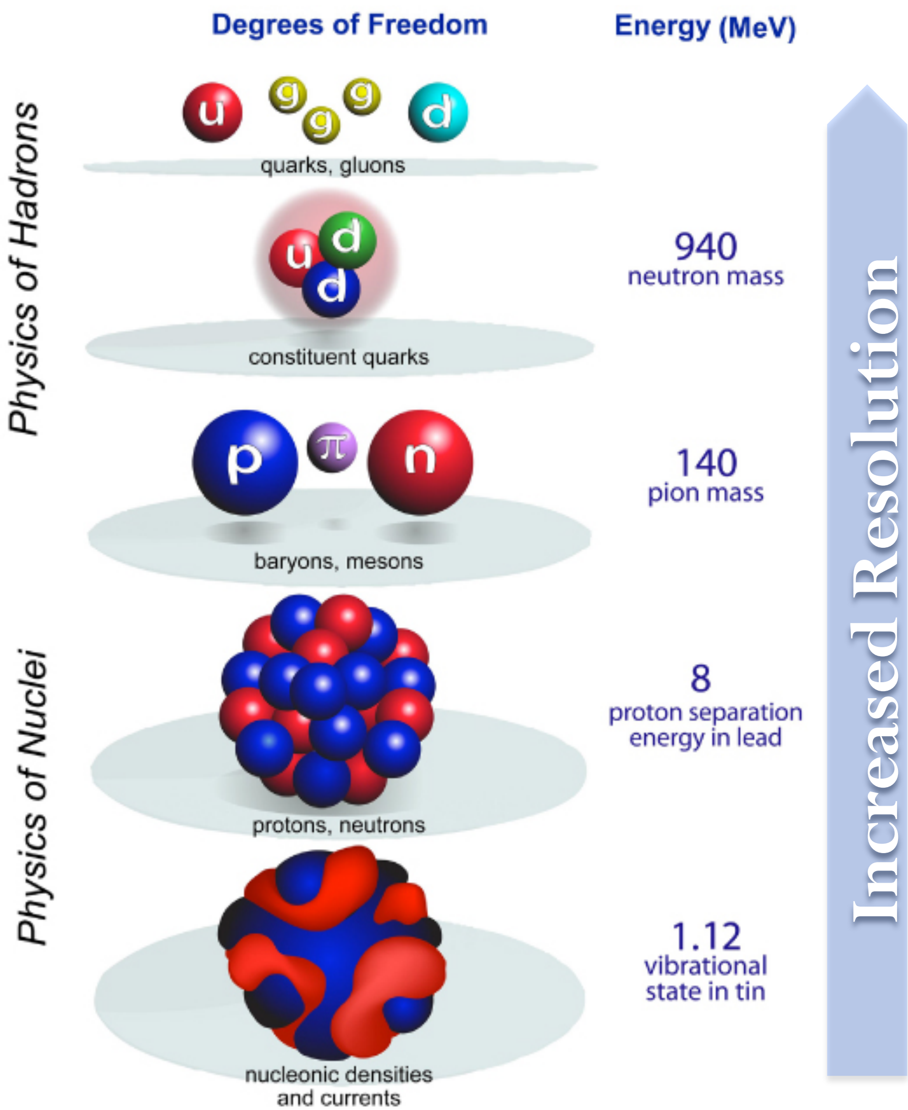
From QCD to Nuclear Interactions

How do we determine interactions between nucleons?

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{3\mathrm{N}}(\Lambda) + V_{4\mathrm{N}}(\Lambda) + \cdots$$



Old view:

Multiple scales complicate life

No meaningful way to connect them

Modern view:

