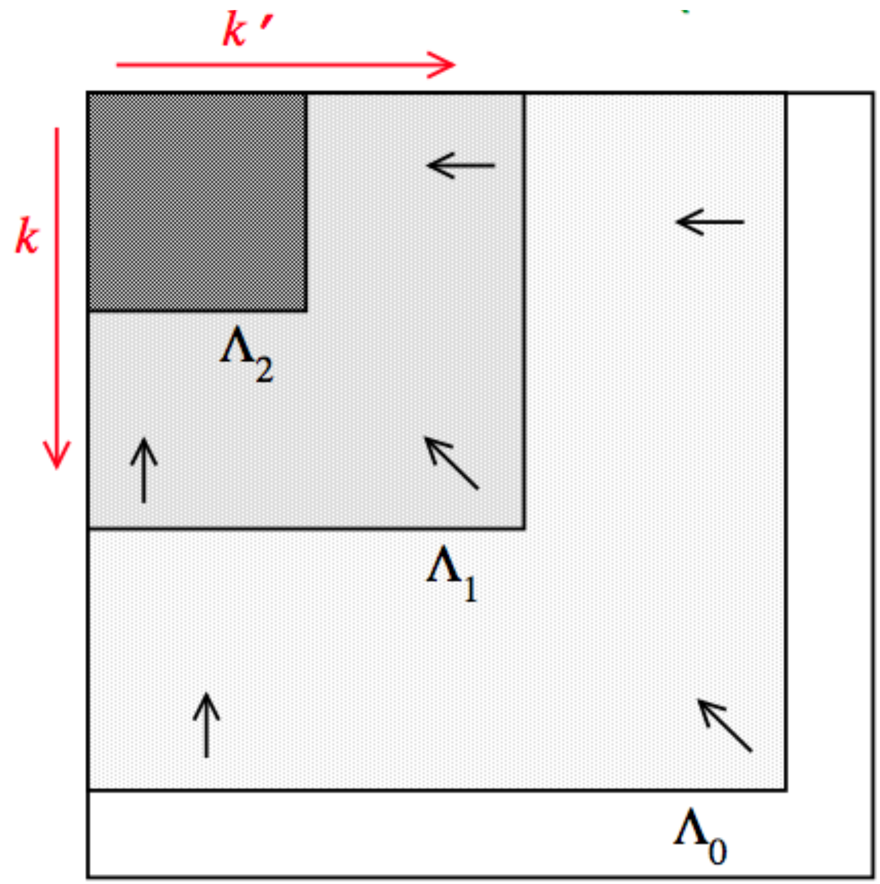
Lower UV cutoff, but preserve low-energy physics!

Leads to **renormalization group equation** for low-momentum interactions

$$\frac{{\rm d}}{{\rm d}\Lambda}V_{{\rm low}\,k}^{\Lambda}(k',k) = \frac{2}{\pi}\frac{V_{{\rm low}\,k}^{\Lambda}(k',\Lambda)T^{\Lambda}(\Lambda,k)}{1-(k/\Lambda)^2}$$

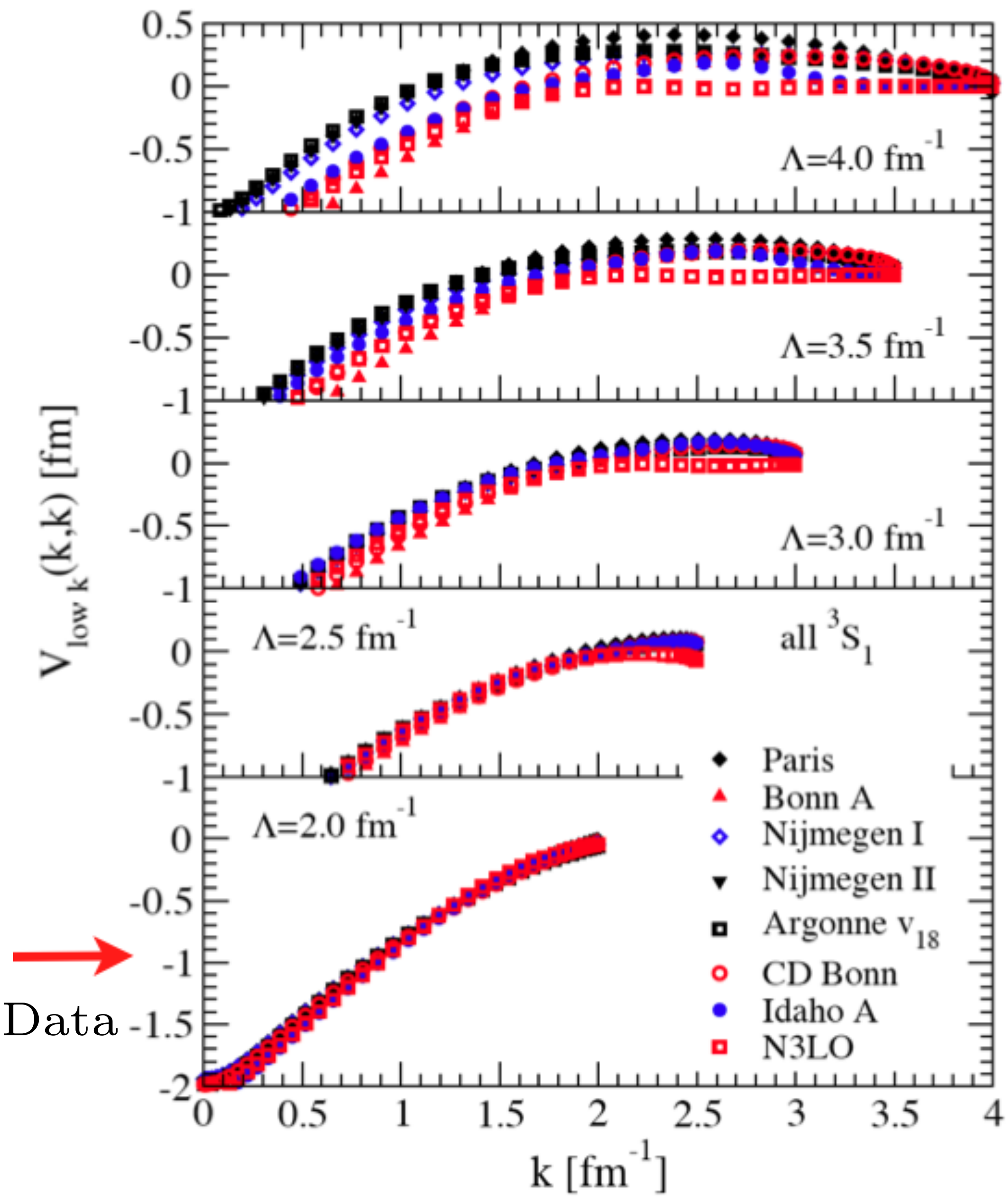
Renormalization of Meson-Exchange Potentials

Run cutoff to lower values – **decouples** high-momentum modes



Start from some initial V_{NN} at high cutoff Λ_0

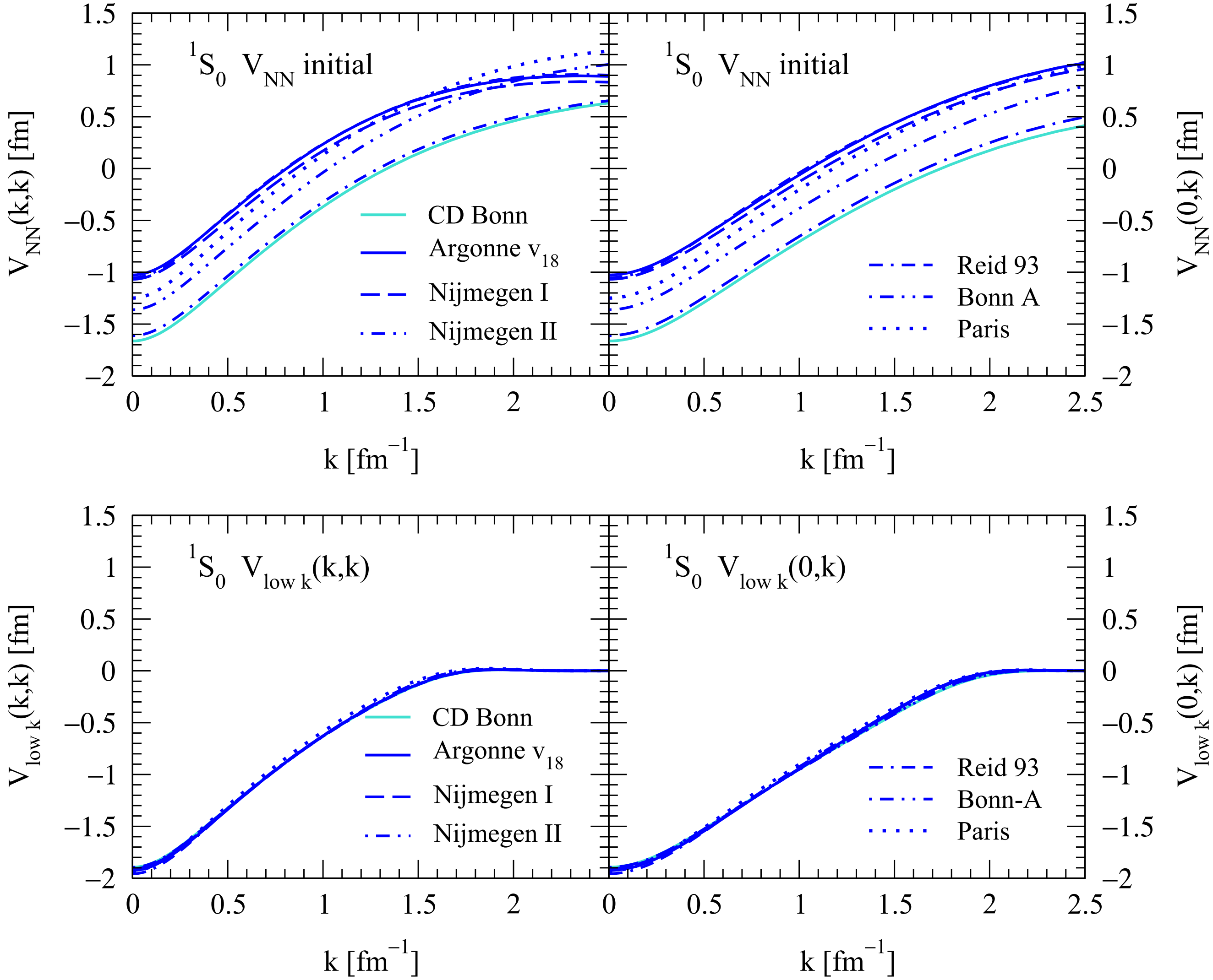
$\Lambda \approx \Lambda_{\mathrm{Data}}$



"Universality" at low momentum

Renormalization of Meson-Exchange Potentials

Diagonal **Off-diagonal**

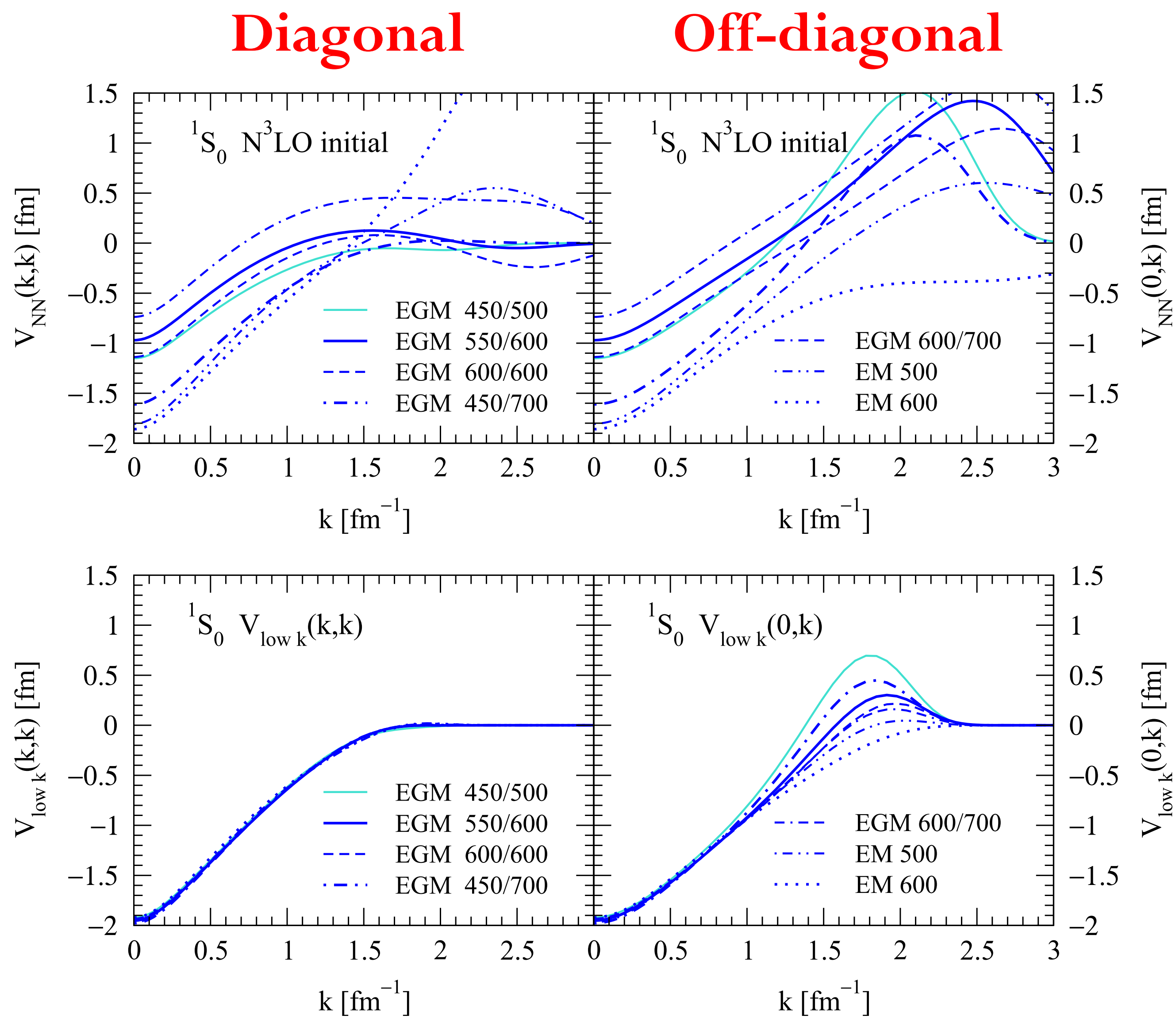


These are all our favorite OBE NN potentials…

These are all our favorite OBE NN potentials… **at low momentum**

Universal collapse in both diagonal/off-diagonal components, most partial waves

Renormalization of Chiral EFT Potentials



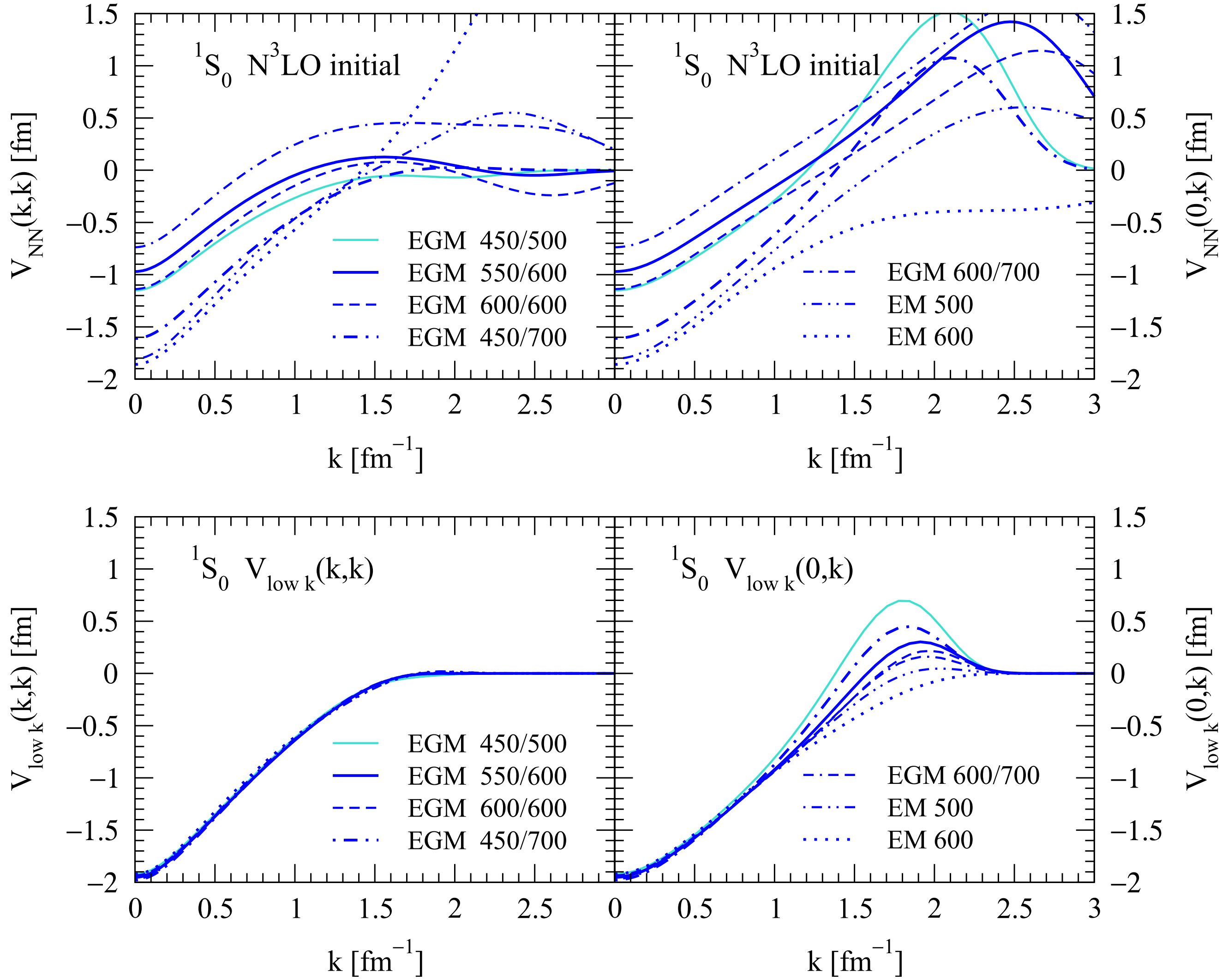
These are all our favorite Chiral EFT NN potentials…

These are all our favorite Chiral EFT NN potentials… **at low momentum**

Differences remain in off-diagonal matrix elements. Why?

Renormalization of Chiral EFT Potentials

Diagonal **Off-diagonal**



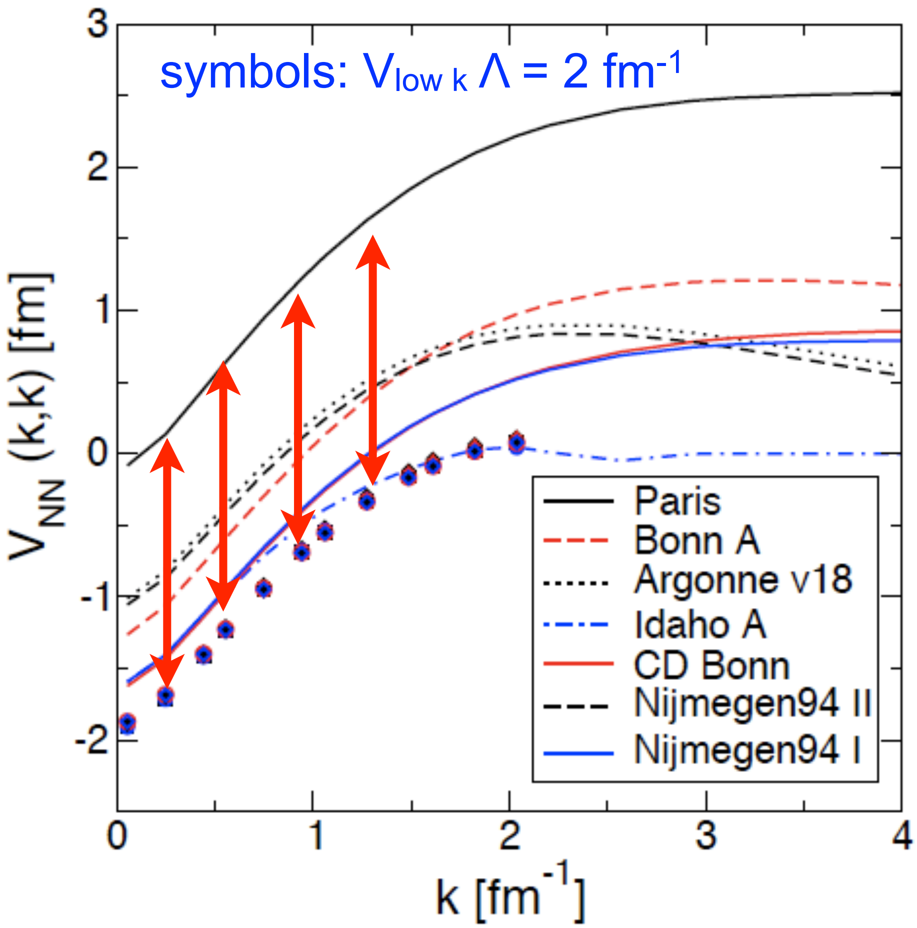
These are all our favorite Chiral EFT NN potentials…

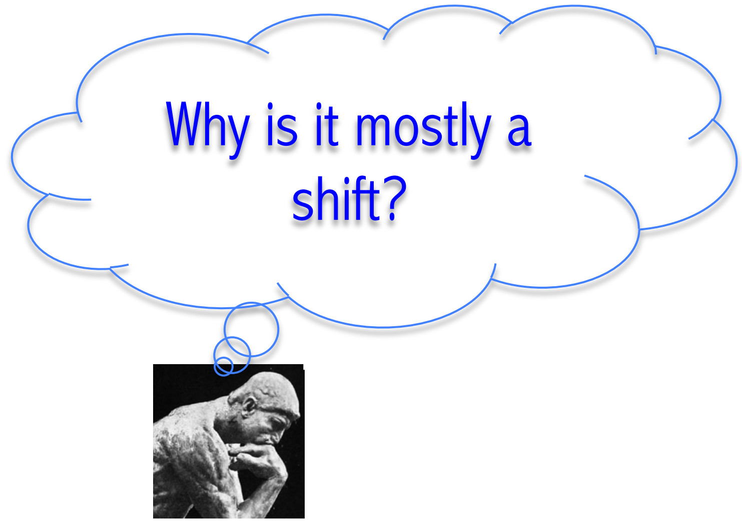
These are all our favorite Chiral EFT NN potentials… **at low momentum**

Differences remain in off-diagonal matrix elements

Sensitive to agreement for phase shifts (not all fit perfectly)

Renormalization of NN Potentials



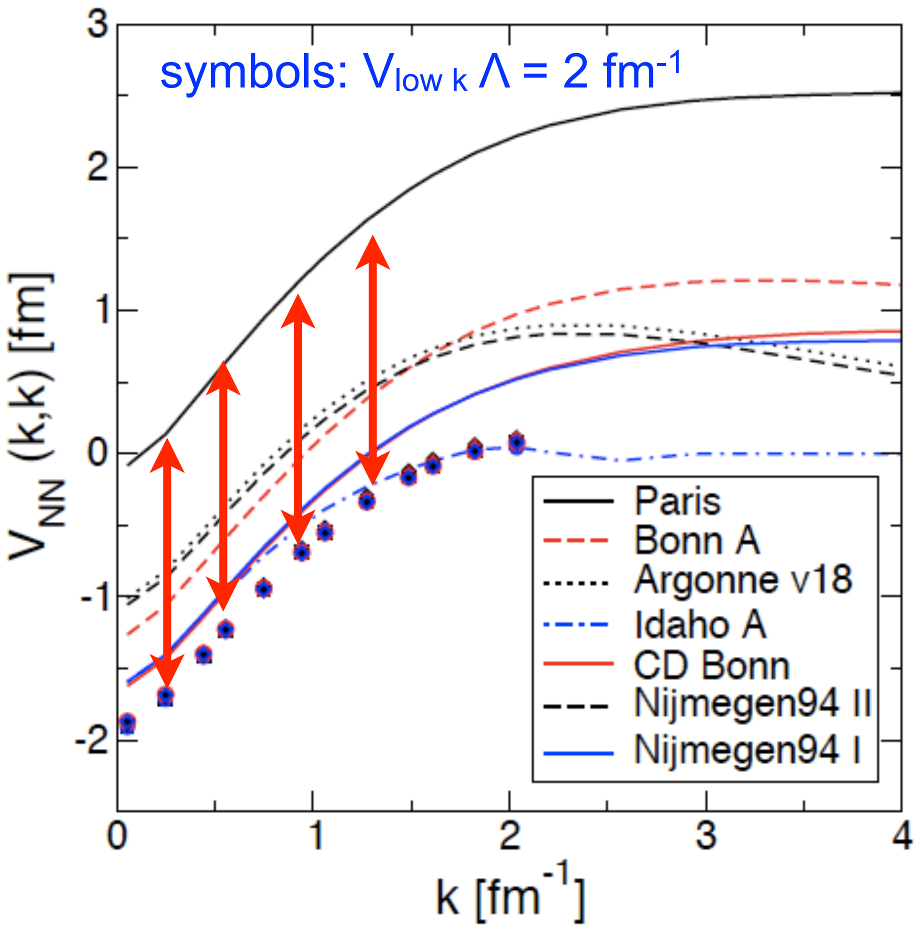


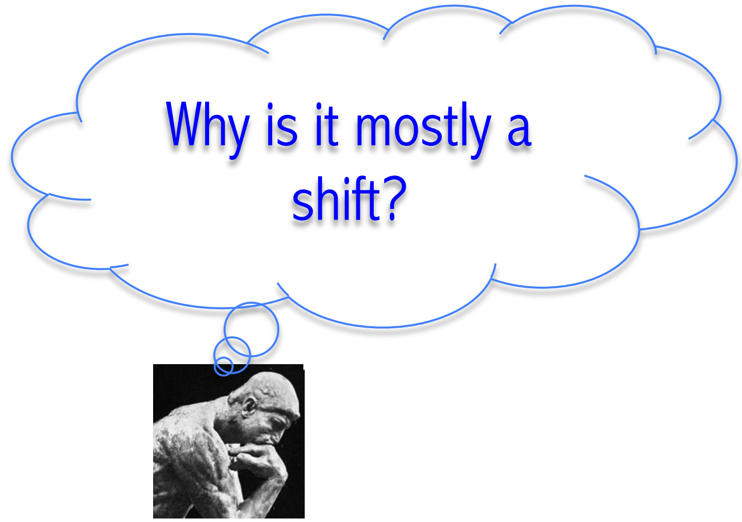
$$V_{\text{eff}} = V_{\text{L}} + \delta V_{\text{c.t.}}(\Lambda)$$

Overall effect of evolving to low momentum

Main effect is shift in momentum space

Renormalization of NN Potentials





$$V_{\rm eff} = V_{\rm L} + \delta V_{\rm c.t.}(\Lambda)$$

Overall effect of evolving to low momentum

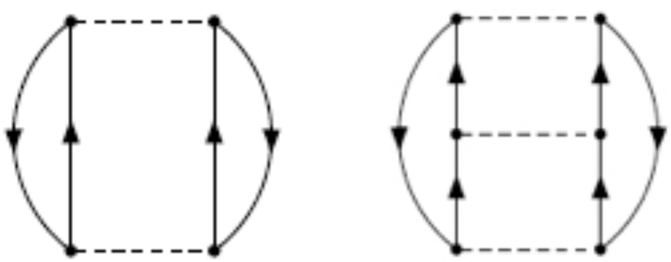
Main effect is shift in momentum space – delta function

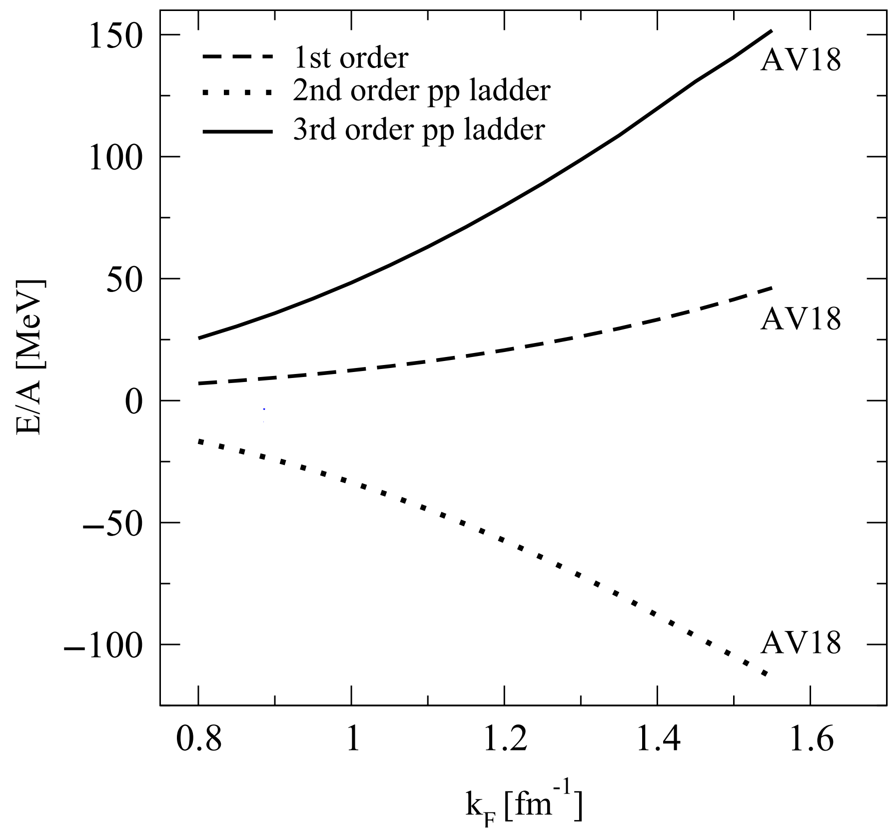
Removes hard core (unconstrained short-range physics)!

