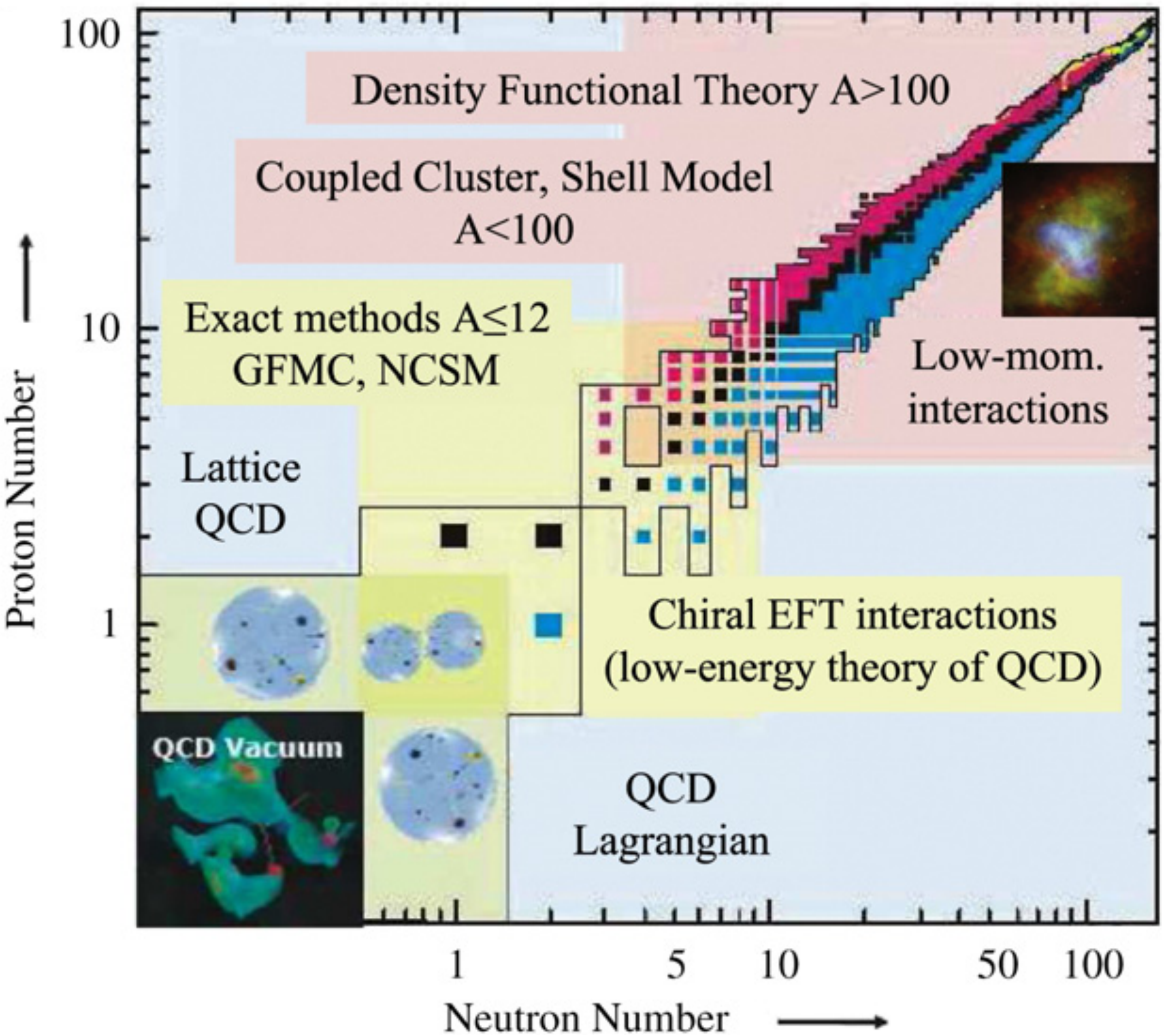
The Challenge of Ab Initio Nuclear Theory

To understand the properties of complex nuclei from first principles



Two significant issues:

Interaction

Not well understood

Not obtainable from QCD

Too “hard” to be useful

Multiple energy scales

Many-body Problem

Not ‘exactly’ solvable above $A \sim 20$

Here we focus on shell model

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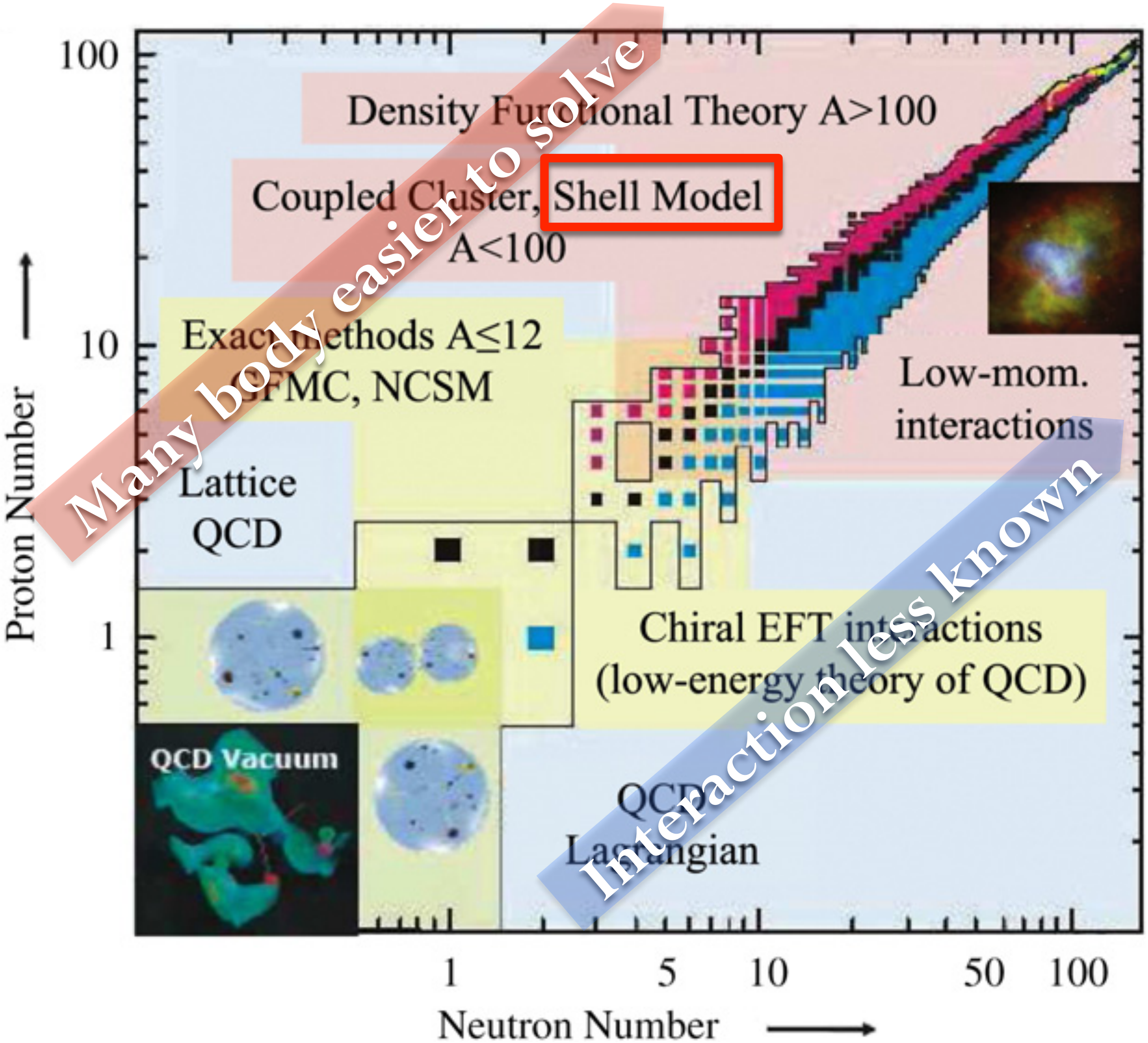

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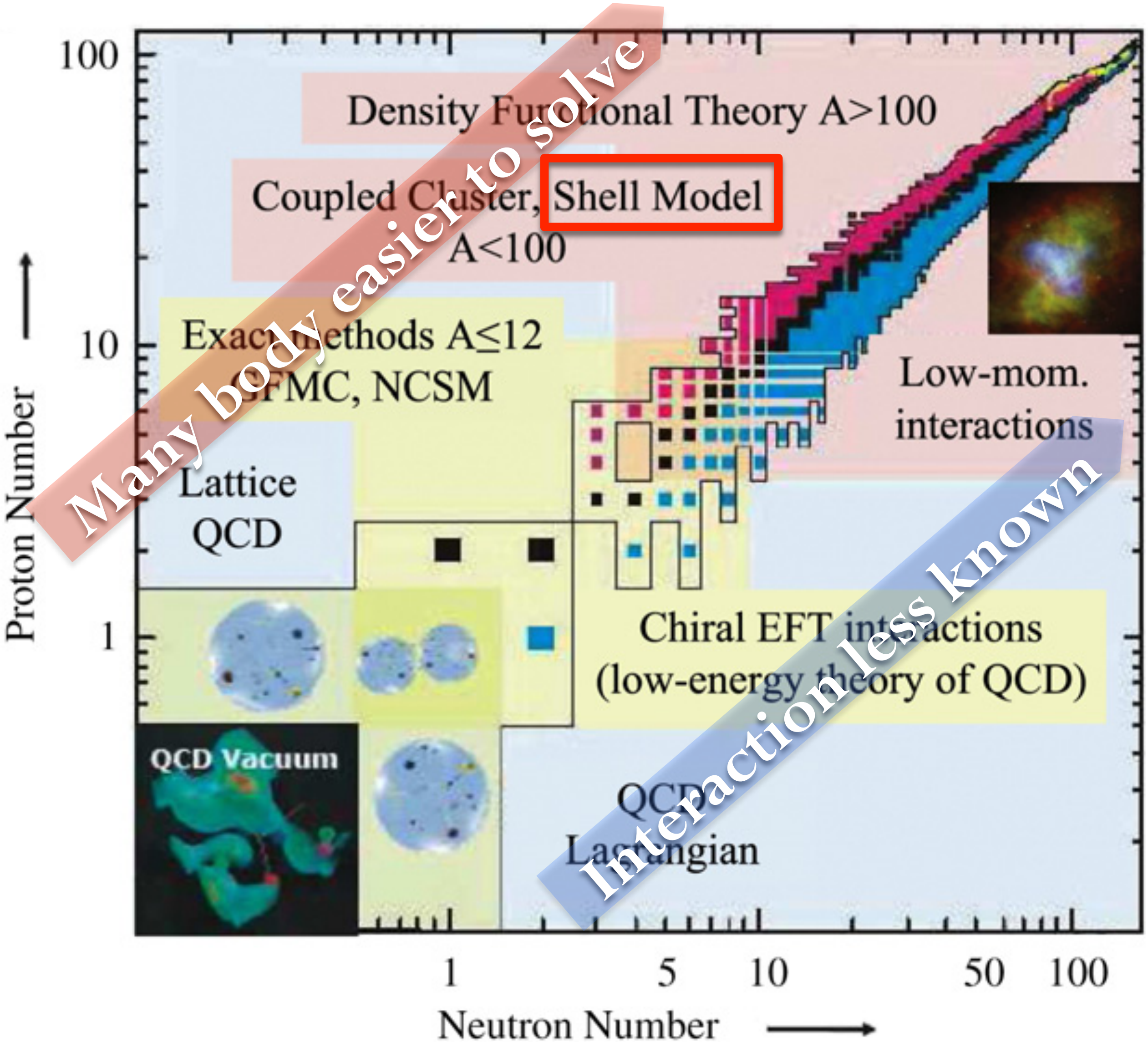

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QCD → NN (3N) forces → Renormalize → "Solve" many-body problem → Predictions

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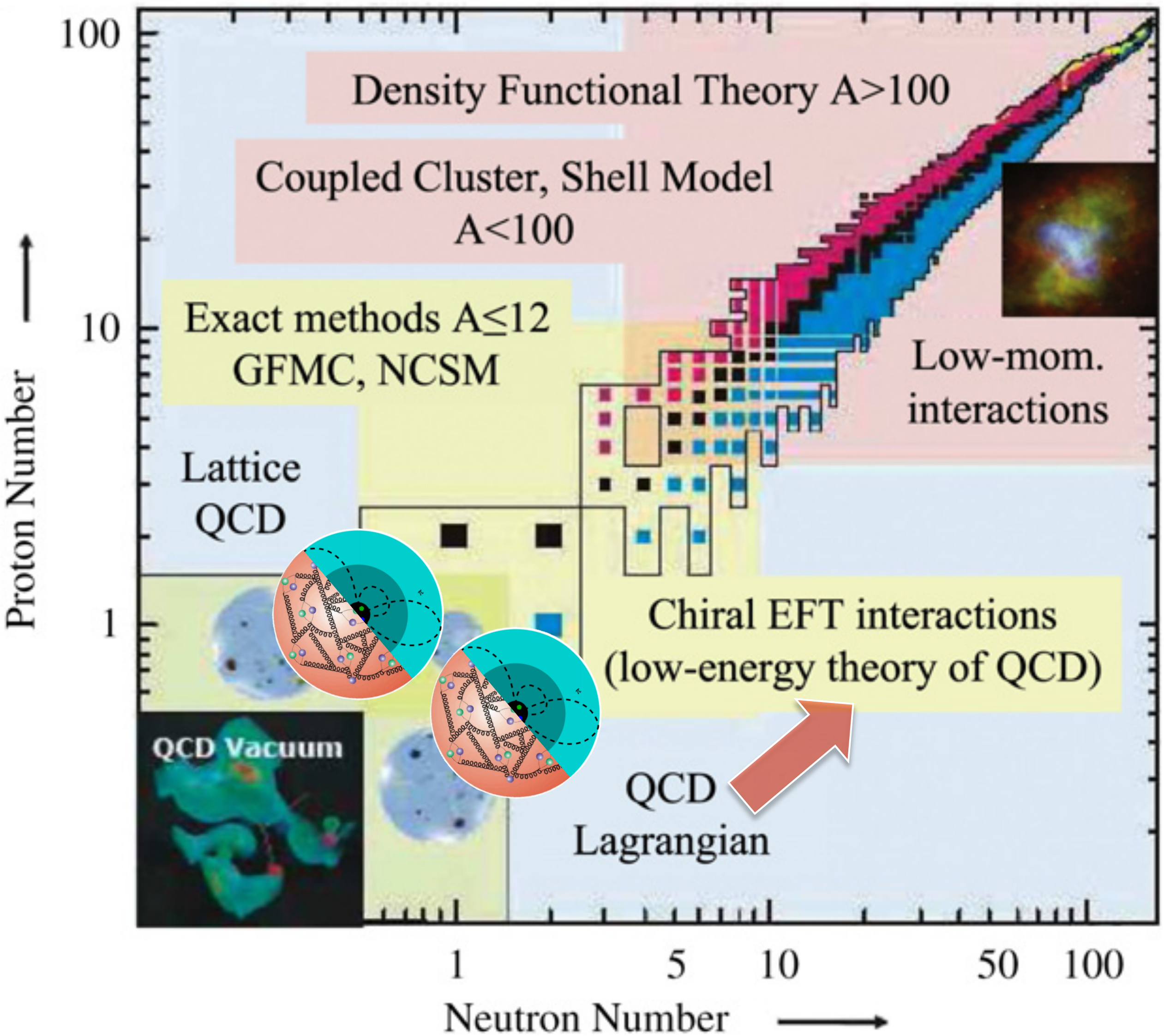

Nucleon-nucleon interaction

Some history

Anatomy of an NN interaction

Construction from QCD?

Ideas of Effective Field Theory

Chiral EFT for nuclear forces

Constraint by data

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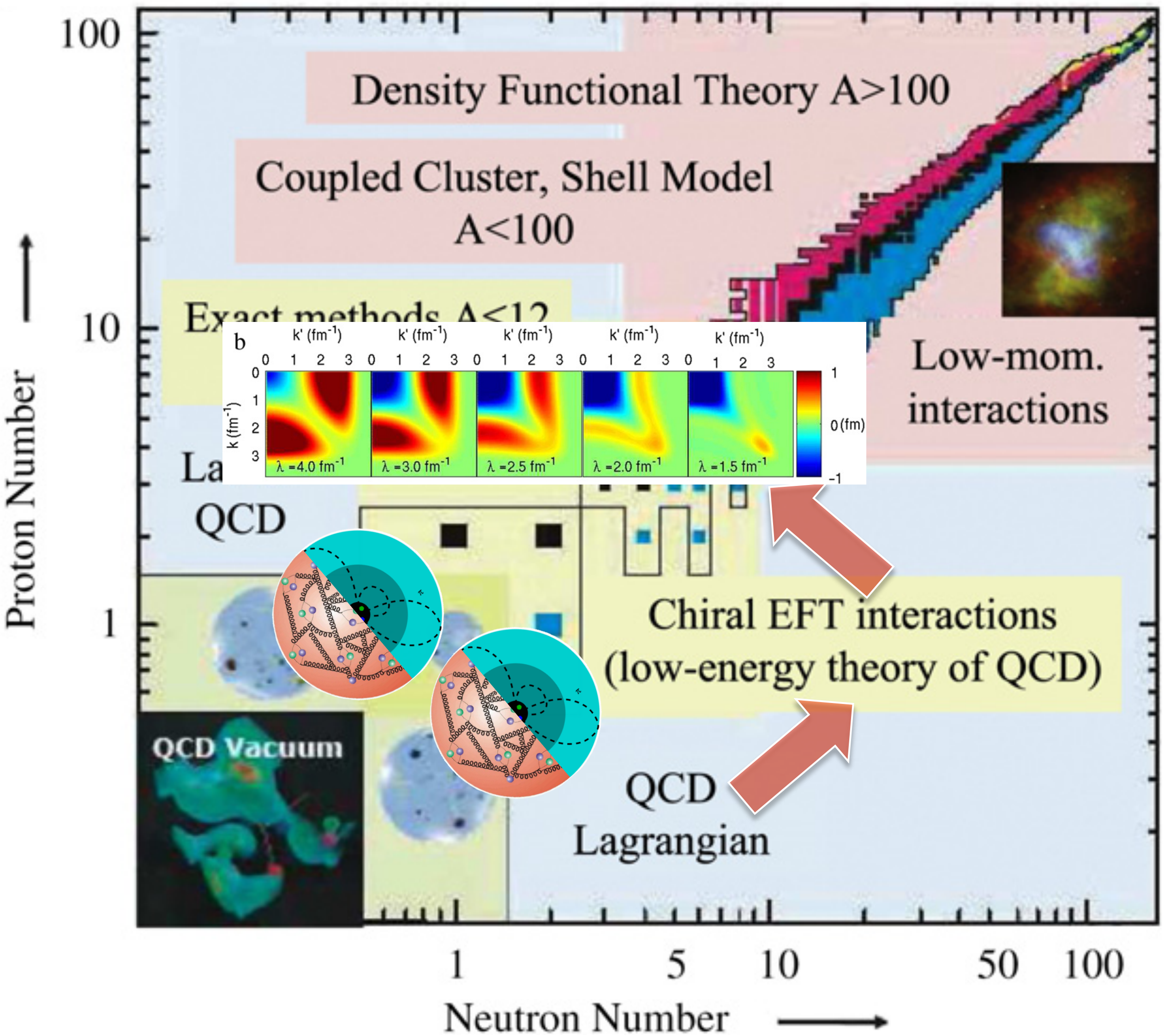

Renormalizing NN Interactions

Basic ideas of RG

Low-momentum interactions

Similarity RG interactions

Benefits of low cutoffs

G-matrix renormalization

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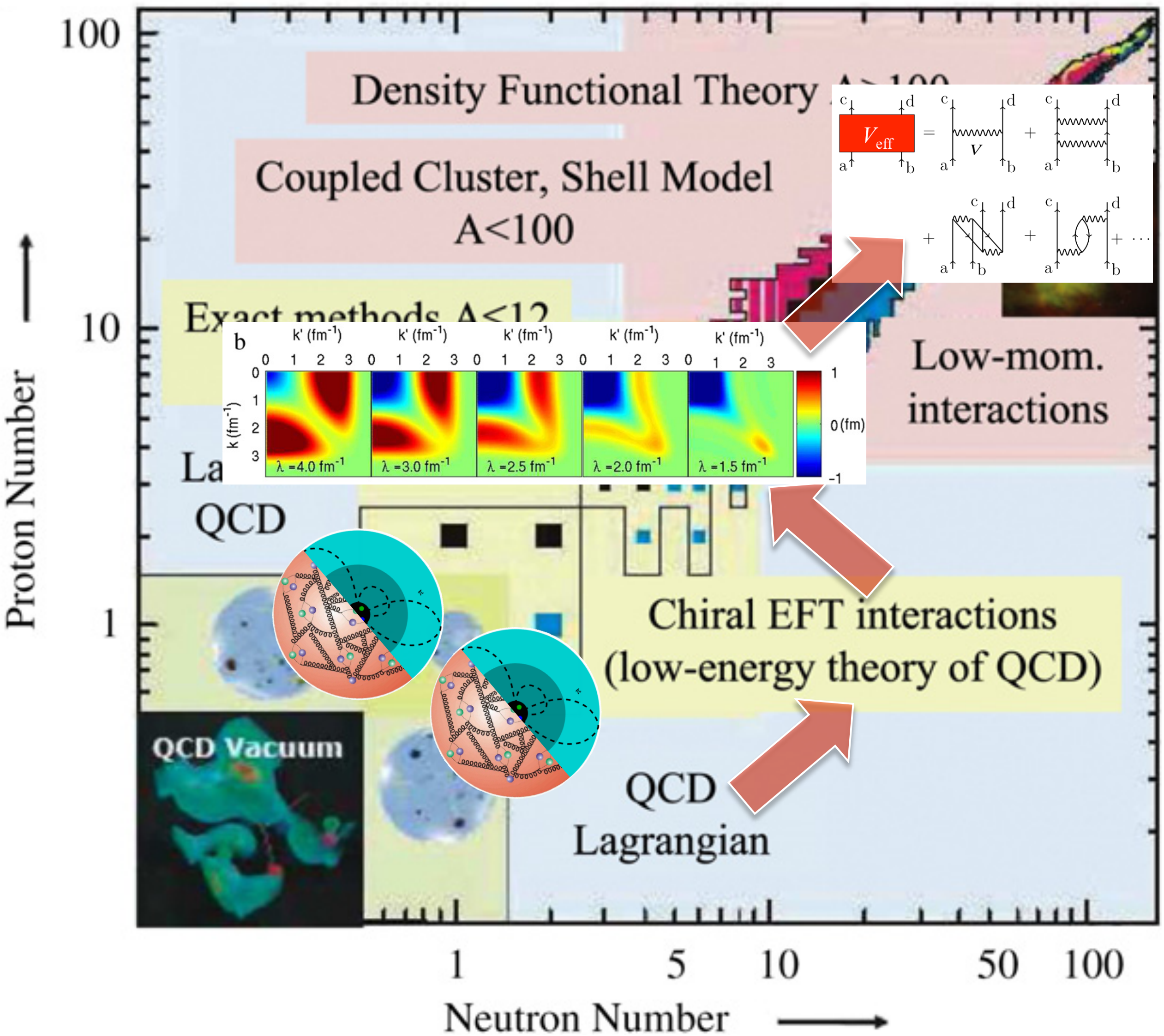