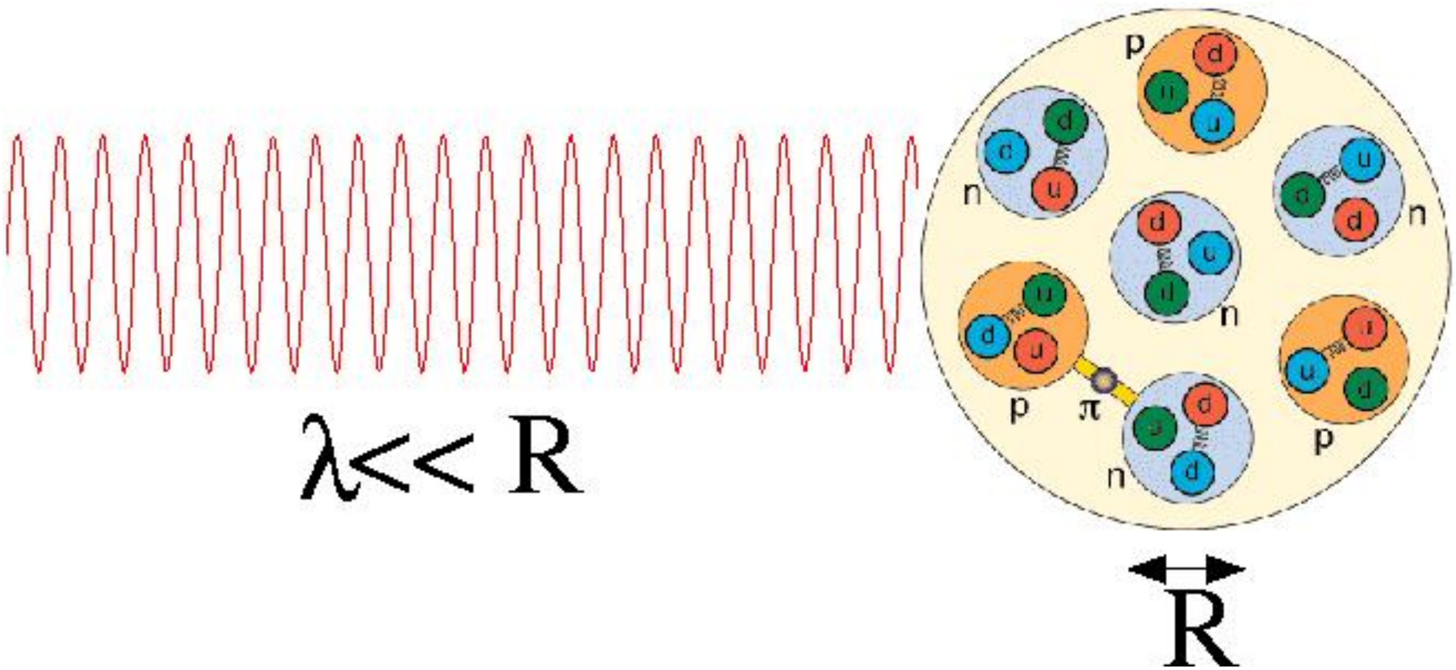
Choose convenient resolution scale

Effective field theory at each scale
connected by RG

Ratio of scales – small parameters

Ideas Behind Effective Theories

Resolution scale and relevant degrees of freedom

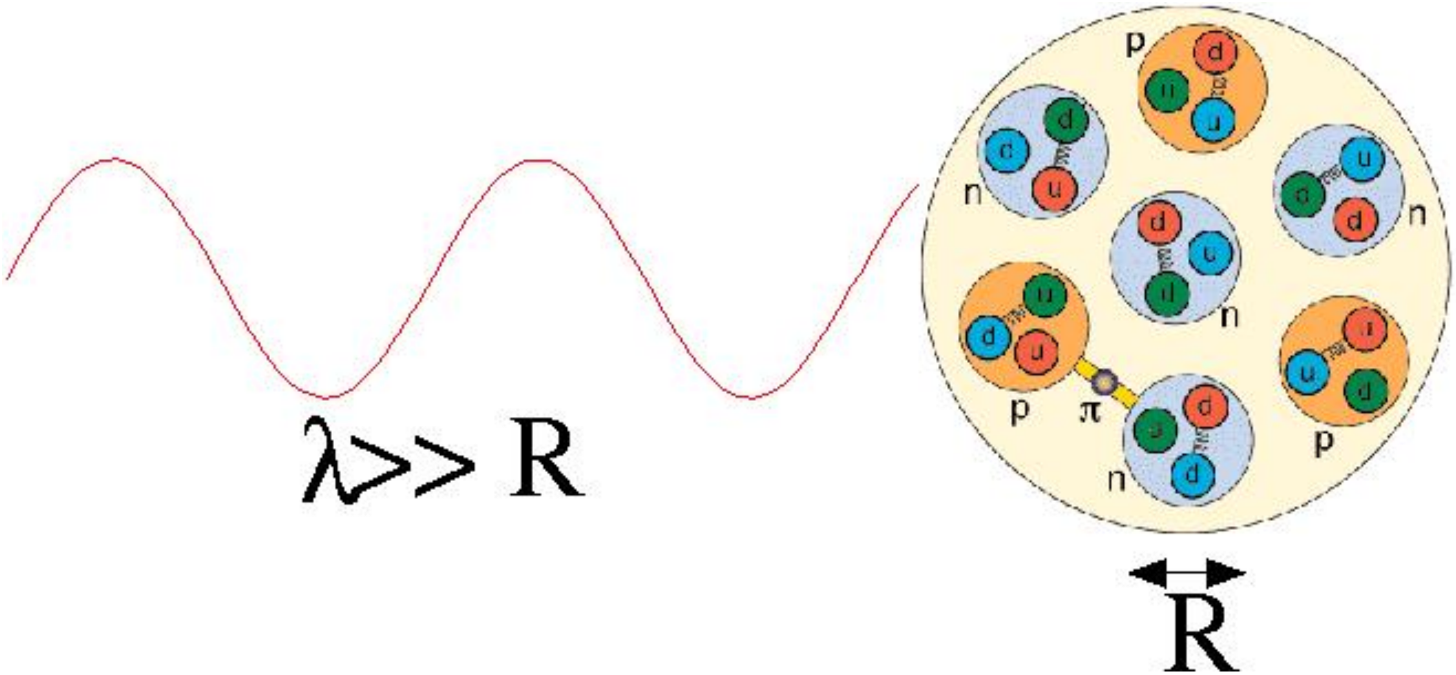


High energy probe resolves fine details

Need high-energy degrees of freedom

Ideas Behind Effective Theories

Resolution scale and relevant degrees of freedom

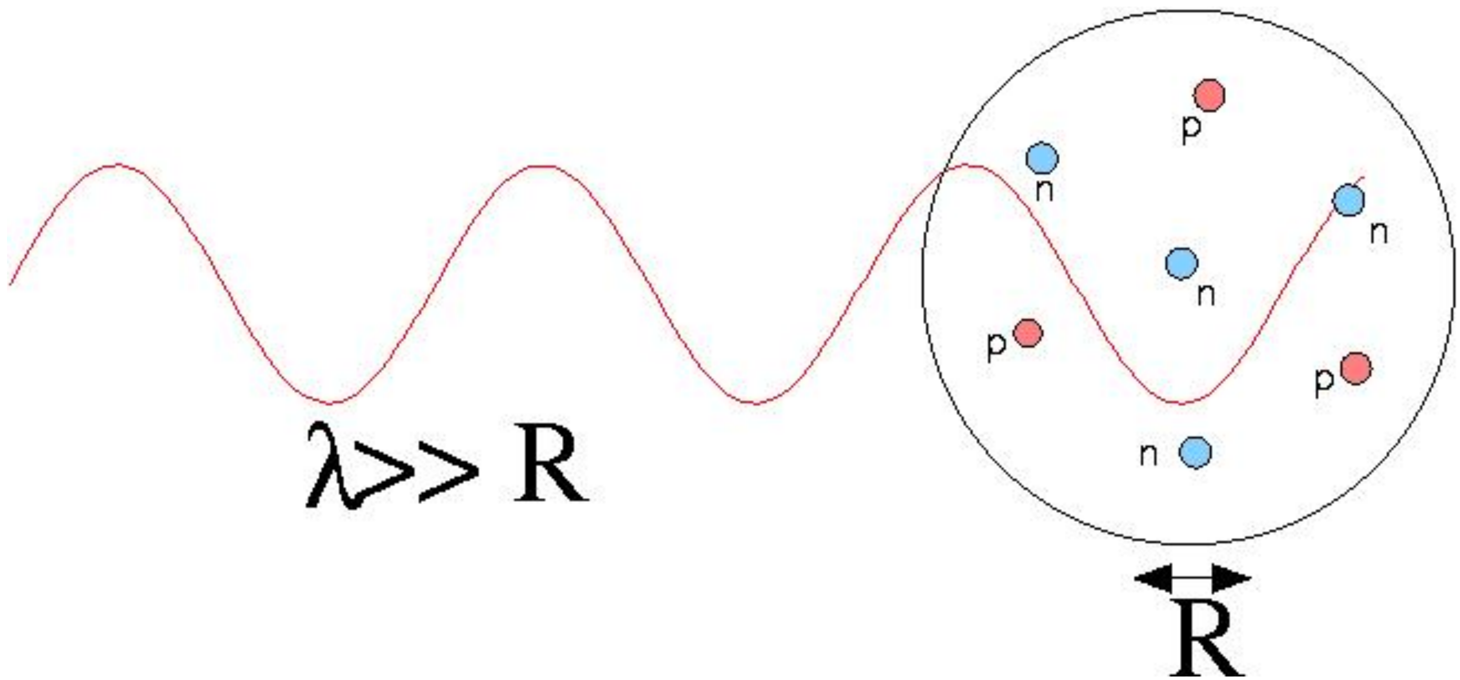


Low-energy probe can't resolve such details

Don't need high-energy degrees of freedom – replace with something simpler

Ideas Behind Effective Theories

Resolution scale and relevant degrees of freedom



Low-energy probe can't resolve such details

Don't need high-energy degrees of freedom – replace with something simpler

Use more convenient dofs, but **preserve low-energy observables!**

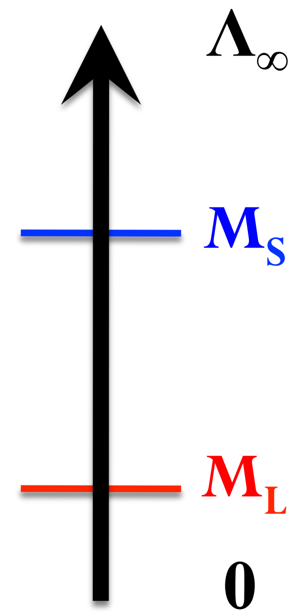
Ideas Behind Effective Theories

Assume underlying theory with cutoff Λ_∞

$$V = V_L + V_S$$

Known **long-distance physics** (like 1π exchange) with some scale M_L

Short-distance physics (ρ,ω exchange) with some scale M_S



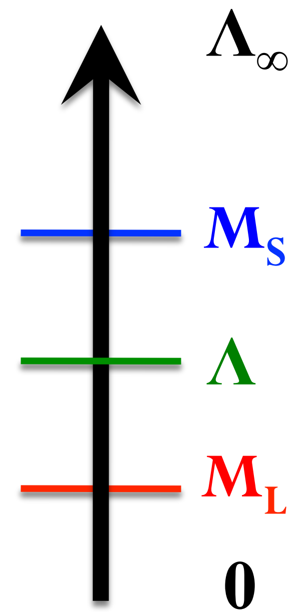
Ideas Behind Effective Theories

Assume underlying theory with cutoff Λ_∞

$$V = V_{\mathrm{L}} + V_{\mathrm{S}}$$

Known **long-distance physics** (like 1π exchange) with some scale M_L

Short-distance physics (ρ,ω exchange) with some scale M_S



And we want a **low-energy *effective* theory** for physics up to some

$$M_{\mathrm{L}} < \Lambda < M_{\mathrm{S}}$$

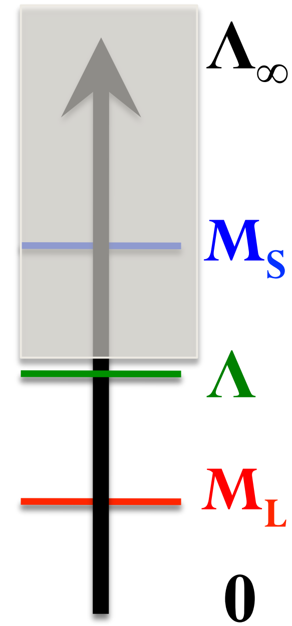
Ideas Behind Effective Theories

Assume underlying theory with cutoff Λ_∞

$$V = V_{\rm L} + V_{\rm S}$$

Known **long-distance physics** (like 1π exchange) with some scale M_L

Short-distance physics (ρ,ω exchange) with some scale M_S



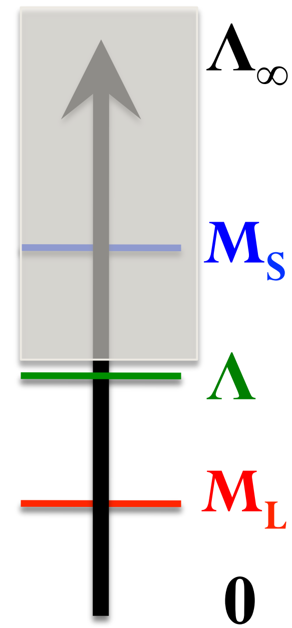
And we want a **low-energy *effective* theory** for physics up to some

$$M_{\rm L} < \Lambda < M_{\rm S}$$

Integrate out states above **Λ** using **Renormalization Group (RG)**