Improvements in Perturbation Theory

Explore improvements in symmetric infinite matter calculations

Order by order in **many-body perturbation theory (MBPT)**





No clear convergence with increasing order in bare potential

Improvements in Perturbation Theory

Explore improvements in symmetric infinite matter calculations

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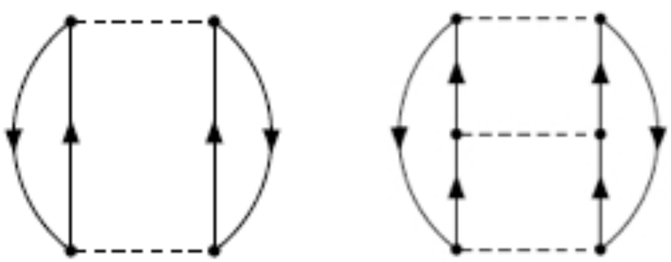
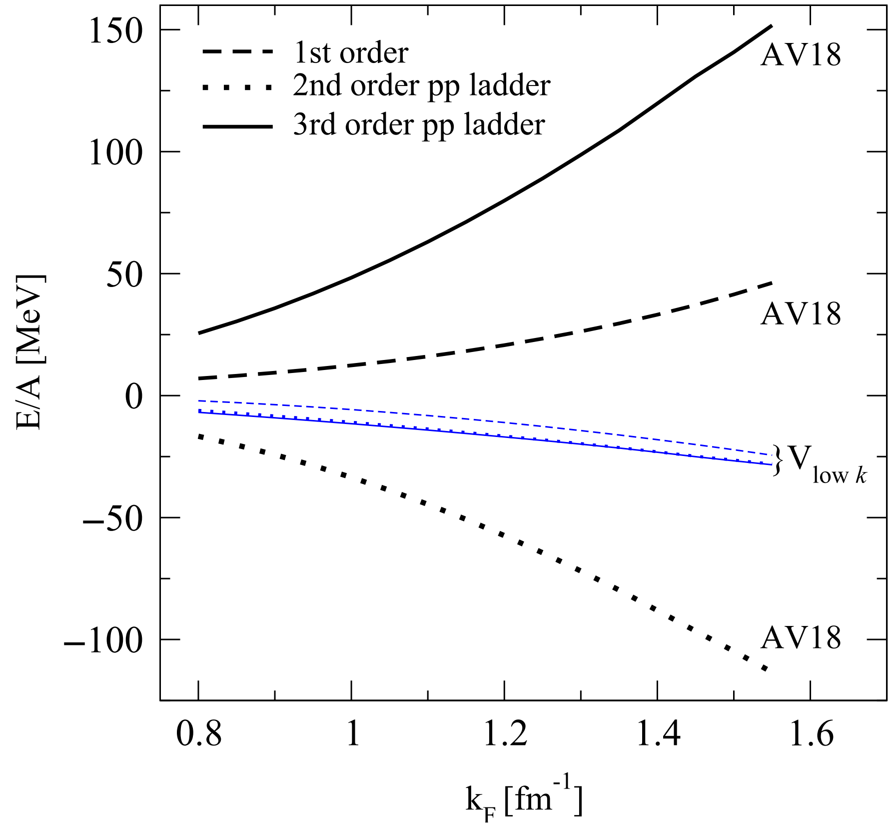




No clear convergence with increasing order in bare potential

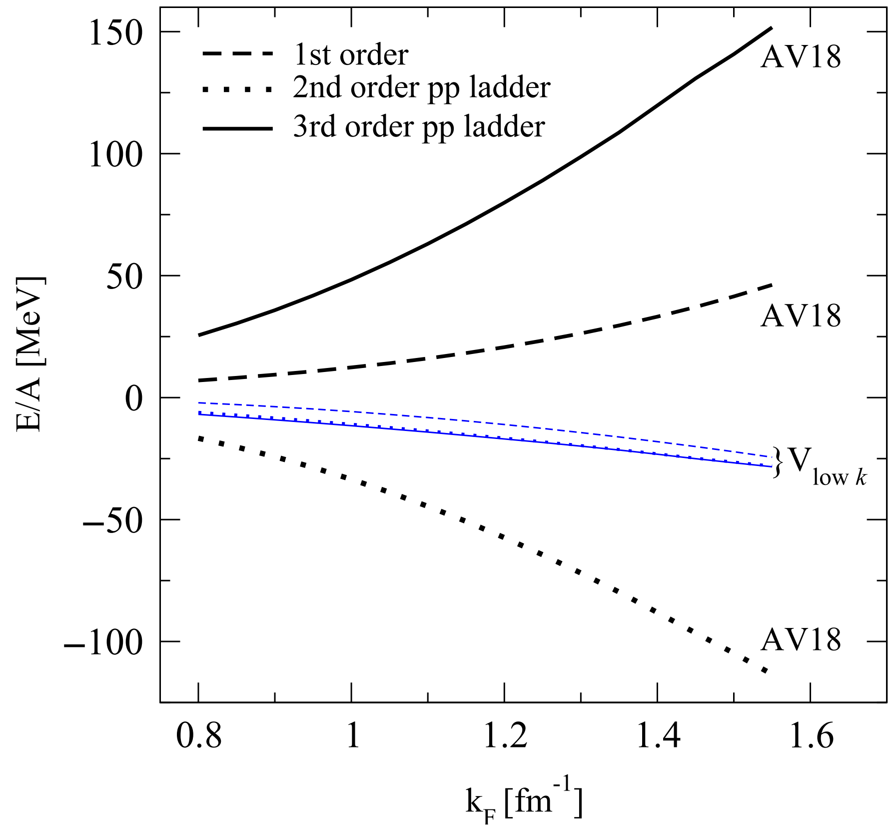
Significant improvement with low-momentum interactions!

Improvements in Perturbation Theory

Explore improvements in symmetric infinite matter calculations

Order by order in **many-body perturbation theory (MBPT)**

Ok, the interactions look perturbative, but something is wrong here…



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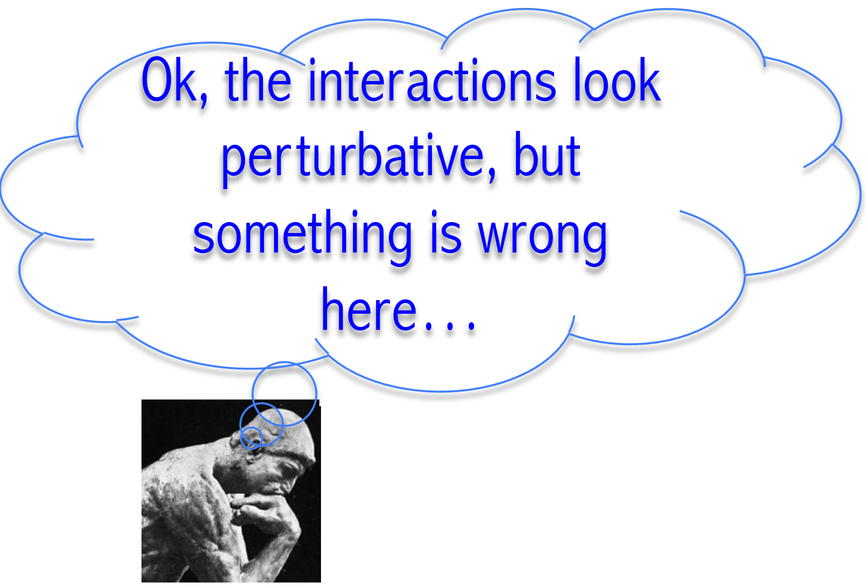
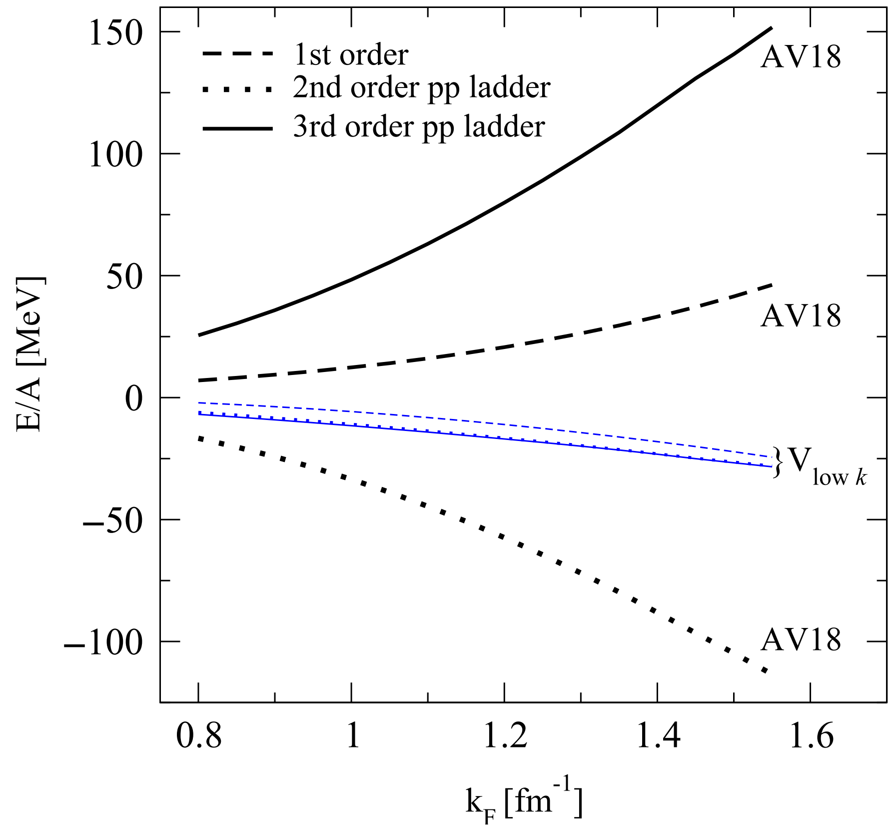




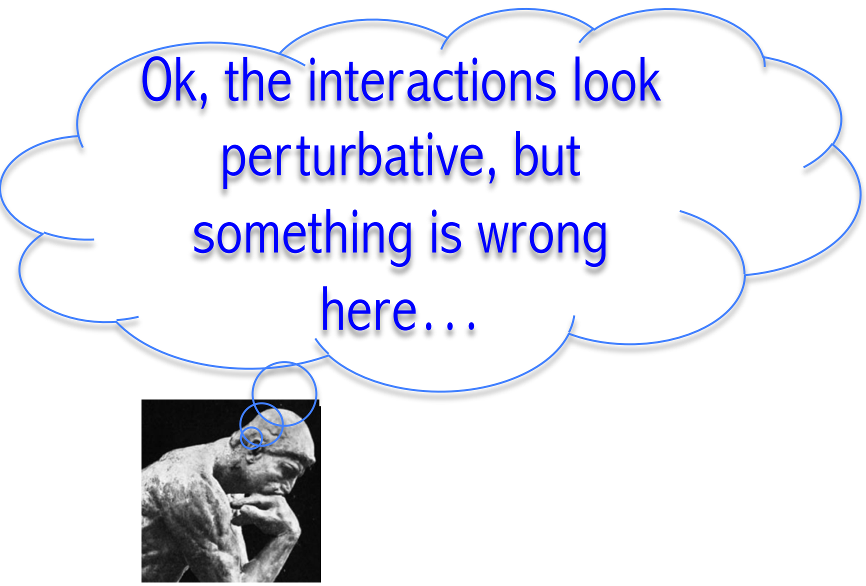
No clear convergence with increasing order in bare potential

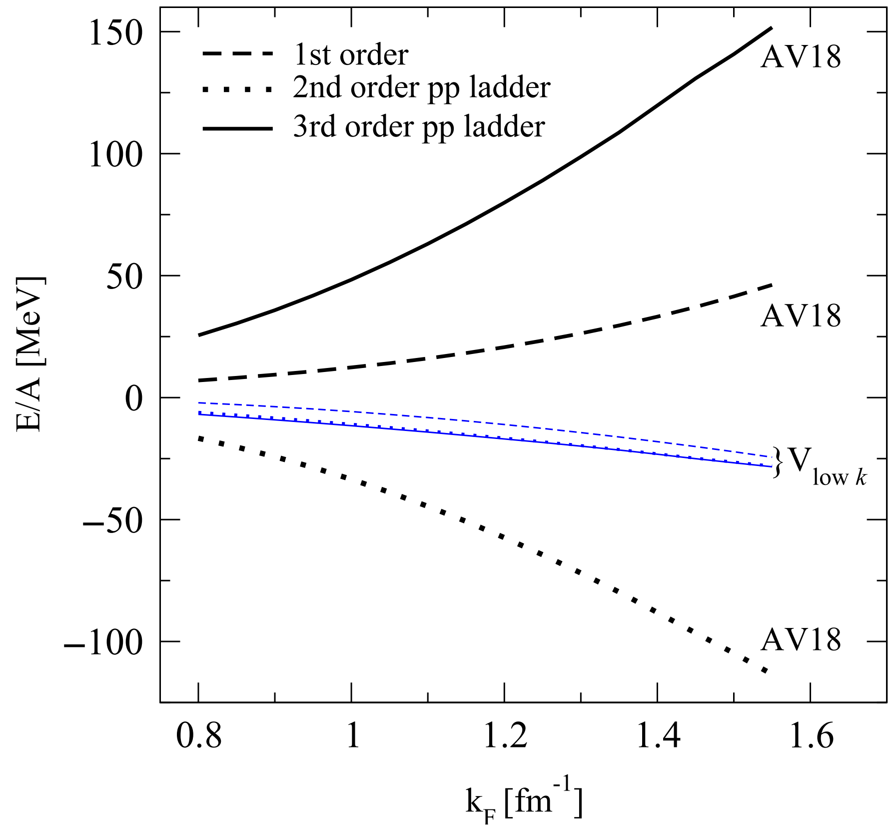
Significant improvement with low-momentum interactions!

Does not saturate – what might be missing?

Improvements in Perturbation Theory

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{\mathrm{3N}}(\Lambda) + V_{\mathrm{4N}}(\Lambda) + \cdots$$





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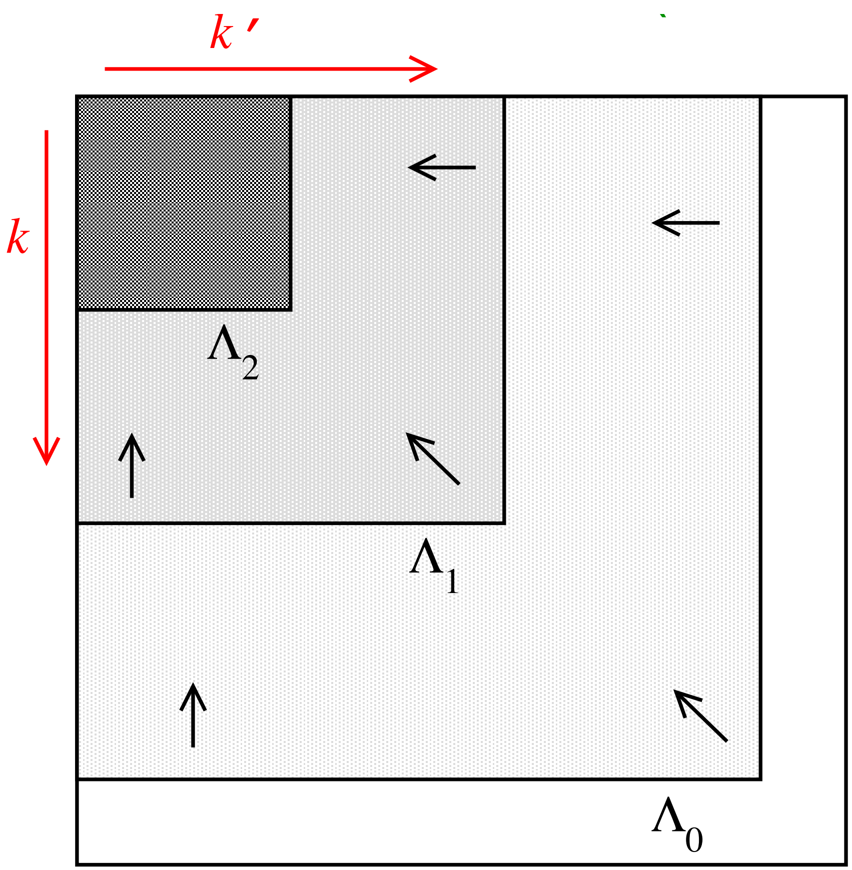
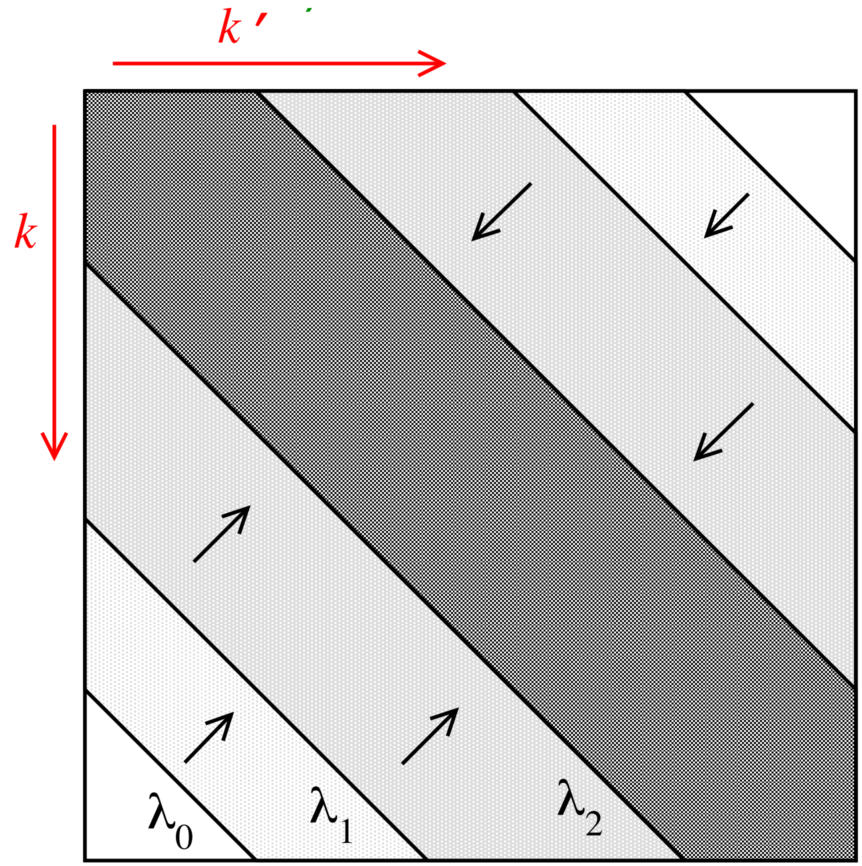
Does not saturate – what might be missing?

Similarity Renormalization Group

Wegner, Glazek/Wilson (1990s)

Complementary method to decouple low from high momenta





Decouples high-momentum

Similarity Renormalization Group

Drives Hamiltonian to band-diagonal

Similarity Renormalization Group

Wegner, Glazek/Wilson (1990s)

Apply a continuous unitary transformation, parameterized by s:

$$H = T + V \to H(s) = U(s) H U^{\dagger}(s) \equiv T + V(s)$$

where differentiating (exercise) yields:

$$\frac{\mathrm{d}H(s)}{\mathrm{d}s} = [\eta(s), H(s)] \quad \text{where} \quad \eta(s) \equiv \frac{\mathrm{d}U(s)}{\mathrm{d}s} U^{\dagger}(s)$$

Never explicitly construct unitary transformation

Instead **choose generator to obtain desired behavior**:

$$\eta(s) = [G(s), H(s)]$$

Many options, e.g.,

$$\eta(s) = [T, H(s)]$$ Drives H(s) to band-diagonal form

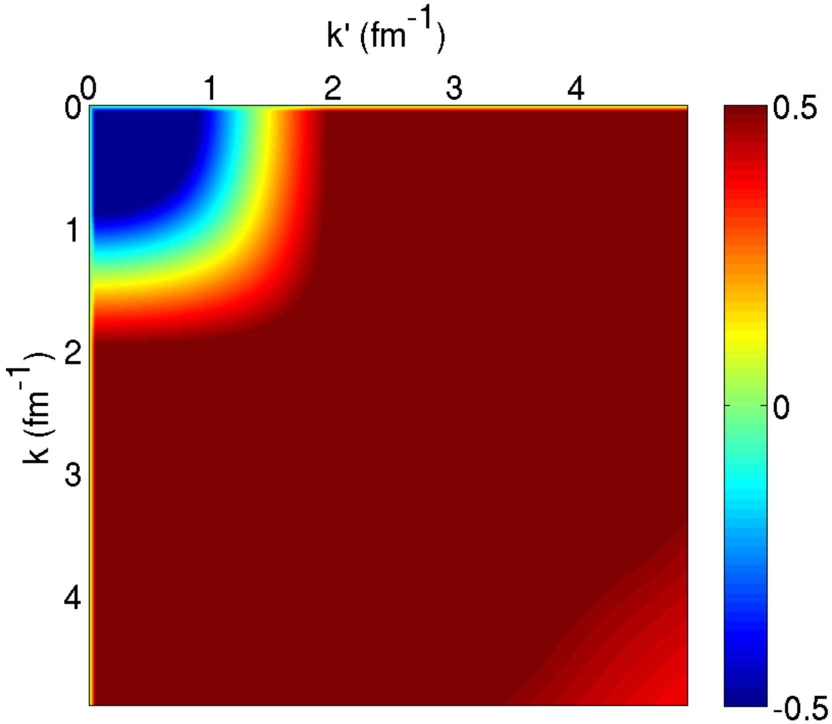
Illustration of SRG Flow

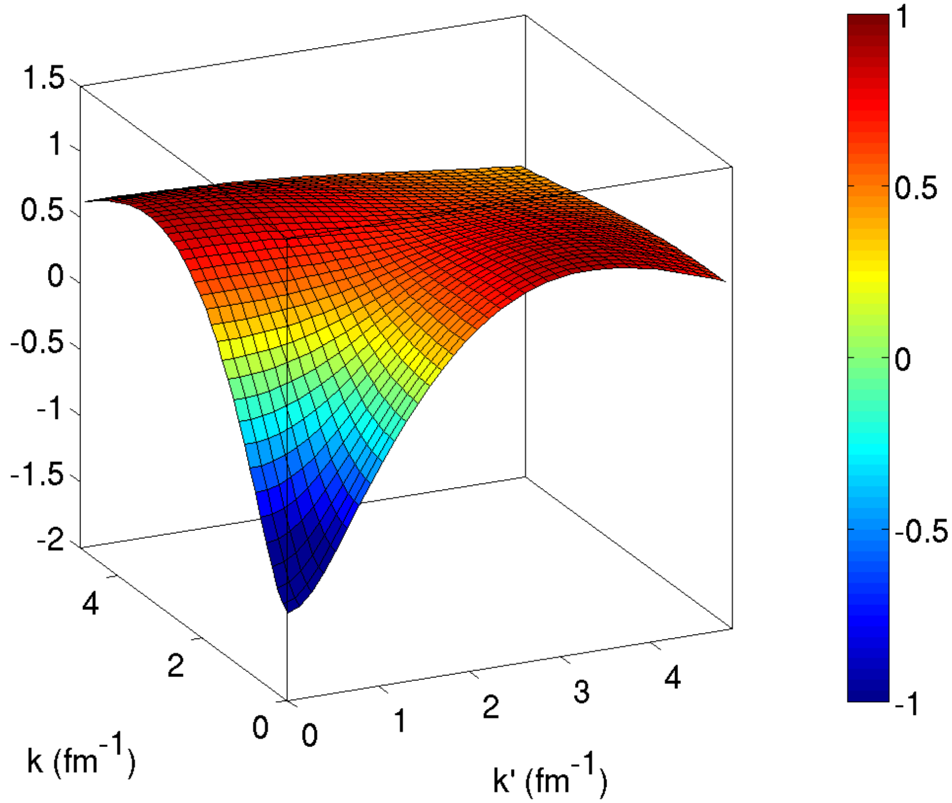
Drive H to band-diagonal form with kinetic-energy generator:

$$\eta(s) = [T, H(s)]$$

With alternate definition of flow parameter: $\lambda^2 = \dfrac{1}{\sqrt{s}}$

Argonne V_{18} $^1S_0$





$\lambda = 8.0\,\mathrm{fm}^{-1}$

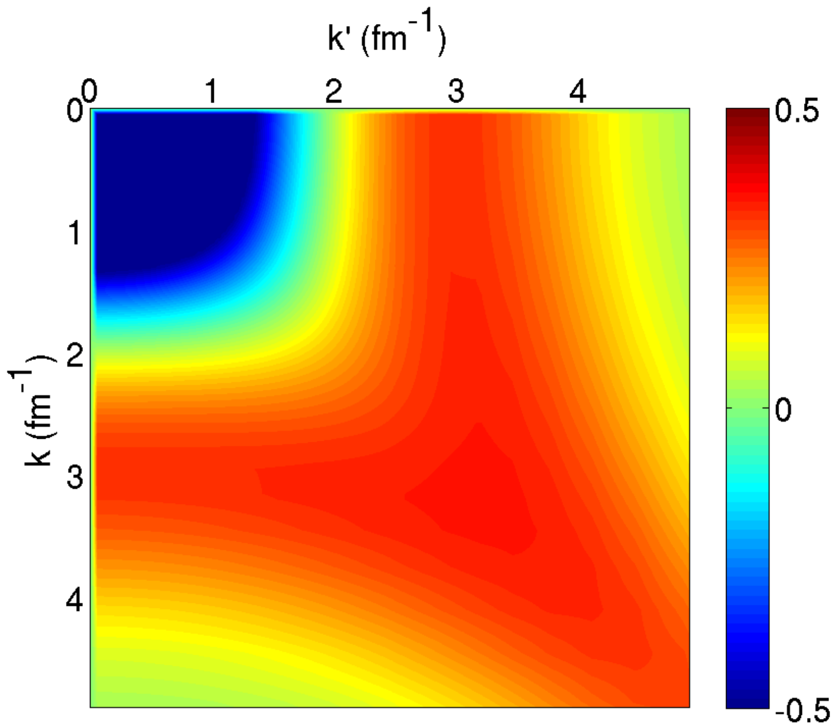
Illustration of SRG Flow

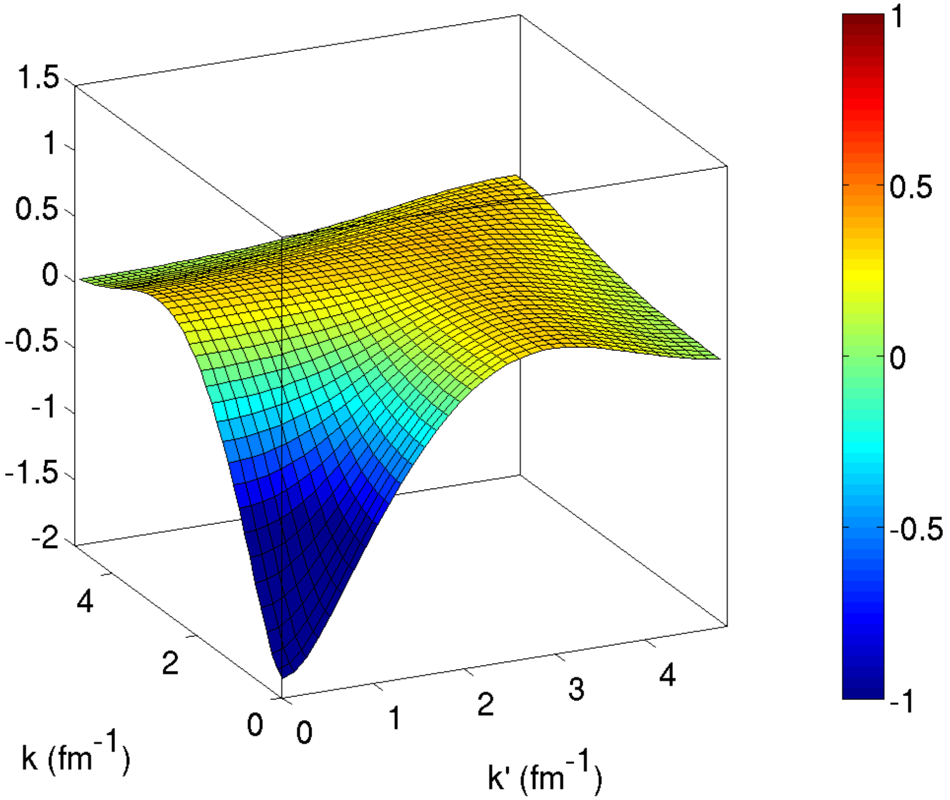
Drive H to band-diagonal form with standard choice:

$$\eta(s) = [T, H(s)]$$

With alternate definition of flow parameter: $\lambda^2 = \frac{1}{\sqrt{s}}$

Argonne V_{18} $^1S_0$





$\lambda = 4.0\,\mathrm{fm}^{-1}$

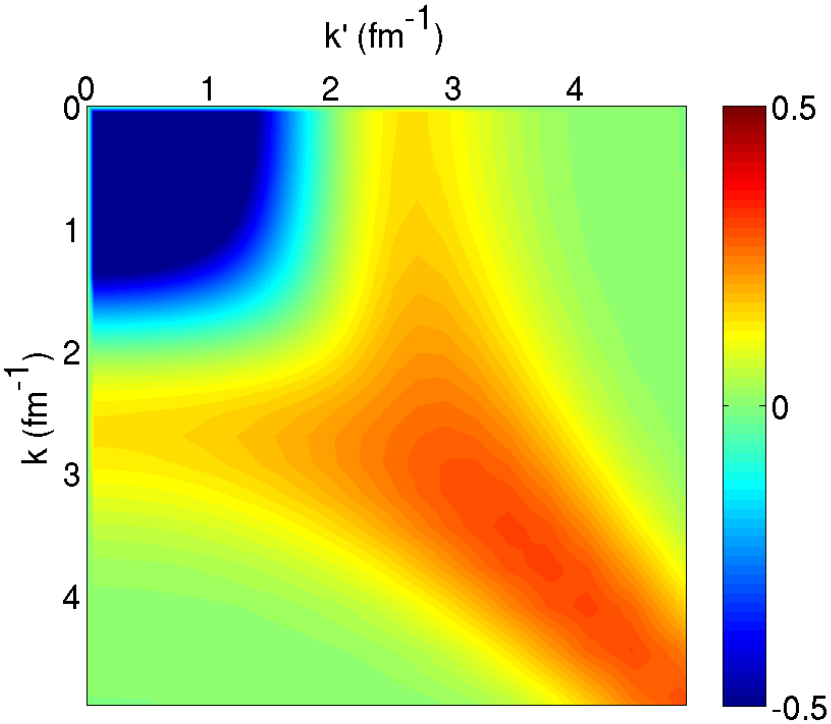
Illustration of SRG Flow

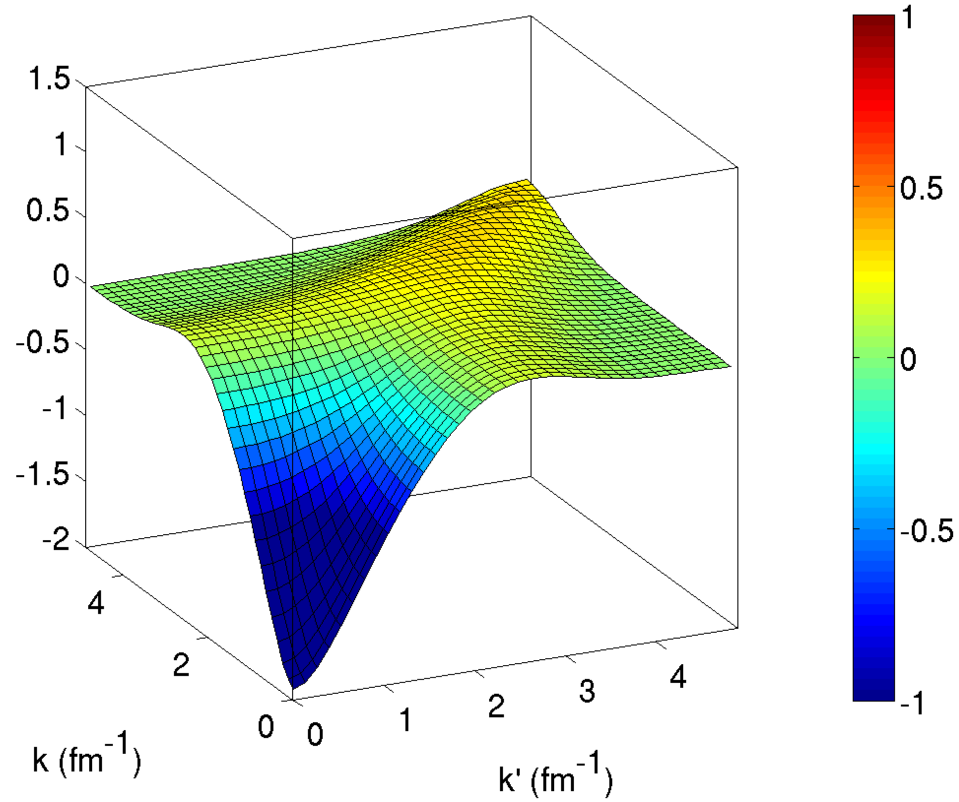
Drive H to band-diagonal form with standard choice:

$$\eta(s) = [T, H(s)]$$

With alternate definition of flow parameter: $\lambda^2 = \frac{1}{\sqrt{s}}$

Argonne V_{18} $^1S_0$





$\lambda = 3.0\,\mathrm{fm}^{-1}$

Illustration of SRG Flow

Drive H to band-diagonal form with standard choice:

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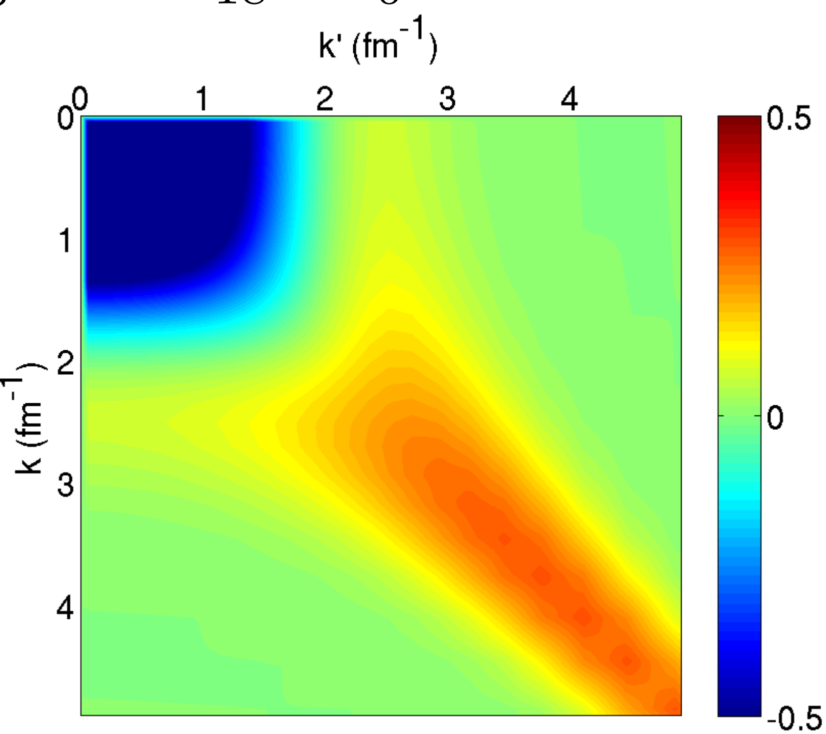
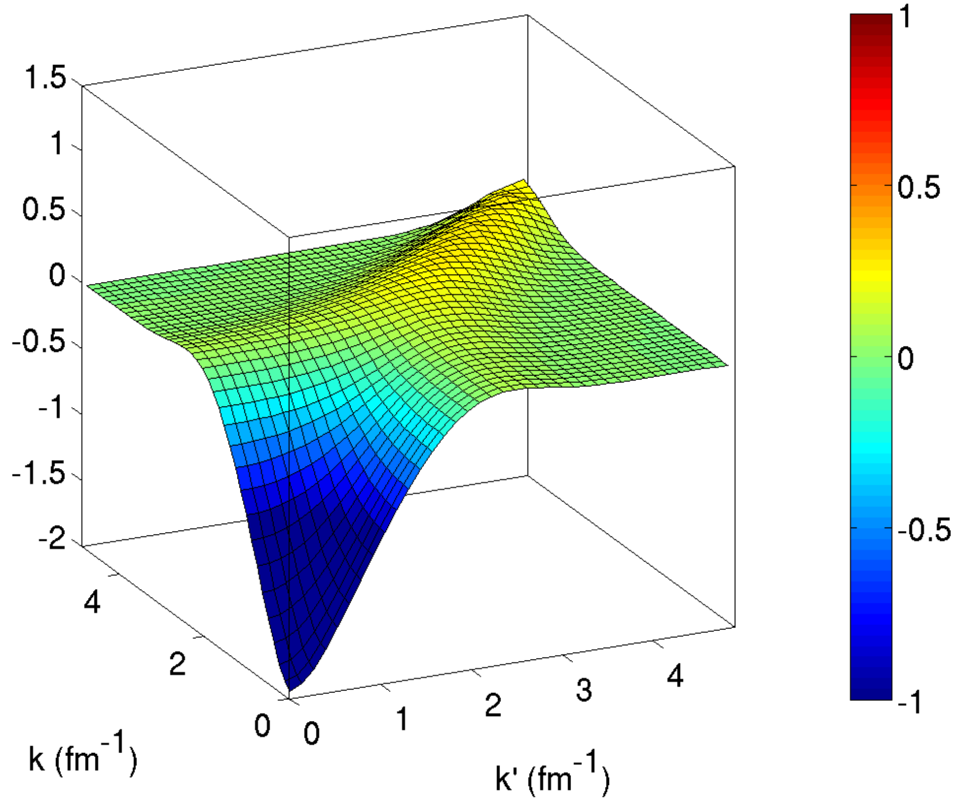
With alternate definition of flow parameter: $\lambda^2 = \frac{1}{\sqrt{s}}$

Argonne V_{18} $^1S_0$





$\lambda = 2.5\,\text{fm}^{-1}$

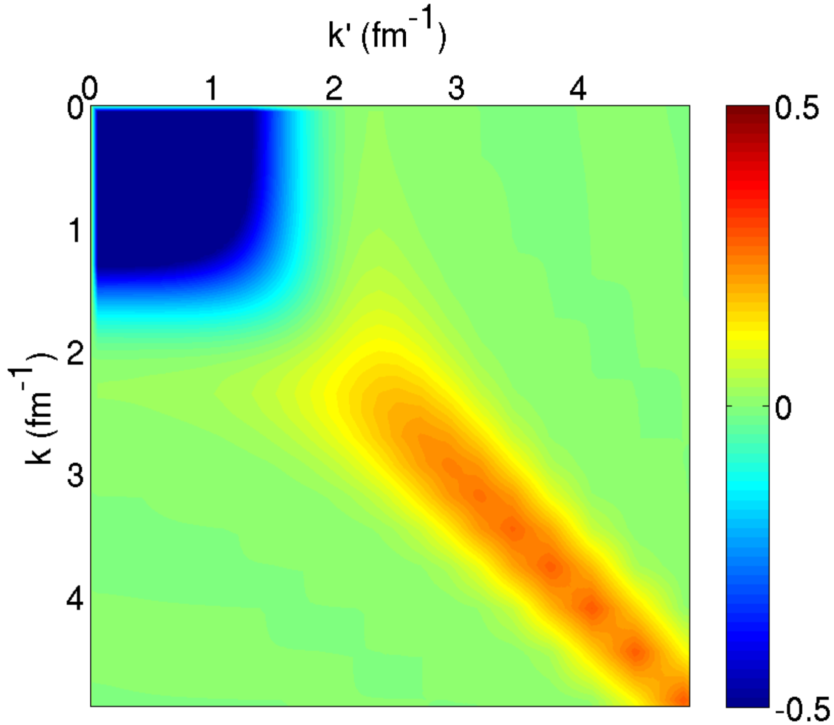
Illustration of SRG Flow

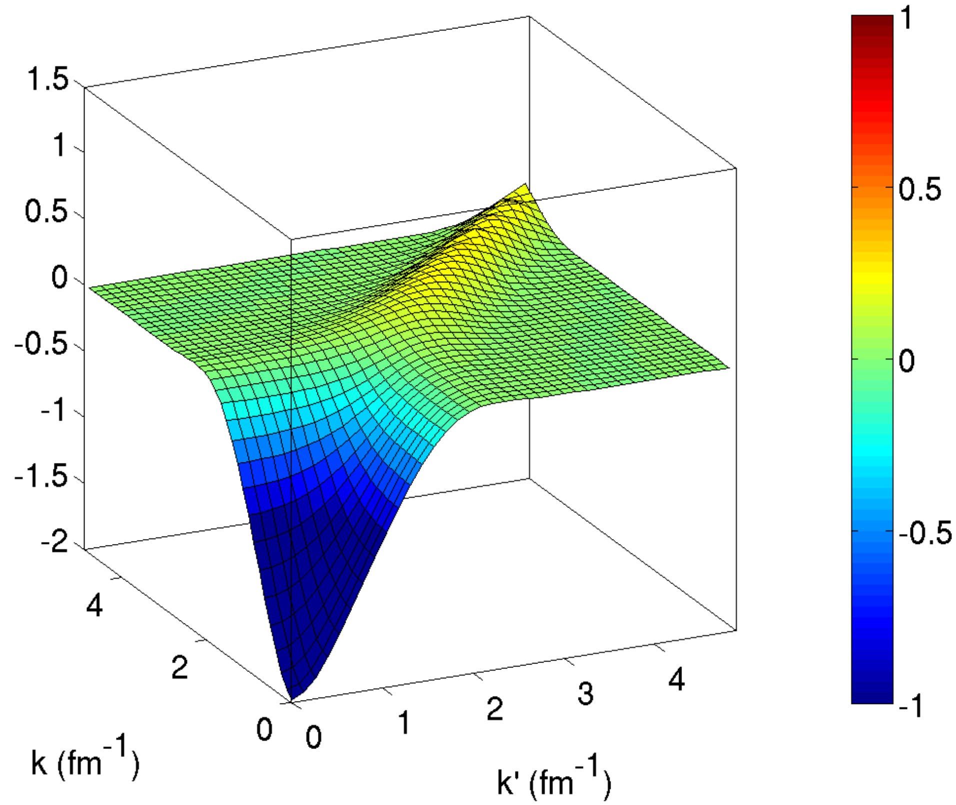
Drive H to band-diagonal form with standard choice:

$$\eta(s) = [T, H(s)]$$

With alternate definition of flow parameter: $\lambda^2 = \dfrac{1}{\sqrt{s}}$

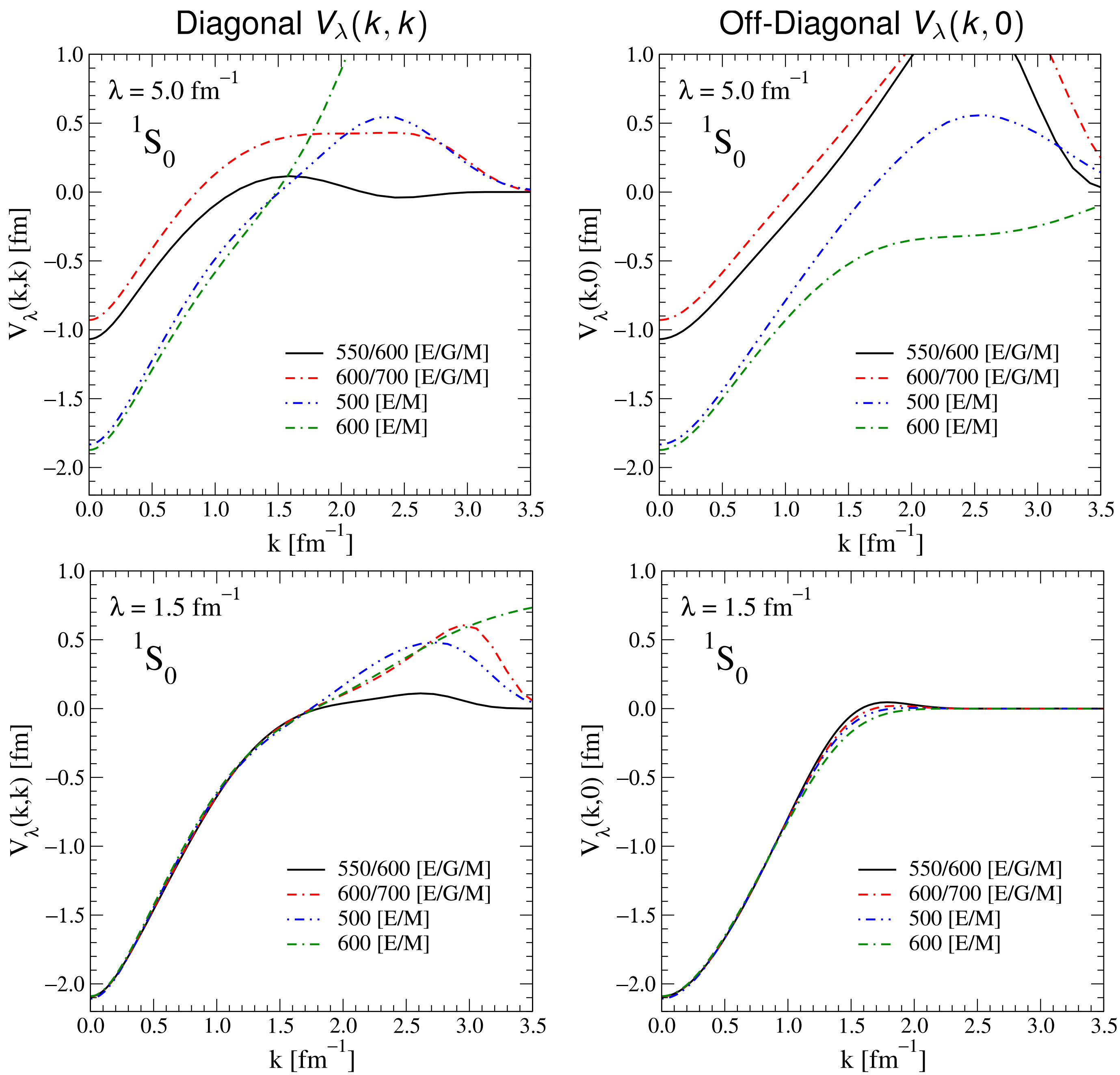
Argonne V_{18} $^1S_0$





$\lambda = 2.0\,\mathrm{fm}^{-1}$

SRG Renormalization of Chiral EFT Potentials



These are all our favorite Chiral EFT NN potentials…

These are all our favorite Chiral EFT NN potentials… **SRG evolved**

Exhibit similar “universal” behavior as low-momentum interactions!

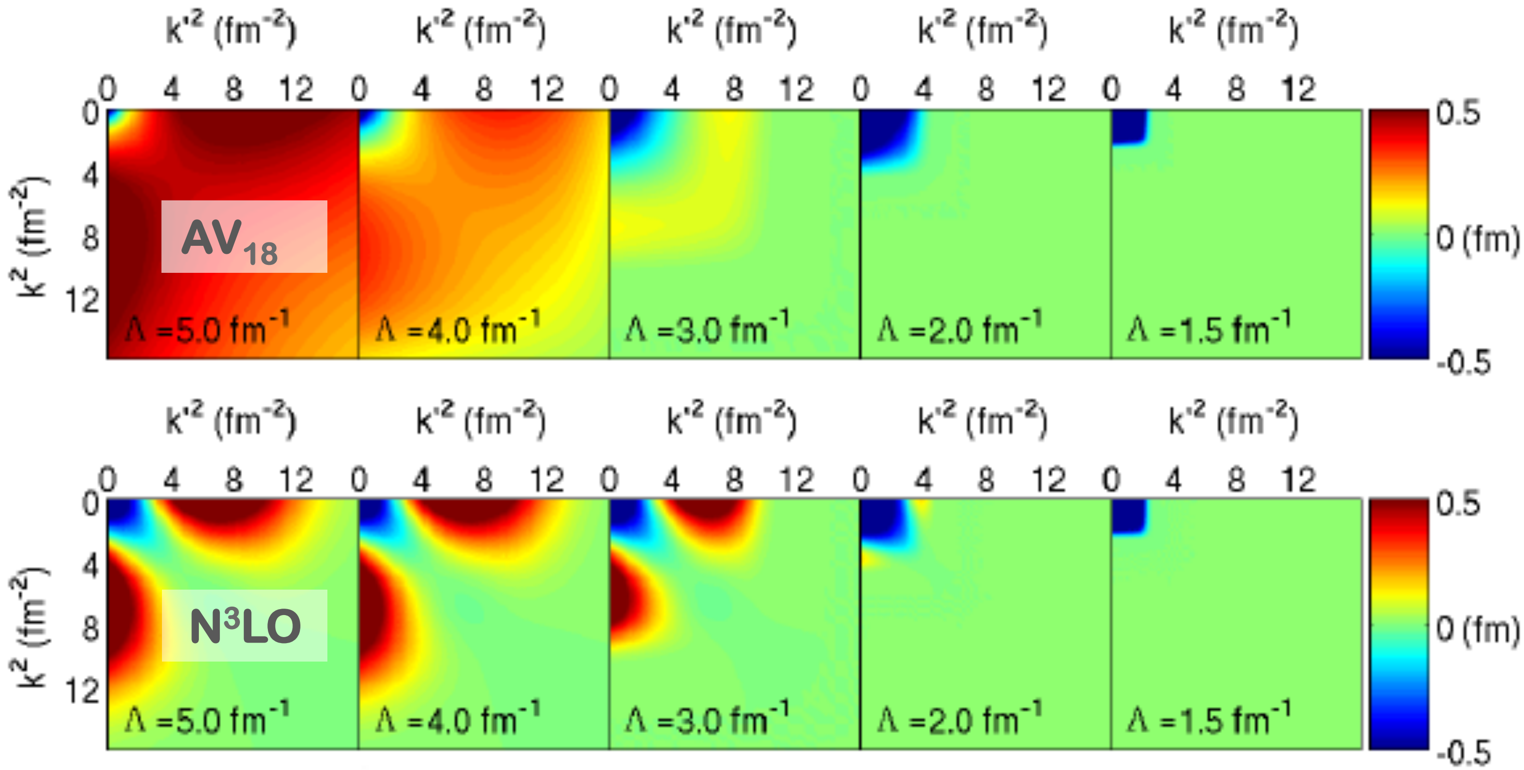
Renormalization of Nuclear Interactions

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{3\mathrm{N}}(\Lambda) + V_{4\mathrm{N}}(\Lambda) + \cdots$$

Evolve momentum resolution scale of chiral interactions from initial Λ_χ

Remove coupling to high momenta, low-energy physics unchanged

Bogner, Kuo, Schwenk, Furnstahl



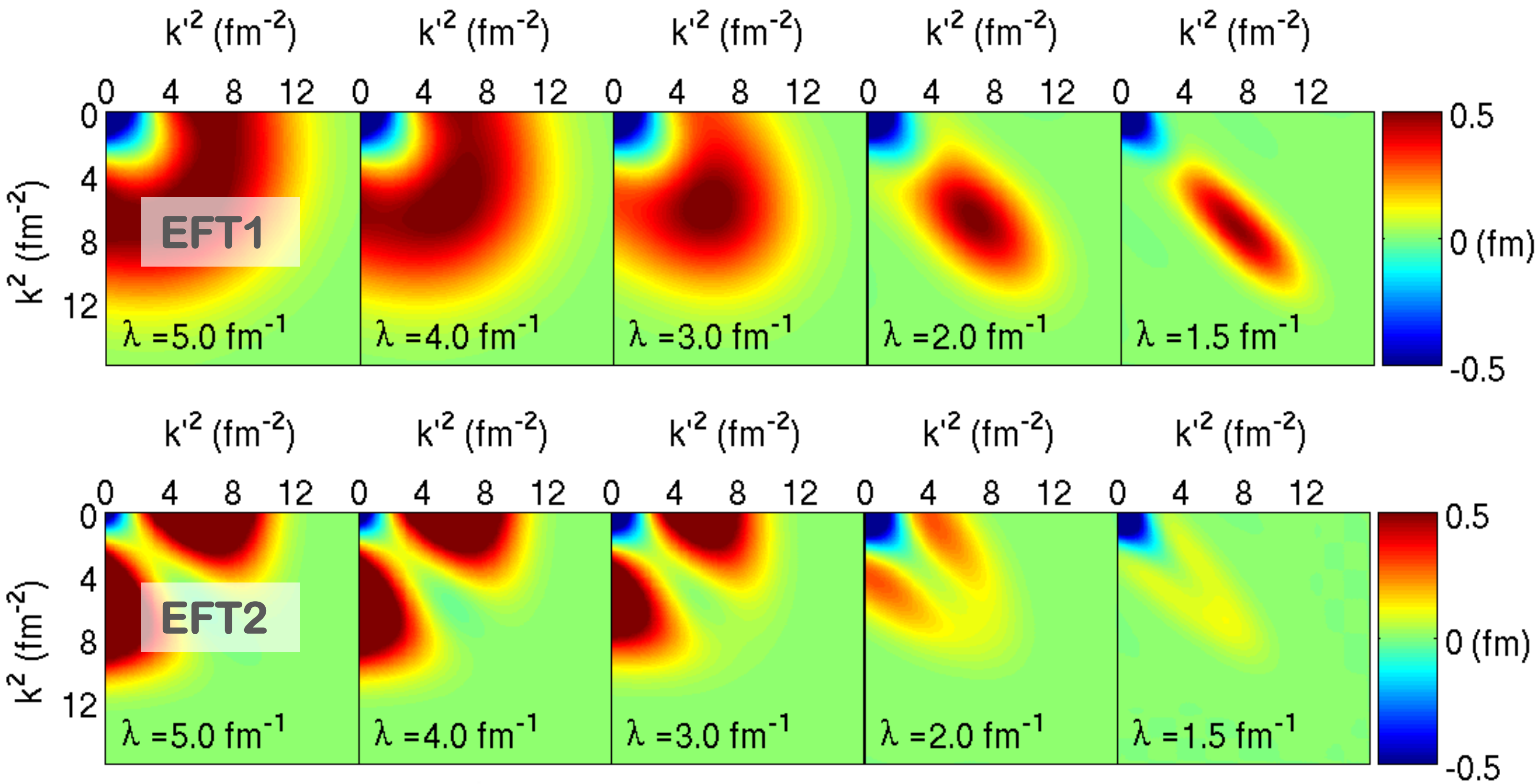
Universal at low-momentum

$V_{\mathrm{low}\,k}(\Lambda)$: lower cutoffs advantageous for nuclear structure calculations

SRG-Evolution of Different Initial Potentials

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{3\mathrm{N}}(\Lambda) + V_{4\mathrm{N}}(\Lambda) + \cdots$$

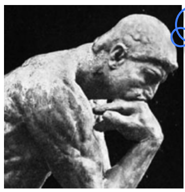
SRG evolution of two different chiral EFT potentials



Lots of pretty pictures, but how does it actually help?