Microscopic Valence-Space Interactions

Model spaces

Many-body perturbation theory (MBPT)

Calculating effective interaction

In-medium Similarity RG

Monopole part of interaction

Deficiencies of this approach

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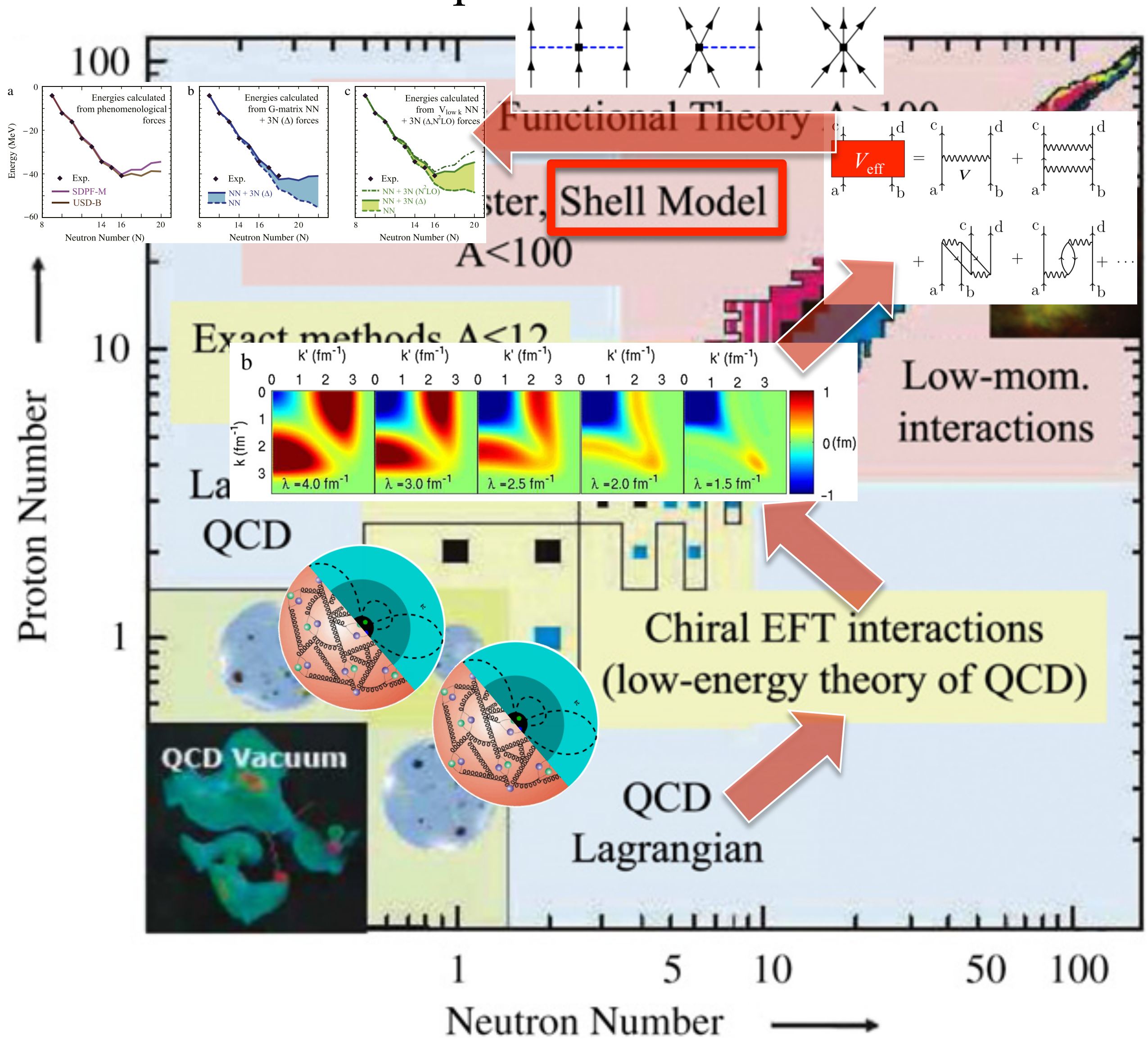

Three-Nucleon Forces

Basic ideas – why needed?

3N from chiral EFT

Implementing in shell model

Relation to monopoles

Predictions/new discoveries

Connections beyond structure

How will we approach this problem:

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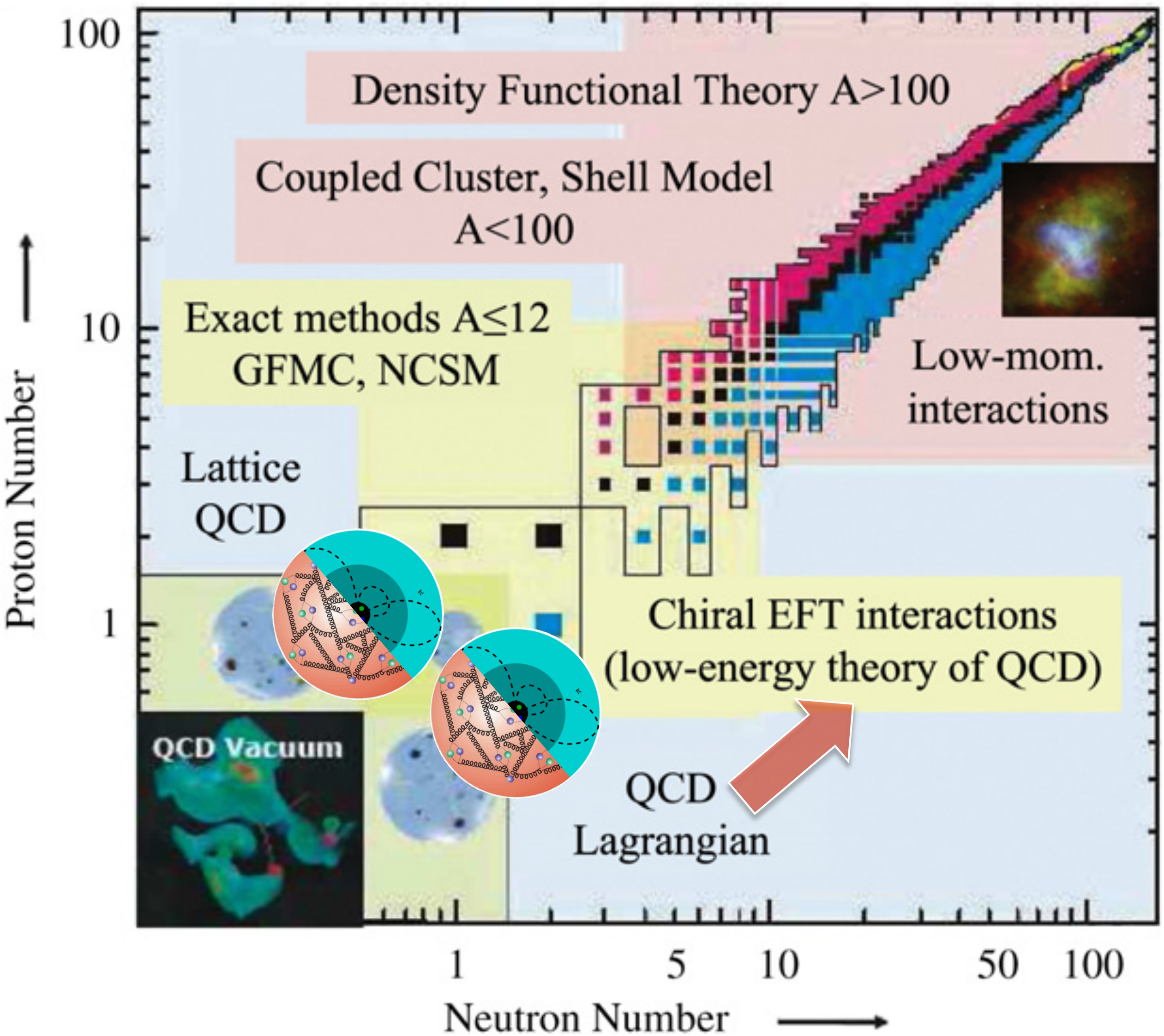
Part I: The Nucleon-Nucleon Interaction

To understand the properties of complex nuclei from first principles



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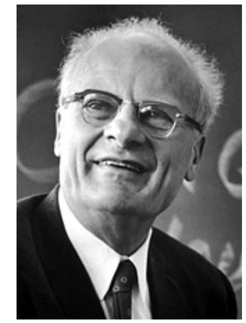
Interaction Between Two Nucleons

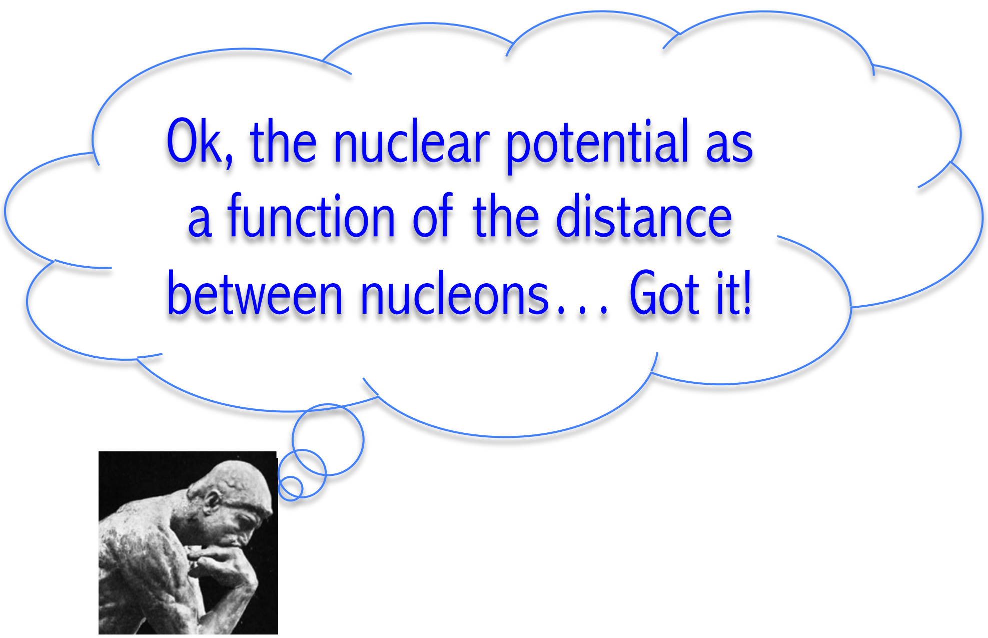
"In the past quarter century physicists have devoted a huge amount of experimentation and mental labor to this problem – probably more man-hours than have been given to any other scientific question in the history of mankind."

–H. Bethe



So let's burn a few more man-hours of mental labor on this!

To start, think to yourself what this should look like, and write it down…



Meson-Exchange Potentials: Yukawa

- First field-theoretical model of nucleon interaction proposed by **Yukawa** 1935
- Postulated nuclear force mediated by (**NEW!**) particle exchange
- Short range (~1fm) of nuclear force $\Longrightarrow$

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  New particle must be massive: $r \sim 1/m; \quad m = ?$

  Hint: $\hbar c \approx 197\,\text{MeV} \cdot \text{fm}$

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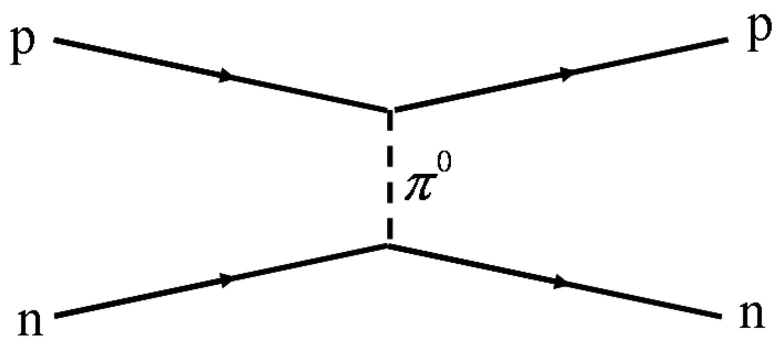

- Pion discovered 1947!

$$V(\vec{r}) = -\frac{f_\pi^2}{m_\pi^2}\left\{\vec{\sigma}_1 \cdot \vec{\sigma}_2 + C_T\left(1 + \frac{3}{m_\alpha r} + \frac{3}{(m_\alpha r)^2}\right) S_{12}(r)\right\} \frac{e^{-m_\pi r}}{m_\pi r}$$

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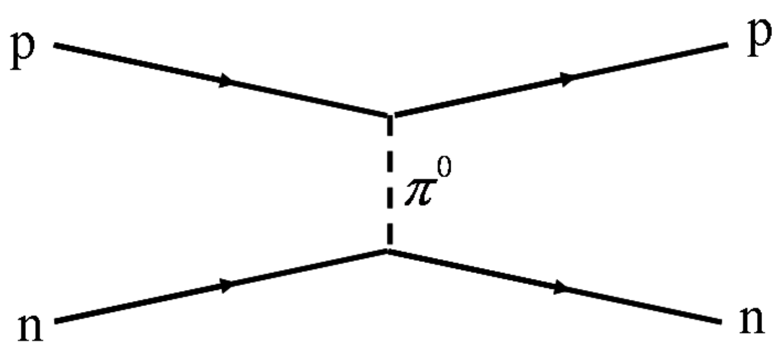

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- **Attractive, "long" range**

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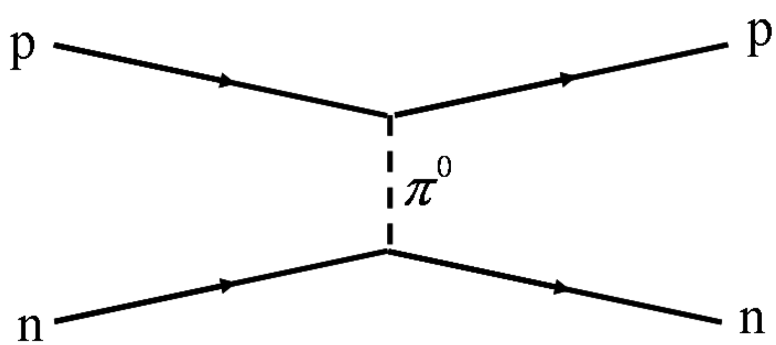

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- Attractive, "long" range, **spin dependent**

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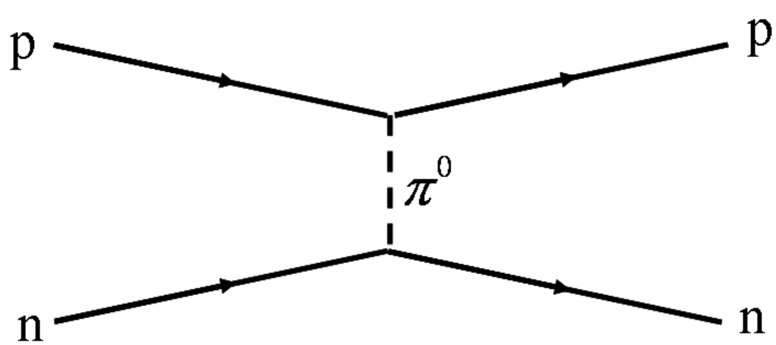

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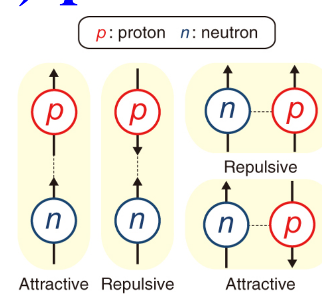
- Attractive, "long" range, spin dependent, **non-central (tensor) part**

  Depends on spin, isospin, orientation of nucleons

  Does not conserve L^2, S^2, but does conserve parity

  $\Longrightarrow$ Mixes different L states (but only differing by 2 units)



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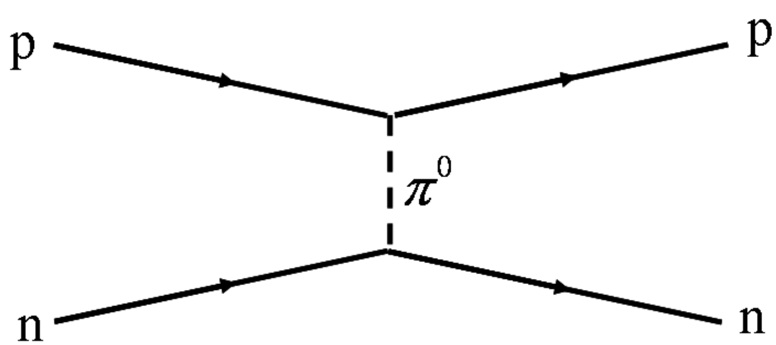

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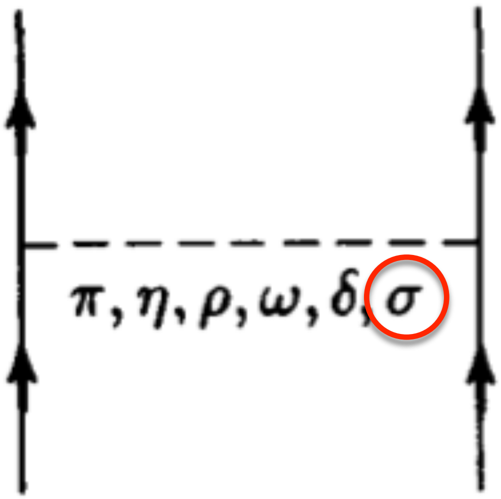
- **Attractive, "long" range, spin dependent, non-central (tensor) part**
- Successful in explaining scattering data, deuteron
- One pion is good, therefore more pions are better…
- Advanced to multi-pion theories in 1950's – **FAILED! Now what??**

One-Boson Exchange Potentials

- Heavy mesons discovered in late 1950s – formed basis for new theories
- Intermediate range – **attractive central, spin-orbit**



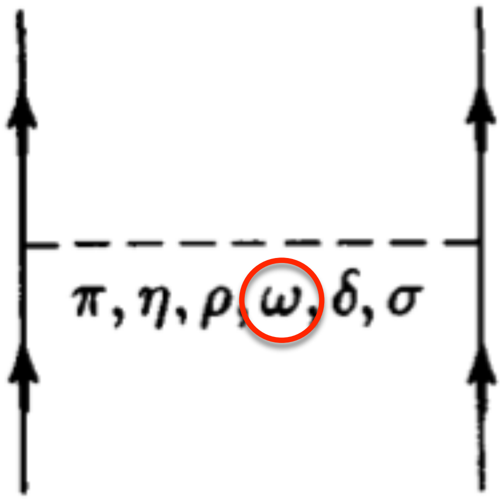
$$V^{\sigma} = g_{\sigma NN}^{2} \frac{1}{\mathbf{k}^2 + m_{\sigma}^2} \left(-1 + \frac{\mathbf{q}^2}{2M_N^2} - \frac{\mathbf{k}^2}{8M_N^2} - \frac{\vec{L}\cdot\vec{S}}{2M_N^2} \right)$$

$$\vec{q}_i \equiv \vec{p}_i^{\,\prime} - \vec{p}_i \qquad \vec{k}_i \equiv \frac{1}{2}\left(\vec{p}_i^{\,\prime} + \vec{p}_i\right)$$

Baryons	Mass (MeV)	Mesons	Mass (MeV)
p, n	938.926	π	138.03
Λ	1116.0	η	548.8
Σ	1197.3	σ	$\approx$ 550.0
Δ	1232.0	ρ	770
Σ^*	1385.0	ω	782.6
		δ	983.0
		K	495.8
		K^*	895.0

One-Boson Exchange Potentials

- Heavy mesons discovered in late 1950s – formed basis for new theories
- Short range; **repulsive central force, attractive spin-orbit**



$$V^{\omega} = g^2_{\omega NN} \frac{1}{\mathbf{k}^2 + m_\omega^2} \left(1 - 3\, \frac{\vec{L}\cdot\vec{S}}{2M_N^2} \right)$$

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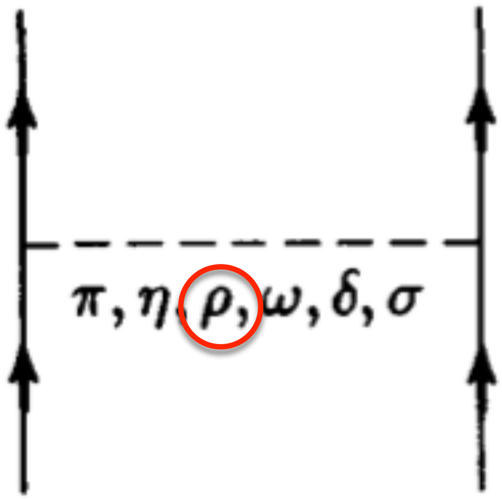
One-Boson Exchange Potentials

- Heavy mesons discovered in late 1950s – formed basis for new theories
- Short range; **tensor force opposite sign of one-pion exchange**



$$V^{\rho} = g^2_{\rho NN} \frac{\mathbf{k}^2}{\mathbf{k}^2 + m^2_{\rho}} \left(-2\, \vec{\sigma}_1 \cdot \vec{\sigma}_2 + S_{12}(\hat{k}) \right) \vec{\tau}_1 \cdot \vec{\tau}_2$$

Baryons	Mass (MeV)	Mesons	Mass (MeV)
p, n	938.926	π	138.03
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Σ	1197.3	σ	≈ 550.0
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Parameterizing the NN Interaction

Starting from any NN-interaction, first solve:

Lipmann-Schwinger scattering T-matrix equation:

$$T^{\alpha}_{ll'}(k,k';K) = V^{\alpha}_{ll'}(k,k') + \frac{2}{\pi}\sum_{l''}\int_0^{\infty} q^2 \mathrm{d}q\, V^{\alpha}_{ll''}(k,q)\frac{q}{k^2-q^2+i\varepsilon}T^{\alpha}_{l''l'}(q,k';K)$$

where $T^{\alpha}_{ll'}(k,k';K) = \langle kK, lL; JST \mid T \mid k'K, l'L; JST\rangle$

Parameterized in partial waves α – in **relative/center of mass frame (K,L)**

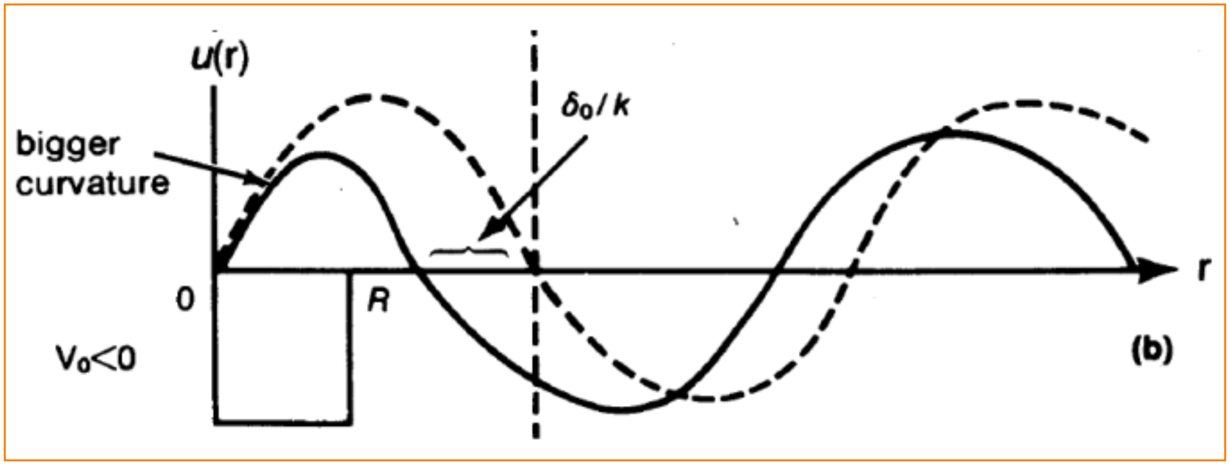
$$\tan\delta(k) = -kT(k,k)$$

Fully-on-shell *T*-matrix directly related to experimental data

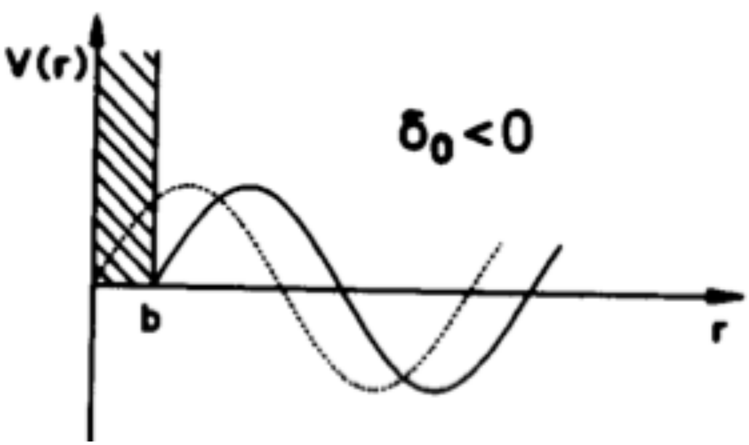
Constraining NN Scattering Phase Shifts

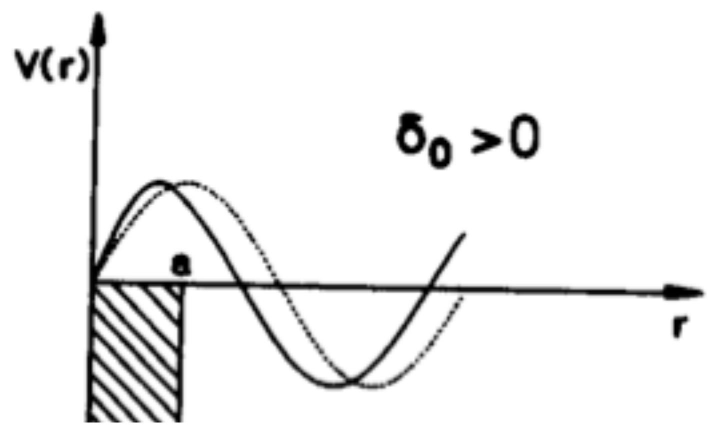
Phase shift is a function of relative momentum k; Figure shows s-wave

Scattering from an attractive well potential



Scattering from repulsive core: phase shift opposite sign





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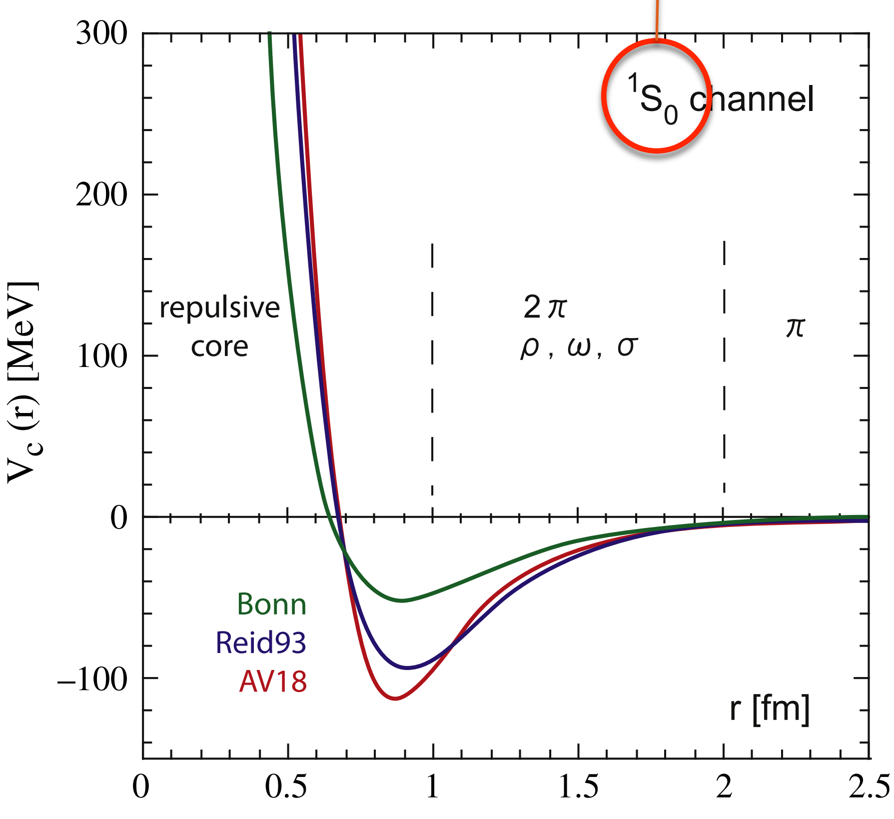
Note intermediate momentum allowed to infinity (but cutoff by regulators)

Coupling of low-to-high momentum in V

Form of NN Interactions

Textbook nuclear potentials in coordinate (**r**) space (distance between nucleons)

$$^{2S+1}L_J$$



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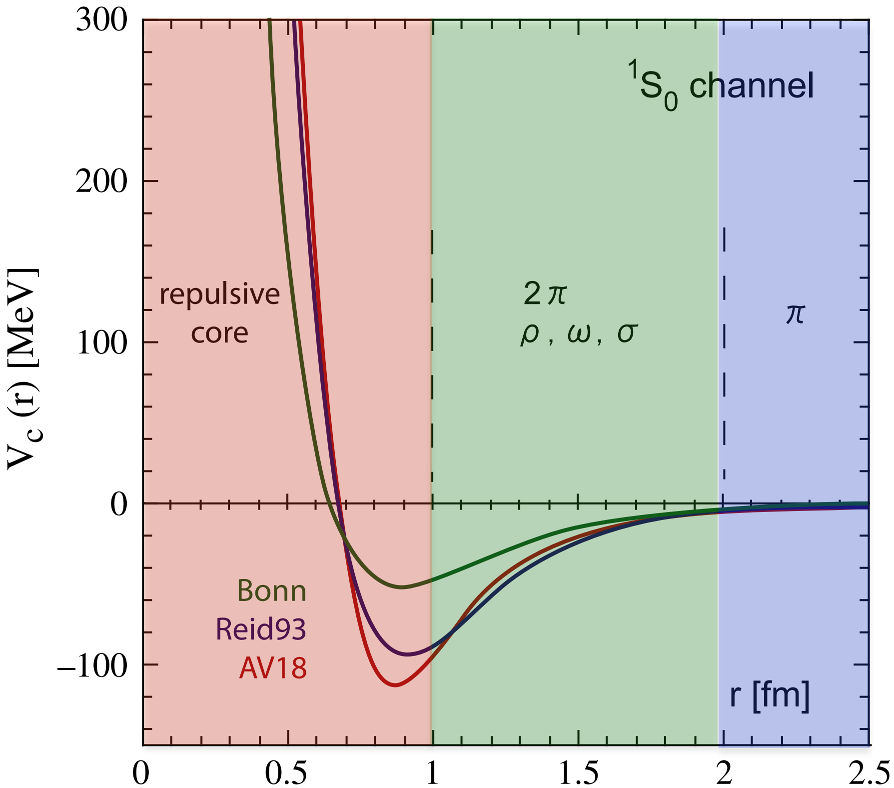
Hard core, **intermediate-range 2π**, **long-range 1π exchange (OPE)**



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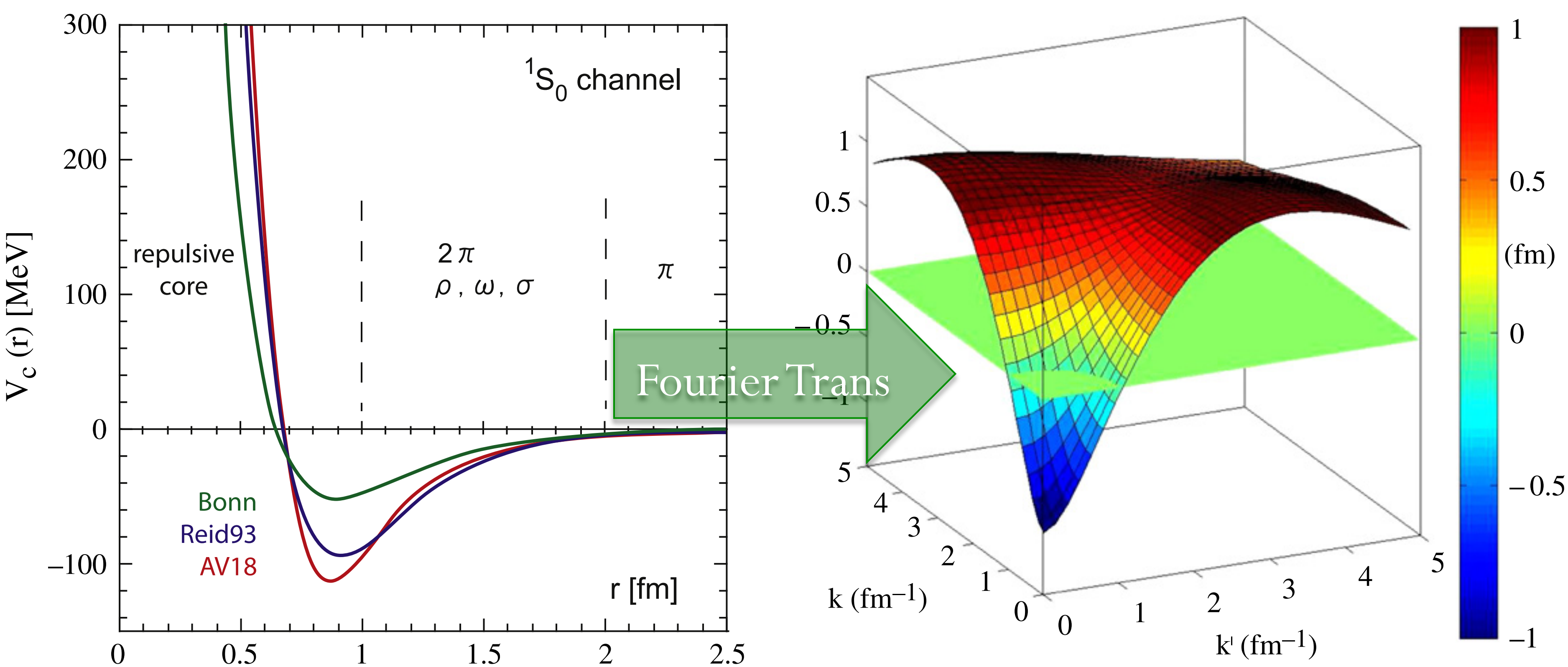
Transform to momentum space via **Fourier Transformation**

Strong high-momentum repulsion, **low-momentum attraction**

$$V_l(k,k') = \frac{2}{\pi}\int_0^\infty r^2 \mathrm{d}r\, j_l(kr)V(r)j_l(k'r)$$



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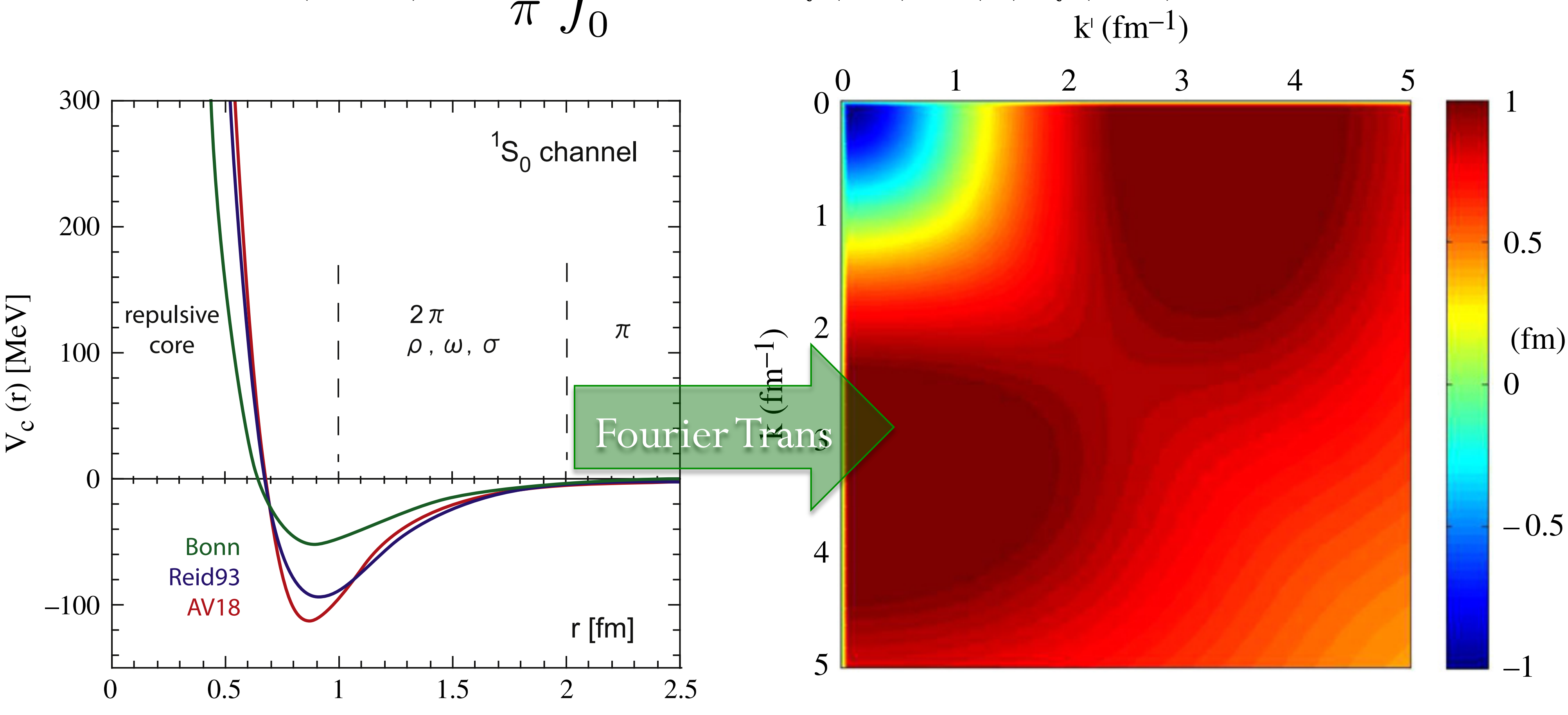

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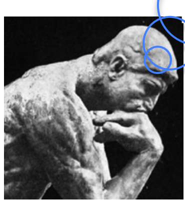
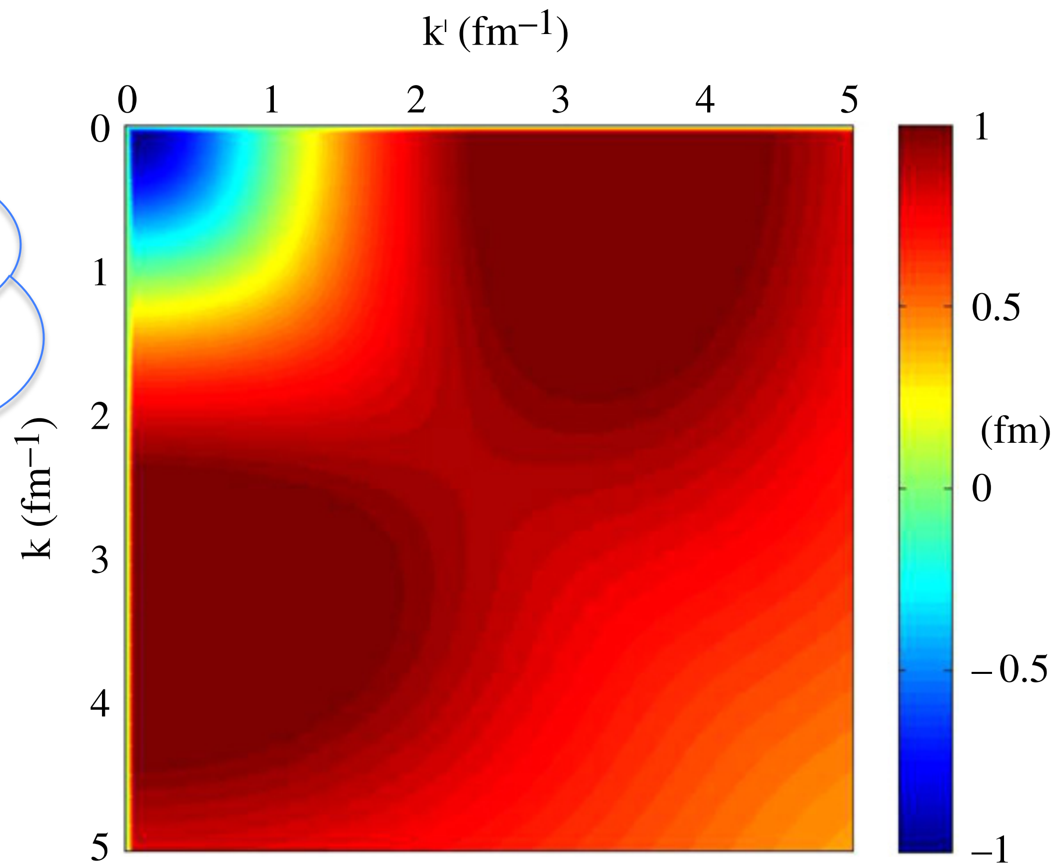
Wait a minute… these potentials can't really go to zero range/infinitely high energies; that would be QCD?



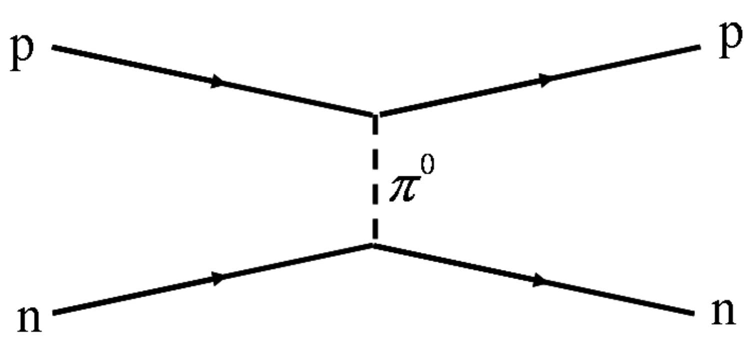


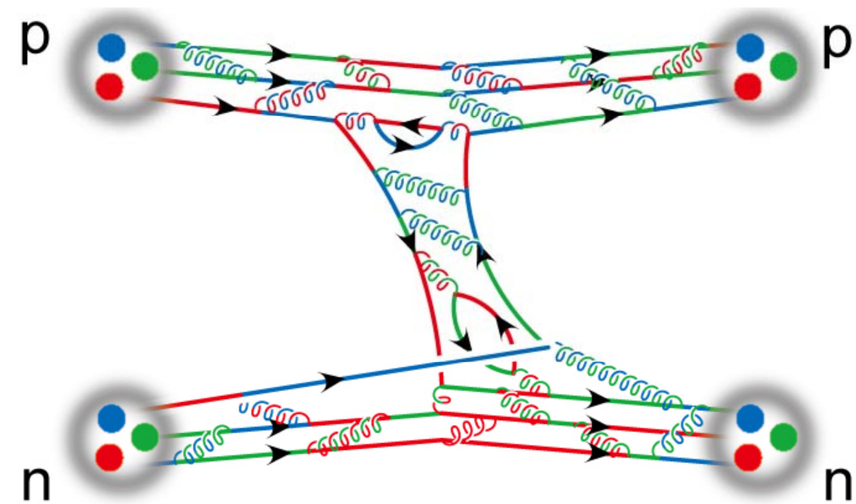
NN Interaction from QCD?