Ideas Behind Effective Theories

Assume underlying theory with cutoff Λ_∞



$$V = V_{\rm L} + V_{\rm S}$$

Known **long-distance physics** (like 1π exchange) with some scale M_L

Short-distance physics (ρ,ω exchange) with some scale M_S

And we want a **low-energy *effective* theory** for physics up to some

$$M_{\rm L} < \Lambda < M_{\rm S}$$

Integrate out states above **Λ** using **Renormalization Group (RG)**

General form of effective theory: $V_{\rm eff} = V_{\rm L} + \delta V_{\rm c.t.}(\Lambda)$

where $\delta V_{\rm c.t.}(\Lambda) = C_0(\Lambda)\delta^3(\mathbf{r}) + C_2(\Lambda)\nabla^2\delta^3(\mathbf{r}) + \cdots$

Also use RG to change resolution scales **within particular EFT**

Ideas Behind Effective Theories

General form of effective theory: $V_{\text{eff}} = V_{\text{L}} + \delta V_{\text{c.t.}}(\Lambda)$

$$\delta V_{\text{c.t.}}(\Lambda) = C_0(\Lambda)\delta^3(\mathbf{r}) + C_2(\Lambda)\nabla^2\delta^3(\mathbf{r}) + \cdots$$

Encodes effects of high-E dof on low-energy observables

Universal; depends only on symmetries

TWO choices:

Short distance structure of "true theory" captured in several numbers

- **Calculate from underlying theory**

When short-range physics is unknown or too complicated

- **Extract from low-energy data**

How do we apply these ideas to nuclear forces?

Chiral Effective Field Theory: Philosophy

"At each scale we have different degrees of freedom and different dynamics. Physics at a larger scale (largely) decouples from physics at a smaller scale… thus a theory at a larger scale remembers only finitely many parameters from the theories at smaller scales, and throws the rest of the details away.

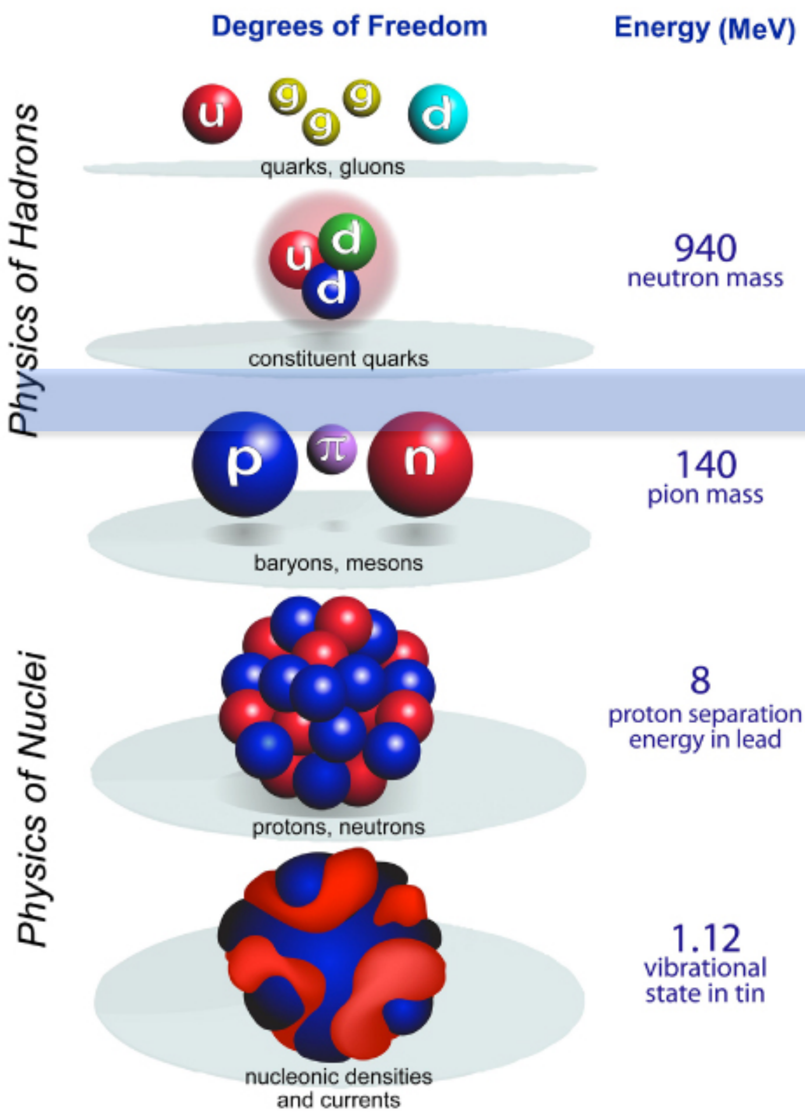
More precisely, when we pass from a smaller scale to a larger scale, we average out irrelevant degrees of freedom… The general aim of the RG method is to explain how this decoupling takes place and why exactly information is transmitted from one scale to another through finitely many parameters."
- ***David Gross***