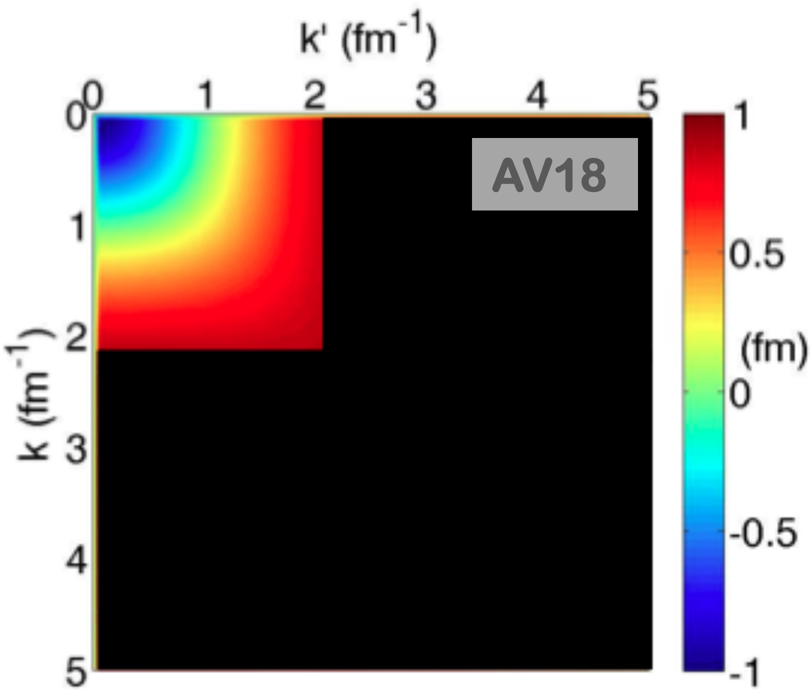
Revisit Low-Pass Filter Idea

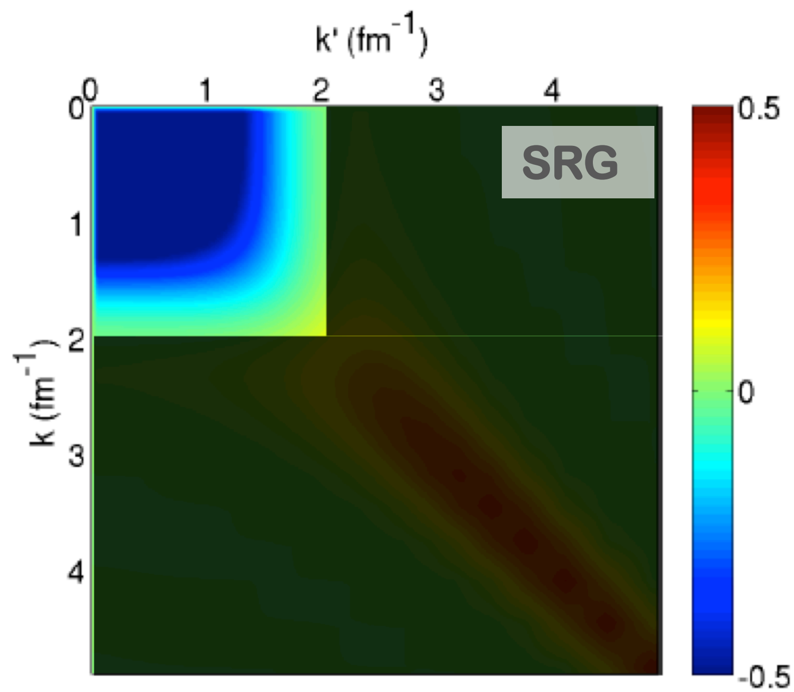
Ok, high momentum is a pain. I wonder what would happen to low-energy observables…



Low-to-high momentum makes life difficult for low-energy nuclear theorists

What's the difference now?





$$V_{\text{filter}}(k',k) \equiv 0; \;\; k,k' > 2.2\,\text{MeV}$$

Revisit Low-Pass Filter Idea

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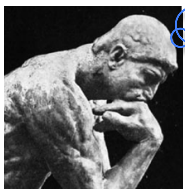
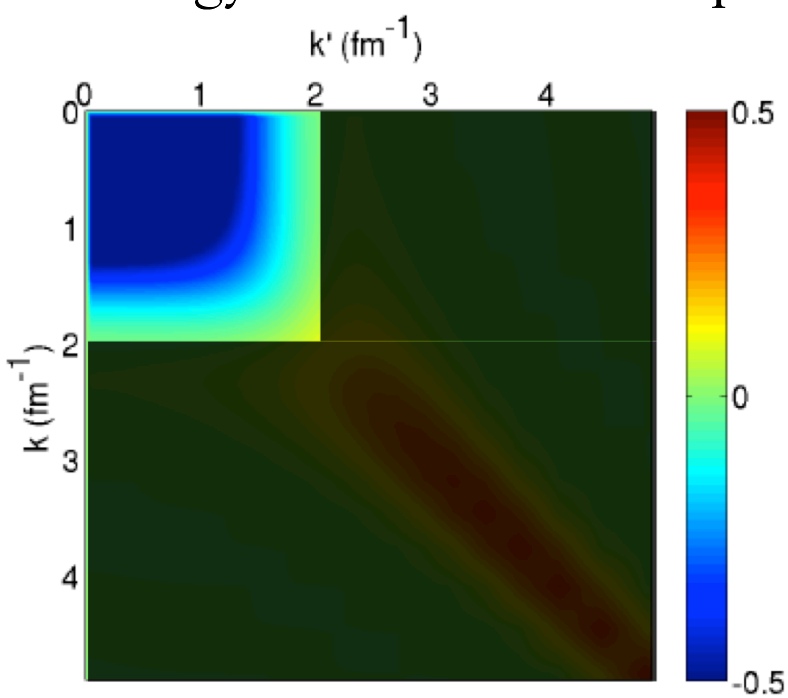
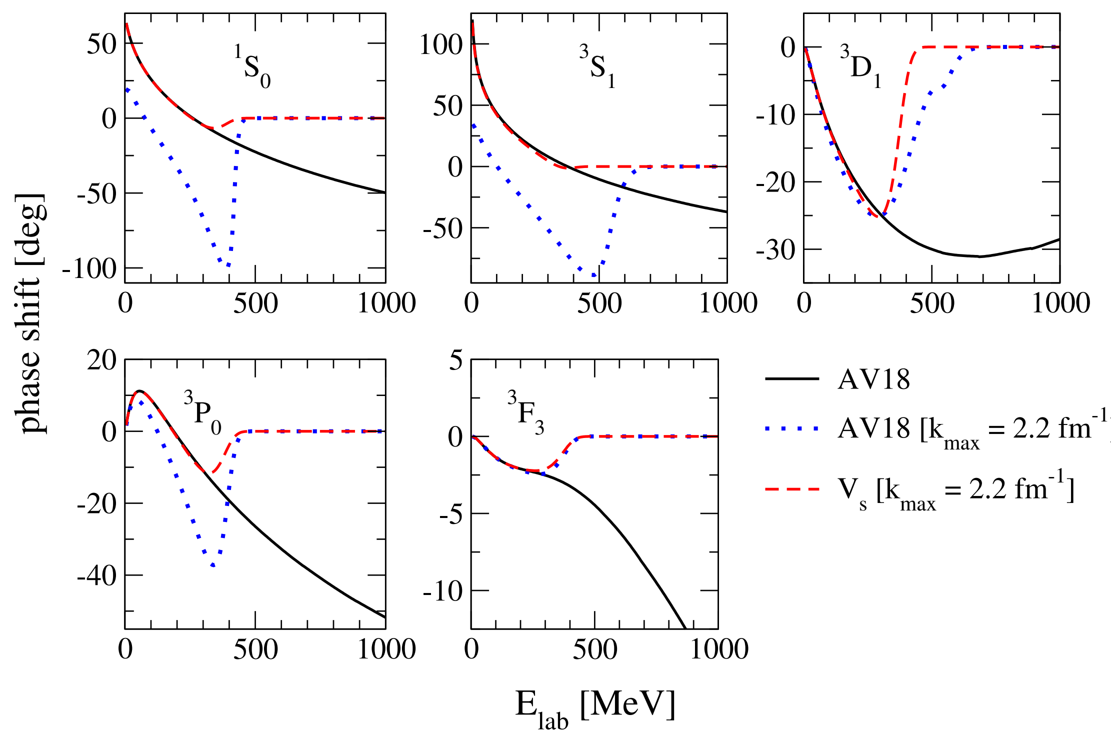


Low-to-high momentum makes life difficult for low-energy nuclear theorists

Low-energy observables were preserved – now sharp cut makes sense!





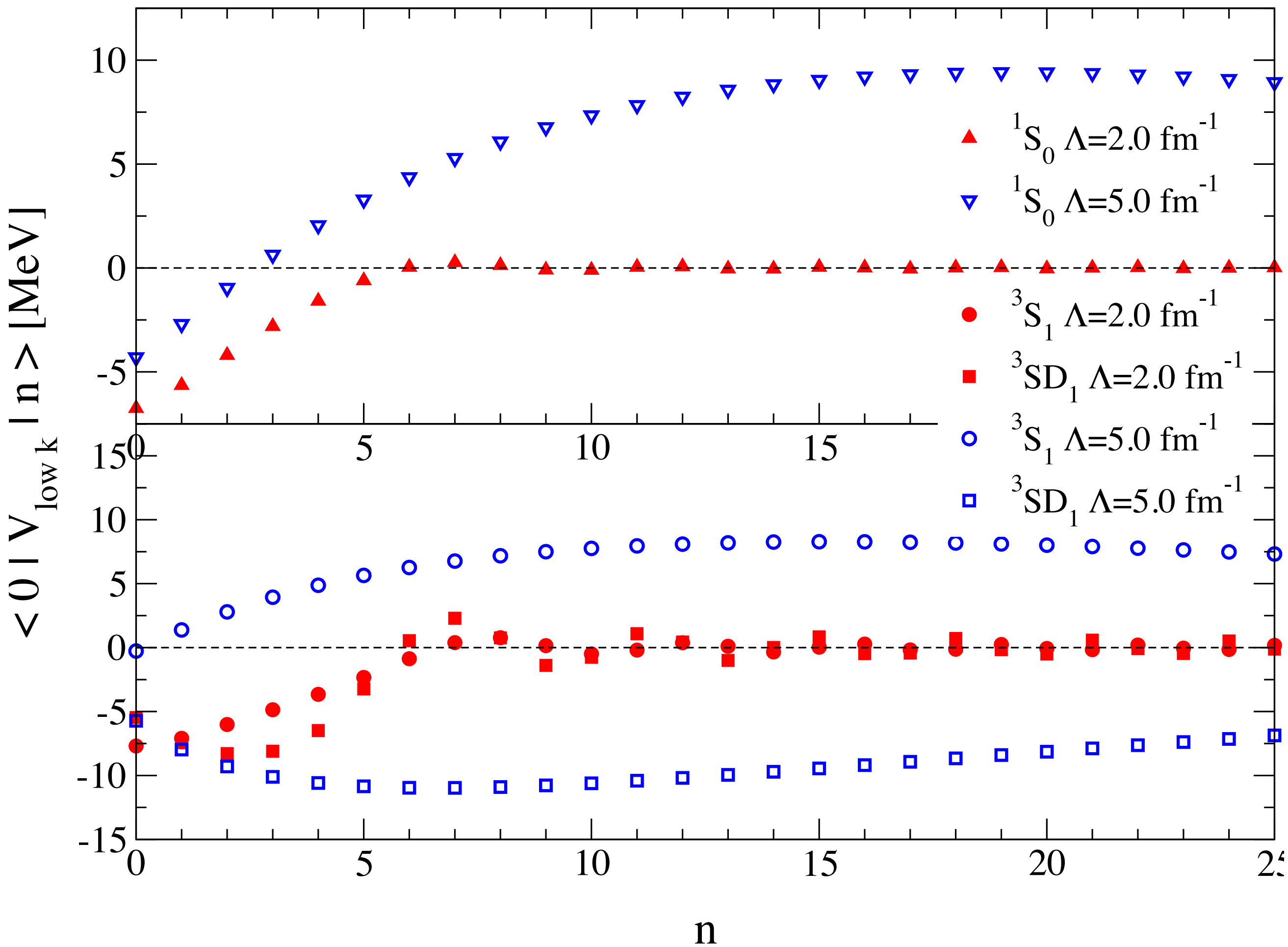
$$V_{\mathrm{filter}}(k', k) \equiv 0;\;\; k, k' > 2.2\,\mathrm{MeV}$$

Benefits of Lower Cutoffs

Often work in HO basis – does this make a difference there?

Removes coupling from low-to-high harmonic oscillator states

Expect to speed convergence in HO basis



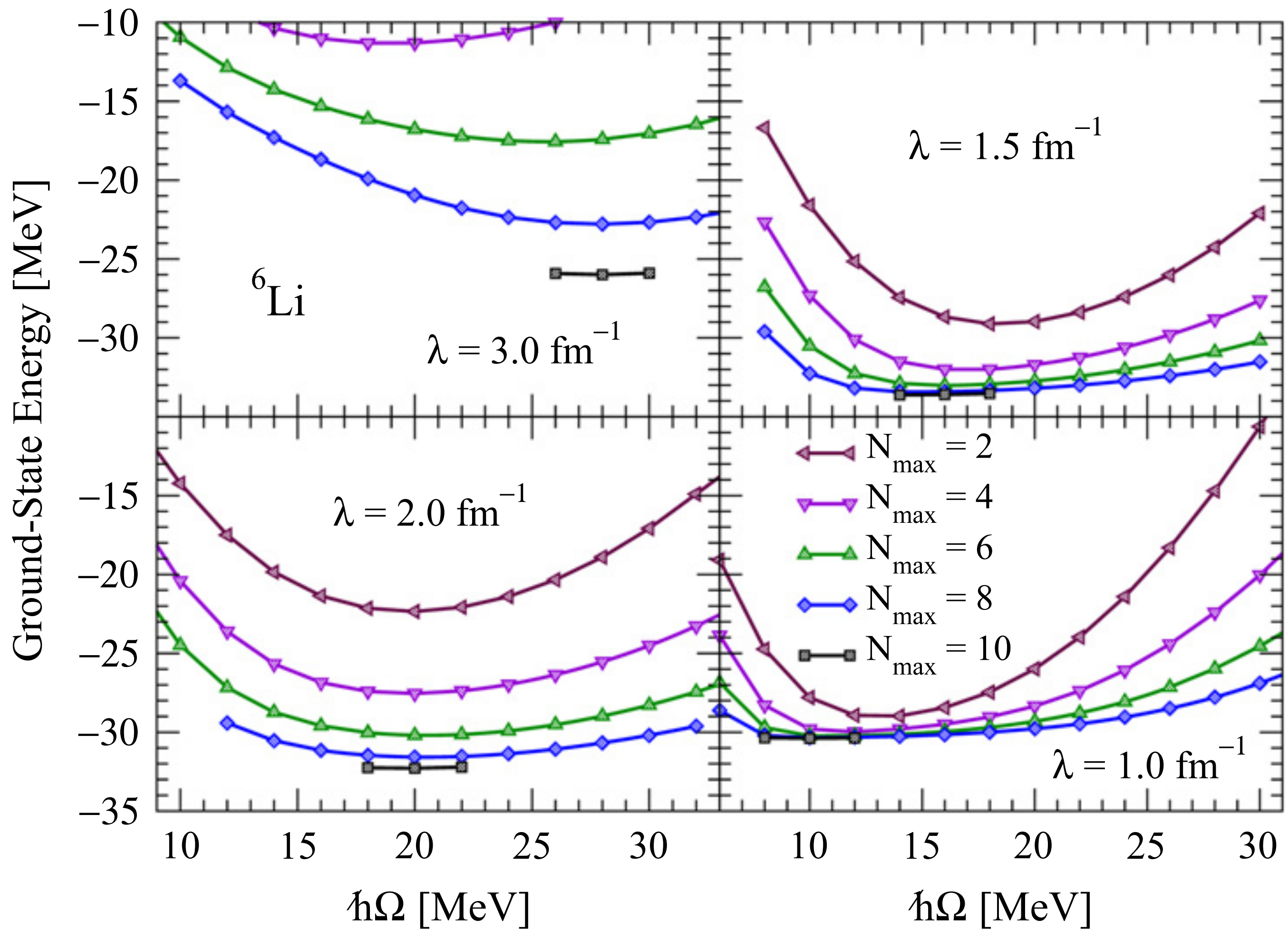
Explicitly see why this causes problems later!

Benefits of Lower Cutoffs

Exactly what happens in **no-core shell model calculations**

Probably equally helpful in normal shell-model calculations?

Come back to this later…



Benefits of Lower Cutoffs

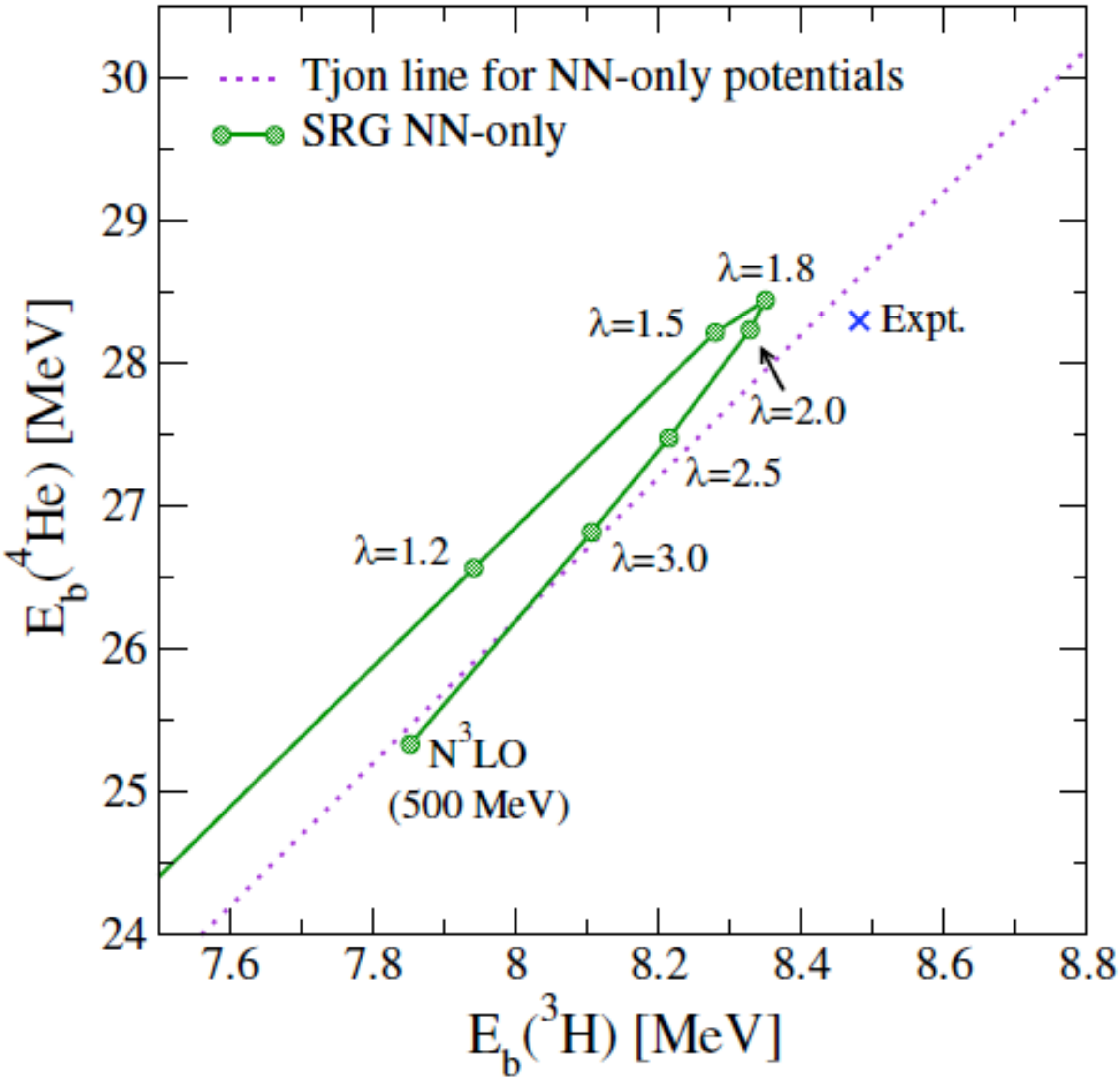
Use cutoff dependence to assess missing physics: return to Tjon line

Varying cutoff moves along line

Still never reaches experiment

Lesson: Variation in physical observables with cutoff indicates missing physics

Tool, not a parameter!



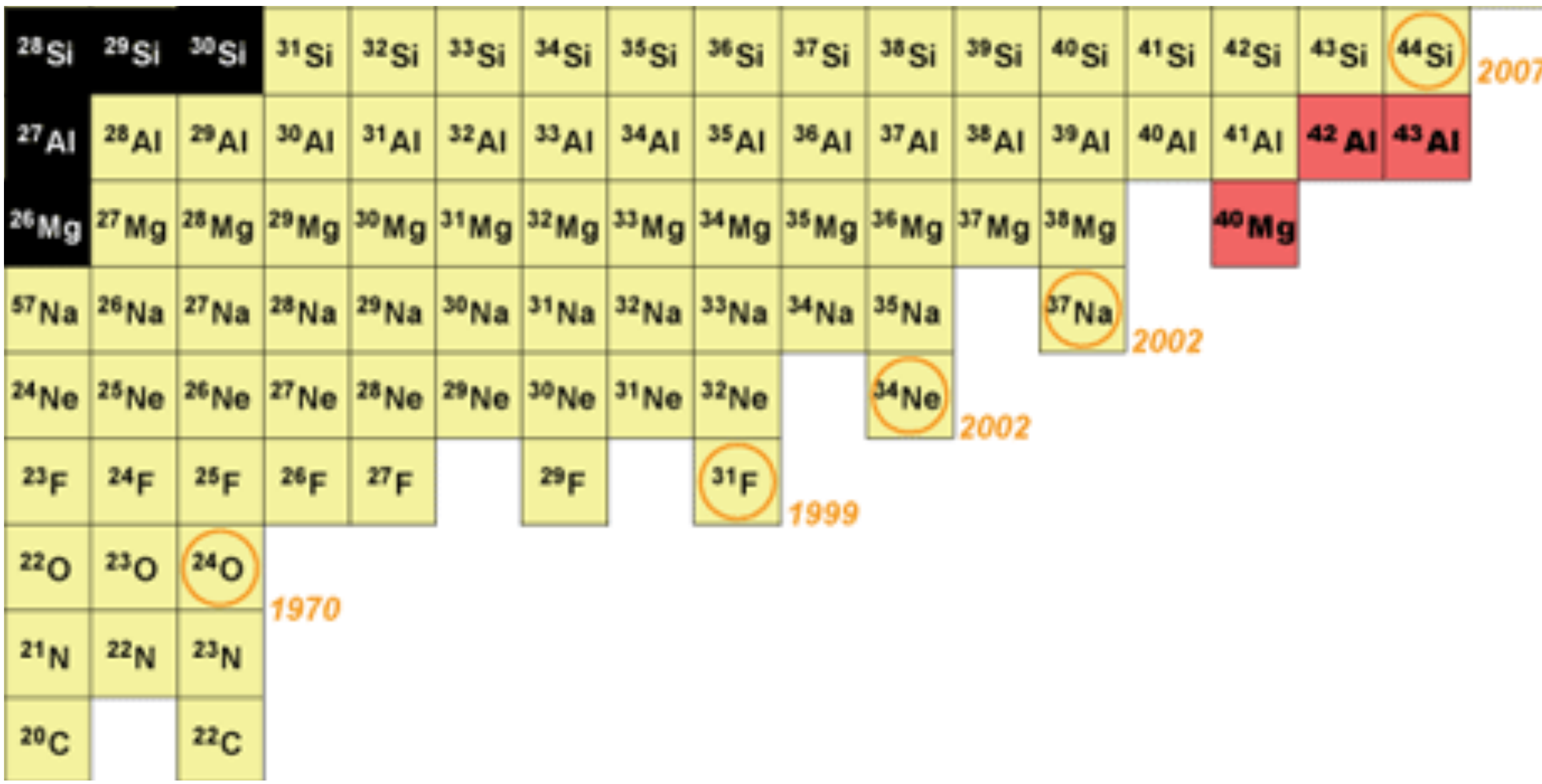
Limits of Nuclear Existence: Oxygen Anomaly

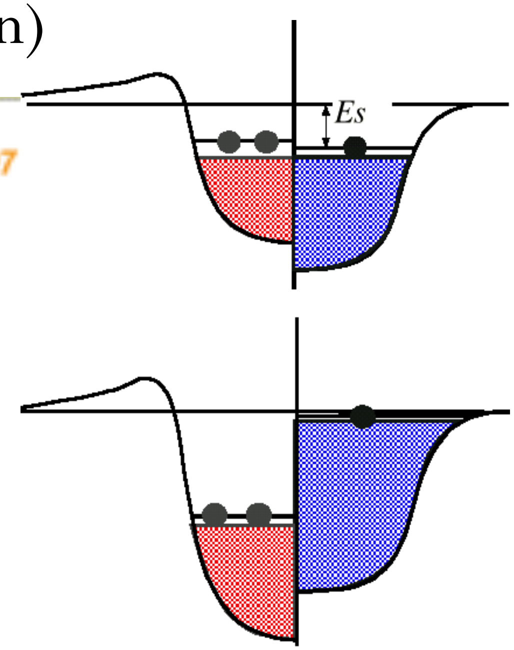
Where is the nuclear dripline?

Limits defined as last isotope with positive neutron separation energy

- Nucleons "drip" out of nucleus

Neutron dripline experimentally established to Z=8 (Oxygen)





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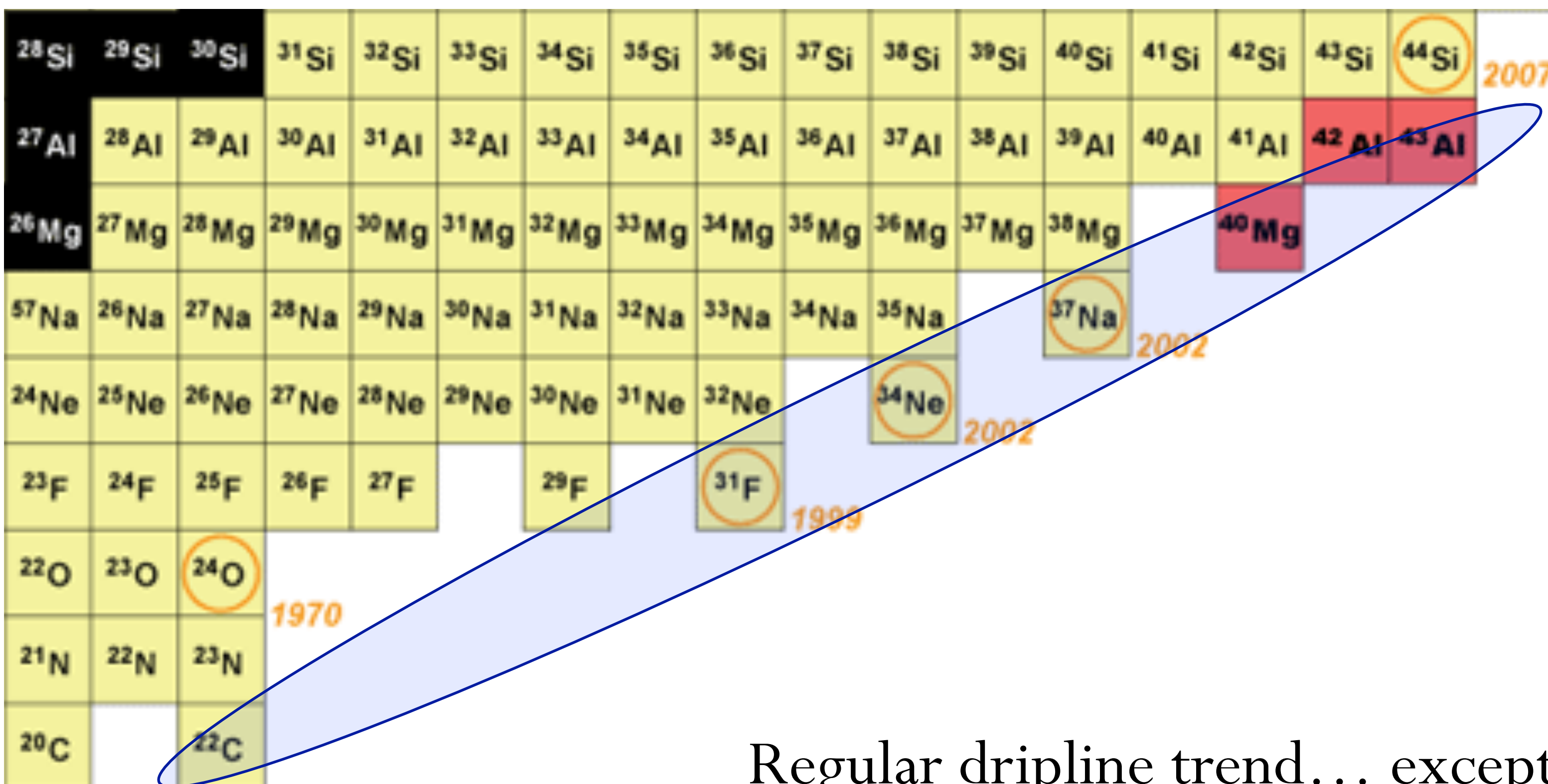