Meson exchange in principle described in Quantum Chromodynamics (QCD)

Low-energy region non-perturbative – treat in the context of **Lattice QCD**

Directly from QCD Lagrangian, solve numerically on discretized space-time





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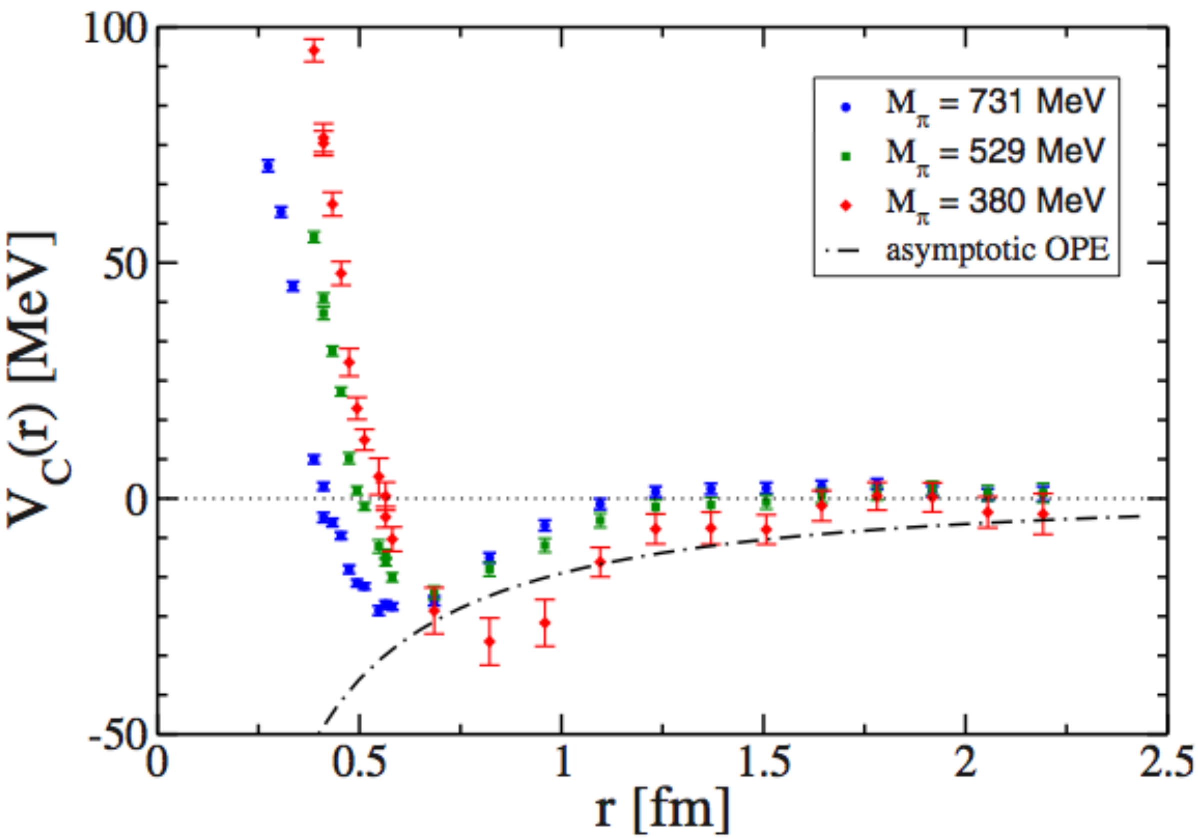

Lattice results give long-range OPE tail, hard core

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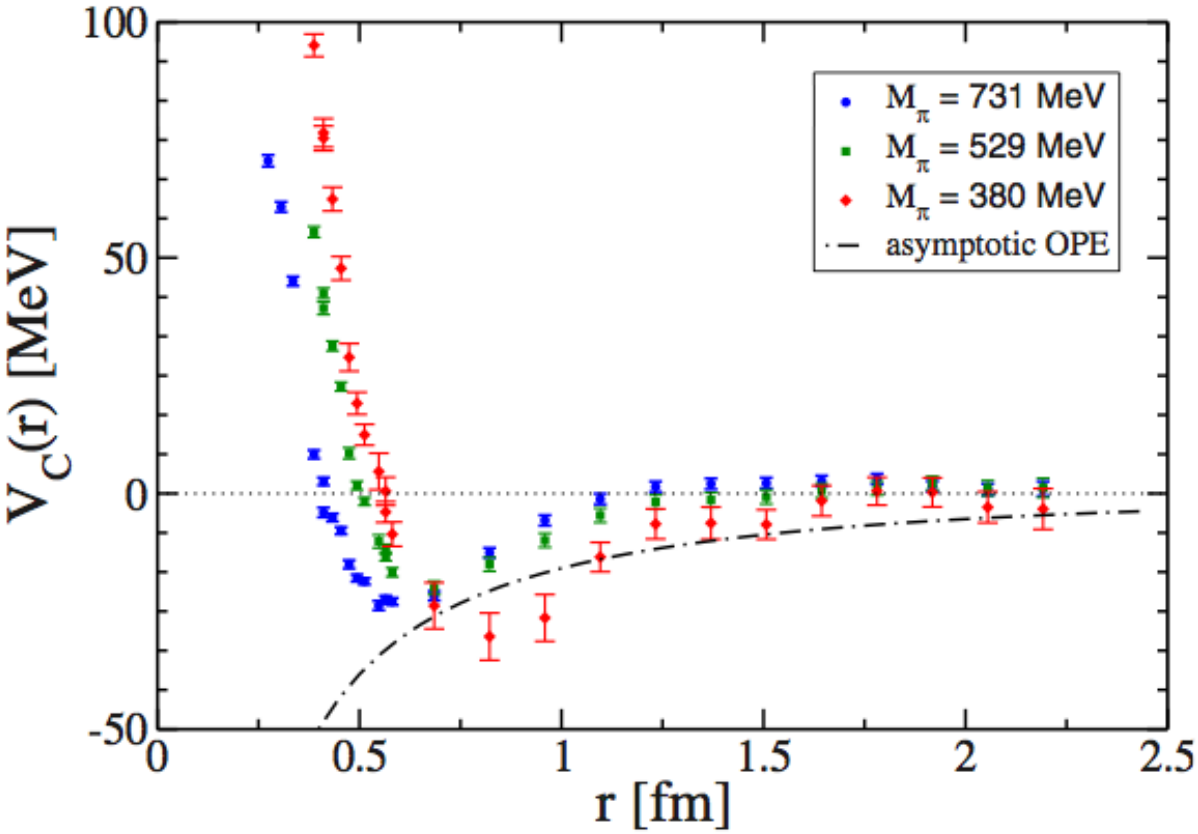
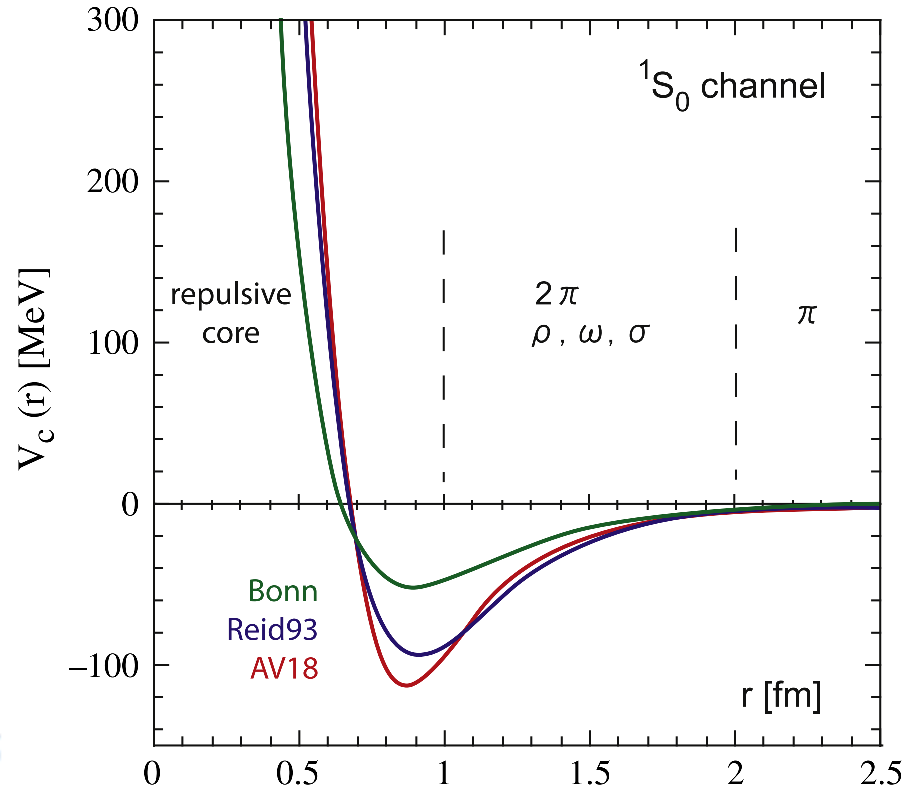




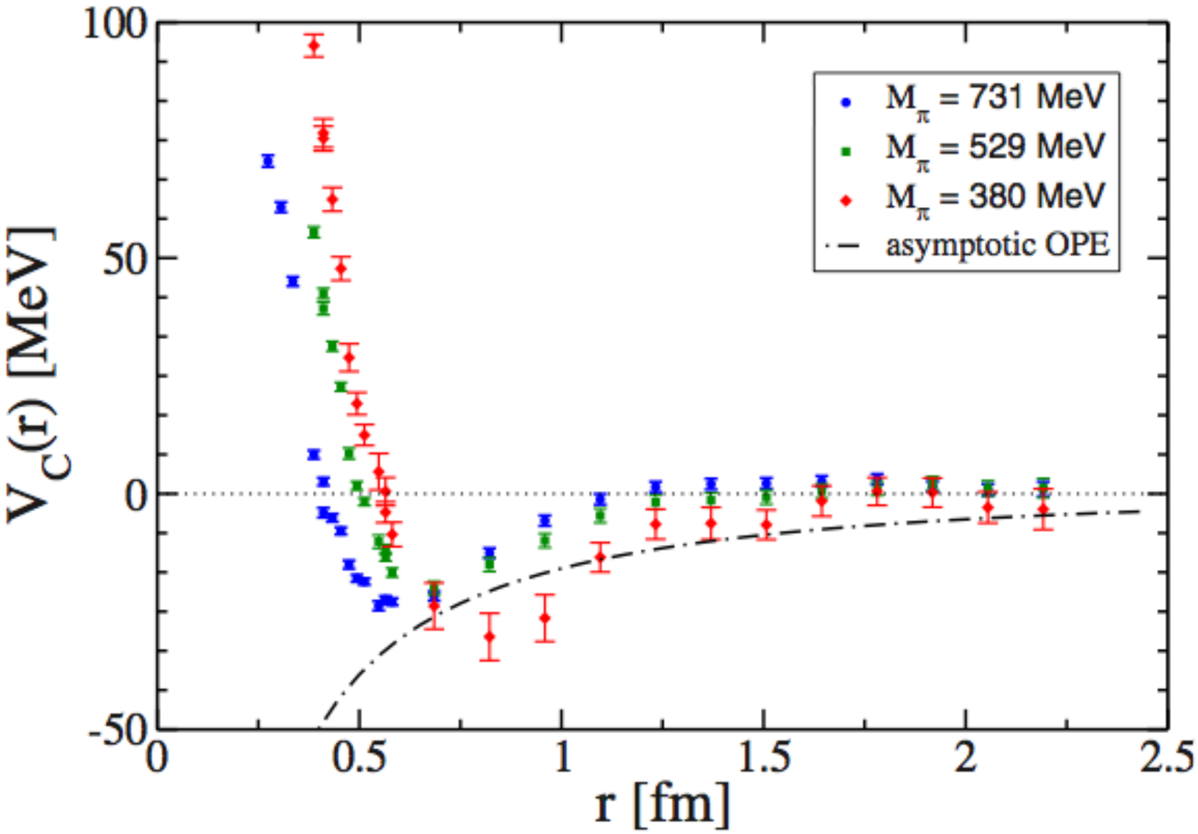
Lattice results give long-range OPE tail, hard core

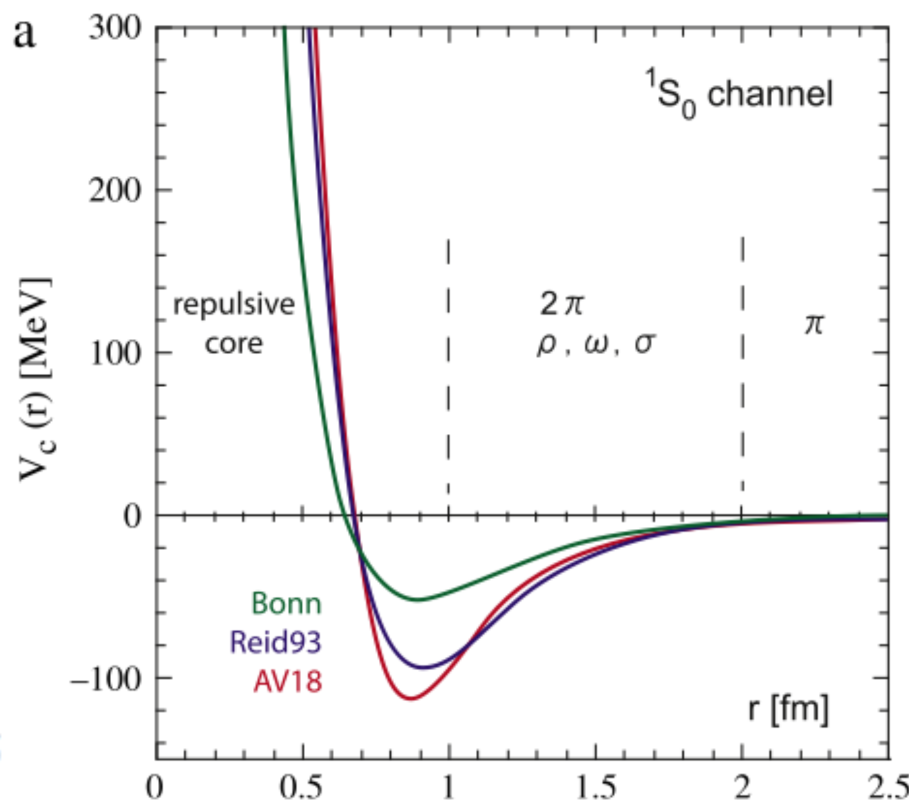
Not yet to physical pion mass – work in progress – so we're done, right?

Unique NN Potential?

What does this tell us in our quest for an NN-potential?

Expected form seems to be confirmed by QCD





OBE Potentials: Summary/Problems

First generation (1960-1990): Paris, Reid, Bonn-A,B,C $\chi^2/\text{dof} \approx 2$

High-precision potentials (1990s): Focus on precision ~40 parameters fit NN data

ArgonneV18, Reid93, Nijmegen, CD-Bonn $\chi^2/\text{dof} \approx 1$

NN problem "solved" !!

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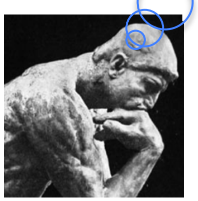
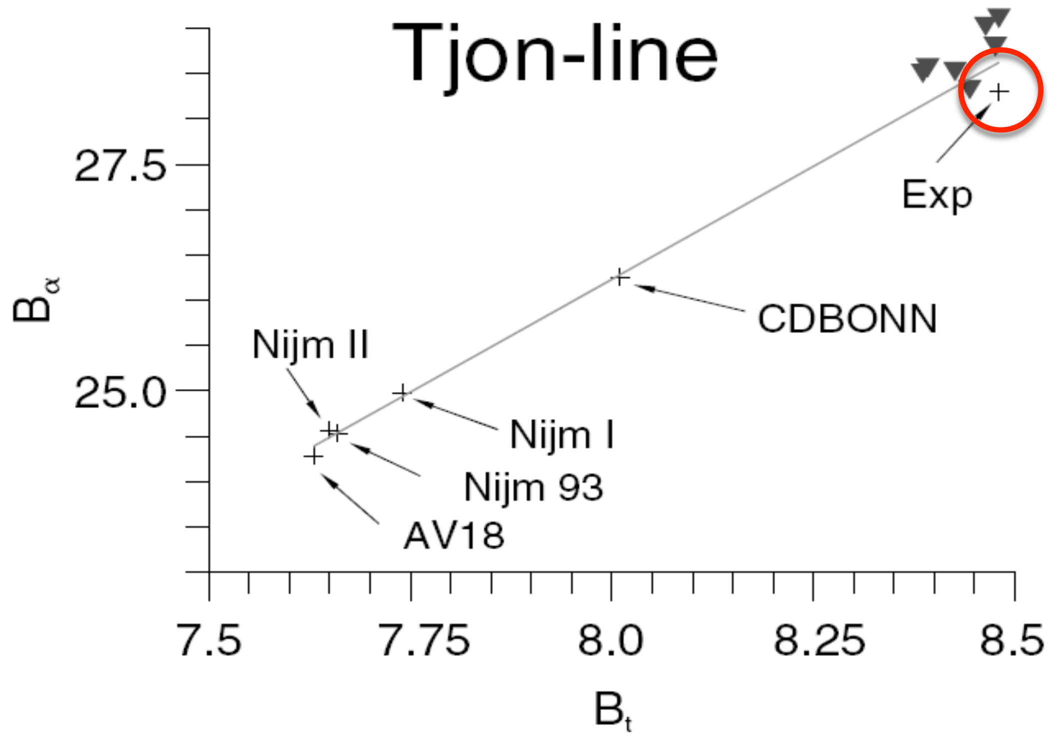
ArgonneV18, Reid93, Nijmegen, CD-Bonn $\chi^2/\text{dof} \approx 1$

NN problem "solved" !!

Hmmm… what if we were to go beyond two nucleons?





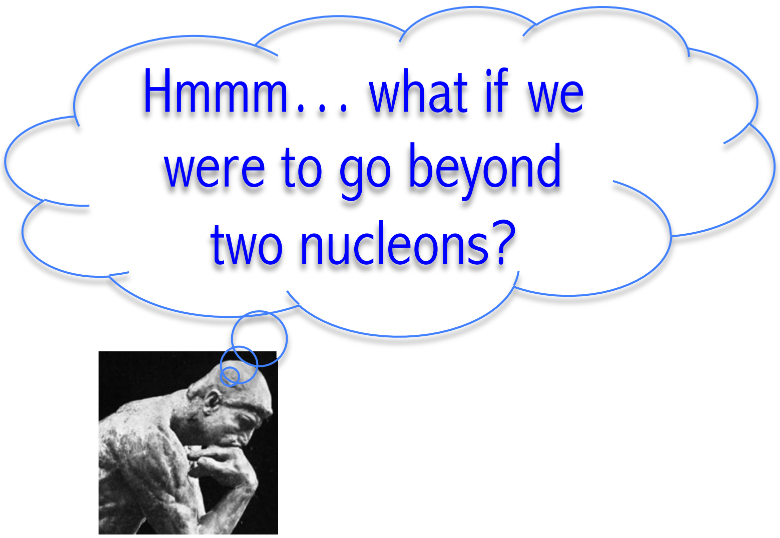
OBE Potentials: Summary/Problems

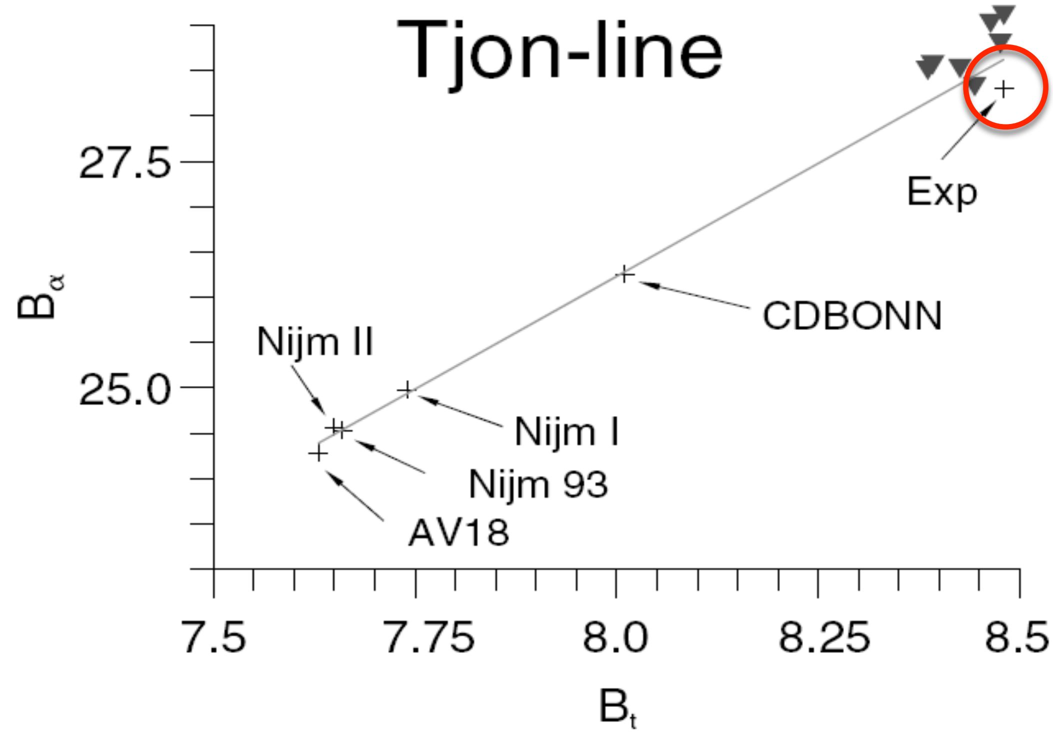
First generation (1960-1990): Paris, Reid, Bonn-A,B,C $\chi^2/\text{dof} \approx 2$

High-precision potentials (1990s): Focus on precision ~40 parameters fit NN data

ArgonneV18, Reid93, Nijmegen, CD-Bonn $\chi^2/\text{dof} \approx 1$

NN problem “solved” !!