"The method in its most general form can.. be understood as a way to arrange in various theories that the degrees of freedom that you're talking about are the relevant degrees of freedom for the problem at hand."
- ***Steven Weinberg***

5 Steps to constructing such a theory for nuclear forces

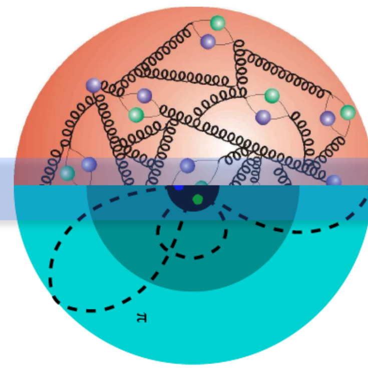
Separation of Scales in Nuclear Physics

Step I: Identify appropriate separation of scales, degrees of freedom



Appropriate separation of scales

Typical momenta in nucleus $Q \sim m_\pi$

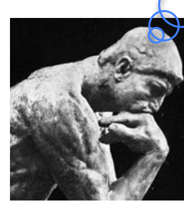


$\Lambda_{\mathrm{chiral}} \sim 700\ \mathrm{MeV}$

Only degrees of freedom relevant to energy scale

Effective theory: only nucleons and pions

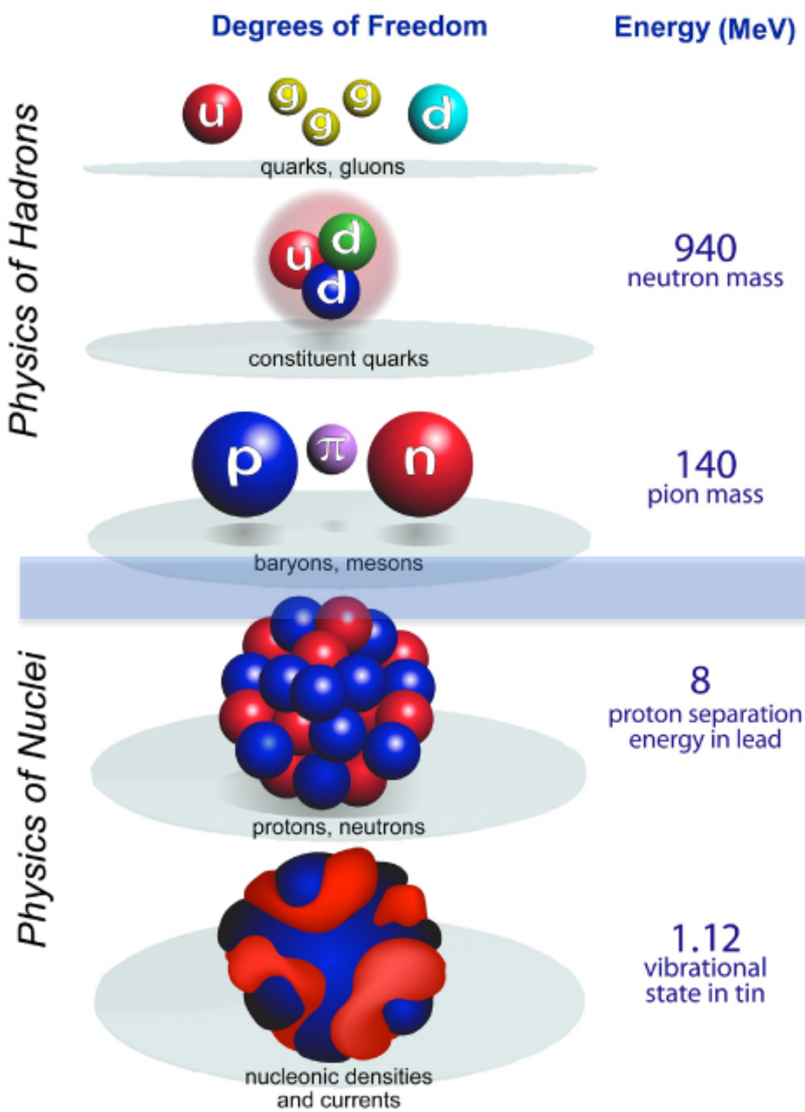
It seems like we're forgetting something…



How low can we go??

Separation of Scales in Nuclear Physics

Step I: Identify appropriate separation of scales, degrees of freedom

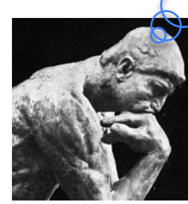


Appropriate separation of scales

Typical momenta in nucleus $Q \sim m_\pi$

Pionless effective theory: only nucleons

Could we make a different separation?



Chiral EFT Symmetries

Step II: Identify relevant symmetries of underlying theory (QCD)

1. SU(3) color symmetry from QCD
   (Nucleons and pions are color singlets)

2. Chiral symmetry: u and d quarks are almost massless
   - Left and right-handed (massless) quarks do not mix: $SU(2)_L$ x $SU(2)_R$ symmetry
   - Explicit symmetry breaking: u and d quarks have a small mass
   - Spontaneous breaking of chiral symmetry (no parity doublets observed in Nature)
     - $SU(2)_L$ x $SU(2)_R$ symmetry spontaneously broken to $SU(2)_V$
     - Pions are the Nambu-Goldstone bosons of spontaneously broken symmetry
     - Low-energy pion Lagrangian completely determined

Missing ingredient in multi-pion-exchange theories of 50's!

Construct Lagrangian based on these symmetries

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi\mathrm{N}} + \mathcal{L}_{\mathrm{NN}}$$

Chiral EFT Lagrangian

Step III: Construct Lagrangian based on identified symmetries

Pion-pion Lagrangian: U is SU(2) matrix parameterized by three pion fields

$$\mathcal{L}_{\pi}^{(0)} = \frac{F^2}{4}\langle \nabla^{\mu} U \nabla_{\mu} U^{\dagger} + \chi_{+} \rangle,$$

Leading-order pion-nucleon

$$\mathcal{L}_{\pi N}^{(0)} = \bar{N}(iv \cdot D + \mathring{g}_A u \cdot S)N,$$

Leading-order nucleon-nucleon (encodes unknown short-range physics)

$$\mathcal{L}_{NN}^{(0)} = -\tfrac{1}{2} C_S (\bar{N}N)(\bar{N}N) + 2C_T (\bar{N}SN) \cdot (\bar{N}SN)$$

EFT Power Counting

Step IV: Design an organized scheme to distinguish more from less important processes: Power Counting

Organize theory in powers of $\left(\frac{Q}{\Lambda_\chi}\right)$ where $Q \sim m_\pi$ typical nuclear momenta

Only valid for small expansion parameters, *i.e.*, low momentum

Irreducible time-ordered diagram has order: $\left(\frac{Q}{\Lambda_\chi}\right)^\nu$