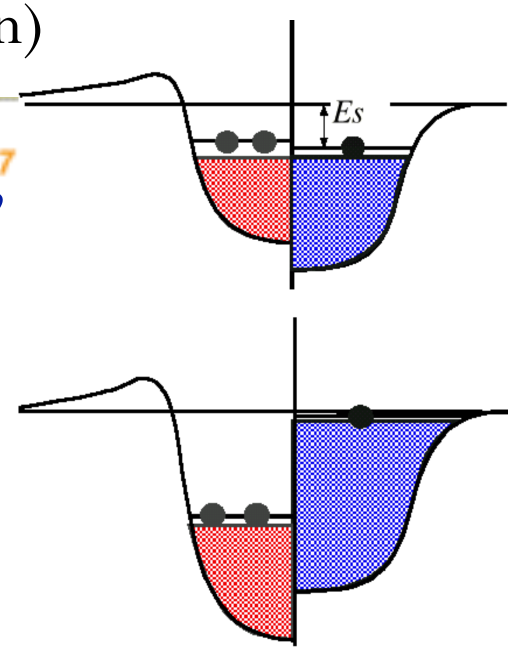

Regular dripline trend… except oxygen

Adding one proton binds 6 additional neutrons

Limits of Nuclear Existence: Oxygen Anomaly

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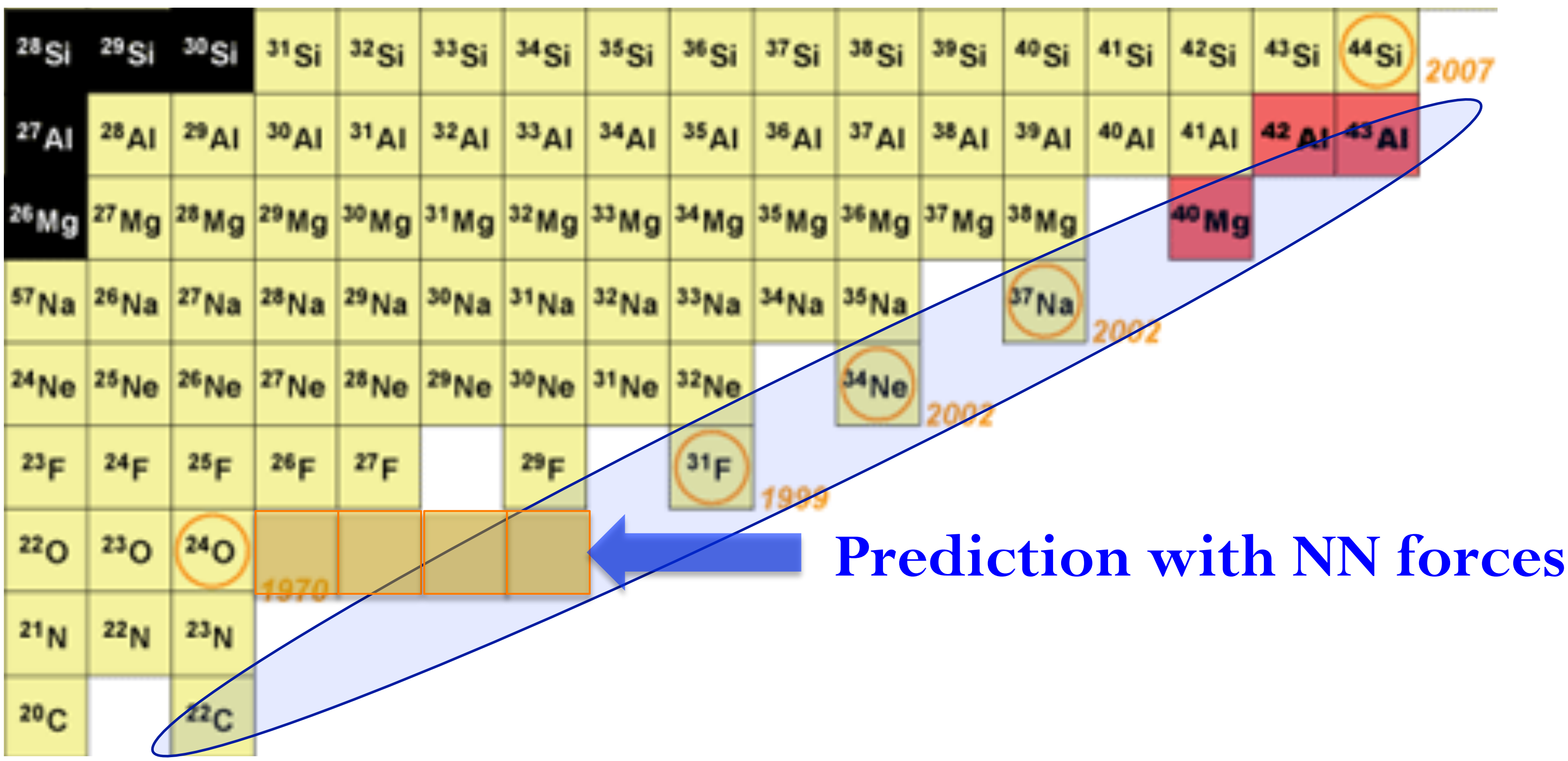
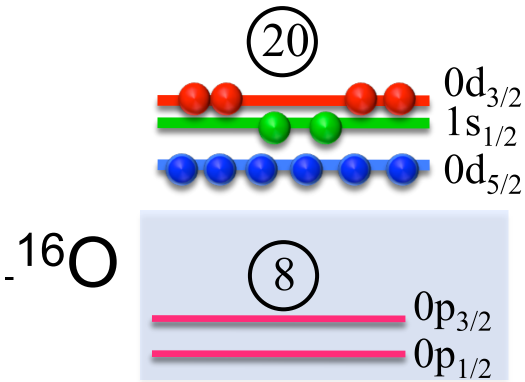


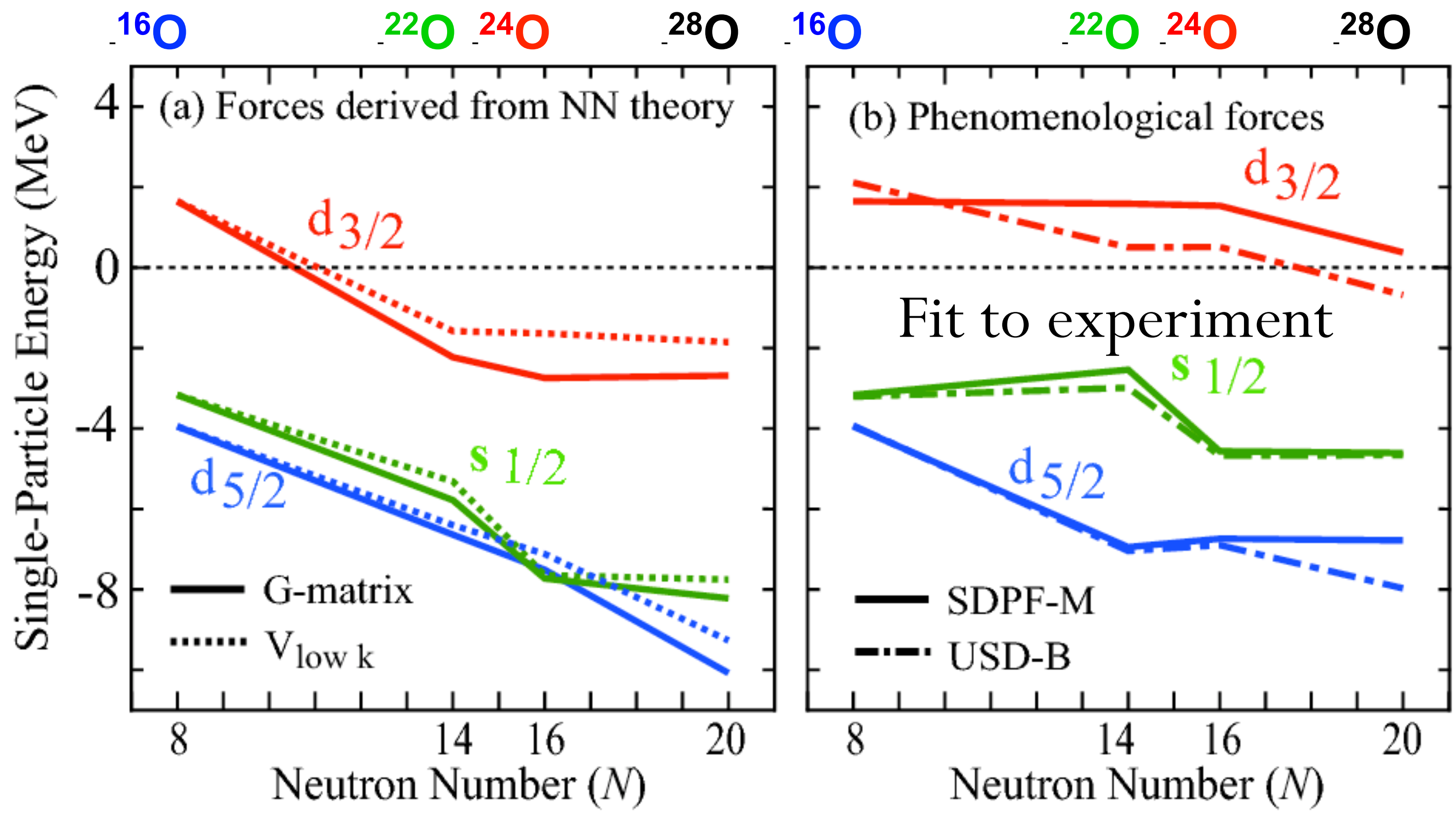
Microscopic picture: **NN-forces too attractive**

Incorrect prediction of dripline

Physics in Oxygen Isotopes

Calculate evolution of *sd*-orbital energies from interactions





Microscopic NN Theories

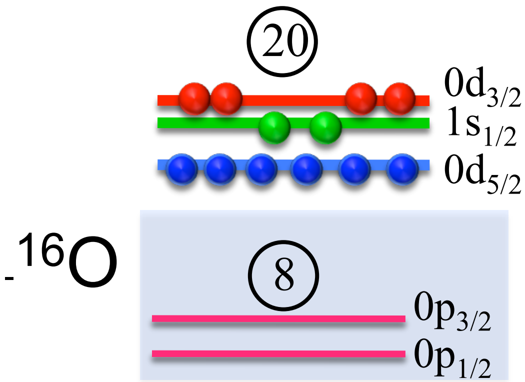
$d_{3/2}$ orbit bound to ^{28}O

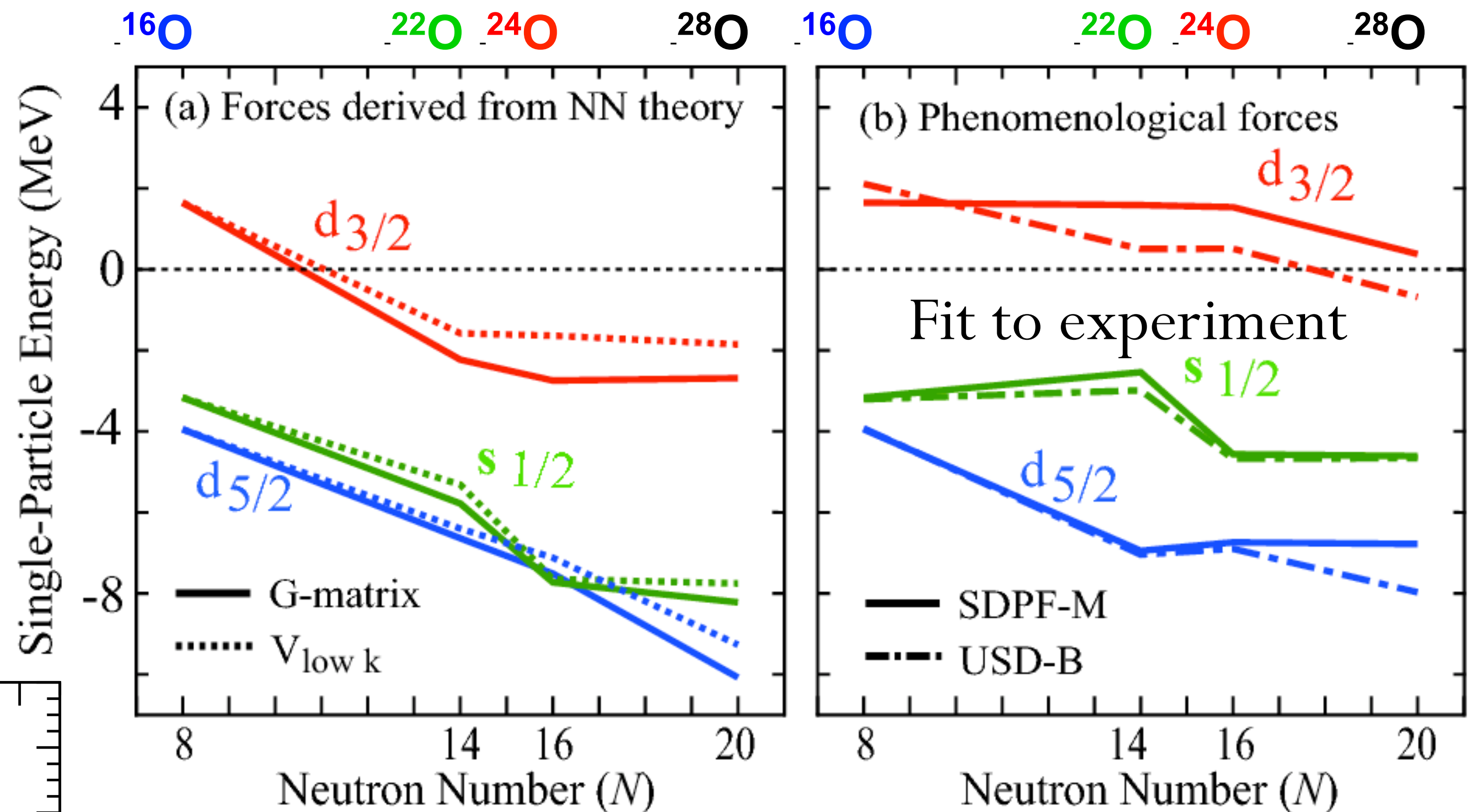
Phenomenological Models

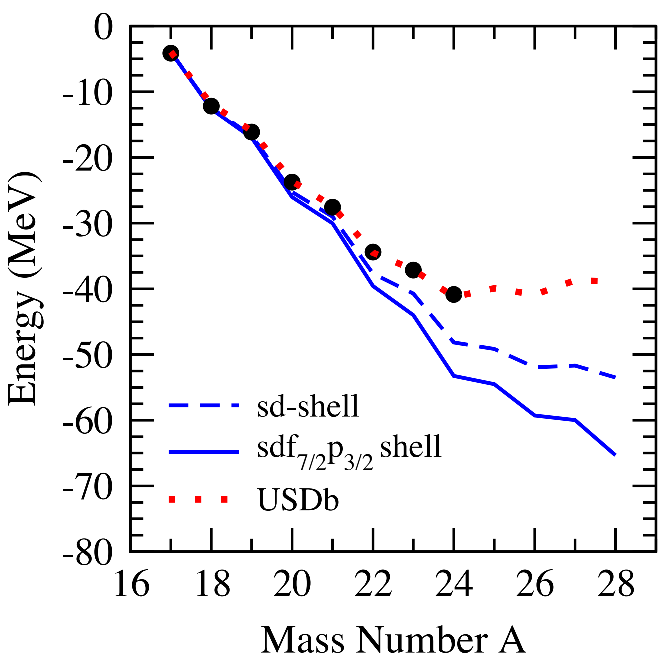
$d_{3/2}$ orbit unbound

Physics in Oxygen Isotopes

Calculate evolution of *sd*-orbital energies from interactions







Microscopic NN Theories	*Phenomenological Models*
$d_{3/2}$ orbit bound to ^{28}O	$d_{3/2}$ orbit unbound
Dripline at ^{28}O	Dripline at ^{24}O

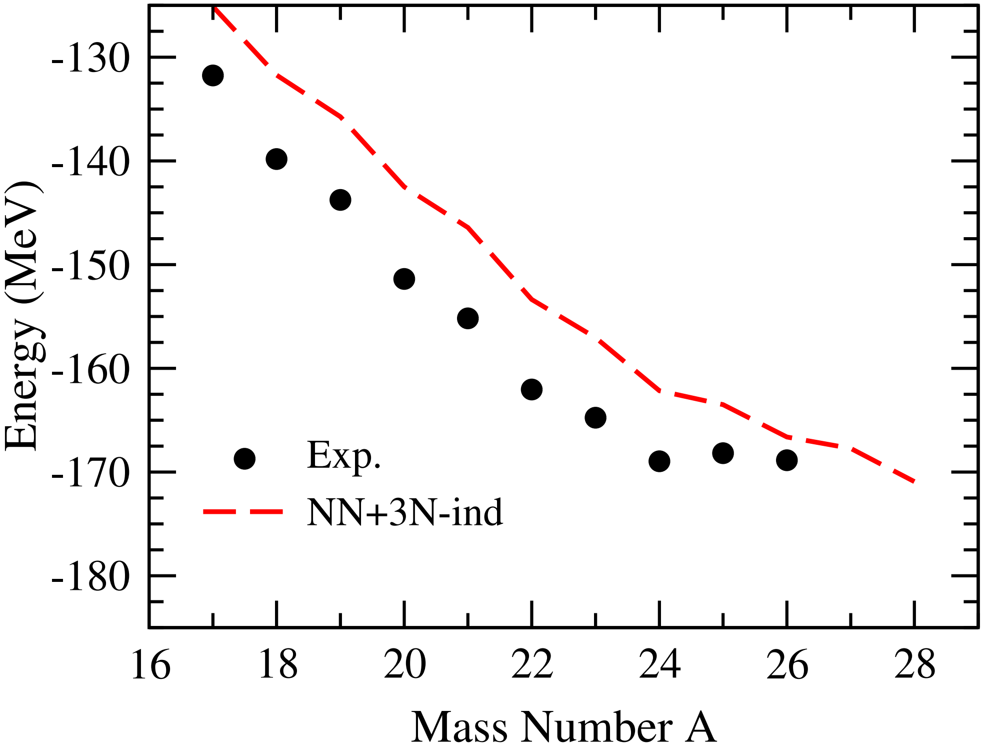
Oxygen anomaly unexplained with NN forces

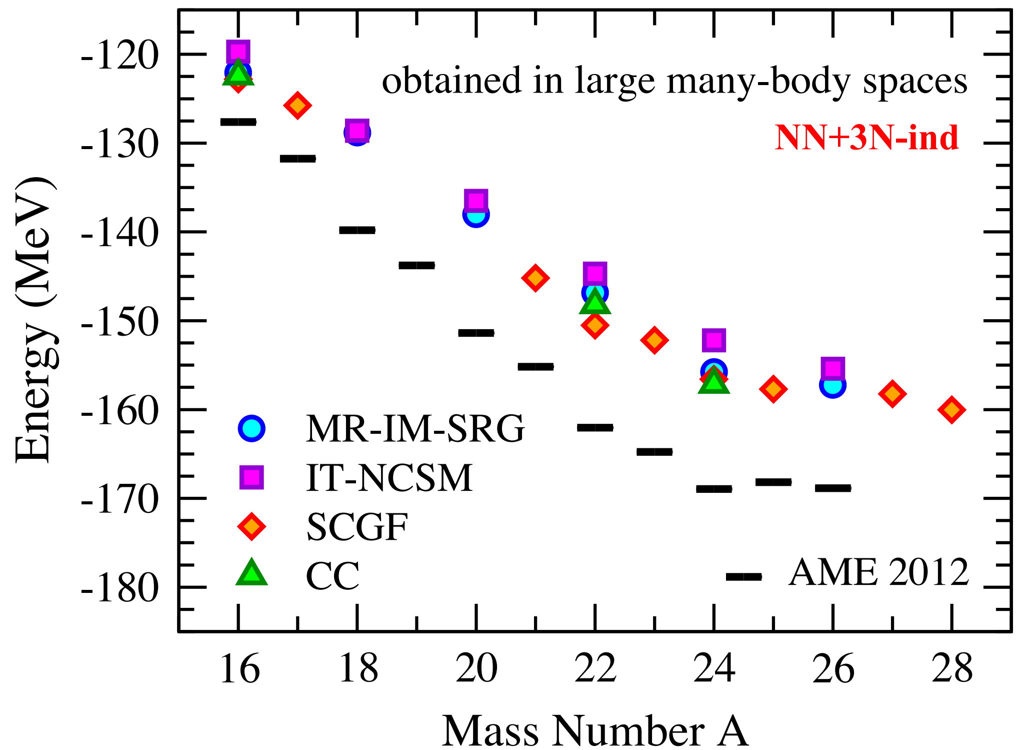
Origin of monopole shifts: Neglected 3N forces

-- **See lecture of A. Poves**

Comparison with Large-Space Methods

Large-space methods with **same SRG-evolved NN+3N-ind forces**



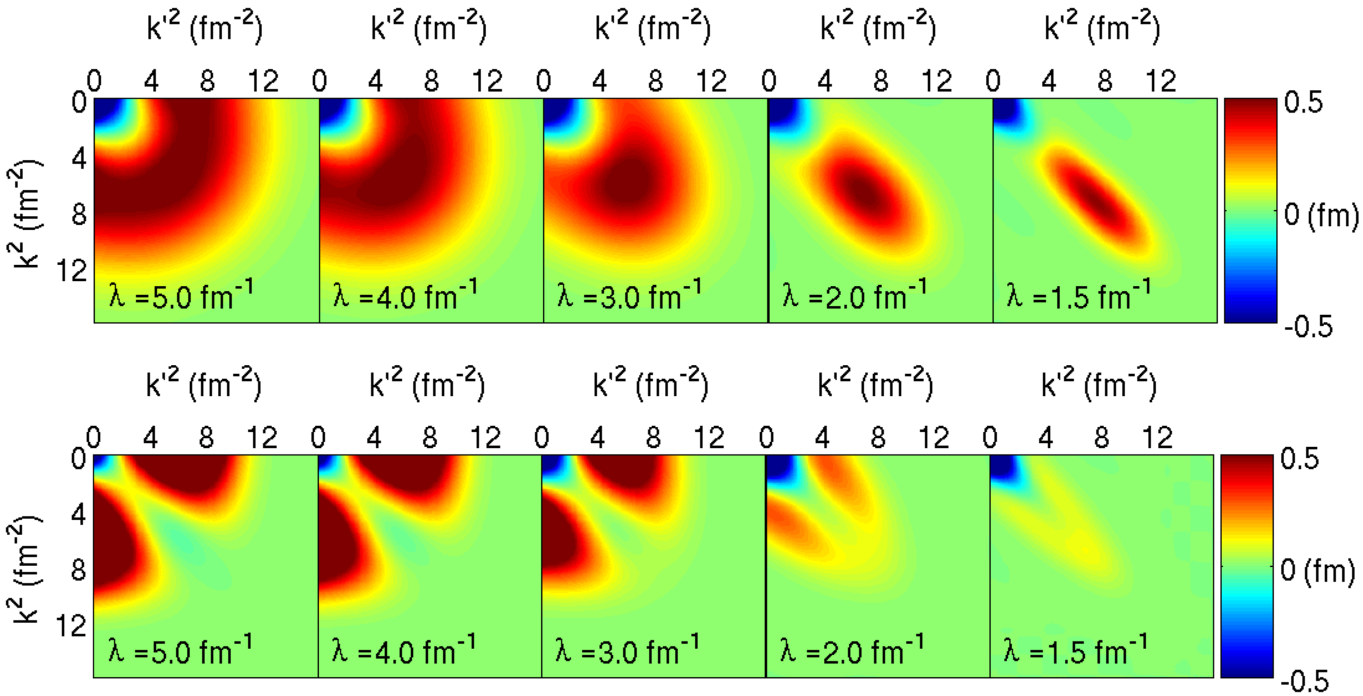


Agreement between all methods with same input forces

No reproduction of dripline in any case

Summary

Low-momentum interactions can be constructed from any V_{NN} via RG



Low-to-high momentum coupling not desirable in low-energy nuclear physics

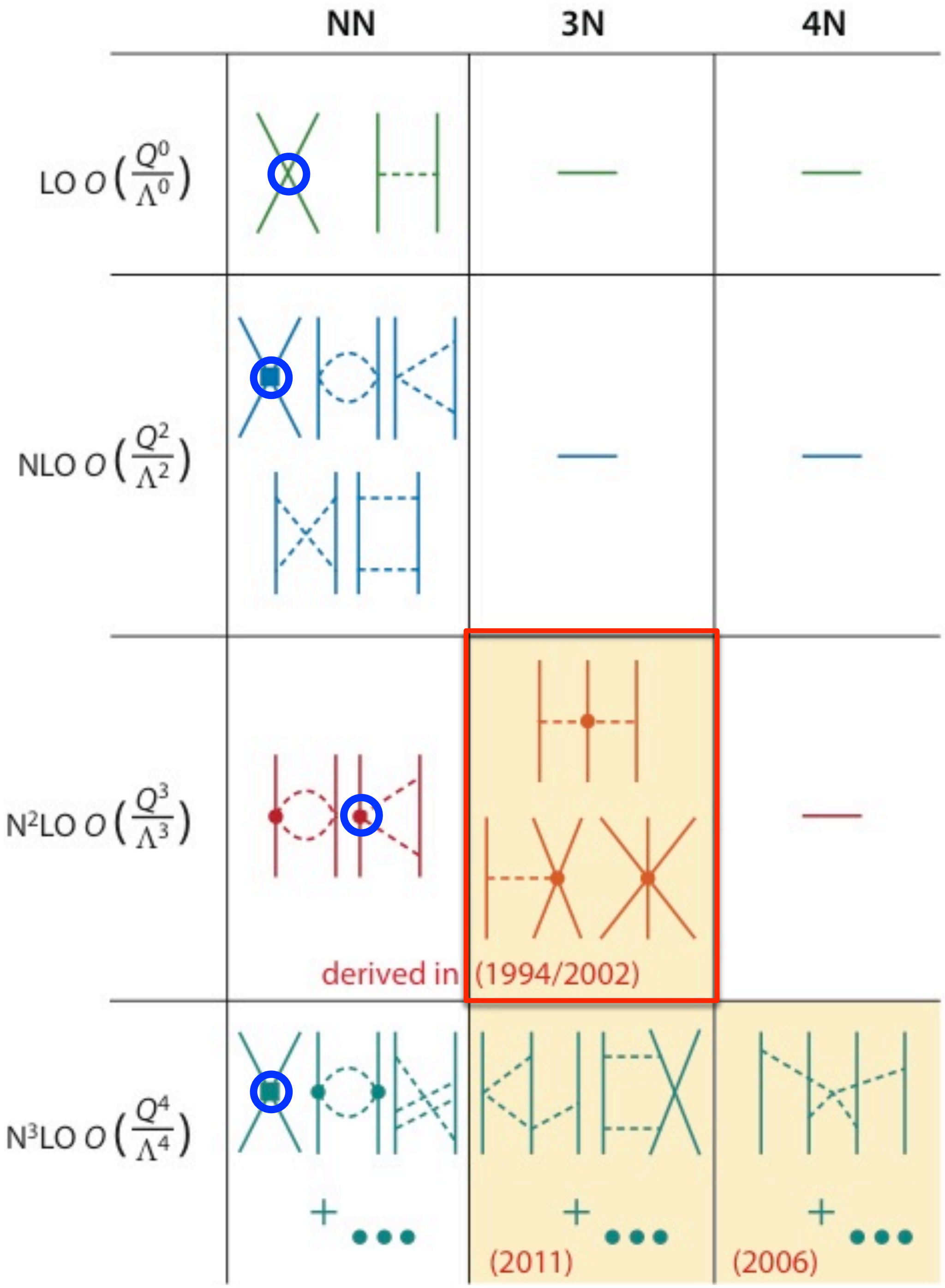
Evolve to low-momentum while preserving low-energy physics

Universality attained near cutoff of data

Low-momentum cutoffs remove low-to-high harmonic oscillator couplings

Cutoff variation assesses missing physics interaction level: tool not a parameter

Chiral Effective Field Theory: Nuclear Forces

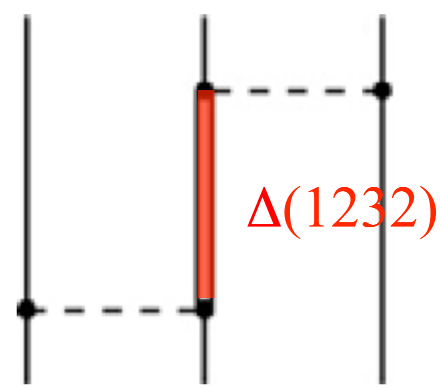


Weinberg, van Kolck, Kaplan, Savage, Wise

Nucleons interact via pion exchanges and contact interactions

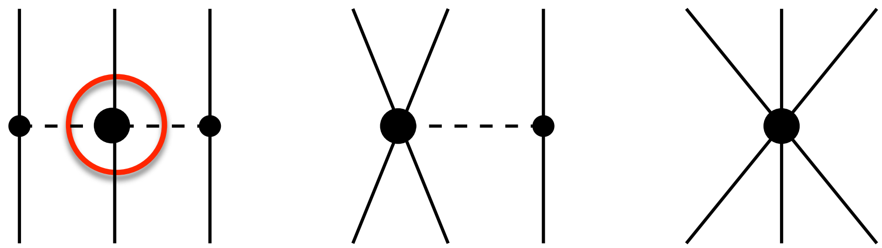
Consistent treatment of NN, 3N,…

NN couplings fit to scattering data



Chiral EFT: N^{2}LO 3N

First non-vanishing 3N contributions: Next-to-next-to-leading order $\nu = 3$



$$V_{3N}^{(3)} = \frac{g_A^2}{8F_\pi^4} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2][q_3^2 + M_\pi^2]} [\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 (-4 c_1 M_\pi^2 + 2 c_3 \vec{q}_1 \cdot \vec{q}_3) + c_4 \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2]$$

$$- \frac{g_A D}{8F_\pi^2} \frac{\vec{\sigma}_3 \cdot \vec{q}_3}{q_3^2 + M_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{q}_3 + \frac{1}{2} E \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3$$