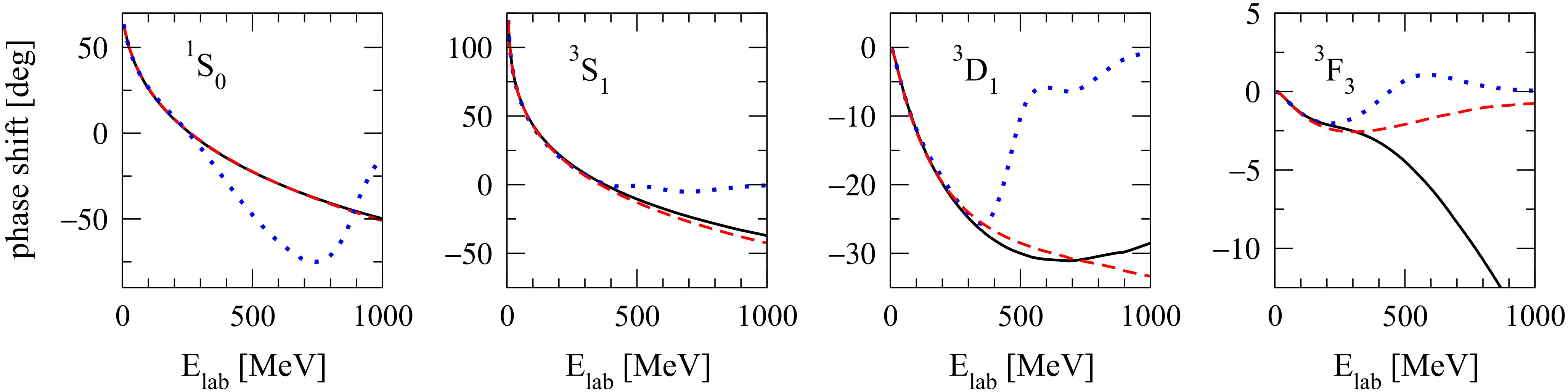


Many successes, but…

1) Difficult (impossible) to assign theoretical error
2) 3N forces (what are those??) not consistent with NN forces
3) No clear connection to QCD
4) Clear **model dependence**…

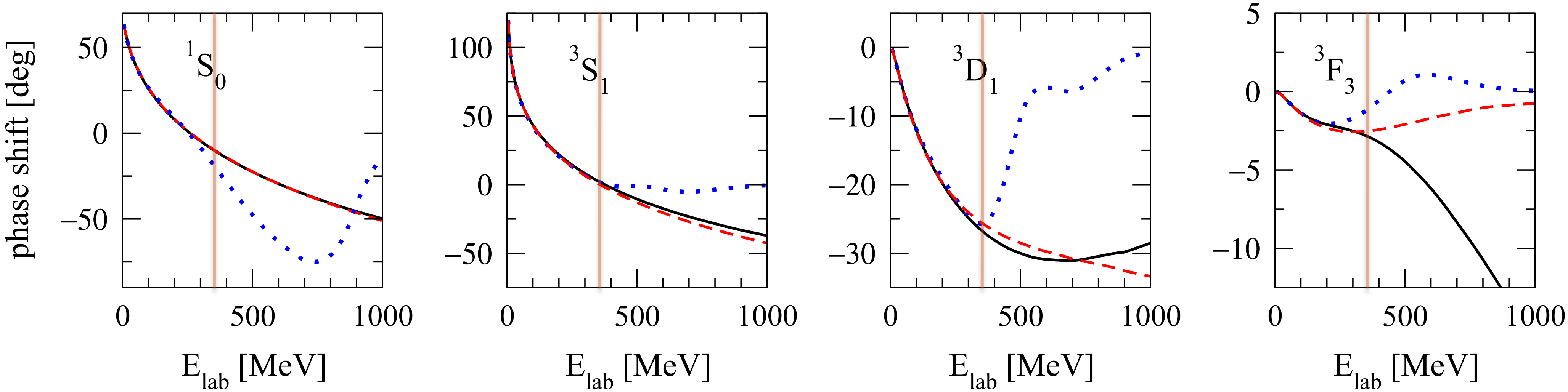
Meson-Exchange Potentials and Phase Shifts

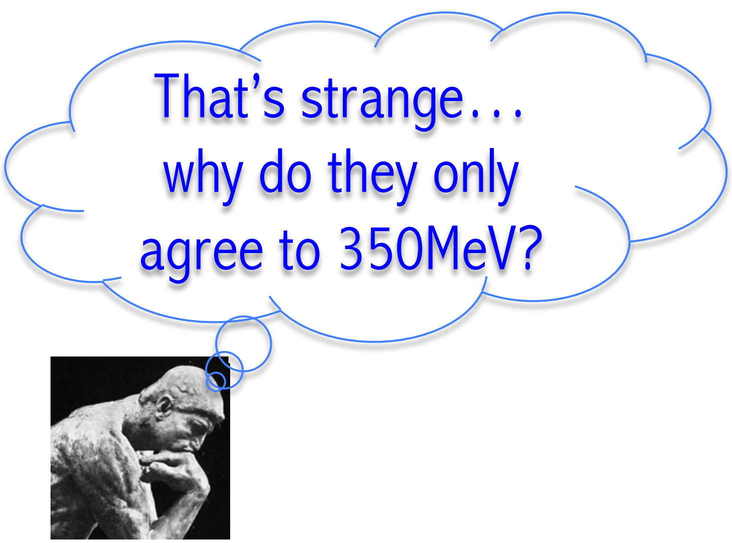
Further model dependence: scattering phase shifts of NN potentials



Meson-Exchange Potentials and Phase Shifts

Further model dependence: scattering phase shifts of NN potentials

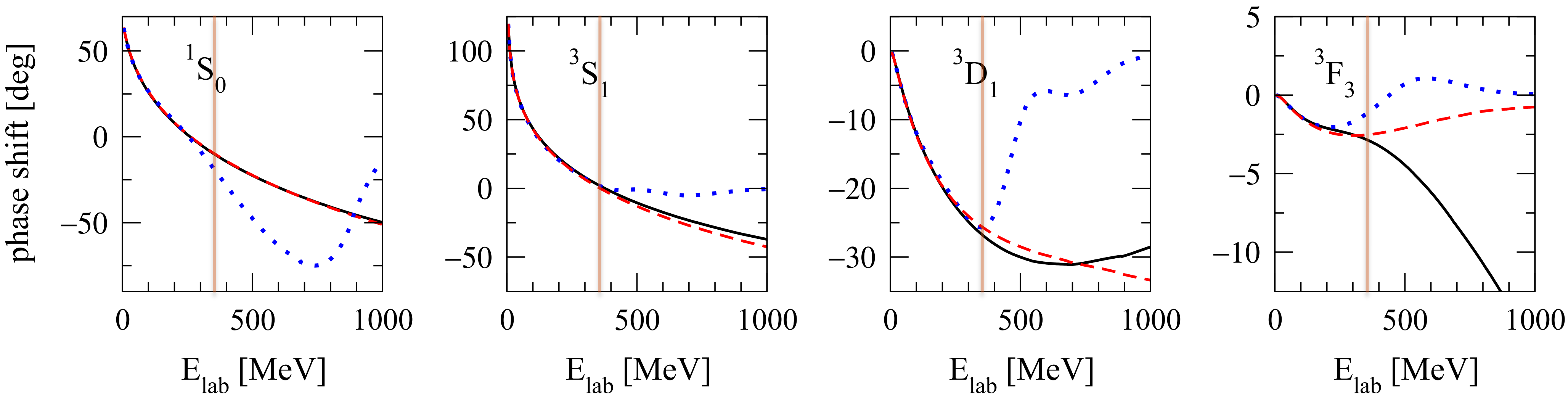


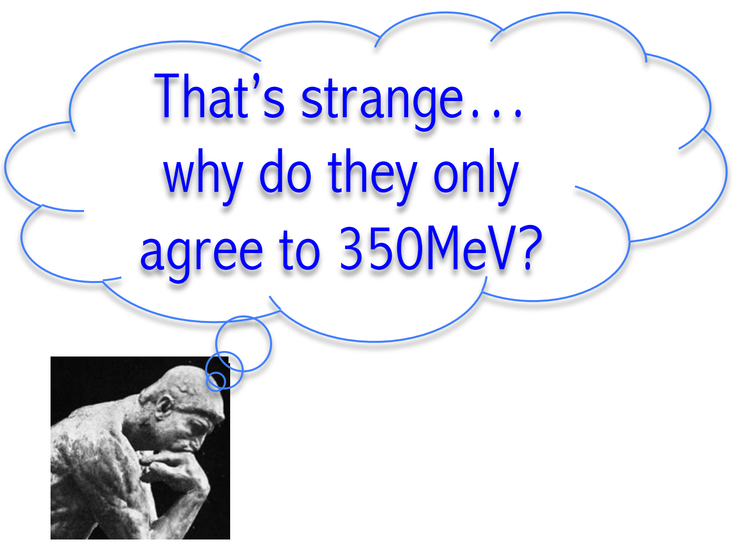


Remember, all have $\chi^2/\text{dof} \approx 1$

Meson-Exchange Potentials and Phase Shifts

Further model dependence: scattering phase shifts of NN potentials





Agree well up to **pion-production threshold** ~350MeV

Most models don't fit phase shifts above this energy – **unconstrained**

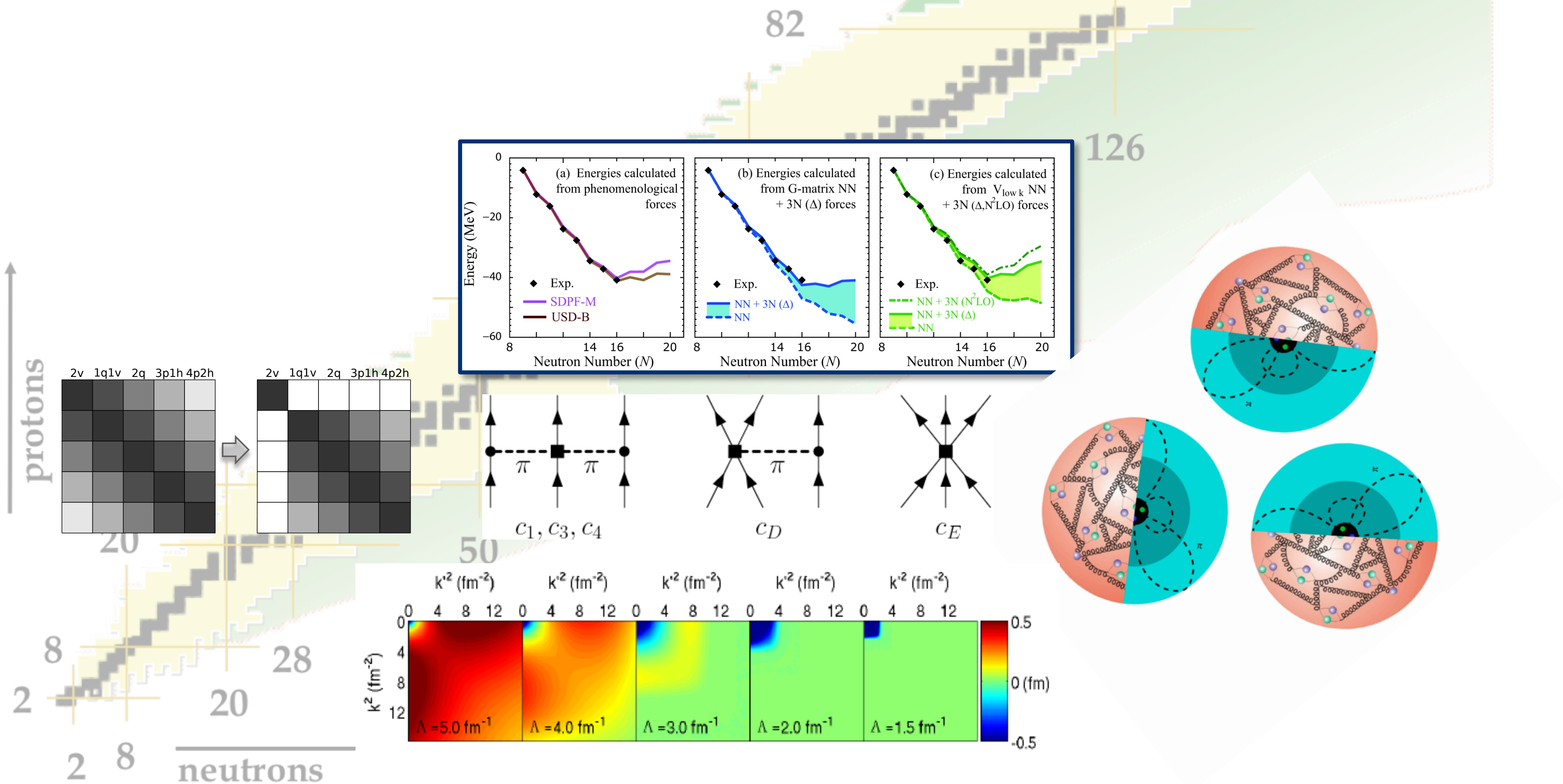
Day 2: Effective Field Theories

Jason D. Holt

NSERC CRSNG

TRIUMF

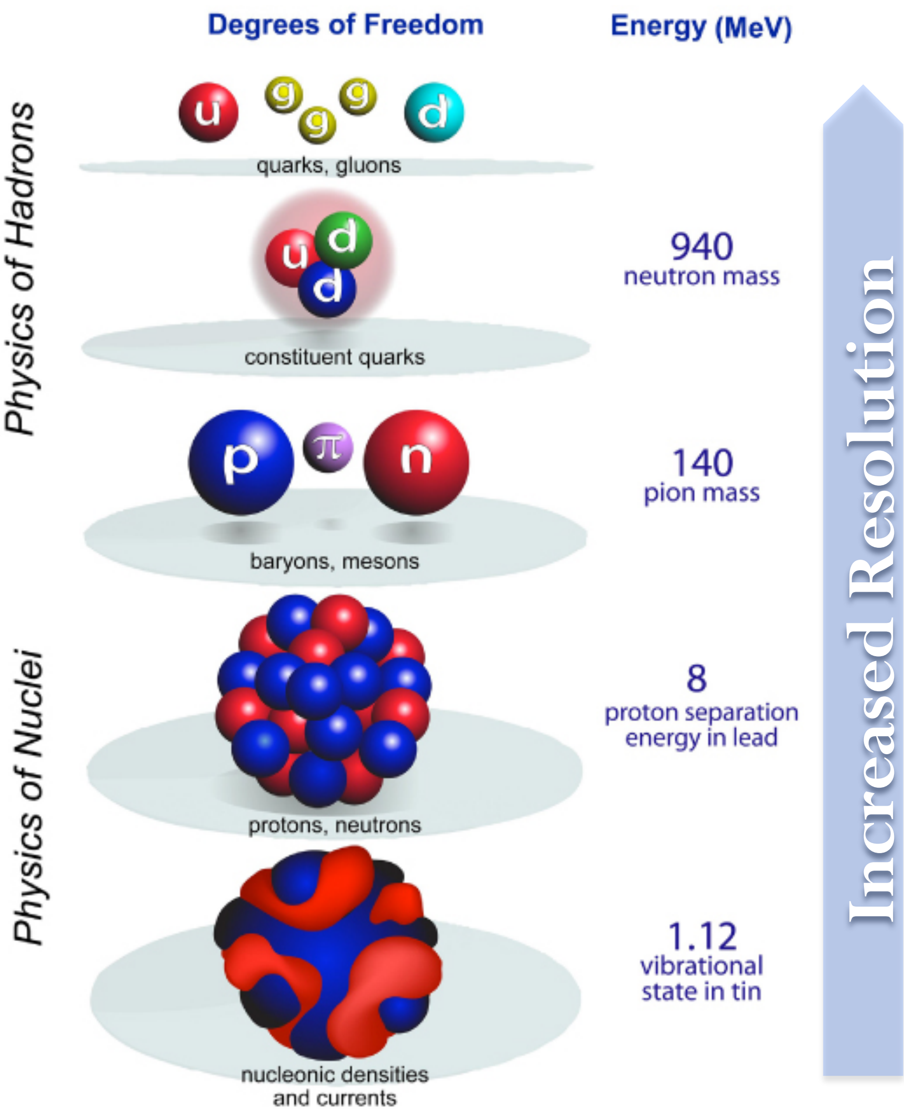
NRC·CNRC



From QCD to Nuclear Interactions

How do we determine interactions between nucleons?

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{3\mathrm{N}}(\Lambda) + V_{4\mathrm{N}}(\Lambda) + \cdots$$



Old view:

Multiple scales complicate life

No meaningful way to connect them

Modern view:

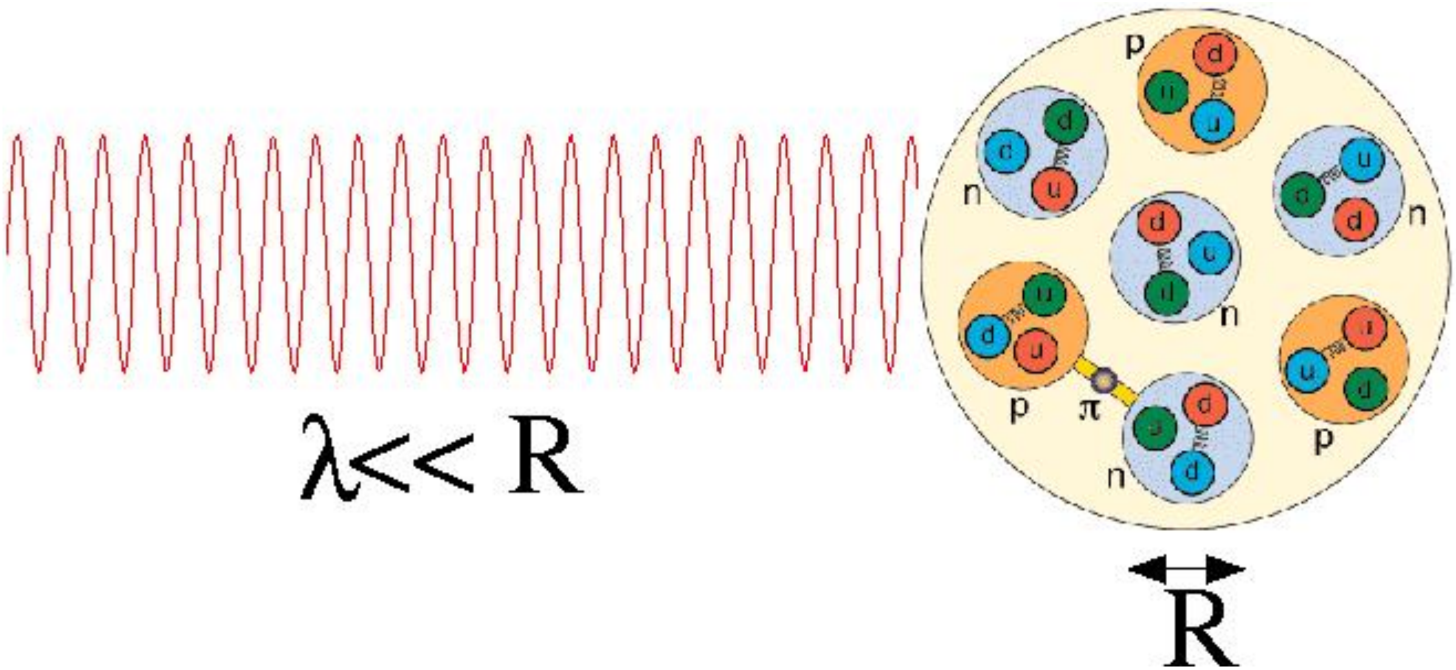
Choose convenient resolution scale

Effective field theory at each scale connected by RG

Ratio of scales – small parameters

Ideas Behind Effective Theories

Resolution scale and relevant degrees of freedom

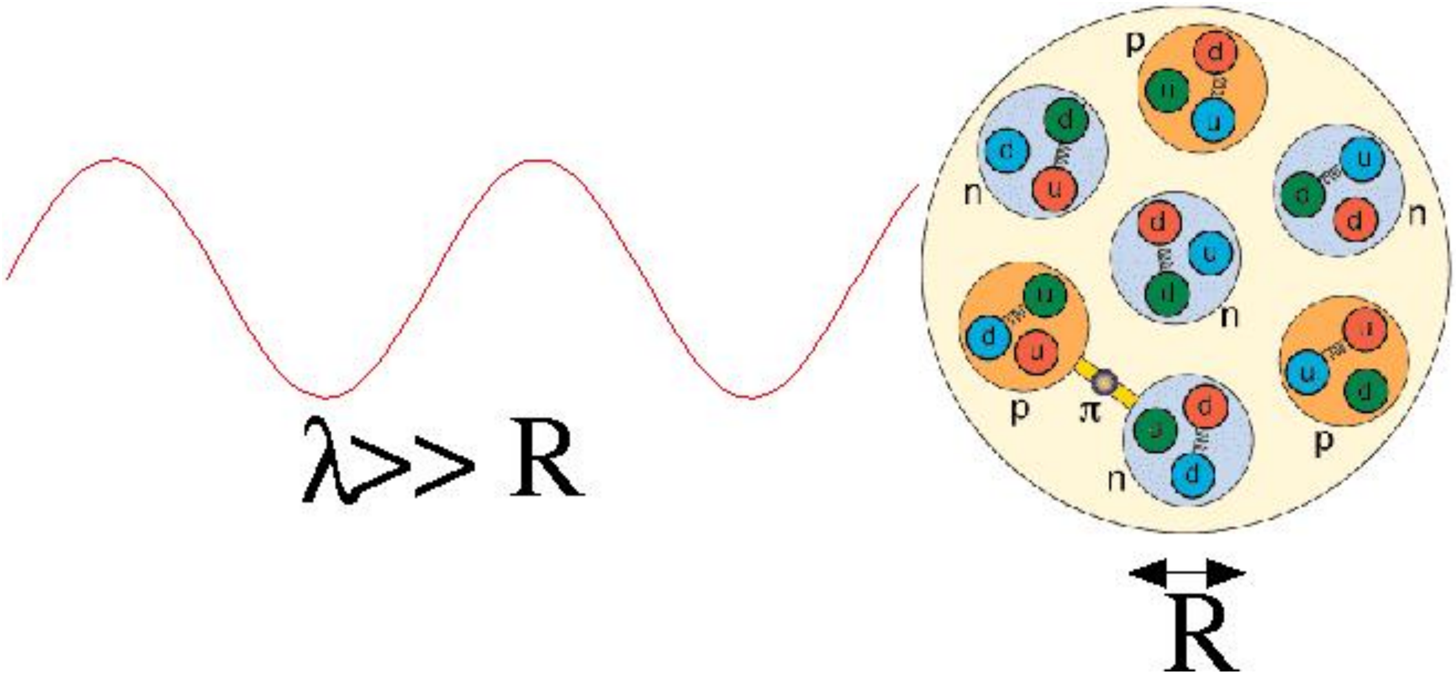


High energy probe resolves fine details

Need high-energy degrees of freedom

Ideas Behind Effective Theories

Resolution scale and relevant degrees of freedom

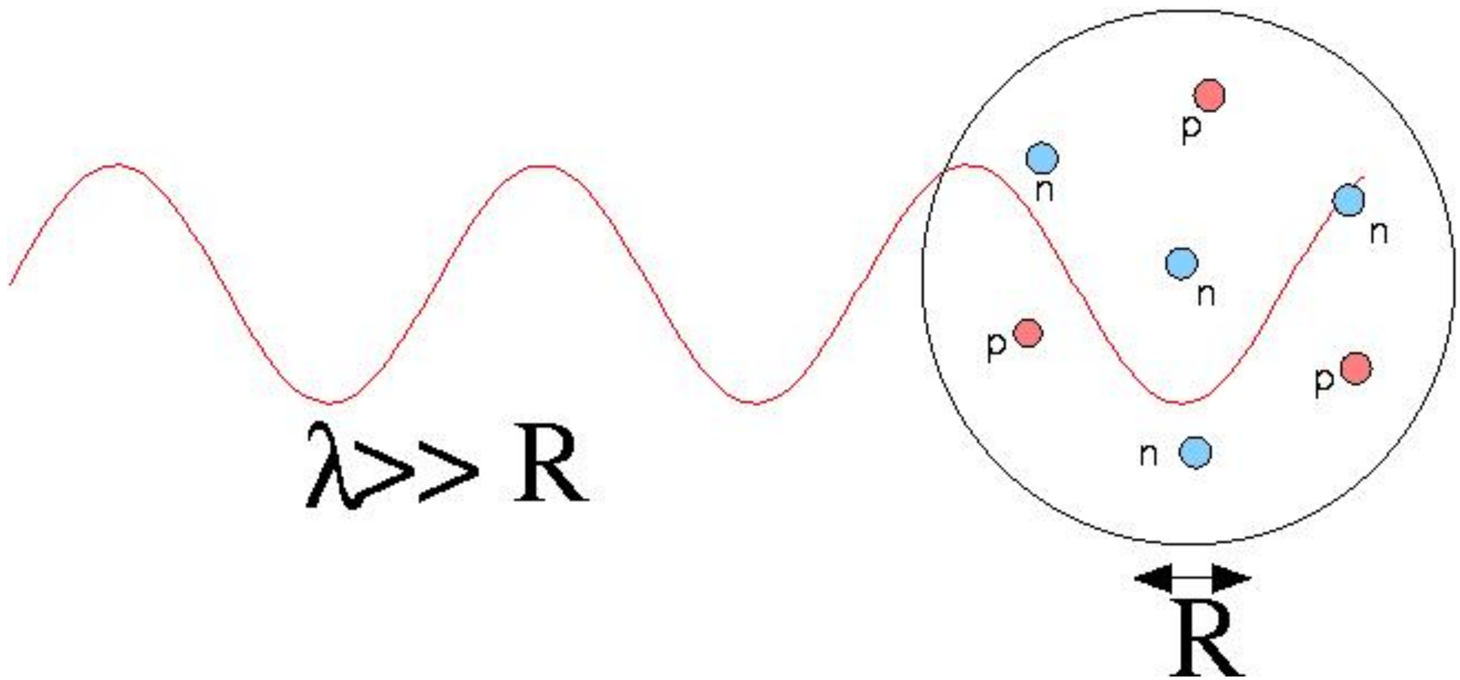


Low-energy probe can't resolve such details

Don't need high-energy degrees of freedom – replace with something simpler

Ideas Behind Effective Theories

Resolution scale and relevant degrees of freedom



Low-energy probe can't resolve such details

Don't need high-energy degrees of freedom – replace with something simpler

Use more convenient dofs, but **preserve low-energy observables!**

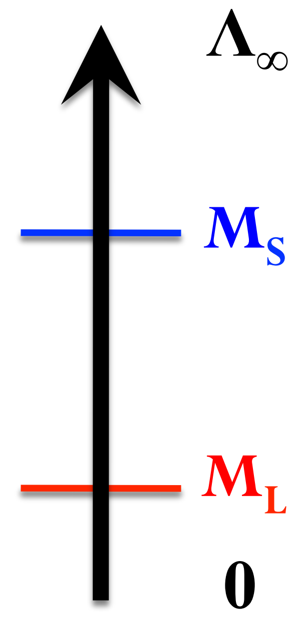
Ideas Behind Effective Theories

Assume underlying theory with cutoff Λ_∞

$$V = V_\mathrm{L} + V_\mathrm{S}$$

Known **long-distance physics** (like 1π exchange) with some scale M_L

Short-distance physics (ρ,ω exchange) with some scale M_S



Ideas Behind Effective Theories

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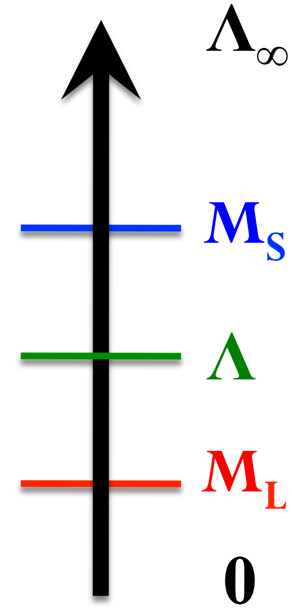
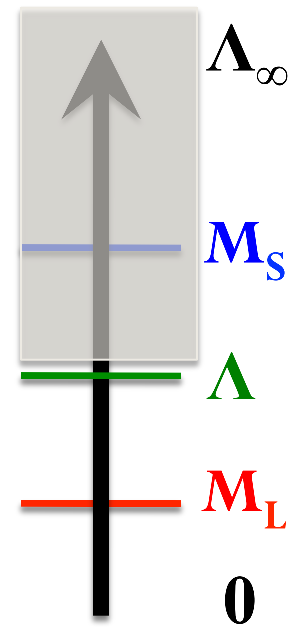


And we want a **low-energy *effective* theory** for physics up to some

$$M_{\rm L} < \Lambda < M_{\rm S}$$

Ideas Behind Effective Theories

Assume underlying theory with cutoff Λ_∞



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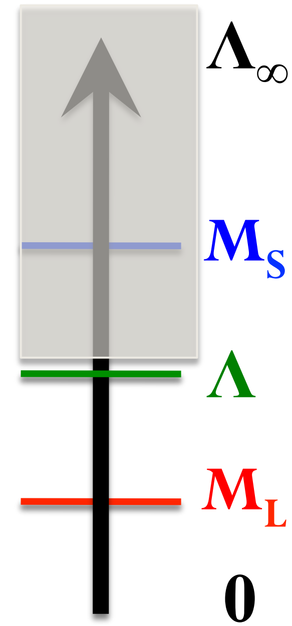
Integrate out states above **Λ** using **Renormalization Group (RG)**

Ideas Behind Effective Theories

Assume underlying theory with cutoff Λ_∞



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And we want a **low-energy *effective* theory** for physics up to some

$$M_{\rm L} < \Lambda < M_{\rm S}$$

Integrate out states above **Λ** using **Renormalization Group (RG)**

General form of effective theory: $V_{\rm eff} = V_{\rm L} + \delta V_{\rm c.t.}(\Lambda)$

where $\delta V_{\rm c.t.}(\Lambda) = C_0(\Lambda)\delta^3(\mathbf{r}) + C_2(\Lambda)\nabla^2\delta^3(\mathbf{r}) + \cdots$

Also use RG to change resolution scales **within particular EFT**

Ideas Behind Effective Theories

General form of effective theory: $V_{\rm eff} = V_{\rm L} + \delta V_{\rm c.t.}(\Lambda)$

$$\delta V_{\rm c.t.}(\Lambda) = C_0(\Lambda)\,\delta^3(\mathbf{r}) + C_2(\Lambda)\,\nabla^2\delta^3(\mathbf{r}) + \cdots$$

Encodes effects of high-E dof on low-energy observables

Universal; depends only on symmetries

TWO choices:

Short distance structure of "true theory" captured in several numbers

- **Calculate from underlying theory**

When short-range physics is unknown or too complicated

- **Extract from low-energy data**

How do we apply these ideas to nuclear forces?

Chiral Effective Field Theory: Philosophy

"At each scale we have different degrees of freedom and different dynamics. Physics at a larger scale (largely) decouples from physics at a smaller scale… thus a theory at a larger scale remembers only finitely many parameters from the theories at smaller scales, and throws the rest of the details away.

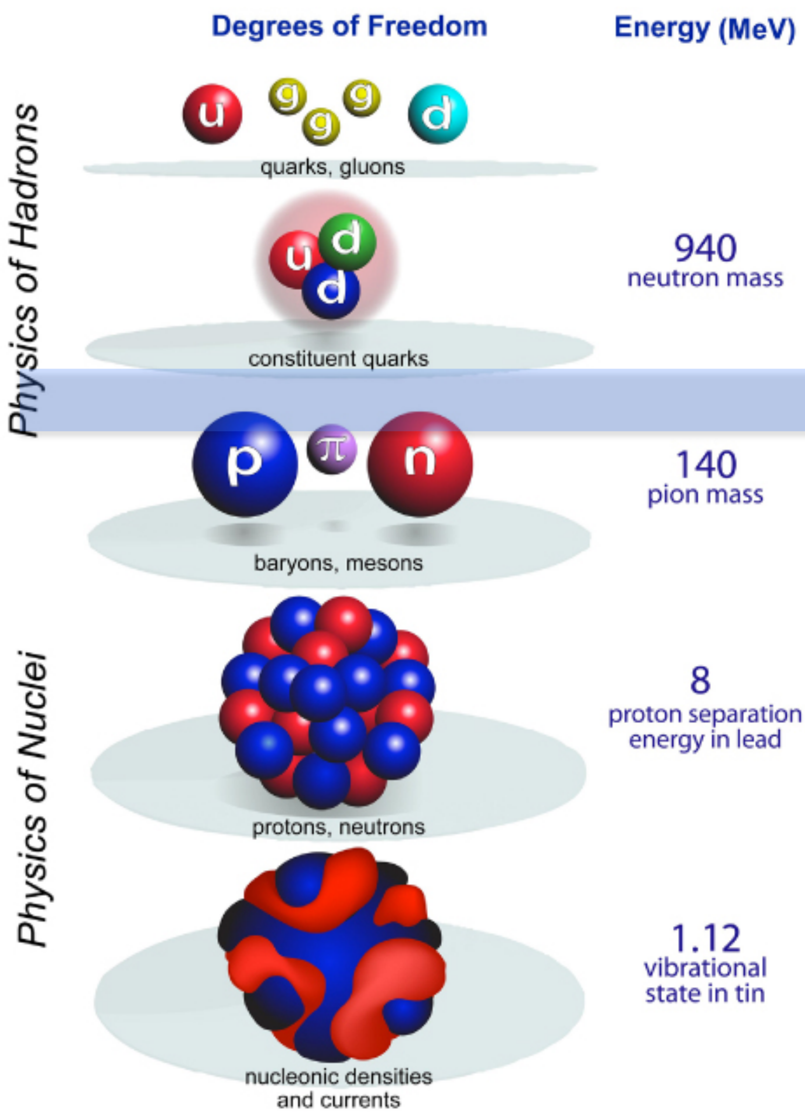
More precisely, when we pass from a smaller scale to a larger scale, we average out irrelevant degrees of freedom… The general aim of the RG method is to explain how this decoupling takes place and why exactly information is transmitted from one scale to another through finitely many parameters."
- ***David Gross***

"The method in its most general form can.. be understood as a way to arrange in various theories that the degrees of freedom that you're talking about are the relevant degrees of freedom for the problem at hand."
- ***Steven Weinberg***

5 Steps to constructing such a theory for nuclear forces

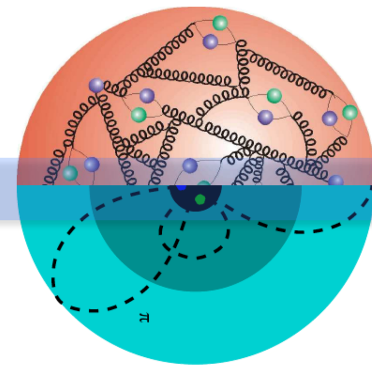
Separation of Scales in Nuclear Physics

Step I: Identify appropriate separation of scales, degrees of freedom



Appropriate separation of scales

Typical momenta in nucleus $Q \sim m_\pi$



$\Lambda_{\mathrm{chiral}} \sim 700\,\mathrm{MeV}$

Only degrees of freedom relevant to energy scale

Effective theory: only nucleons and pions

How low can we go??

Chiral EFT Symmetries

Step II: Identify relevant symmetries of underlying theory (QCD)

1. SU(3) color symmetry from QCD
   (Nucleons and pions are color singlets)

2. Chiral symmetry: u and d quarks are almost massless
   - Left and right-handed (massless) quarks do not mix: $SU(2)_L \times SU(2)_R$ symmetry
   - Explicit symmetry breaking: u and d quarks have a small mass
   - Spontaneous breaking of chiral symmetry (no parity doublets observed in Nature)
     - $SU(2)_L \times SU(2)_R$ symmetry spontaneously broken to $SU(2)_V$
     - Pions are the Nambu-Goldstone bosons of spontaneously broken symmetry
     - Low-energy pion Lagrangian completely determined

Missing ingredient in multi-pion-exchange theories of 50's!

Construct Lagrangian based on these symmetries

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi\text{N}} + \mathcal{L}_{\text{NN}}$$

Chiral EFT Lagrangian

Step III: Construct Lagrangian based on identified symmetries

Pion-pion Lagrangian: U is SU(2) matrix parameterized by three pion fields

$$\mathcal{L}_{\pi}^{(0)} = \frac{F^2}{4}\langle \nabla^\mu U \nabla_\mu U^\dagger + \chi_+ \rangle,$$

Leading-order pion-nucleon

$$\mathcal{L}_{\pi N}^{(0)} = \bar{N}(iv \cdot D + \mathring{g}_A u \cdot S)N,$$

Leading-order nucleon-nucleon (encodes unknown short-range physics)

$$\mathcal{L}_{NN}^{(0)} = -\tfrac{1}{2}C_S(\bar{N}N)(\bar{N}N) + 2C_T(\bar{N}SN)\cdot(\bar{N}SN)$$

EFT Power Counting

Step IV: Design an organized scheme to distinguish more from less important processes: Power Counting

Organize theory in powers of $\left(\frac{Q}{\Lambda_\chi}\right)$ where $Q \sim m_\pi$ typical nuclear momenta

Only valid for small expansion parameters, *i.e.*, low momentum

Irreducible time-ordered diagram has order: $\left(\frac{Q}{\Lambda_\chi}\right)^\nu$

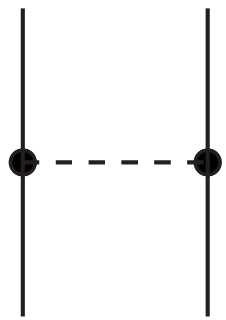
$$\nu = -4 + 2N + 2L + \sum_i V_i \Delta_i \qquad \Delta_i = d_i + \frac{1}{2}n_i - 2 \text{ “Chiral dimension”}$$

$N =$ Number of nucleons
$L =$ Number of pion loops
$V_i =$ Number of vertices of type i
$d =$ Number of derivatives or insertions of m_π
$n =$ Number of nucleon field operators

Chiral EFT: Lowest Order (LO)

Step V: Calculate Feynmann diagrams to the desired accuracy

Leading order (LO) $\nu = 0$



One-pion exchange

$$V_{NN}^{(0)} = -\frac{g_A^2}{4F_\pi^2}\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^2+M_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$$

$g_A = 1.26$

$F_\pi = 92.4\ \mathrm{MeV}$

$\vec{q}_i \equiv \vec{p}_i' - \vec{p}_i \qquad \vec{k}_i \equiv \frac{1}{2}(\vec{p}_i' + \vec{p}_i)$

Chiral EFT: Lowest Order (LO)

Step V: Calculate Feynmann diagrams to the desired accuracy

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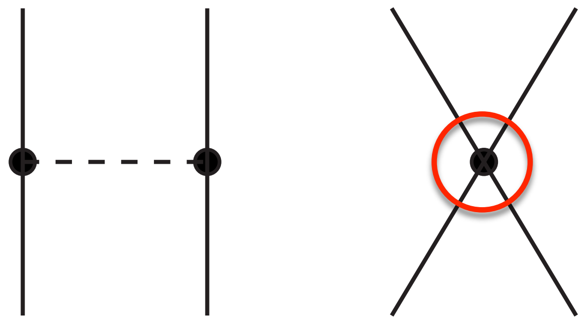


One-pion exchange

NN contact interaction

$$V_{NN}^{(0)} = -\frac{g_A^2}{4F_\pi^2}\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^2+M_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + C_S + C_T\vec{\sigma}_1\cdot\vec{\sigma}_2$$

$g_A = 1.26$

$F_\pi = 92.4$ MeV

Two **low-energy constants (LECs)**: C_S, C_T

$\vec{q}_i \equiv \vec{p}_i' - \vec{p}_i \qquad \vec{k}_i \equiv \frac{1}{2}(\vec{p}_i' + \vec{p}_i)$

Chiral EFT

Step V: Calculate Feynmann diagrams to the desired accuracy

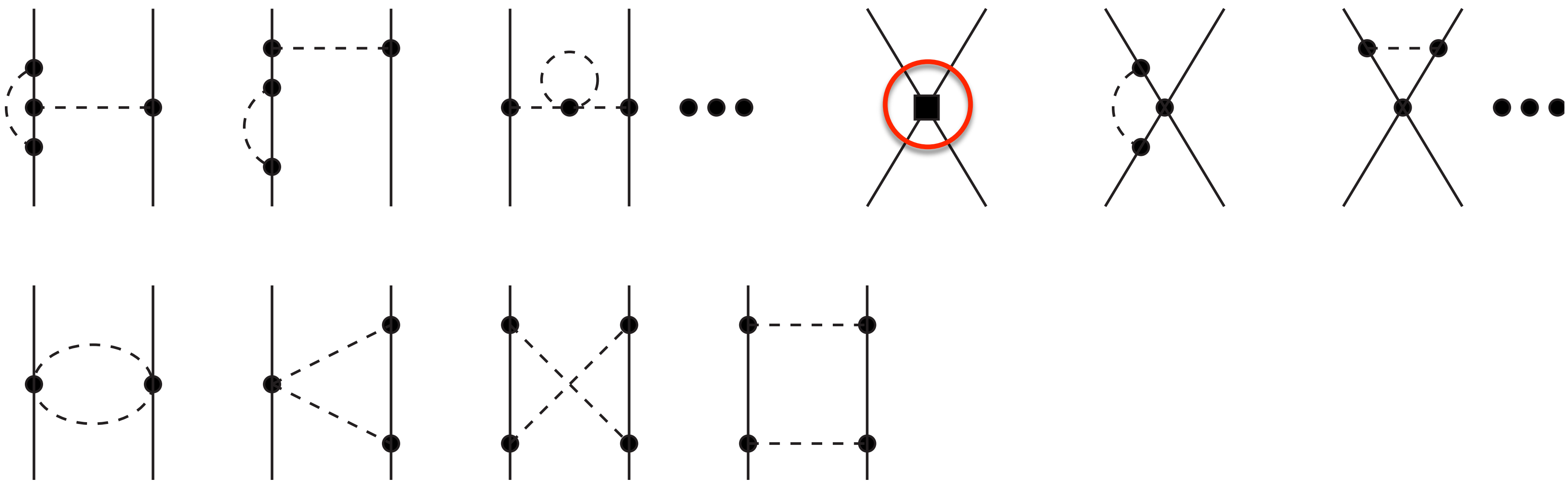
Question: What will $\nu = 1$ look like?