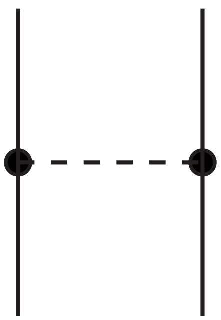
$$\nu = -4 + 2N + 2L + \sum_i V_i \Delta_i \qquad \Delta_i = d_i + \frac{1}{2} n_i - 2 \text{ “Chiral dimension”}$$

$N =$ Number of nucleons
$L =$ Number of pion loops
$V_i =$ Number of vertices of type i
$d =$ Number of derivatives or insertions of m_π
$n =$ Number of nucleon field operators

Chiral EFT: Lowest Order (LO)

Step V: Calculate Feynmann diagrams to the desired accuracy

Leading order (LO) $\nu = 0$



One-pion exchange

$$V_{NN}^{(0)} = -\frac{g_A^2}{4F_\pi^2}\frac{\vec{\sigma}_1 \cdot \vec{q}\,\vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^2 + M_\pi^2}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

$g_A = 1.26$

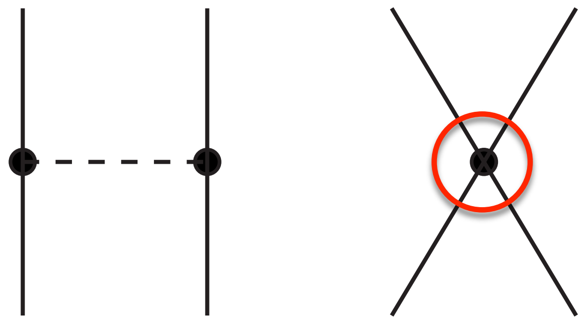
$F_\pi = 92.4$ MeV

$\vec{q}_i \equiv \vec{p}_i' - \vec{p}_i \qquad \vec{k}_i \equiv \frac{1}{2}(\vec{p}_i' + \vec{p}_i)$

Chiral EFT: Lowest Order (LO)

Step V: Calculate Feynmann diagrams to the desired accuracy

Leading order (LO) $\nu = 0$



One-pion exchange
NN contact interaction

$$V_{NN}^{(0)} = -\frac{g_A^2}{4F_\pi^2}\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^2+M_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + C_S + C_T\vec{\sigma}_1\cdot\vec{\sigma}_2$$

$g_A = 1.26$

$F_\pi = 92.4$ MeV

Two **low-energy constants (LECs)**: C_S, C_T

$$\vec{q}_i \equiv \vec{p}_i^{\,\prime} - \vec{p}_i \qquad \vec{k}_i \equiv \frac{1}{2}\left(\vec{p}_i^{\,\prime} + \vec{p}_i\right)$$

Chiral EFT

Step V: Calculate Feynmann diagrams to the desired accuracy

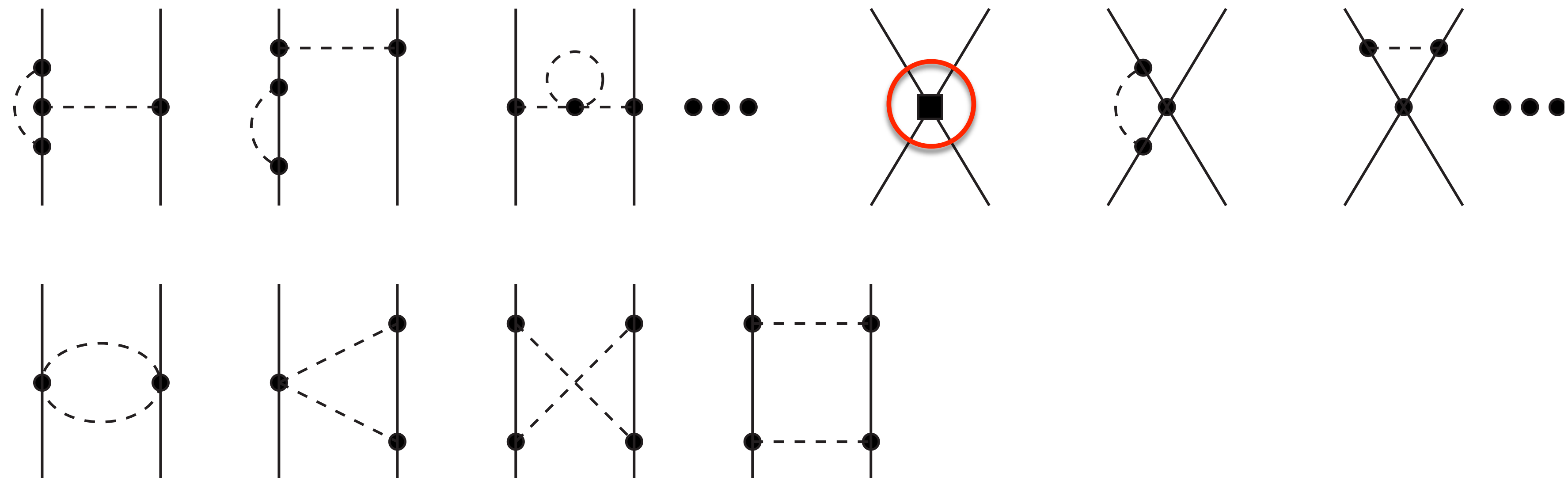
Question: What will $\nu = 1$ look like?

Answer: No contribution at this order

Chiral EFT: NLO

Step V: Calculate Feynmann diagrams to the desired accuracy

Next-to-leading order (NLO) $\nu = 2$



Higher order contact interaction: 7 new LECs, spin-orbit

$$+ C_1 \vec{q}^2 + C_2 \vec{k}^2 + (C_3 \vec{q}^2 + C_4 \vec{k}^2)\vec{\sigma}_1 \cdot \vec{\sigma}_2$$

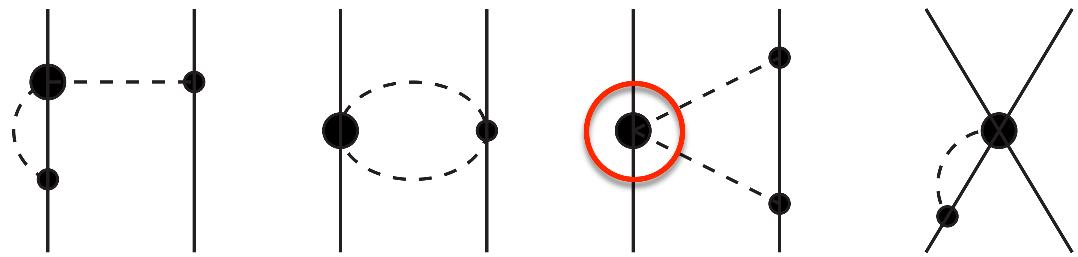
$$+ iC_5 \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{q} \times \vec{k} + C_6 \vec{q} \cdot \vec{\sigma}_1 \vec{q} \cdot \vec{\sigma}_2$$