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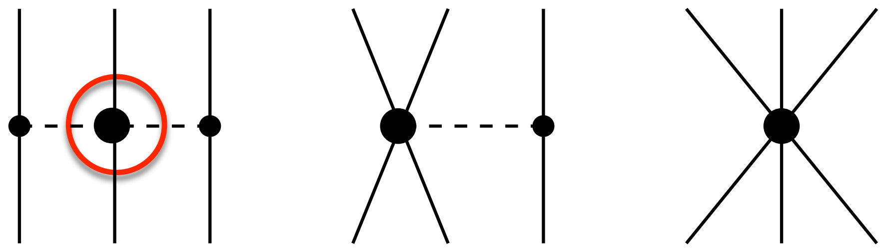

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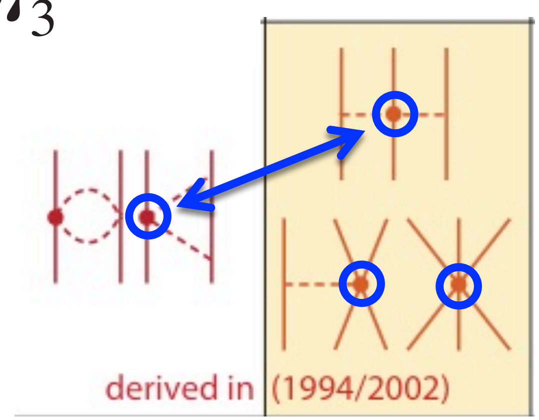
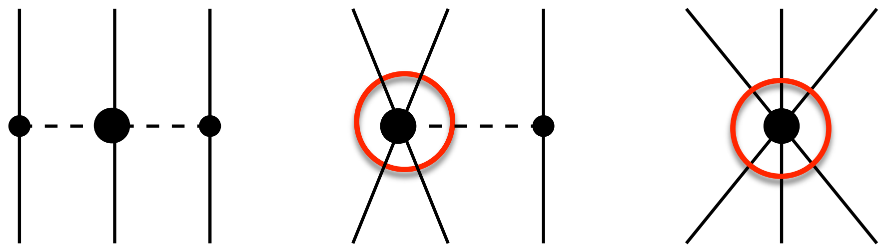
Three undetermined πN couplings from NN fit



Chiral EFT: N^{2}LO 3N

First non-vanishing 3N contributions: Next-to-next-to-leading order $\nu = 3$



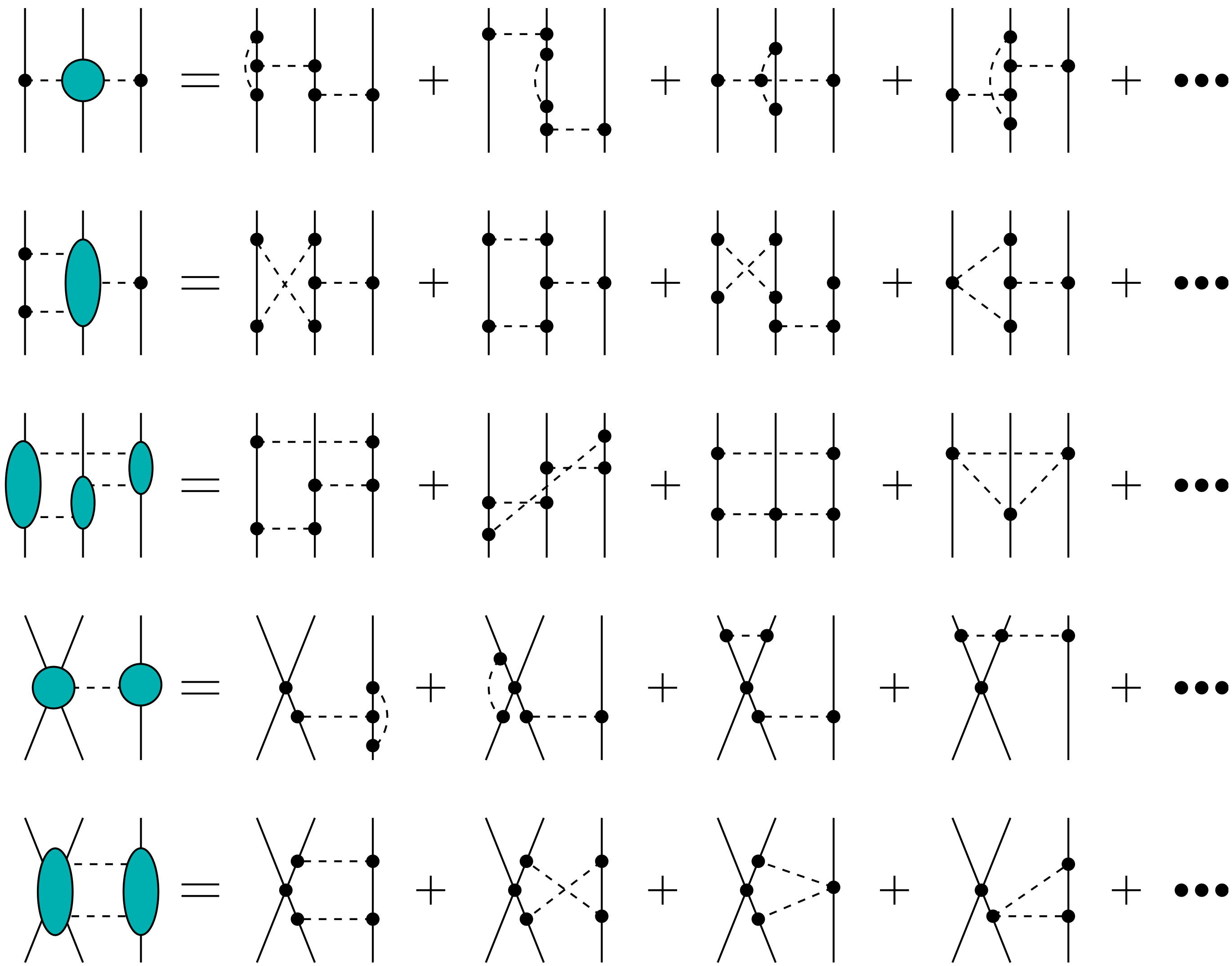
$$V^{(3)}_{1\pi,\,\mathrm{cont}} = -\sum_{i\neq j\neq k}\left(\frac{g_A}{8F_\pi}\right)^2 D \frac{\left(\vec{\sigma}_j\cdot\vec{q}_j\right)}{\left(\vec{q}_j^{\,2}+M_\pi^2\right)}\left(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\right)\left(\vec{\sigma}_i\cdot\vec{\sigma}_j\right)$$

$$V^{(3)}_{\mathrm{cont}} = \frac{1}{2}\sum_{j\neq k} E\left(\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k\right)$$

Two new couplings cD, cE

Chiral EFT: N^{3}LO 3N

Next-to-next-to-next-to-leading order $\nu = 4$



Good news: **no new constants**   Bad news: well, there's all this

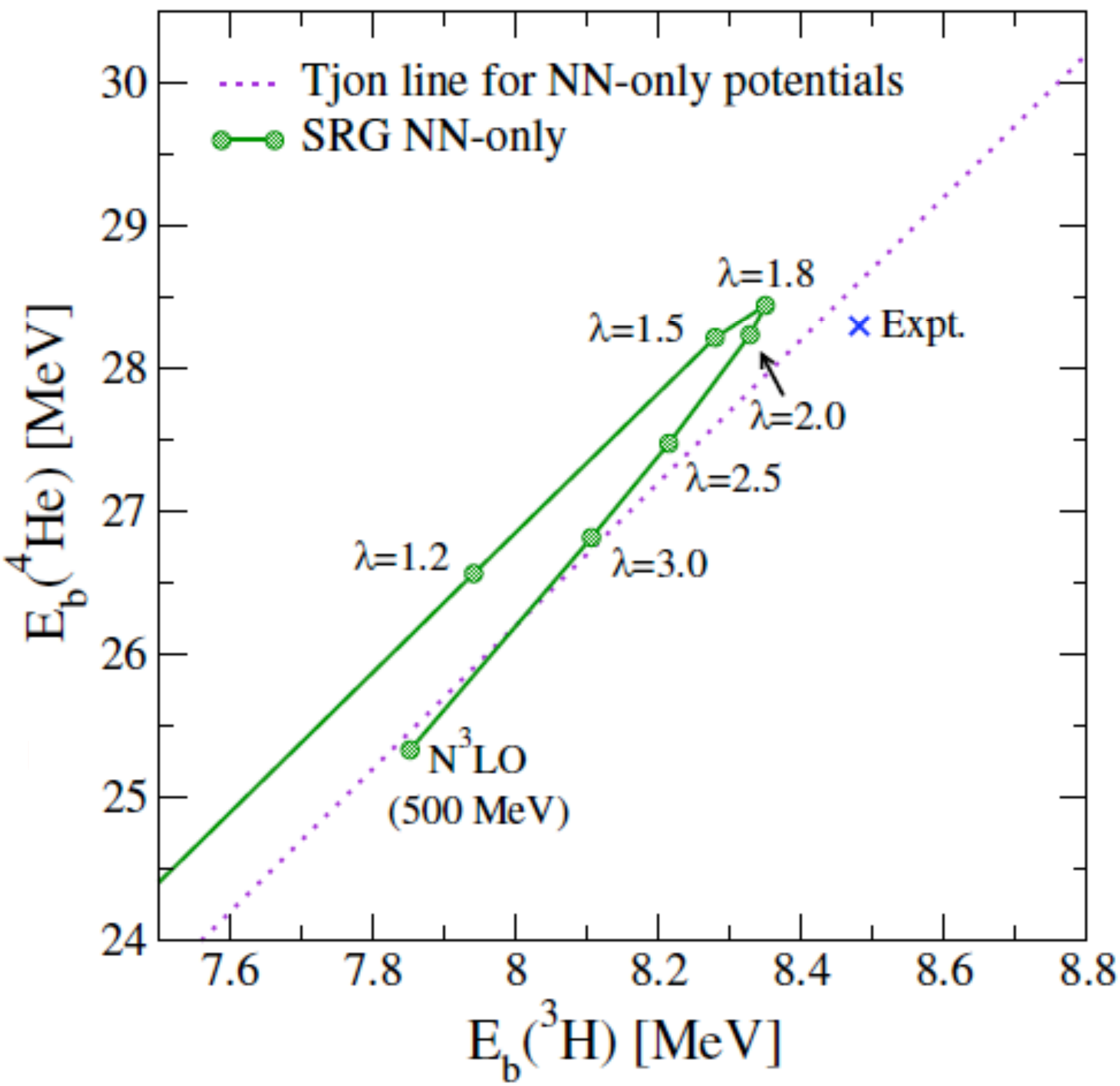
Benefits of Lower Cutoffs

Use cutoff dependence to assess missing physics: return to Tjon line

Varying cutoff moves along line

Still never reaches experiment

Tool, not a parameter!



Benefits of Lower Cutoffs

Use cutoff dependence to assess missing physics: return to Tjon line

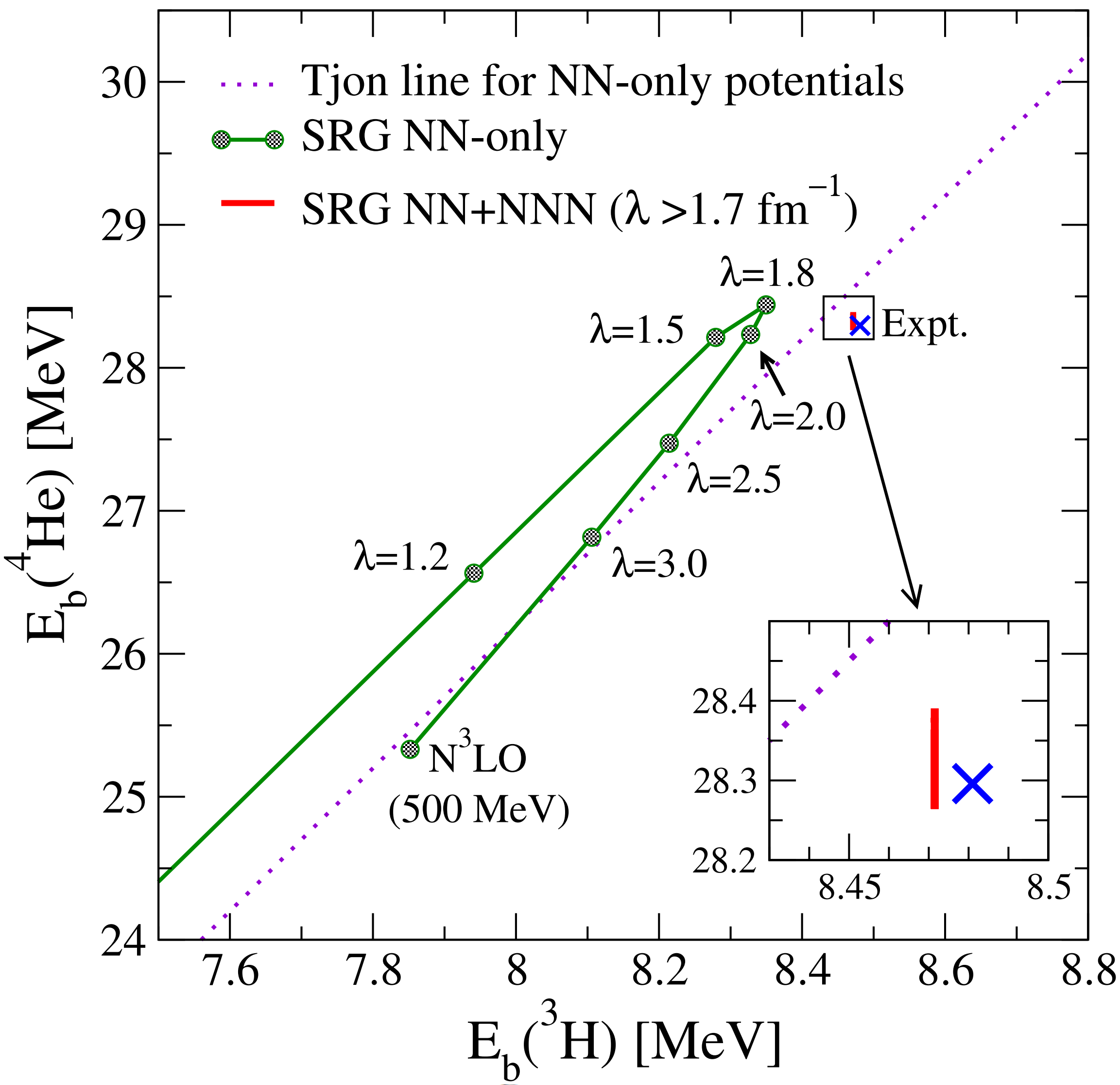
Varying cutoff moves along line

Still never reaches experiment

Tool, not a parameter!

Including 3N reaches expt.

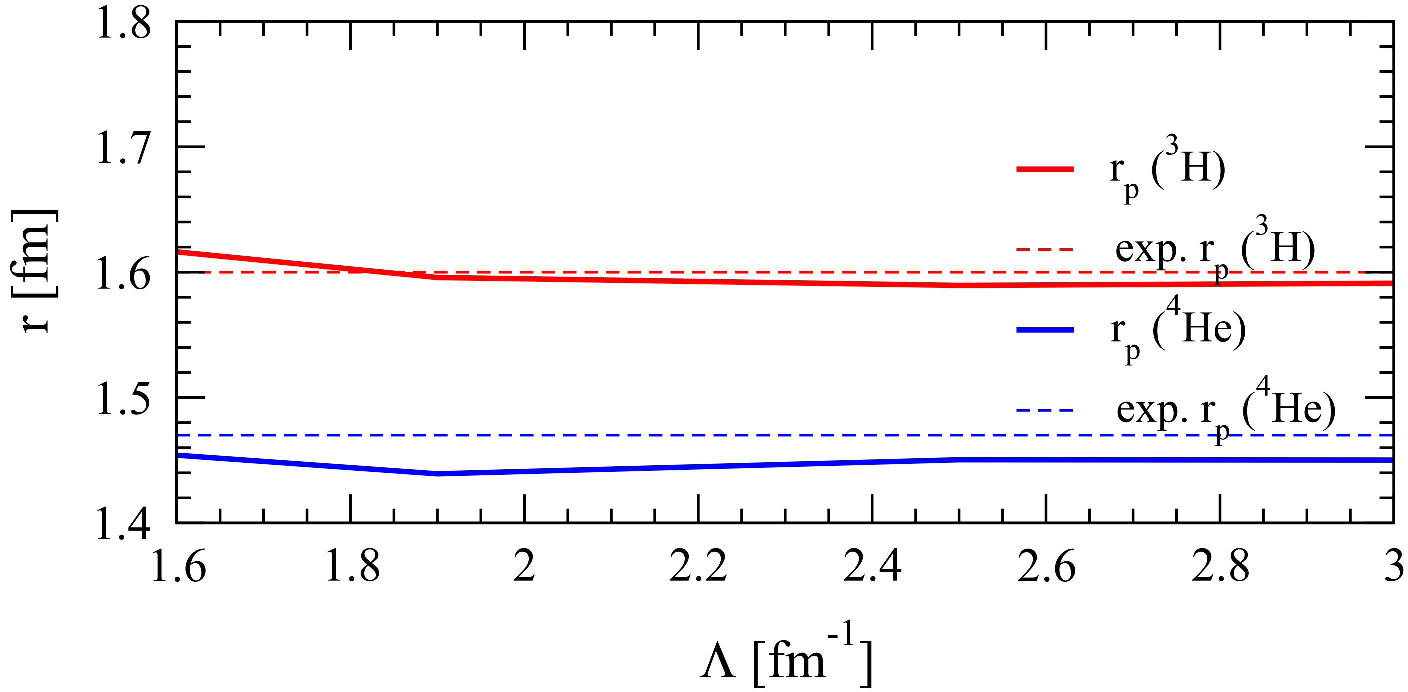
Why not perfect fit?



Cutoff Variation with 3N Forces

Use cutoff variation to assess missing physics in few body systems

Radii of triton and alpha particle calculated from NN+3N forces

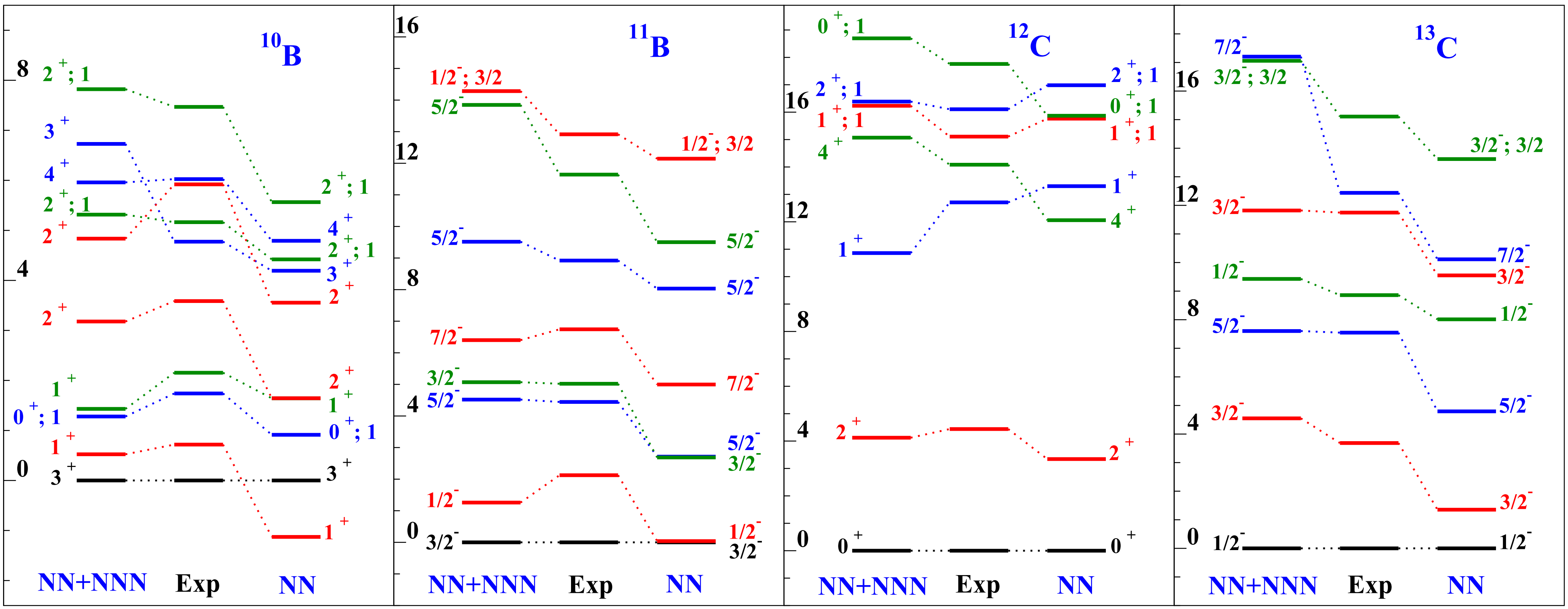


Minimal cutoff variation

Chiral Three-Body Forces in Light Nuclei

Importance of chiral 3N forces established in light nuclei

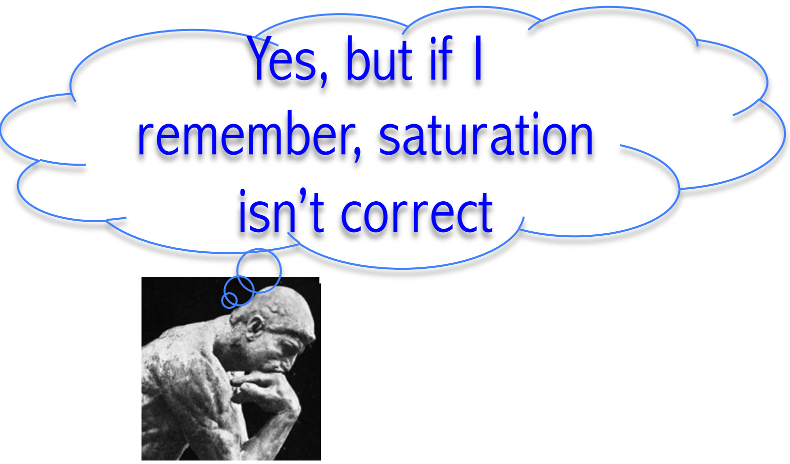
Converged NCSM (Navratil 2007)

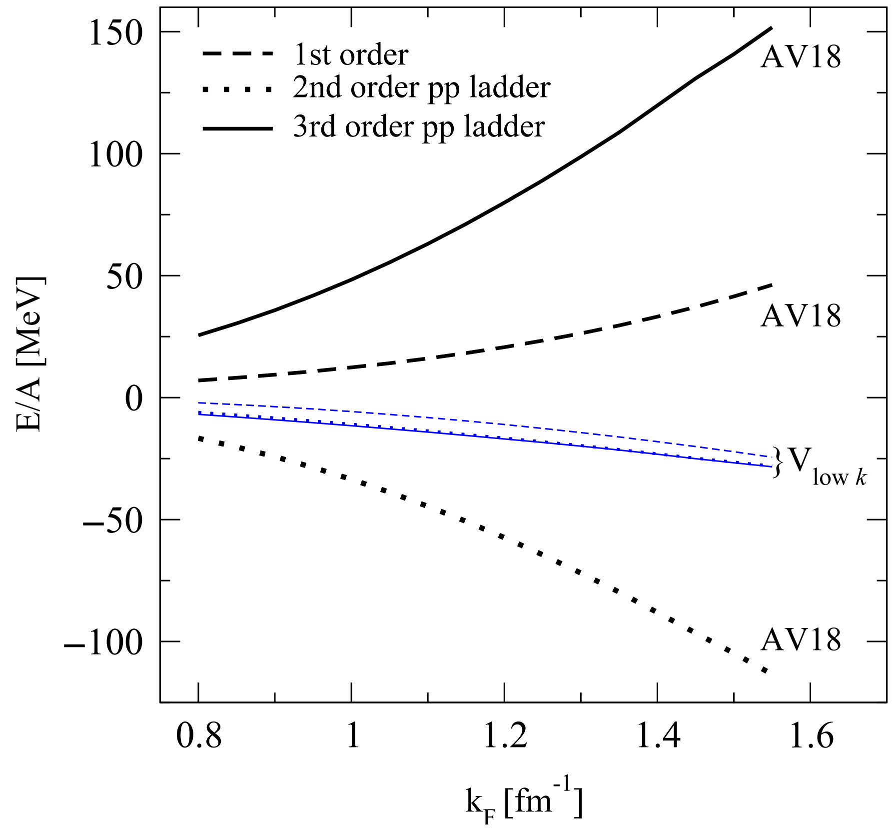


They work! What about nuclear matter?

Perturbative in Symmetric Nuclear Matter?

$$H\left(\Lambda\right) = T + V_{\mathrm{NN}}\left(\Lambda\right) + V_{3\mathrm{N}}\left(\Lambda\right) + V_{4\mathrm{N}}\left(\Lambda\right) + \cdots$$

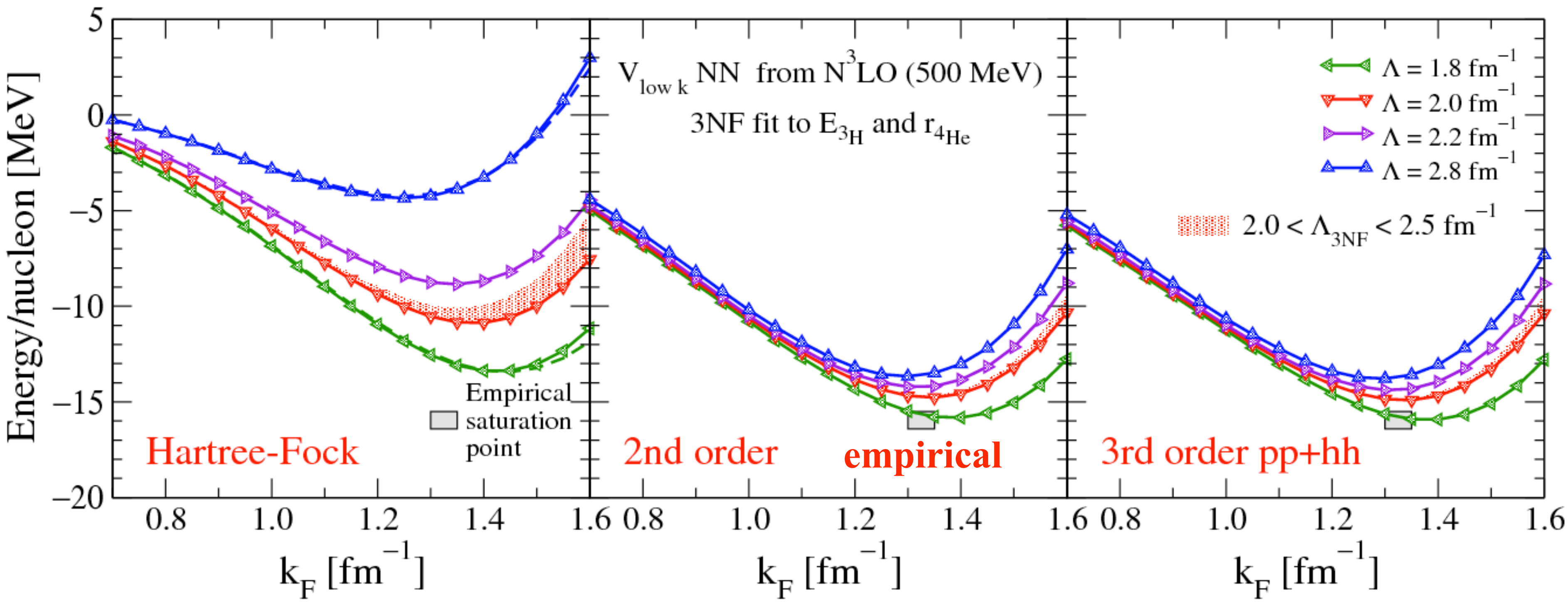




Significant improvement with low-momentum interactions!