Answer: No contribution at this order

Chiral EFT: NLO

Step V: Calculate Feynmann diagrams to the desired accuracy

Next-to-leading order (NLO) $\nu = 2$



Higher order contact interaction: 7 new LECs, spin-orbit

$$+ C_1 \vec{q}^2 + C_2 \vec{k}^2 + (C_3 \vec{q}^2 + C_4 \vec{k}^2) \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

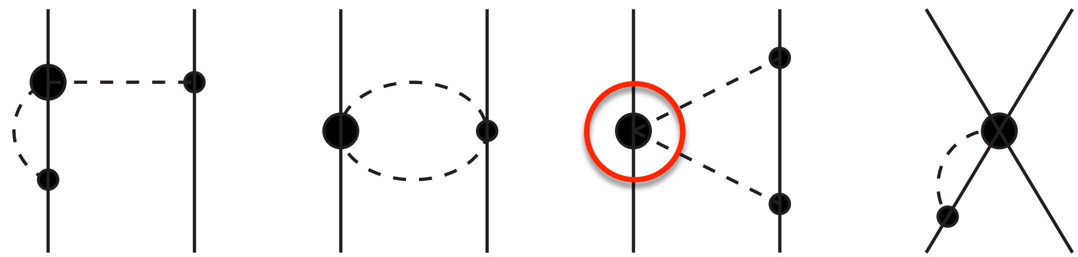
$$+ i C_5 \frac{1}{2} (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{q} \times \vec{k} + C_6 \vec{q} \cdot \vec{\sigma}_1 \vec{q} \cdot \vec{\sigma}_2$$

$$+ C_7 \vec{k} \cdot \vec{\sigma}_1 \vec{k} \cdot \vec{\sigma}_2,$$

Chiral EFT: N^{2}LO

Step V: Calculate Feynmann diagrams to the desired accuracy

Next-to-next-to-leading order (N^{2}LO) $\nu = 3$



3 new couplings from ππNN vertex – not LECs!

$$V_{NN}^{(3)} = -\frac{3g_A^2}{16\pi F_\pi^4}[2M_\pi^2(2c_1 - c_3) - c_3\vec{q}^2]$$

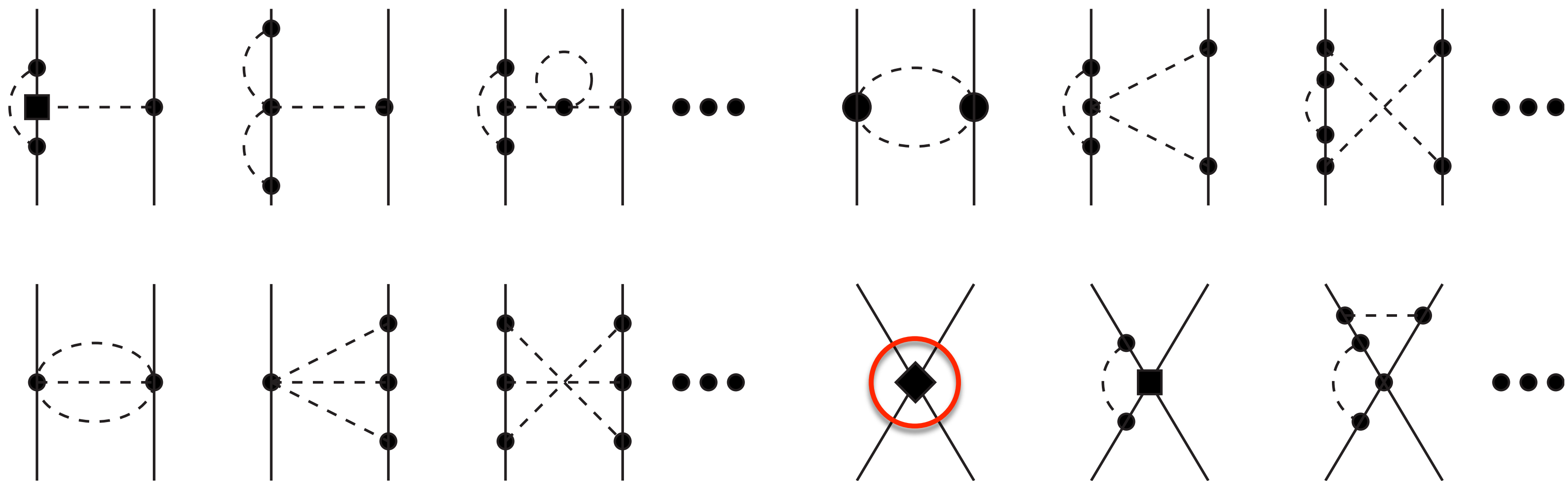
$$\times(2M_\pi^2 + \vec{q}^2)A^{\tilde{\Lambda}}(q) - \frac{g_A^2 c_4}{32\pi F_\pi^4}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2(4M_\pi^2$$

$$+ q^2)A^{\tilde{\Lambda}}(q)(\vec{\sigma}_1 \cdot \vec{q}\vec{\sigma}_2 \cdot \vec{q} - \vec{q}^2\vec{\sigma}_1 \cdot \vec{\sigma}_2),$$

Chiral EFT: N^{3}LO

Step V: Calculate Feynmann diagrams to the desired accuracy

Next-to-next-to-next-to-leading order $\nu = 4$



Higher order contact interaction: 15 new LECs

Regularization of Chiral potentials

Remember: constructing potential involves solving **L-S equation**

All NN potentials cutoff loop momenta at some value > 1GeV

Impose exponential regulator, Λ, in Chiral EFT potentials – **not in integral**

$$T^{\alpha}(k,k') = V^{\alpha}(k,k') + \frac{2}{\pi}\sum_{l''}\int_0^{\infty} q^2 \mathrm{d}q\, V^{\alpha}(k,q)\frac{q}{k^2-q^2+i\varepsilon}T^{\alpha}(q,k')$$

$$V(k,k') \rightarrow e^{(-k'/\Lambda)^{2n}}V(k,k')e^{(-k/\Lambda)^{2n}}$$

LECs will depend on regularization approach and Λ

Infinitely many ways to do this

$\Longrightarrow$ **Infinitely many chiral potentials!**

Indeed, many on the market – some fit well to phase shifts, others not

Chiral EFT:
Resulting fits to Phase shifts

Systematic improvement of chiral EFT potentials fit to phase shifts

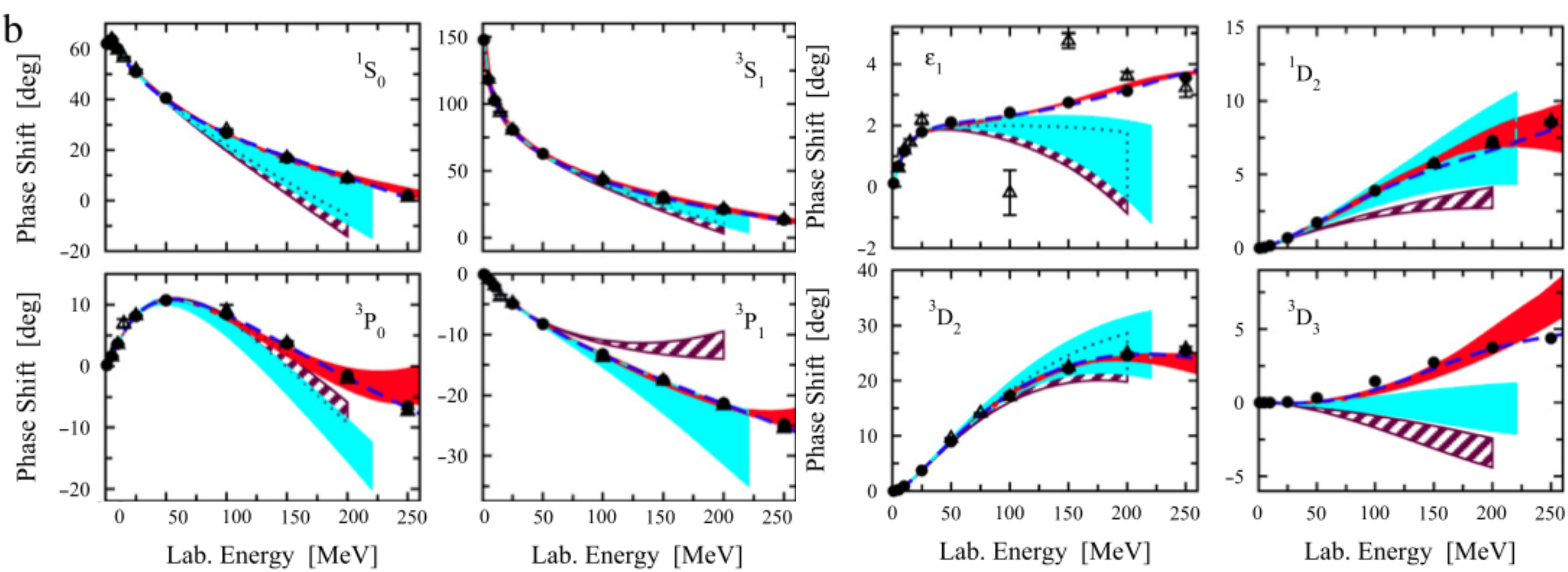
Cutoff variation – information about missing physics

NLO: dashed band 9 Parameters

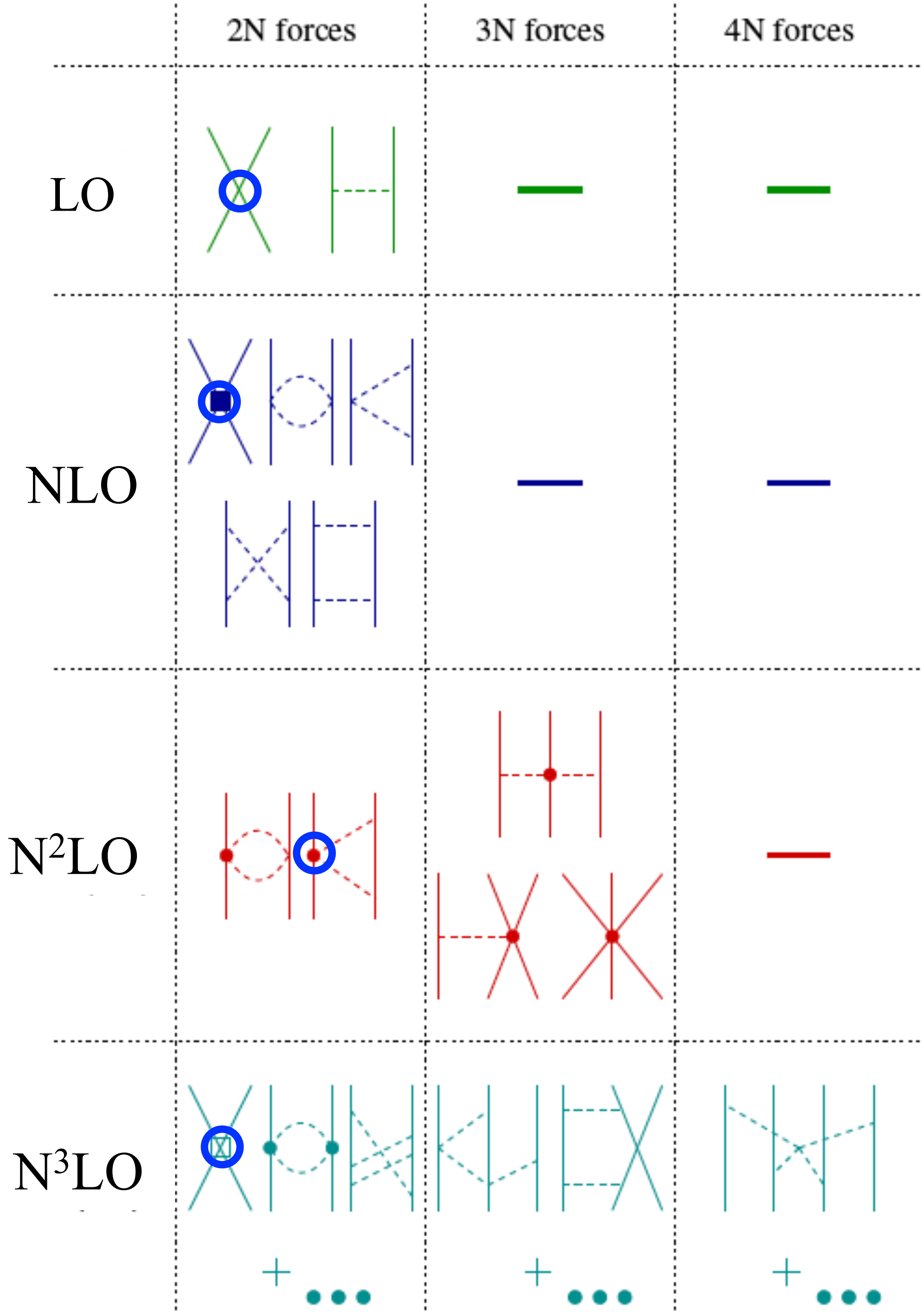
N^2LO: light band 12 Parameters

N^3LO: dark band 27 Parameters

Generally decreasing error and increasing accuracy – not entirely… (exercise)



Chiral Effective Field Theory: Nuclear Forces



Meson exchange potentials were an admirable effort

Using ideas of **effective field theory:**

Nucleons interact via pion exchanges and contact interactions

Lower momentum
Systematic – can assign error
Connected to QCD

Hierarchy: $V_{NN} > V_{3N} > \ldots$

Consistent treatment of NN, 3N, … electroweak operators

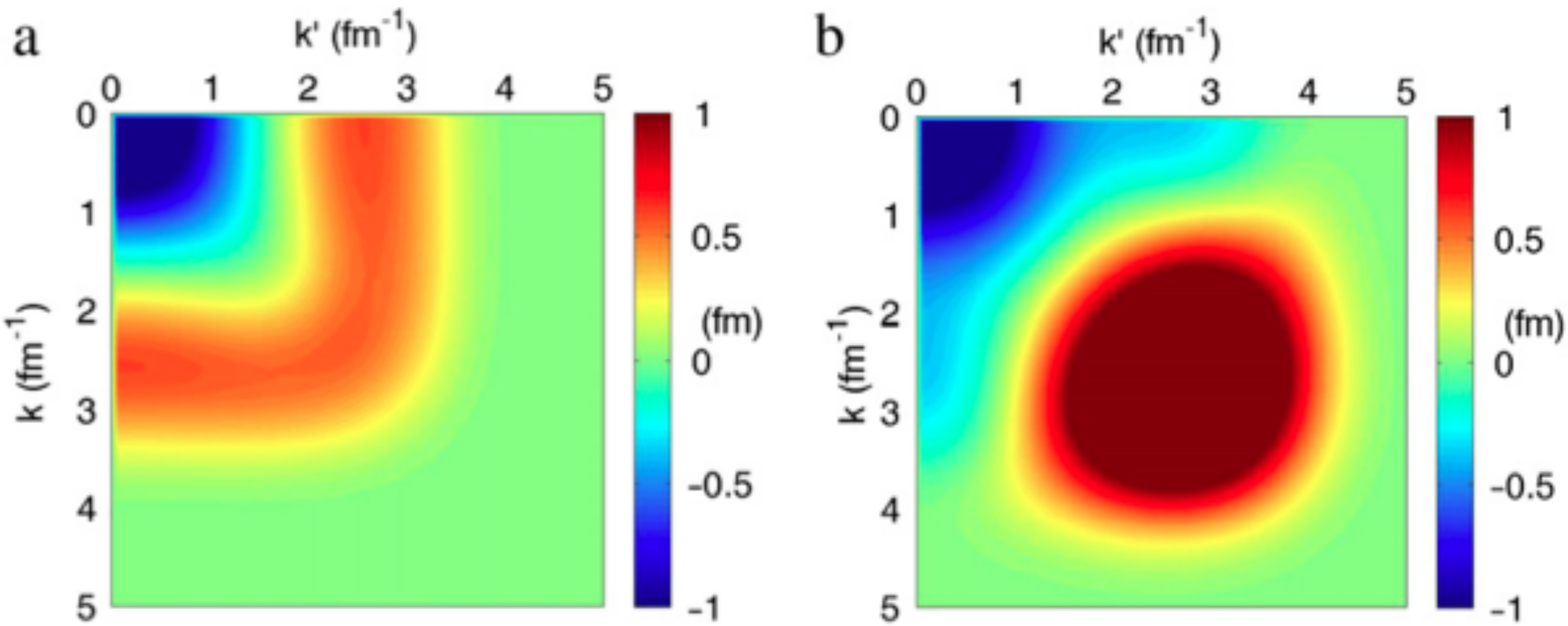
Couplings fit to experiment once

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Meissner,…

Chiral NN Potentials

Two chiral potentials with regulators of 500MeV and 600MeV

Still low-to-high momentum coupling: poor convergence, non perturbative, etc.



How do these compare to the potential you drew?

Lesson: Infinitely many phase-shift equivalent potentials

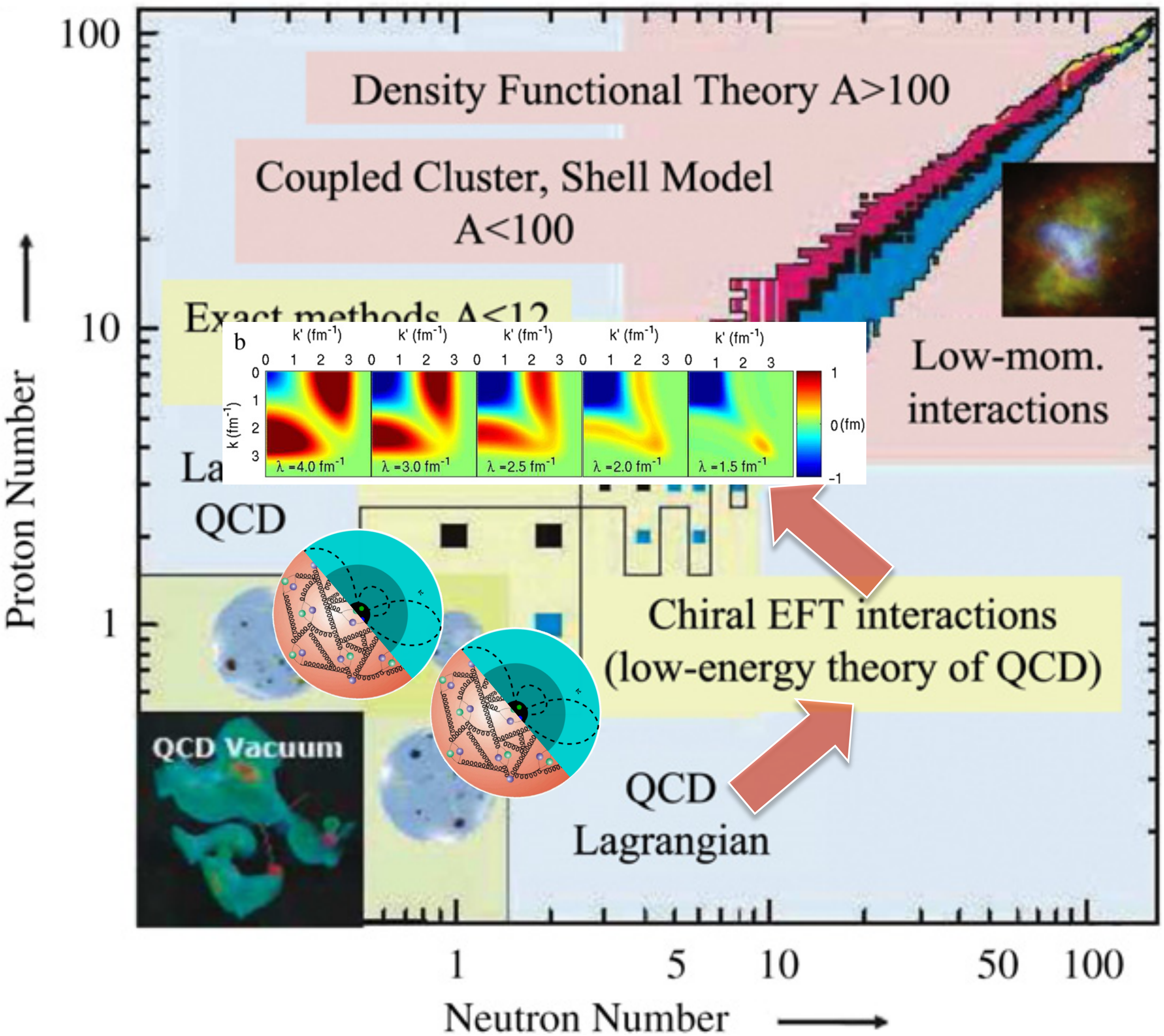
$$E_n = \langle \Psi_n | H | \Psi_n \rangle = \left(\langle \Psi_n | U^\dagger \right) U H U^\dagger \left(U | \Psi_n \rangle \right) = \left\langle \tilde{\Psi}_n \middle| \tilde{H} \middle| \tilde{\Psi}_n \right\rangle$$

NN interaction not observable

Low-to-high momentum makes life difficult for low-energy nuclear theorists

Part II: (S)RG and Low-Momentum Interactions

To understand the properties of complex nuclei from first principles



Renormalizing NN Interactions

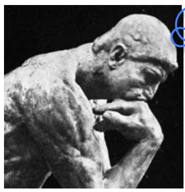
- Basic ideas of RG
- Low-momentum interactions
- Similarity RG interactions
- Benefits of low cutoffs
- G-matrix renormalization

How will we approach this problem:

QCD → NN (3N) forces → Renormalize → "Solve" many-body problem → Predictions

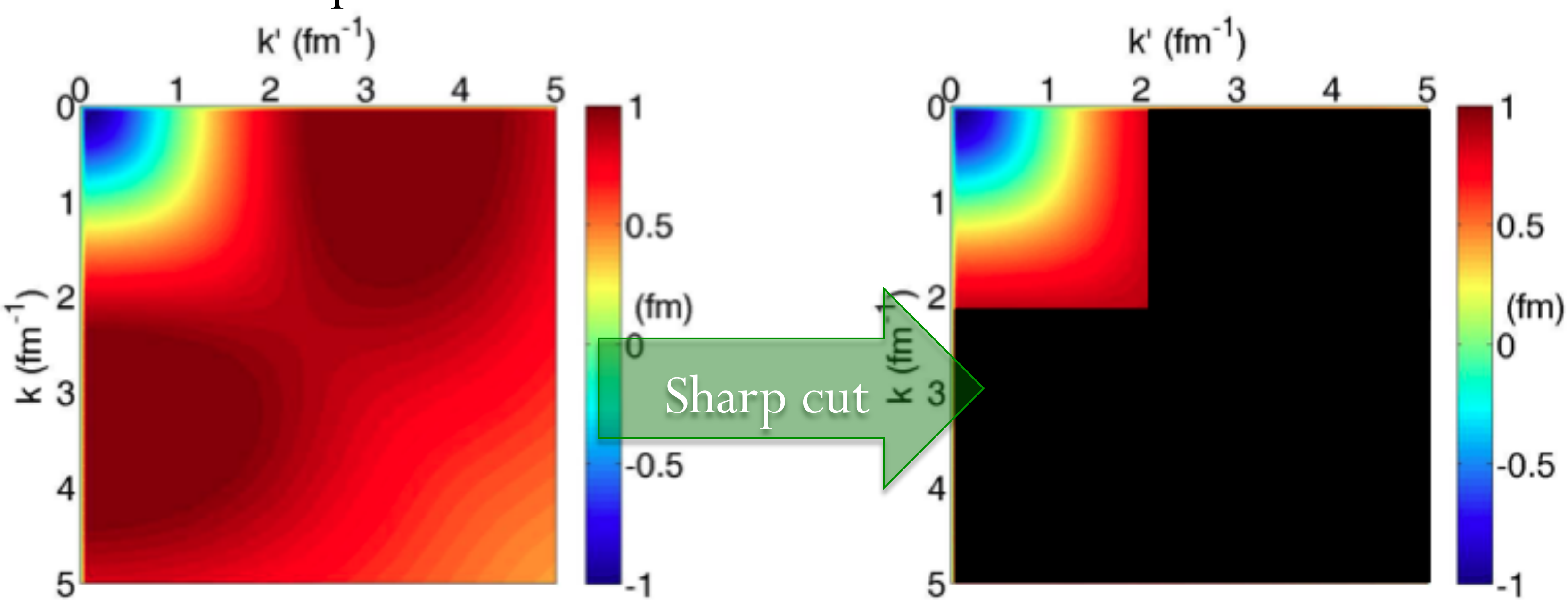
Renormalization of Meson-Exchange Potentials

Ok, high momentum is a pain. I wonder what would happen to low-energy observables…



Low-to-high momentum makes life difficult for low-energy nuclear theorists, so let's get rid of it

Can we just make a sharp cut and see if it works?