$$+ C_7 \vec{k} \cdot \vec{\sigma}_1 \vec{k} \cdot \vec{\sigma}_2,$$

Chiral EFT: N^2LO

Step V: Calculate Feynmann diagrams to the desired accuracy

Next-to-next-to-leading order (N^2LO) $\nu = 3$



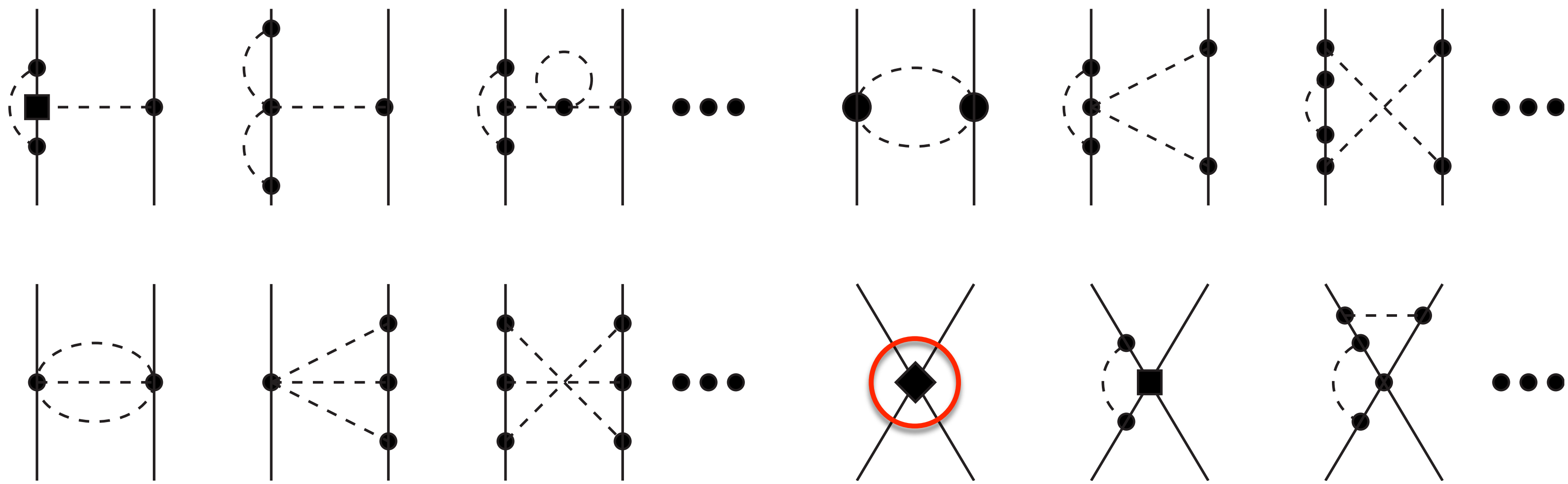
3 new couplings from ππNN vertex – not LECs!

$$V_{NN}^{(3)} = -\frac{3g_A^2}{16\pi F_\pi^4}[2M_\pi^2(2c_1 - c_3) - c_3\vec{q}^2]$$
$$\times(2M_\pi^2 + \vec{q}^2)A^{\tilde{\Lambda}}(q) - \frac{g_A^2 c_4}{32\pi F_\pi^4}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2(4M_\pi^2$$
$$+ q^2)A^{\tilde{\Lambda}}(q)(\vec{\sigma}_1 \cdot \vec{q}\vec{\sigma}_2 \cdot \vec{q} - \vec{q}^2\vec{\sigma}_1 \cdot \vec{\sigma}_2),$$

Chiral EFT: N^3LO

Step V: Calculate Feynmann diagrams to the desired accuracy

Next-to-next-to-next-to-leading order $\nu = 4$



Higher order contact interaction: 15 new LECs

Question: Consider two high-precision NN potentials from chiral EFT with different cutoffs. How will the solutions of the nuclear many-body problem depend on the cutoff?

1. There will be (almost) no cutoff dependence in the two-body system.
2. There will be (almost) no cutoff dependence in many-body systems as Nature must be cutoff independent.
3. The cutoff dependence measures missing contributions from higher orders.
4. The cutoff dependence measures missing short-range contributions from higher orders.

Regularization of Chiral potentials

Remember: constructing potential involves solving **L-S equation**

All NN potentials cutoff loop momenta at some value > 1GeV

Impose exponential regulator, Λ, in Chiral EFT potentials – **not in integral**

$$T^{\alpha}(k,k') = V^{\alpha}(k,k') + \frac{2}{\pi}\sum_{l''}\int_0^{\infty} q^2 \mathrm{d}q\, V^{\alpha}(k,q)\frac{q}{k^2 - q^2 + i\varepsilon}T^{\alpha}(q,k')$$

$$V(k,k') \rightarrow e^{(-k'/\Lambda)^{2n}} V(k,k') e^{(-k/\Lambda)^{2n}}$$

LECs will depend on regularization approach and Λ

Infinitely many ways to do this

$\Longrightarrow$ **Infinitely many chiral potentials!**

Indeed, many on the market – some fit well to phase shifts, others not

Chiral EFT:
Resulting fits to Phase shifts

Systematic improvement of chiral EFT potentials fit to phase shifts

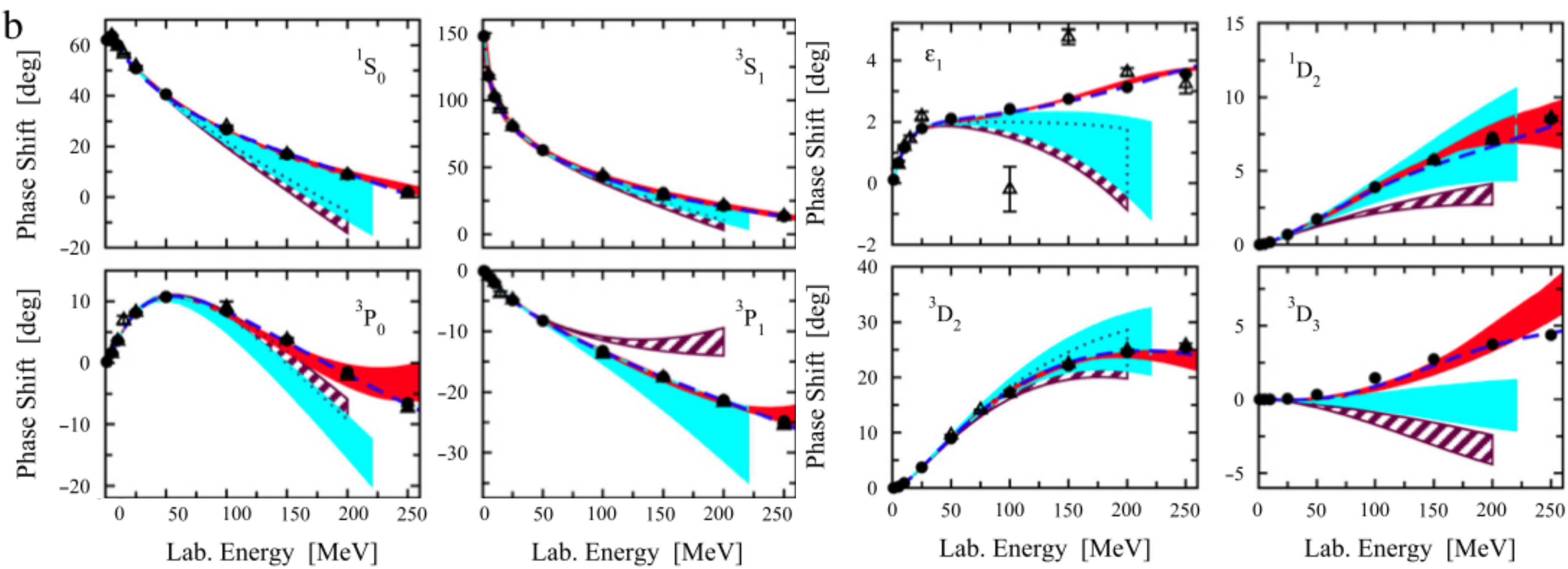
Cutoff variation – information about missing physics