Perturbative in Symmetric Nuclear Matter?

$$H(\Lambda) = T + V_{\rm NN}(\Lambda) + V_{\rm 3N}(\Lambda) + V_{\rm 4N}(\Lambda) + \cdots$$

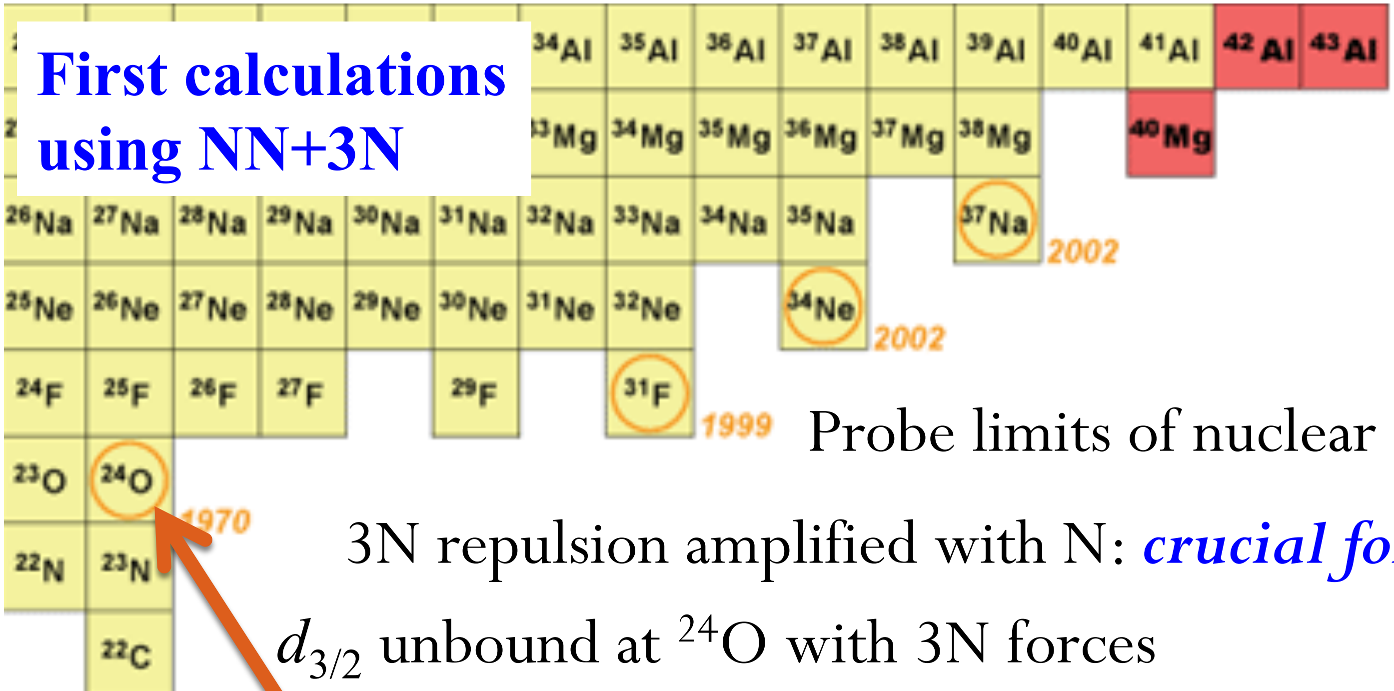


Now NN+3N-fit remain perturbative and reproduce saturation!

Minor but non-negligible cutoff variation

Oxygen Anomaly

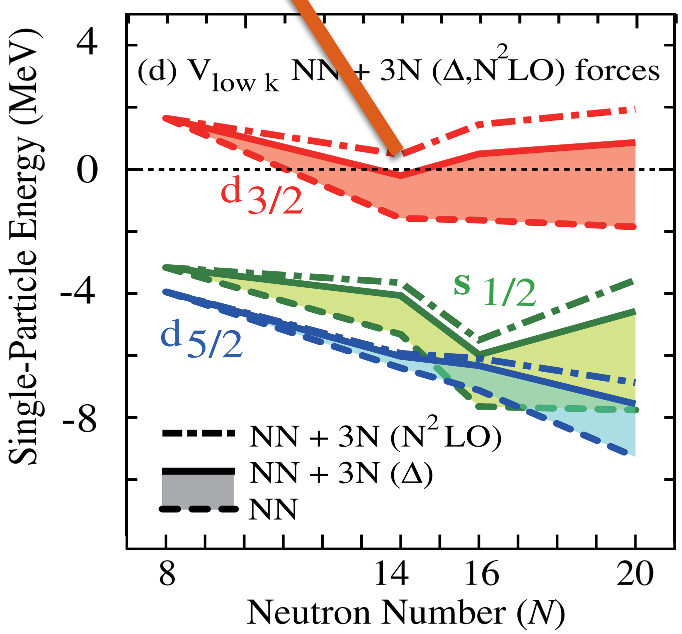
First calculations using NN+3N



Probe limits of nuclear existence with 3N forces

3N repulsion amplified with N: ***crucial for neutron-rich nuclei***

$d_{3/2}$ unbound at ^{24}O with 3N forces



Otsuka, Suzuki, JDH, Schwenk, Akaishi, PRL (2010)

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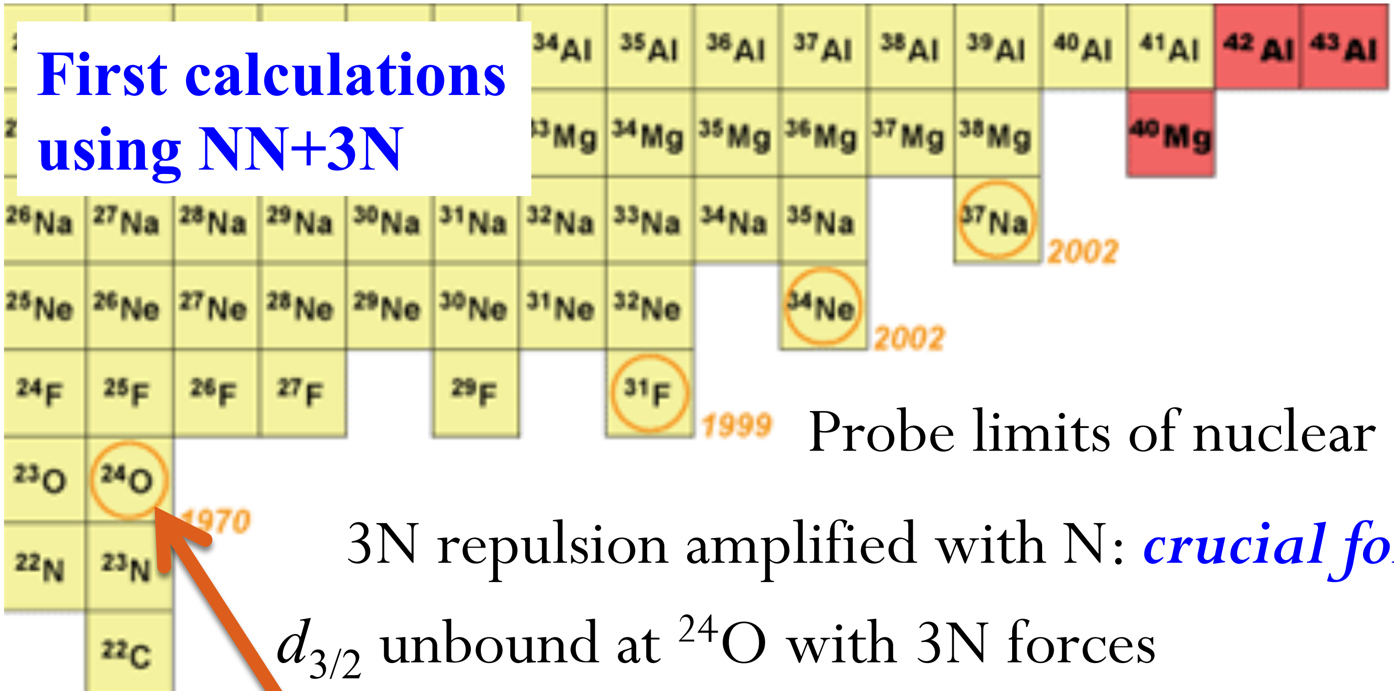

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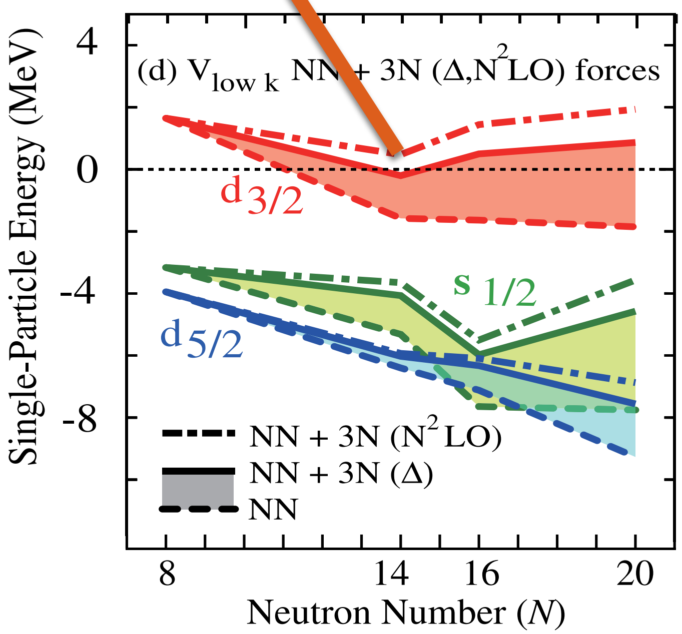
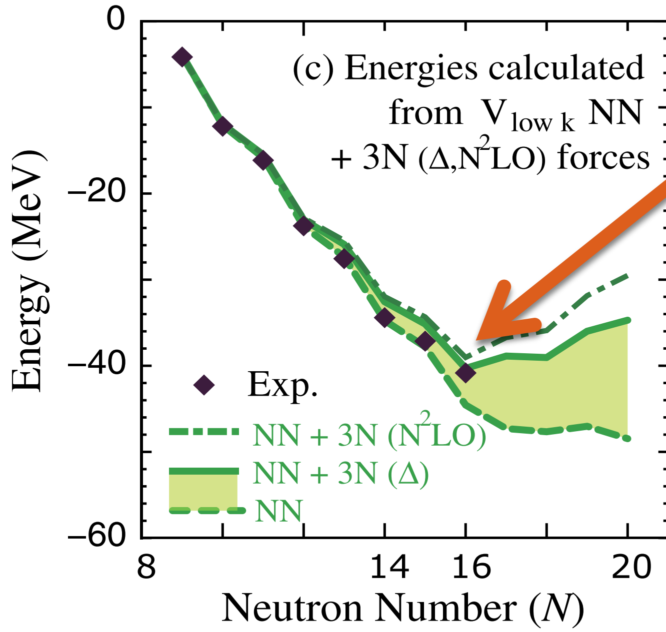




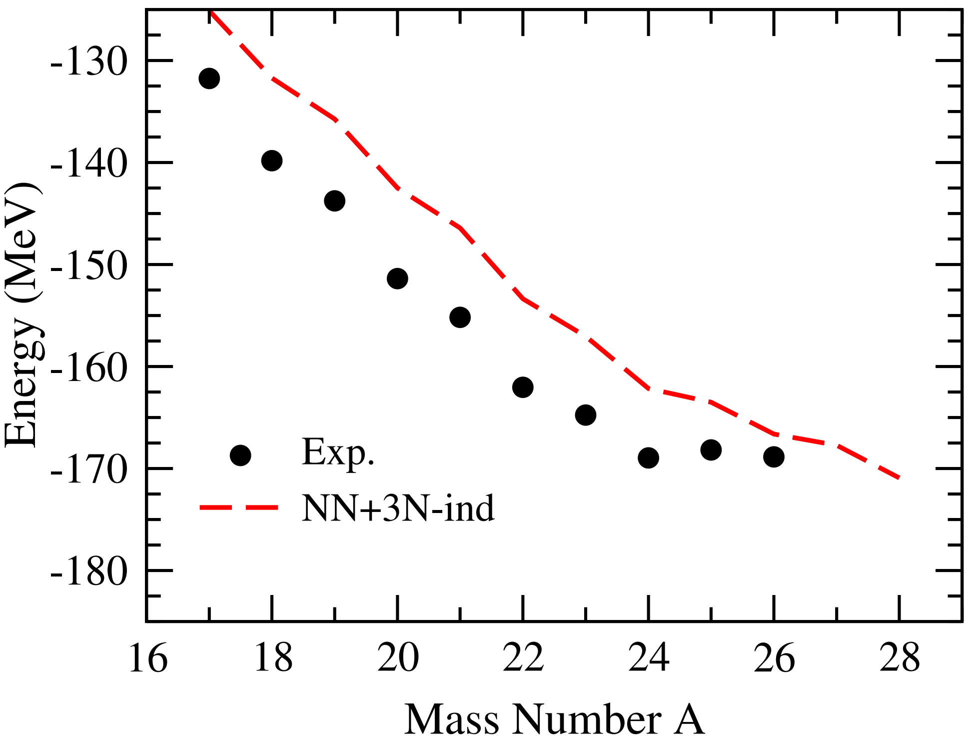
Isotopes unbound beyond ^{24}O

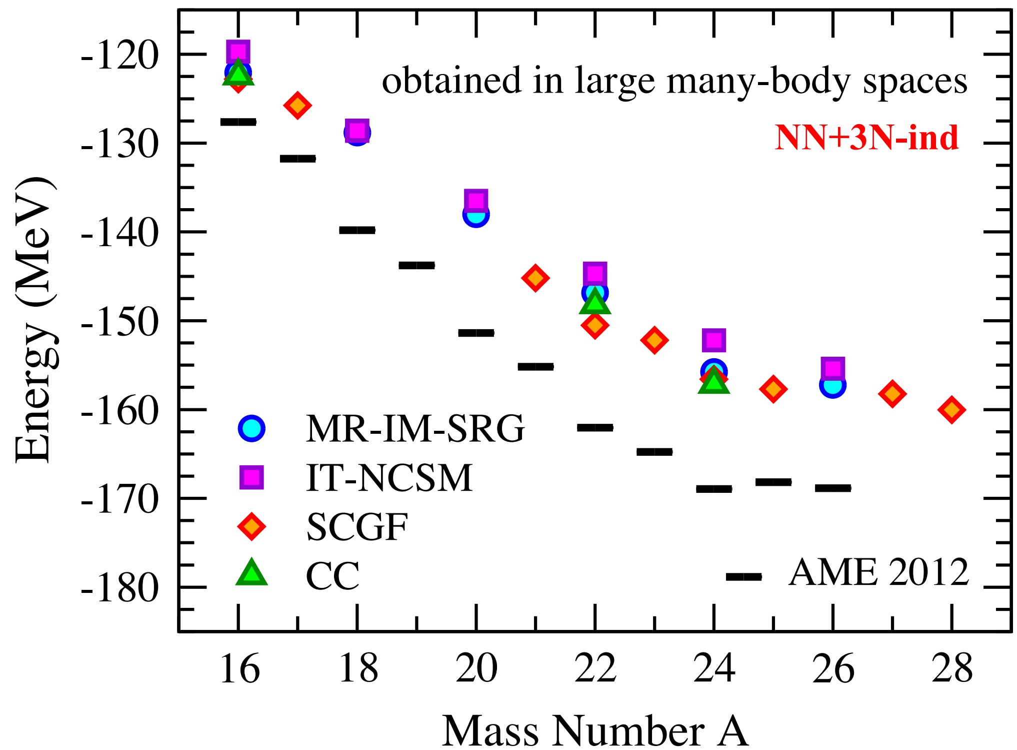
First microscopic explanation of oxygen anomaly

Otsuka, Suzuki, JDH, Schwenk, Akaishi, PRL (2010)

Comparison with Large-Space Methods

Large-space methods with **same SRG-evolved NN+3N-ind forces**



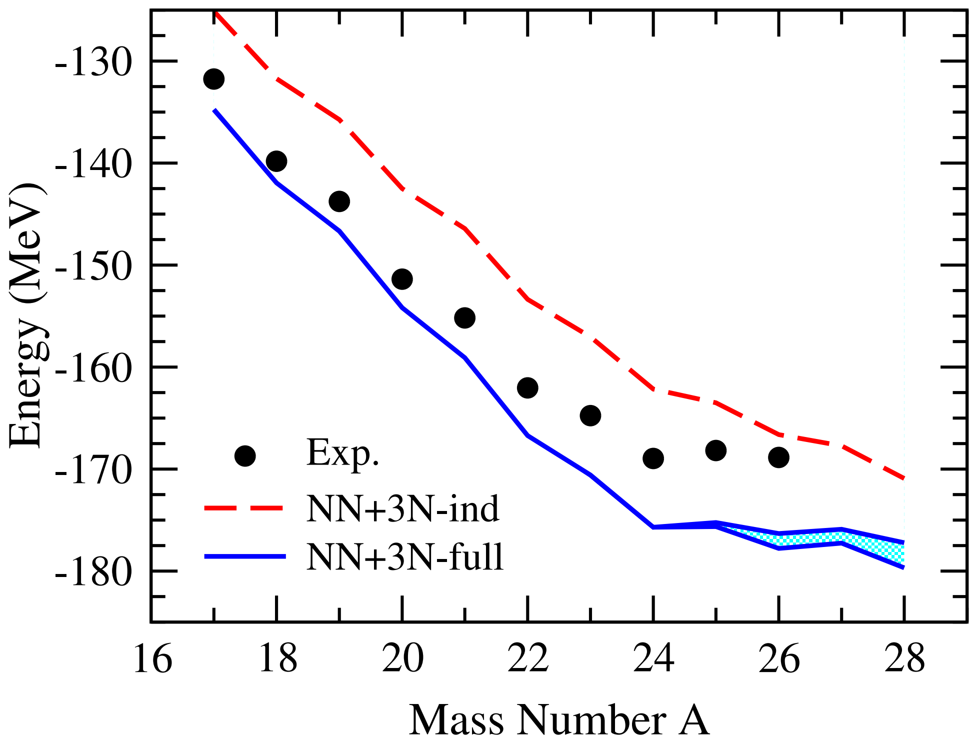


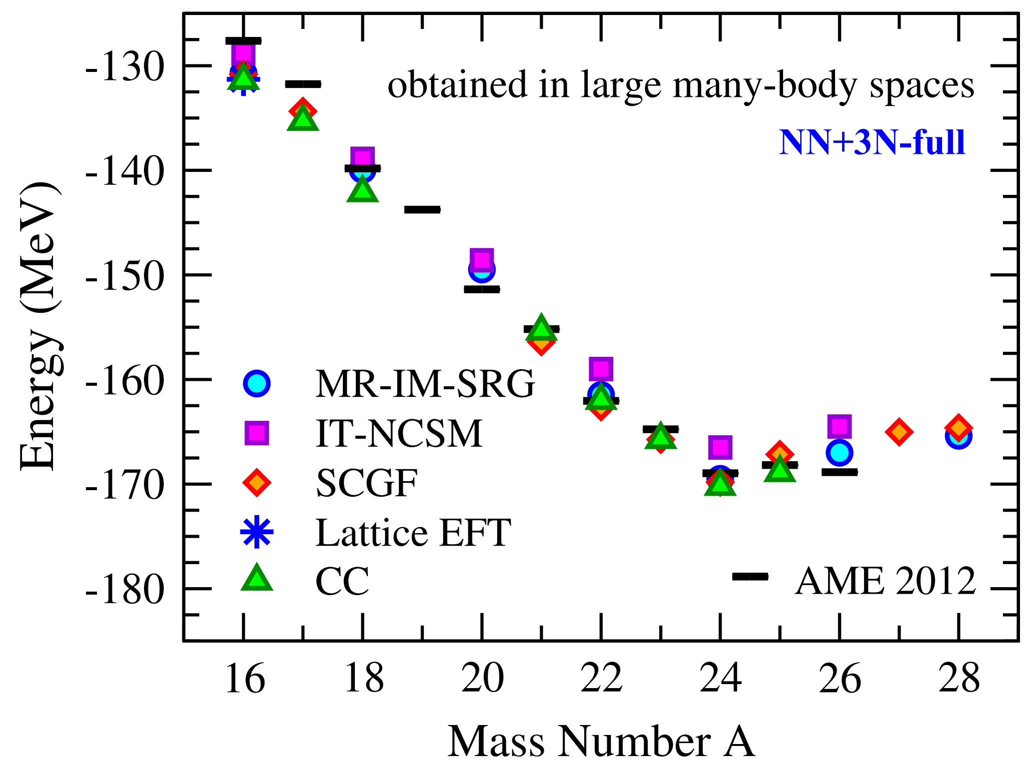
Agreement between all methods with same input forces

No reproduction of dripline in any case

Comparison with Large-Space Methods

Large-space methods with **same SRG-evolved NN+3N-full forces**





Agreement between all methods with same input forces

Clear improvement with NN+3N-full

Validates valence-space results

Further Readings

Lepage, nucl-th/9706029 (1997)

Epelbaum, Hammer, Meißner, Rev. Mod. Phys. (2009)

Machleidt, Entem, Phys. Rep. (2011)

Bogner, Furnstahl, Schwenk, Prog. Part. Nucl. Phys. (2010)

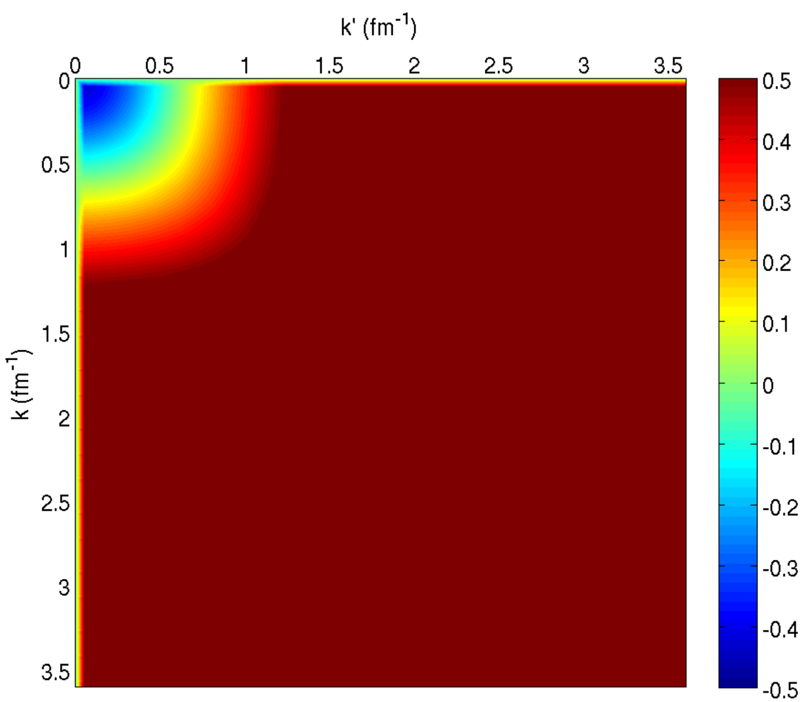
Hebeler, Holt, Menendez, Schwenk, Ann. Rev. Nucl. Part. Sci. (2015)

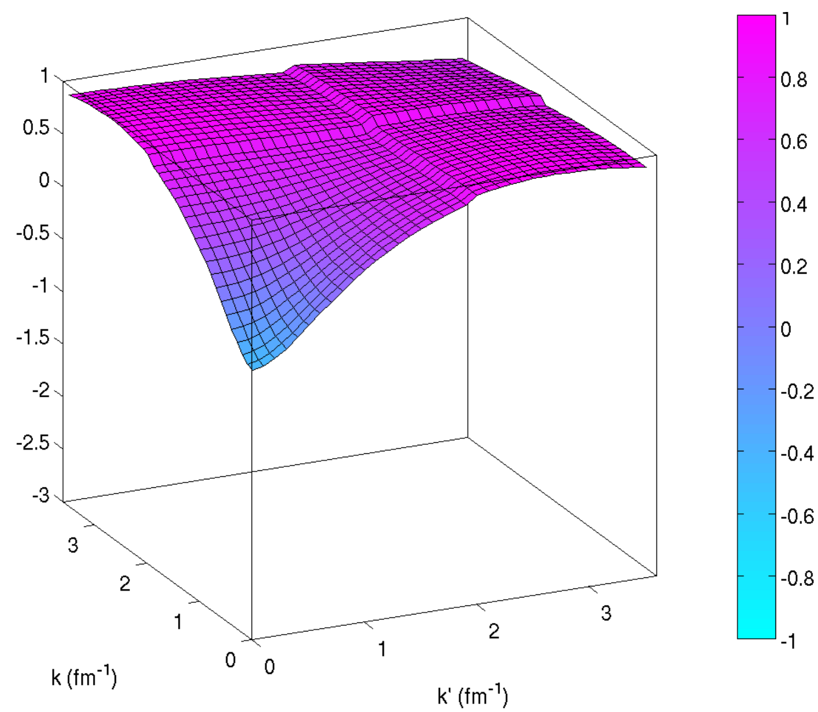
Other Generator Choices: Block Diagonal

Create block diagonal form like V_{lowk}?

$$G(s) = H_{\text{BD}} = \begin{pmatrix} PH(s)P & 0 \\ 0 & QH(s)Q \end{pmatrix}$$

With alternate definition of flow parameter: $\lambda^2 = \frac{1}{\sqrt{s}}$





Argonne V_{18} $^3S_1$

$\lambda = 10.0\,\text{fm}^{-1}$

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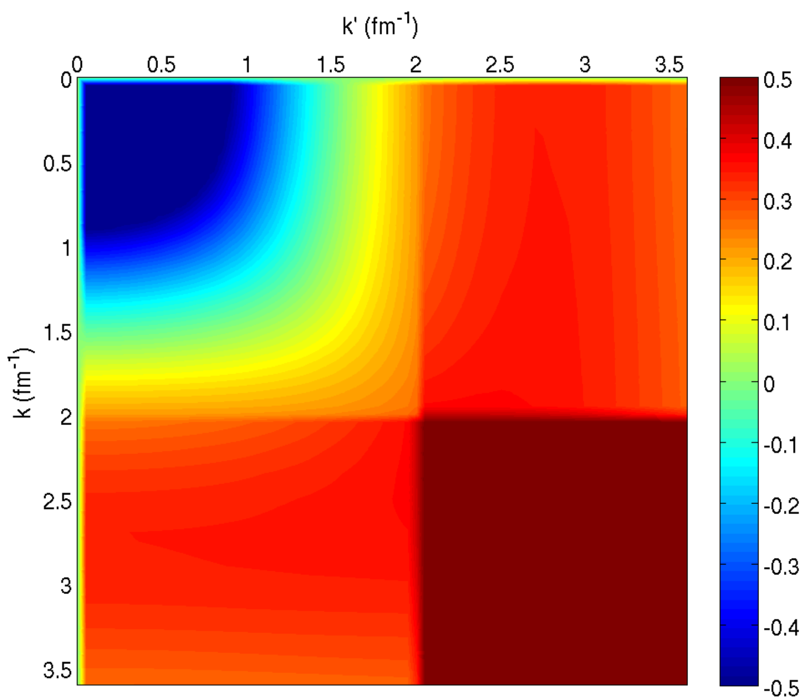
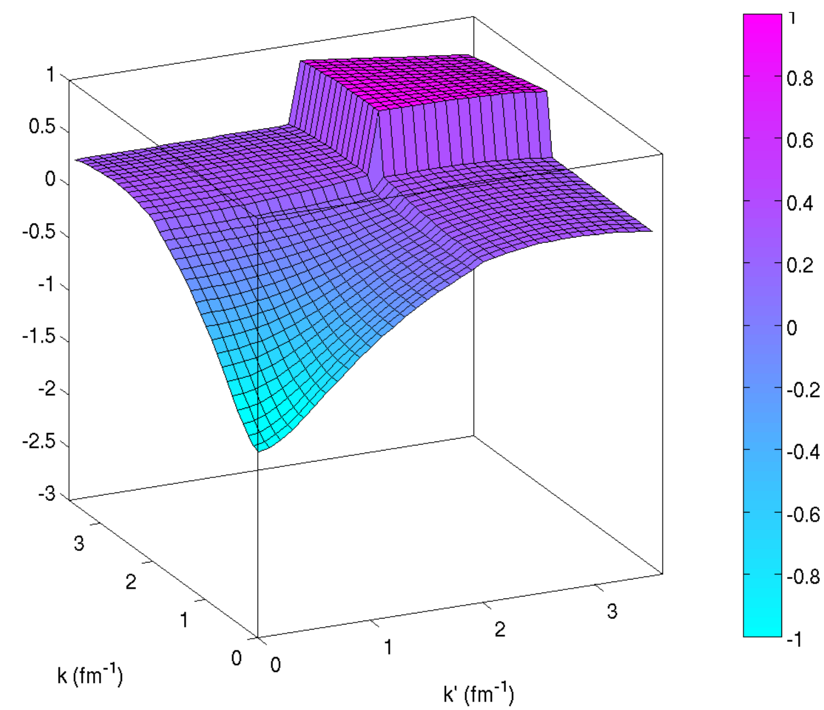




Argonne V_{18} $^3S_1$

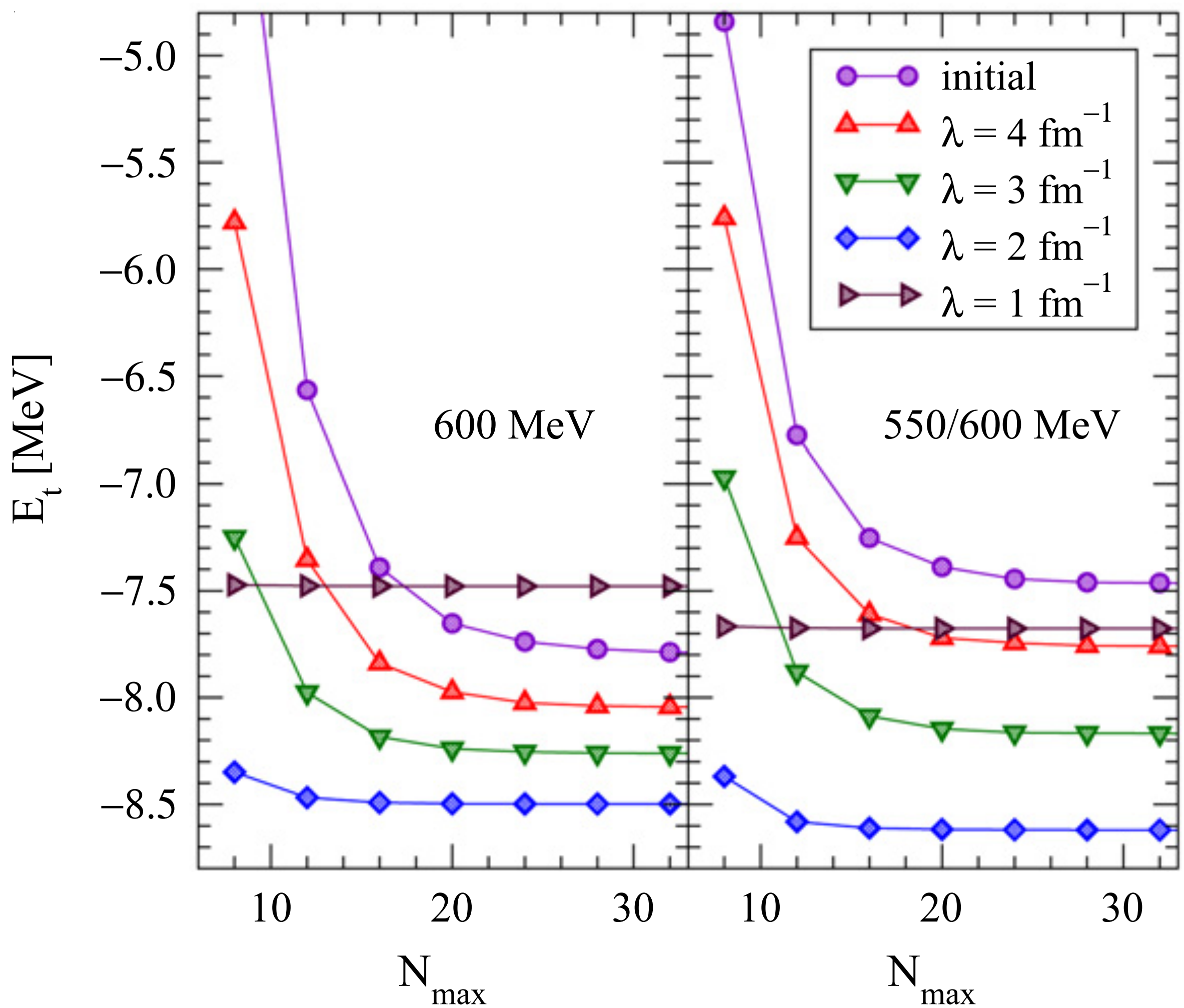
$\lambda = 5.0\,\text{fm}^{-1}$

Benefits of Lower Cutoffs

Triton binding energy - again clearly improved convergence behavior

Clear dependence on cutoff – more than one, look closely…

What is the source(s)?