NLO: dashed band 9 Parameters

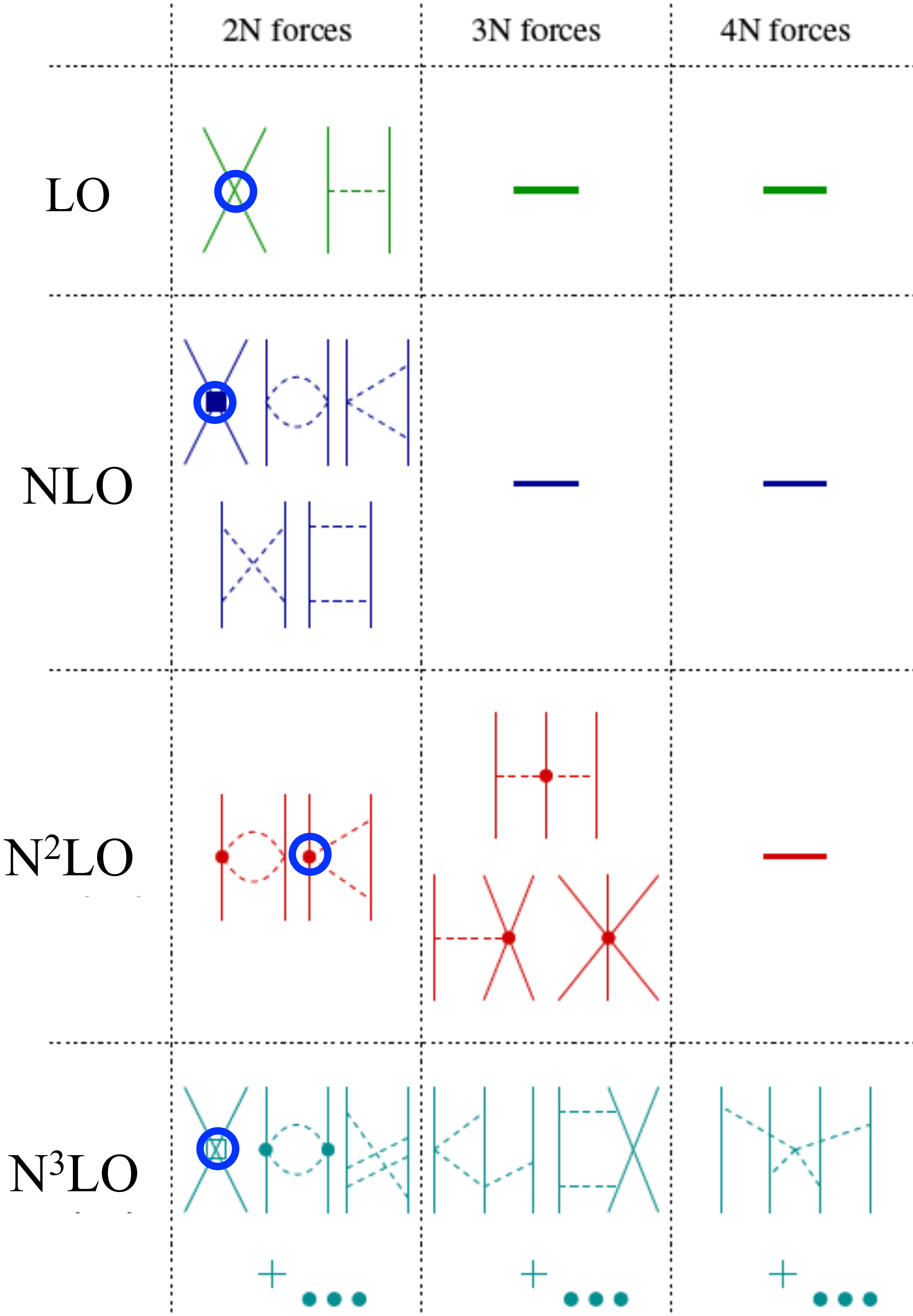
N^2LO: light band 12 Parameters

N^3LO: dark band 27 Parameters

Generally decreasing error and increasing accuracy – not entirely… (exercise)



Chiral Effective Field Theory: Nuclear Forces



Meson exchange potentials were an admirable effort

Using ideas of **effective field theory:**

Nucleons interact via pion exchanges and contact interactions

Lower momentum

Systematic – can assign error

Connected to QCD

Hierarchy: $V_{NN} > V_{3N} > \ldots$

Consistent treatment of NN, 3N, … electroweak operators

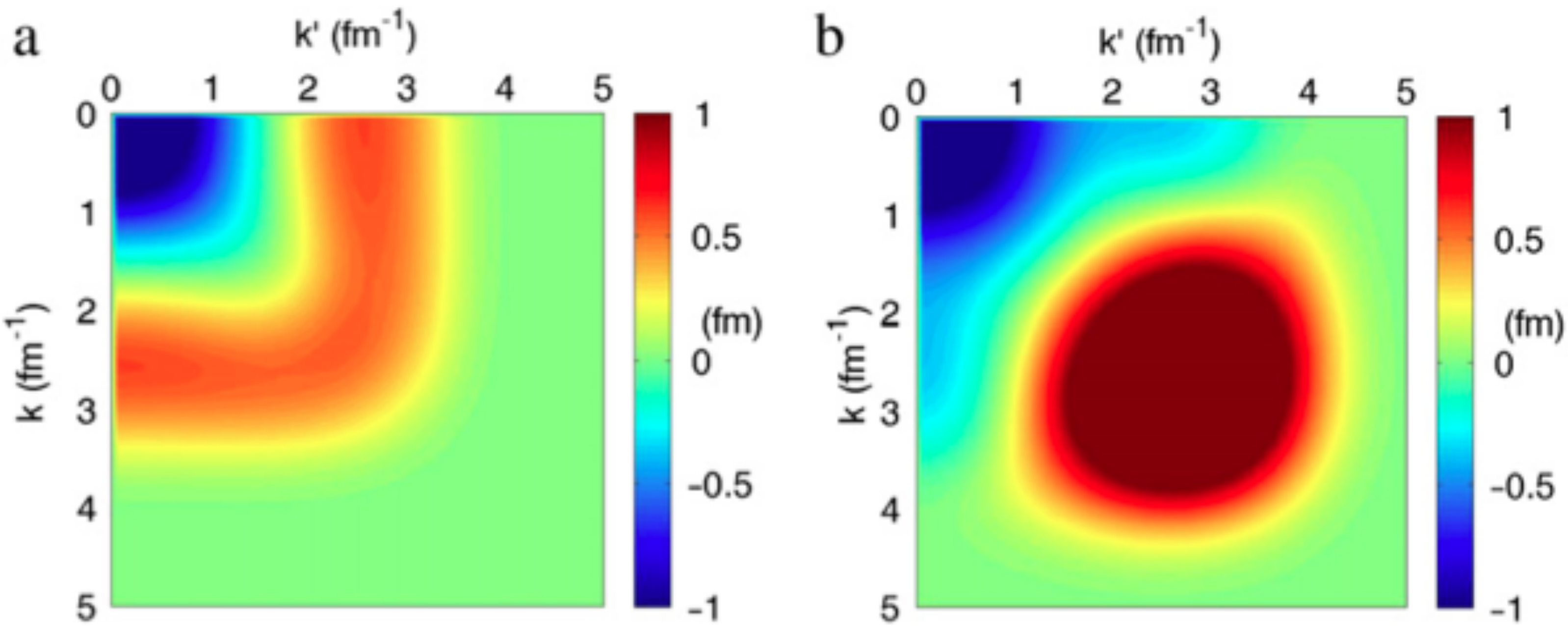
Couplings fit to experiment once

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Meissner,…

Chiral NN Potentials

Two chiral potentials with regulators of 500MeV and 600MeV

Still low-to-high momentum coupling: poor convergence, non perturbative, etc.