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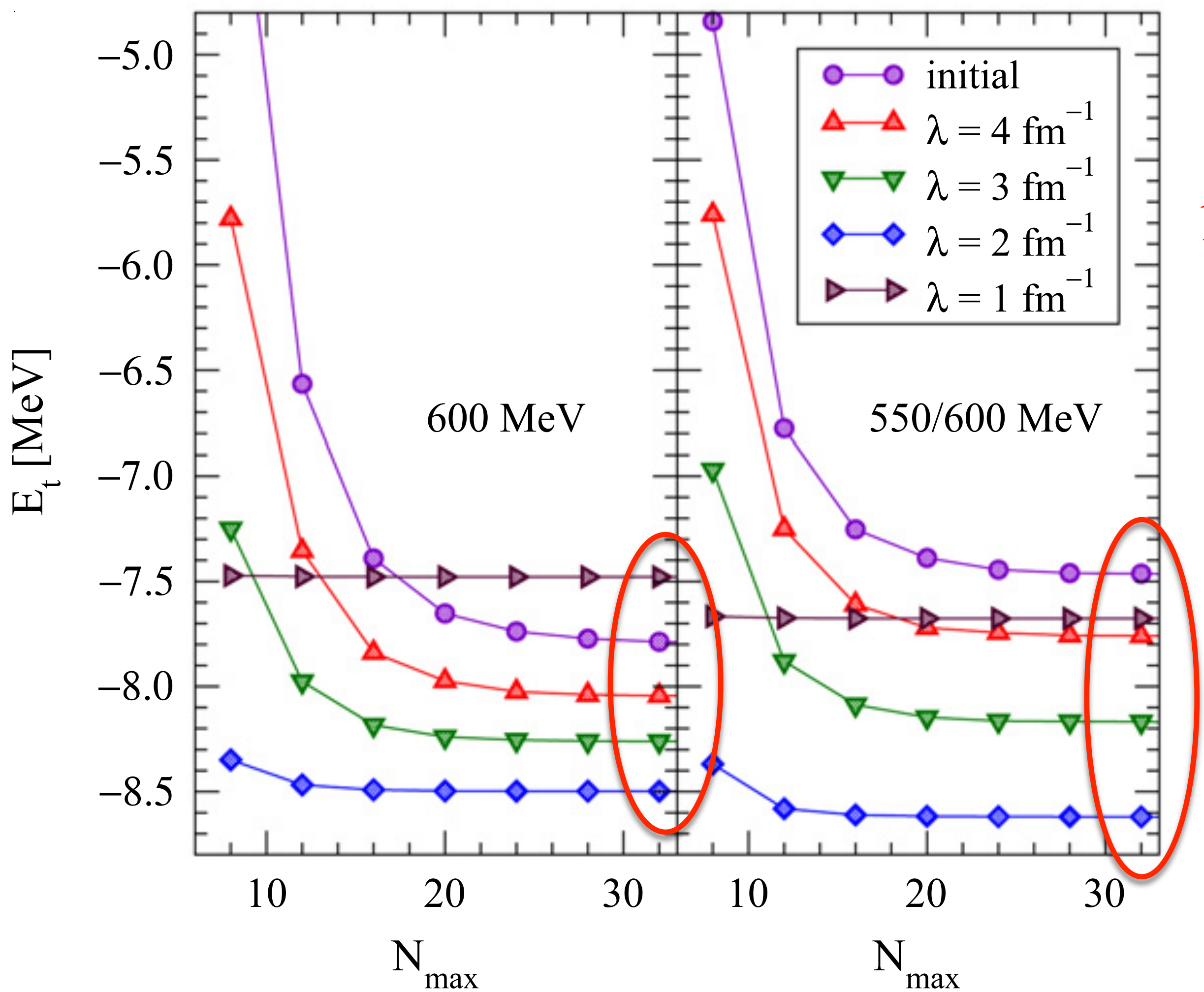

1) SRG cutoff dependence

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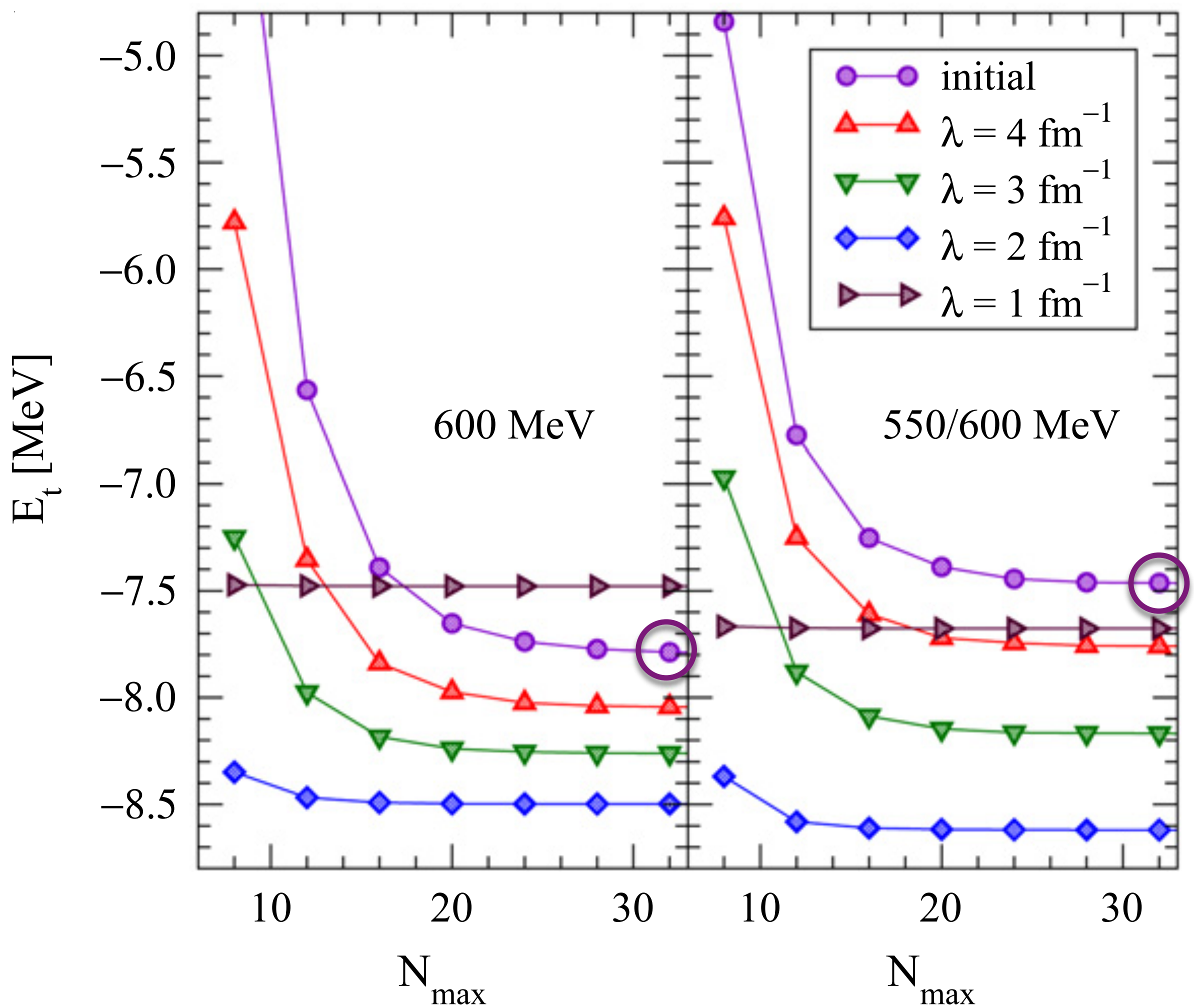

1) SRG cutoff dependence

2) Initial cutoff dependence

Something missing in each case!

Case 1: Price of Low Cutoffs = Induced Forces

Life Lesson: no free lunch ☹

Consider Hamiltonian with only two-body forces:

$$H = T + V_{\mathrm{NN}}$$

And $\eta(s) = [T, H(s)]$

$$\frac{\mathrm{d}H(s)}{\mathrm{d}s} = [\eta(s), H(s)] = [[T, T + V(s)], T + V(s)]$$

Simply expand with creation/annihilation operators:

Case 1: Price of Low Cutoffs = Induced Forces

Life Lesson: no free lunch ☹

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And $\eta(s) = [T, H(s)]$

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Simply expand with creation/annihilation operators:

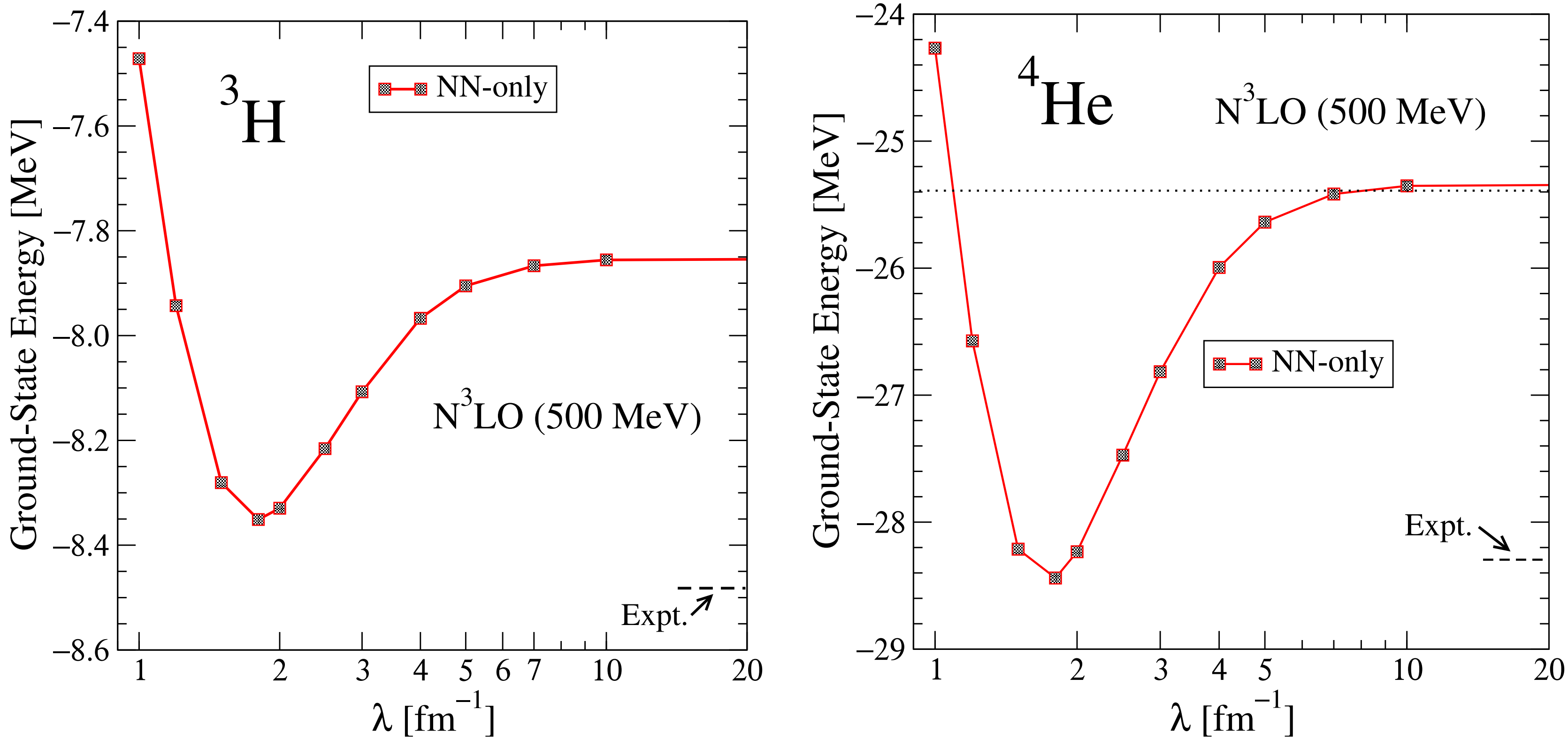
$$\frac{\mathrm{d}V(s)}{\mathrm{d}s} = \left[\left[\sum a^\dagger a, \sum a^\dagger a^\dagger a a\right], \sum a^\dagger a^\dagger a a\right] = \cdots + \sum a^\dagger a^\dagger a^\dagger a a a + \cdots$$

Three-body terms will appear even when initial 3-body forces absent

Call these **induced 3N forces (3N-ind)**

Induced 3N Forces

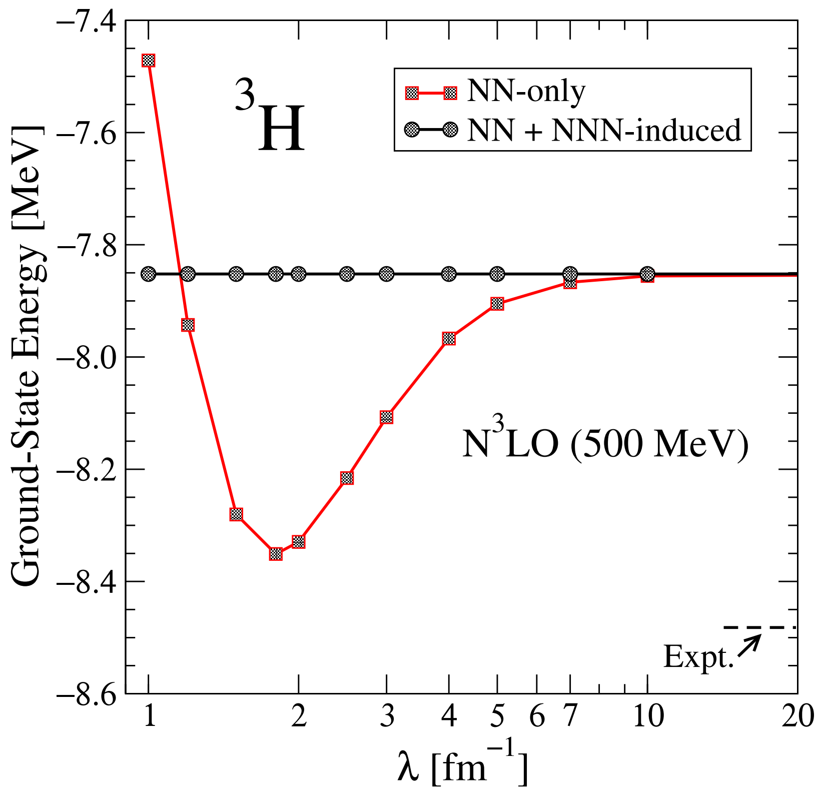
Effect of including 3N-ind? Exactly initial V_{NN} **up to neglected 4N-ind**

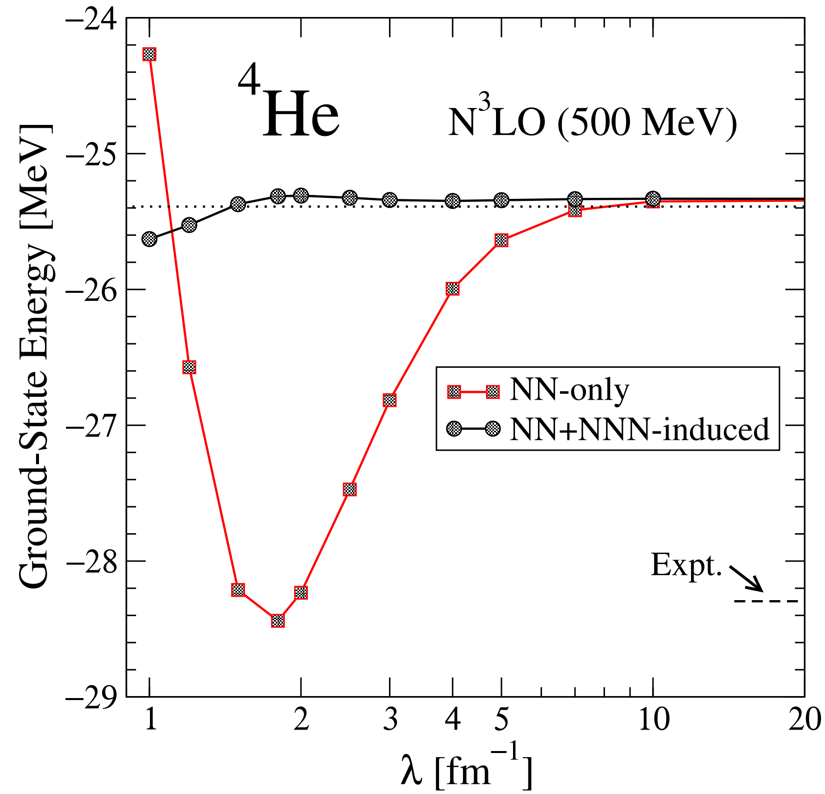


NN-only clear cutoff dependencs

Induced 3N Forces

Effect of including 3N-ind? Exactly initial $V_{\rm NN}$ **up to neglected 4N-ind**





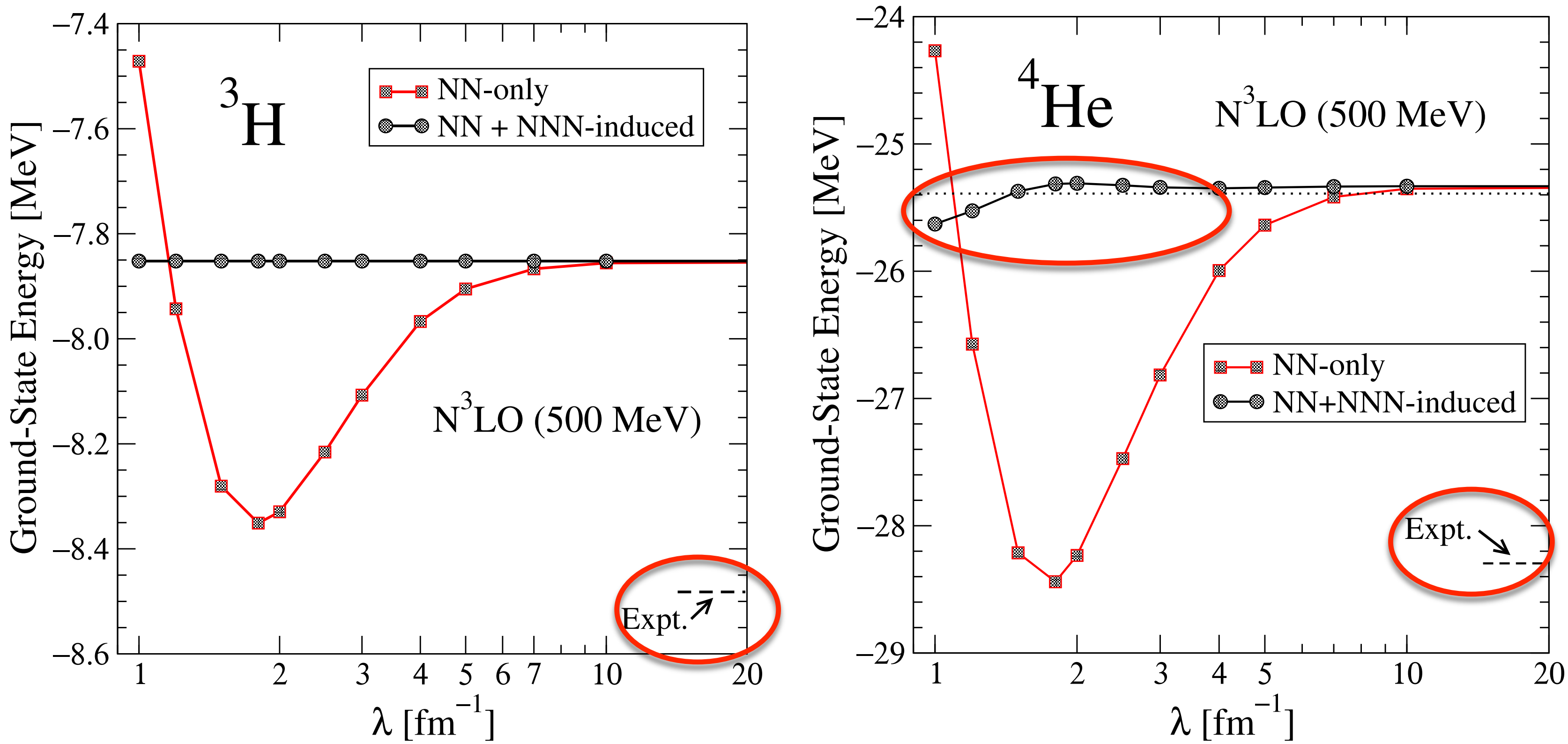
NN-only clear cutoff dependencs

3N-induced – dramatic reduction in cutoff dependence!

Lesson: SRG cutoff variation a sign of neglected induced forces

Induced 3N Forces

Effect of including 3N-ind? Exactly initial V_{NN} **up to neglected 4N-ind**



NN-only clear cutoff dependencs

3N-induced – dramatic reduction in cutoff dependence!

Lesson: SRG cutoff variation a sign of neglected induced forces

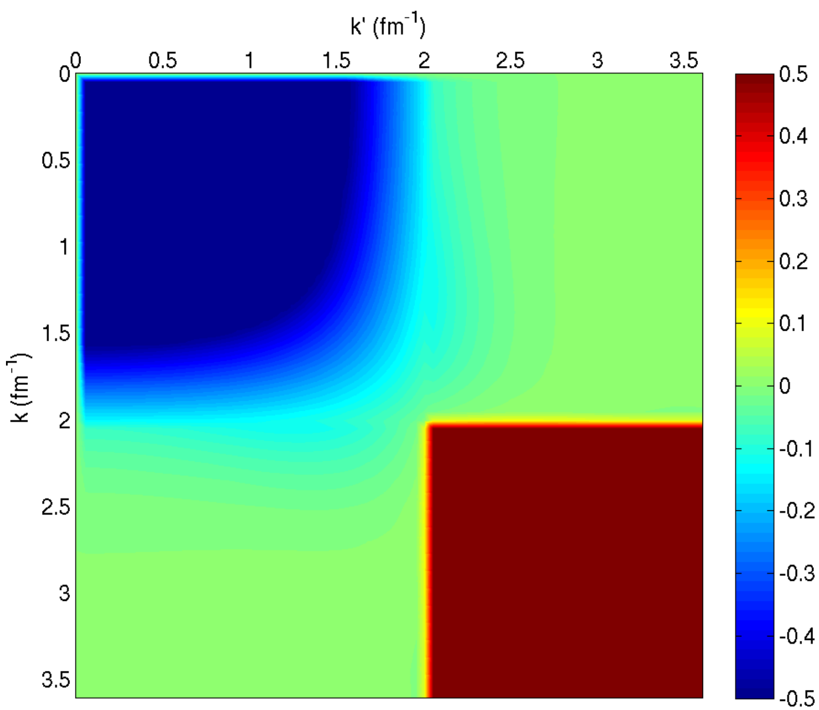
Still far from experiment and remaining (minor) cutoff dependence!

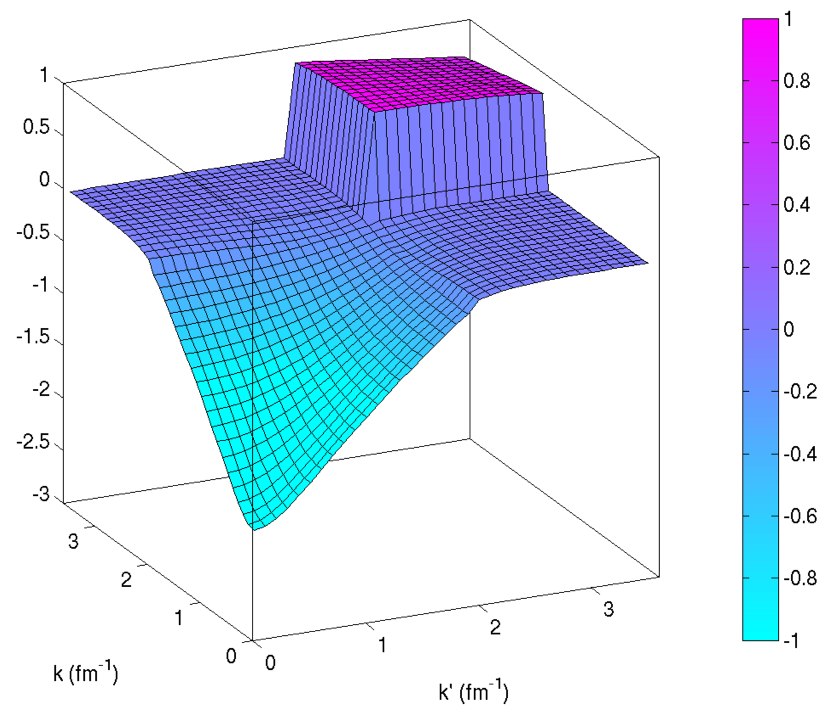
Other Generator Choices: Block Diagonal

Create block diagonal form like V_{lowk}?

$$G(s) = H_{\text{BD}} = \begin{pmatrix} PH(s)P & 0 \\ 0 & QH(s)Q \end{pmatrix}$$

With alternate definition of flow parameter: $\lambda^2 = \frac{1}{\sqrt{s}}$





Argonne V_{18} $^3S_1$ $\qquad$ $\lambda = 2.0\,\text{fm}^{-1}$