How do these compare to the potential you drew?

Lesson: Infinitely many phase-shift equivalent potentials

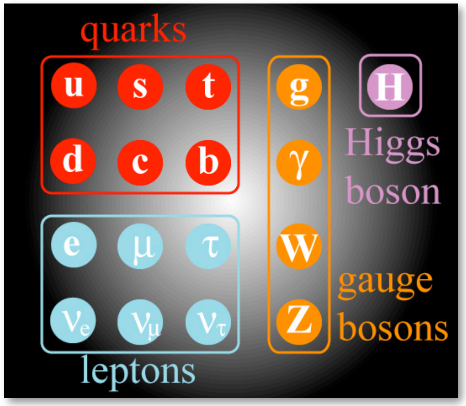
$$E_n = \langle \Psi_n | H | \Psi_n \rangle = \left(\langle \Psi_n | U^\dagger \right) U H U^\dagger \left(U | \Psi_n \rangle \right) = \left\langle \tilde{\Psi}_n \middle| \tilde{H} \middle| \tilde{\Psi}_n \right\rangle$$

NN interaction not observable

Low-to-high momentum makes life difficult for low-energy nuclear theorists

Chiral EFT Symmetries

Step II: Identify relevant symmetries of underlying theory (QCD)



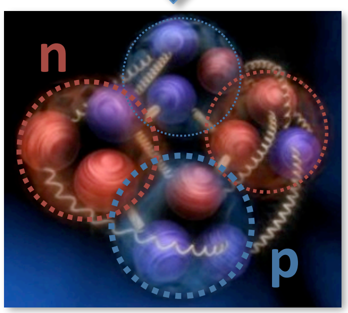
Quark/gluon (high energy) dynamics

$$\mathcal{L} = -\frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a} + \bar{q}_L i \gamma_\mu D^\mu q_L + \bar{q}_R i \gamma_\mu D^\mu q_R - \bar{q} \mathcal{M} q$$

Approximate **chiral symmetry**
(left- and right-handed quarks transform independently)

QCD chiral symmetry
quarks
left handed
right handed

Weinberg



Nucleon/pion (low energy) dynamics

$$\mathcal{L}_{eff} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \dots$$

Compatible with explicit and spontaneous **chiral symmetry breaking**

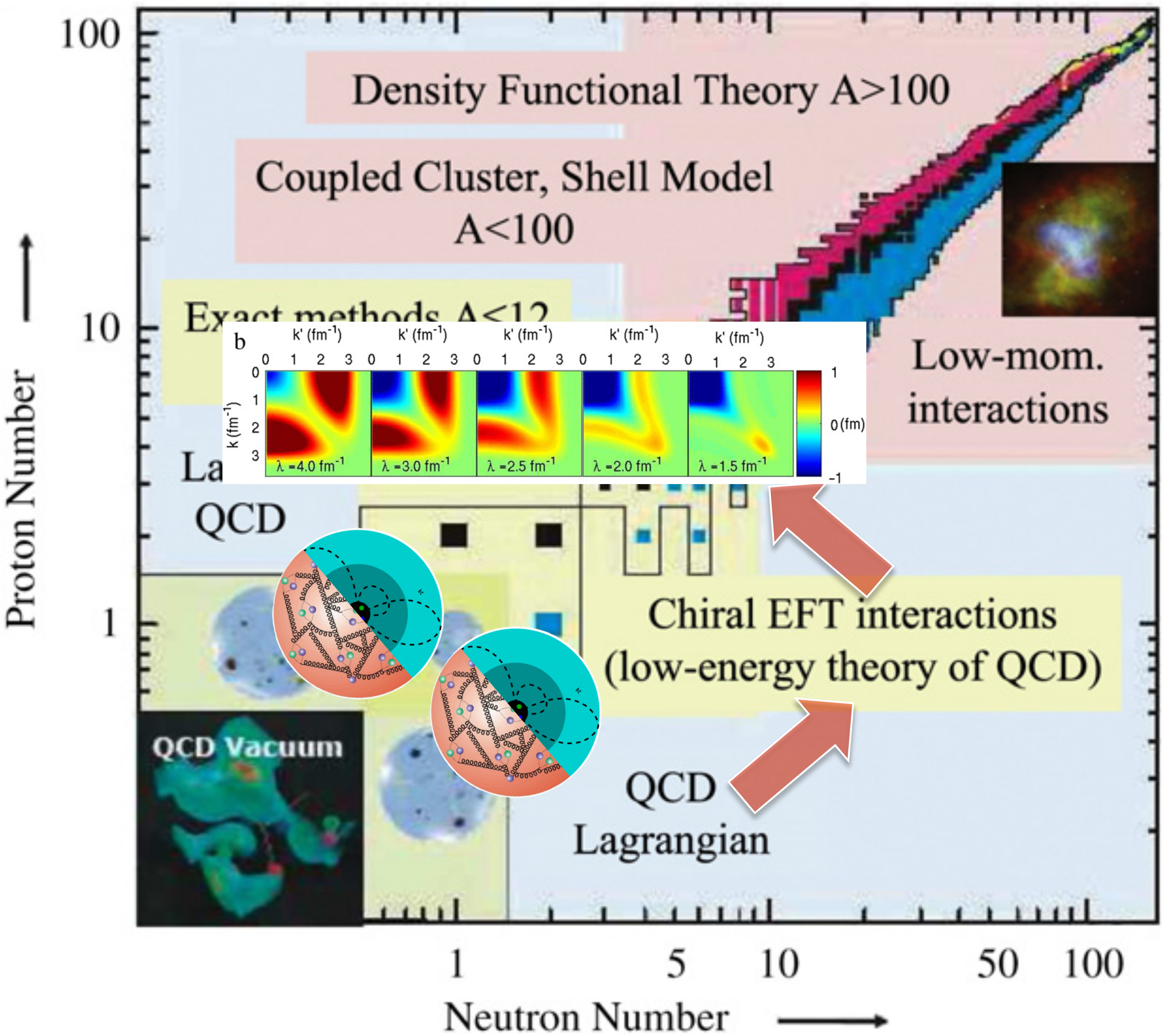
Missing ingredient in multi-pion-exchange theories of 50's!

Construct Lagrangian based on these symmetries

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi \text{N}} + \mathcal{L}_{\text{NN}}$$

Part II: (S)RG and Low-Momentum Interactions

To understand the properties of complex nuclei from first principles



Renormalizing NN Interactions

Basic ideas of RG

Low-momentum interactions

Similarity RG interactions

Benefits of low cutoffs

G-matrix renormalization

How will we approach this problem:

QCD → NN (3N) forces → Renormalize → "Solve" many-body problem → Predictions