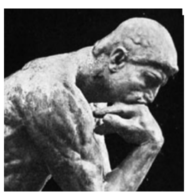
$$V_{\rm filter}(k', k) \equiv 0; \;\; k, k' > 2.2\,{\rm MeV}$$

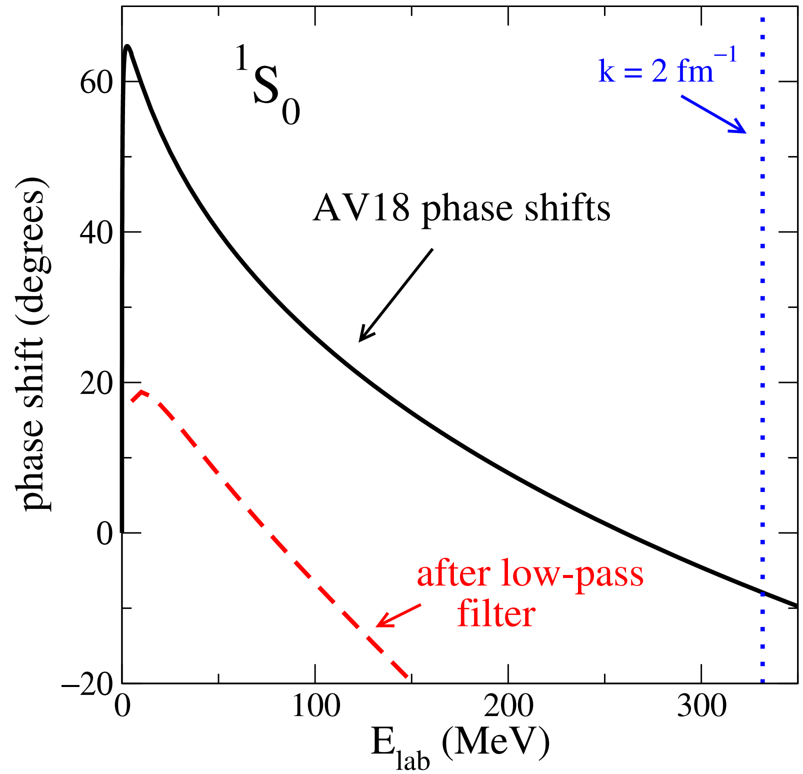
Renormalization of Meson-Exchange Potentials

Can we just make a sharp cut?

Nope! Low-energy physics is not correct

Glad I didn't bet money on that… I wonder what went wrong





Renormalization of Meson-Exchange Potentials

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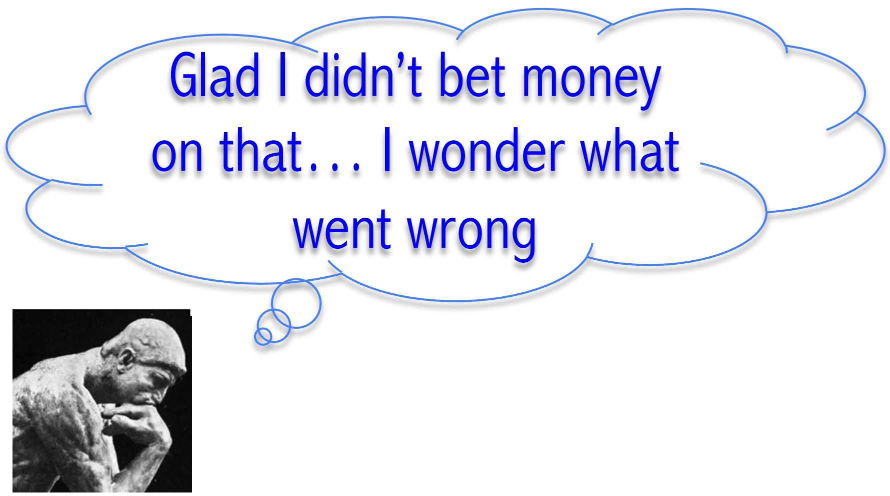
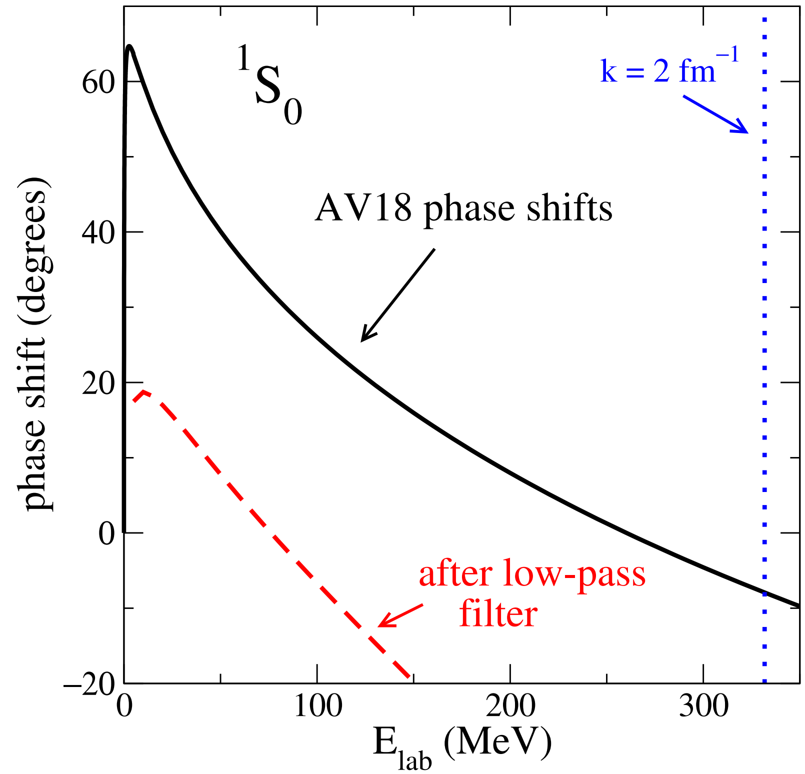




Phase shifts involve couplings of low-to-high momenta

$$\langle k|V|k'\rangle + \sum_{q=0}^{\Lambda} \frac{\langle k|V|q\rangle\langle q|V|k'\rangle}{\epsilon_{k'} - \epsilon_q} + \sum_{q=\Lambda}^{\infty} \frac{\langle k|V|q\rangle\langle q|V|k'\rangle}{\epsilon_{k'} - \epsilon_q}$$

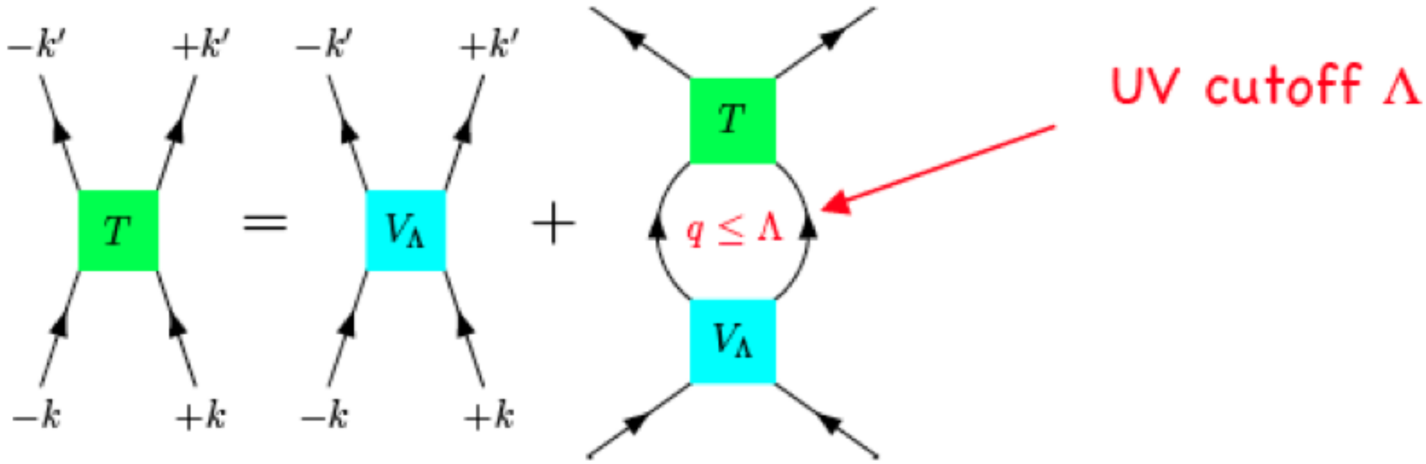
Lesson: Must ensure low-energy physics is preserved!

Renormalization of Meson-Exchange Potentials

To do properly, from T-matrix equation, define **low-momentum** equation:

$$T^{\alpha}(k,k') = V_{\rm NN}^{\alpha}(k,k') + \frac{2}{\pi}\int_0^{\infty} q^2 {\rm d}q\, \frac{V_{\rm NN}^{\alpha}(k,q)T^{\alpha}(q,k')}{k^2-q^2+i\varepsilon}$$

$$\to V_{{\rm low}\,k}^{\Lambda}(k,k') + \frac{2}{\pi}\int_0^{\Lambda} q^2 {\rm d}q\, \frac{V_{{\rm low}\,k}^{\Lambda}(k,q)T(q,k')}{k^2-q^2+i\varepsilon}$$



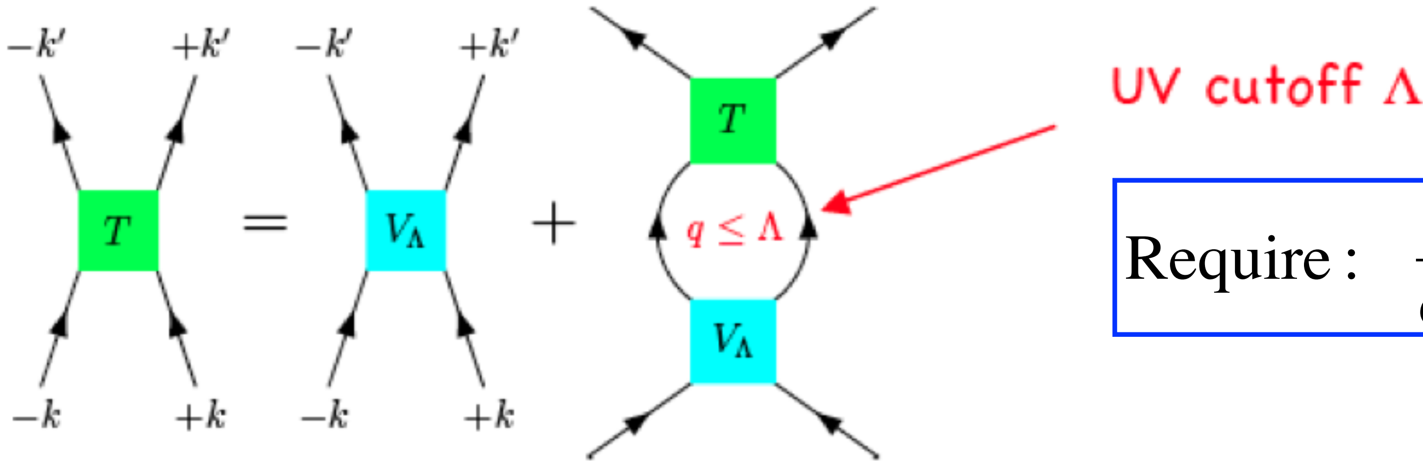
Lower UV cutoff, but preserve low-energy physics!

Renormalization of Meson-Exchange Potentials

To do properly, from T-matrix equation, define **low-momentum** equation:

$$T^{\alpha}(k,k') = V^{\alpha}_{\rm NN}(k,k') + \frac{2}{\pi}\int_0^{\infty} q^2 {\rm d}q \, \frac{V^{\alpha}_{\rm NN}(k,q)T^{\alpha}(q,k')}{k^2-q^2+i\varepsilon}$$

$$\rightarrow V^{\Lambda}_{{\rm low}\,k}(k,k') + \frac{2}{\pi}\int_0^{\Lambda} q^2 {\rm d}q \, \frac{V^{\Lambda}_{{\rm low}\,k}(k,q)T(q,k')}{k^2-q^2+i\varepsilon}$$



$$\text{Require:} \quad \frac{\rm d}{{\rm d}\Lambda} T = 0$$

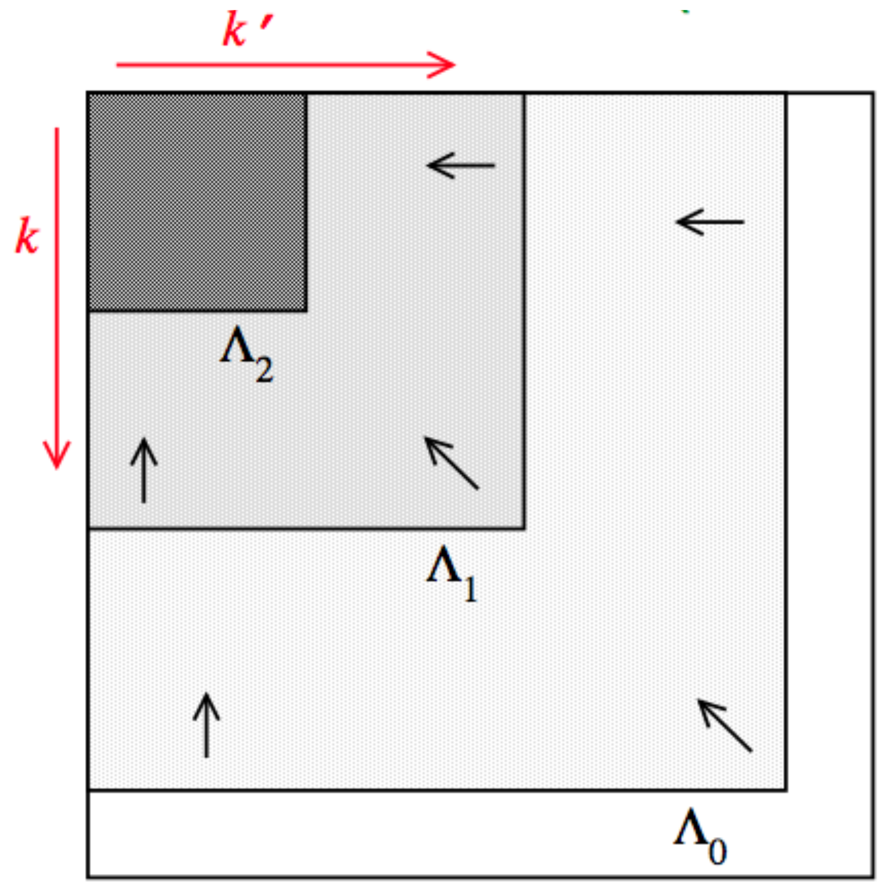
Lower UV cutoff, but preserve low-energy physics!

Leads to **renormalization group equation** for low-momentum interactions

$$\frac{\rm d}{{\rm d}\Lambda} V^{\Lambda}_{{\rm low}\,k}(k',k) = \frac{2}{\pi} \frac{V^{\Lambda}_{{\rm low}\,k}(k',\Lambda)T^{\Lambda}(\Lambda,k)}{1-(k/\Lambda)^2}$$

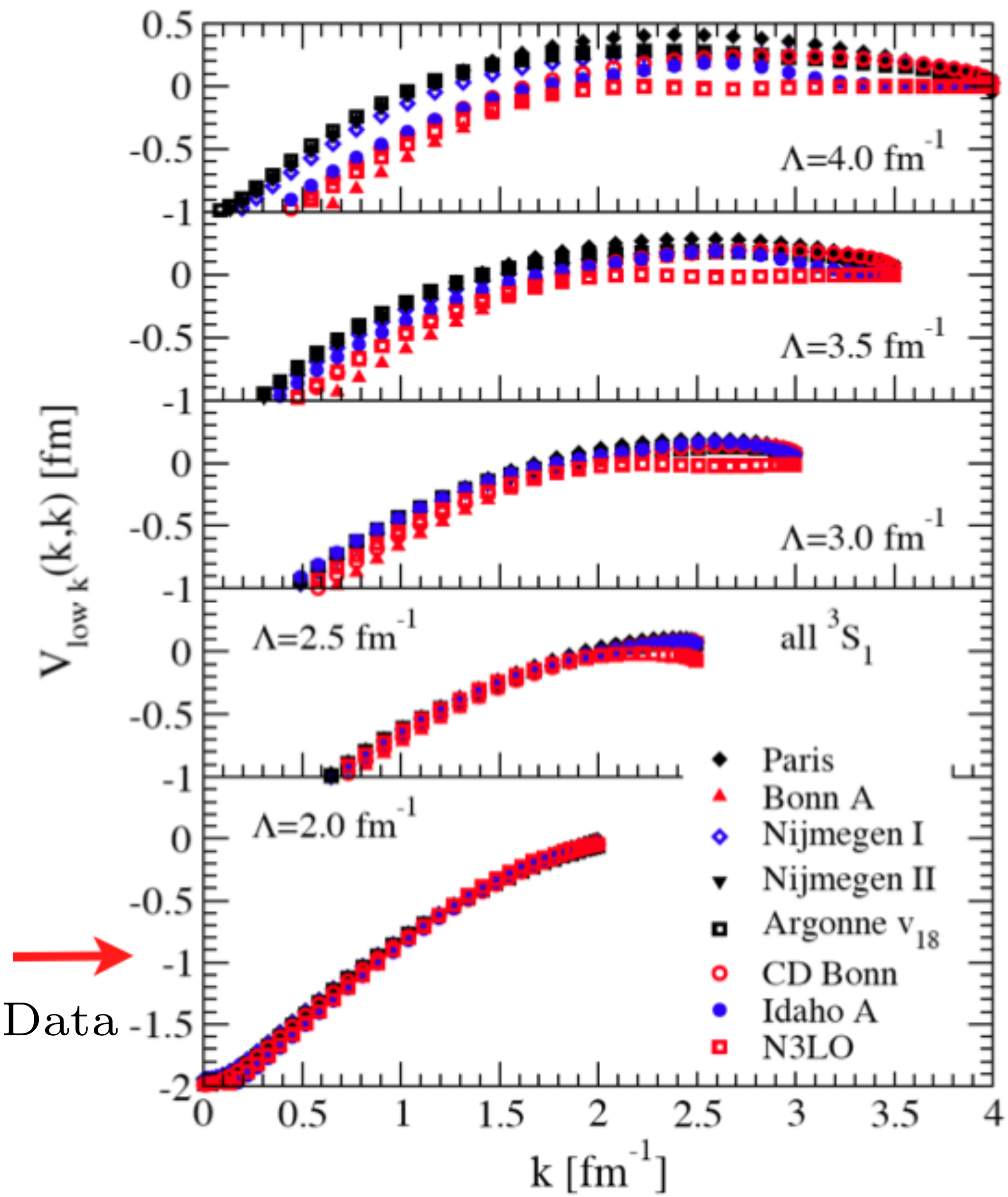
Renormalization of Meson-Exchange Potentials

Run cutoff to lower values – **decouples** high-momentum modes



Start from some initial V_{NN}
at high cutoff Λ_0

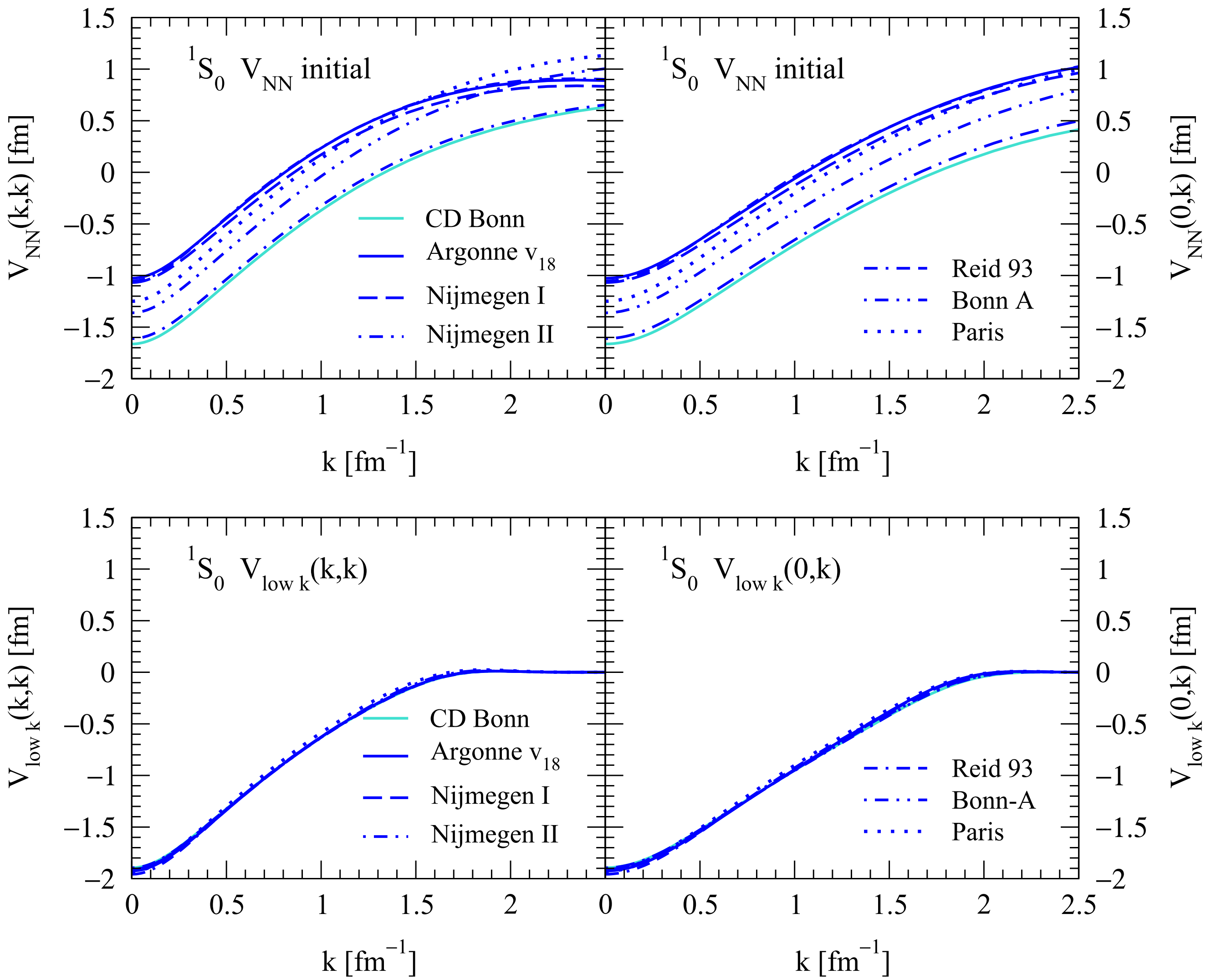
$\Lambda \approx \Lambda_{\mathrm{Data}}$



"Universality" at low momentum

Renormalization of Meson-Exchange Potentials

Diagonal **Off-diagonal**

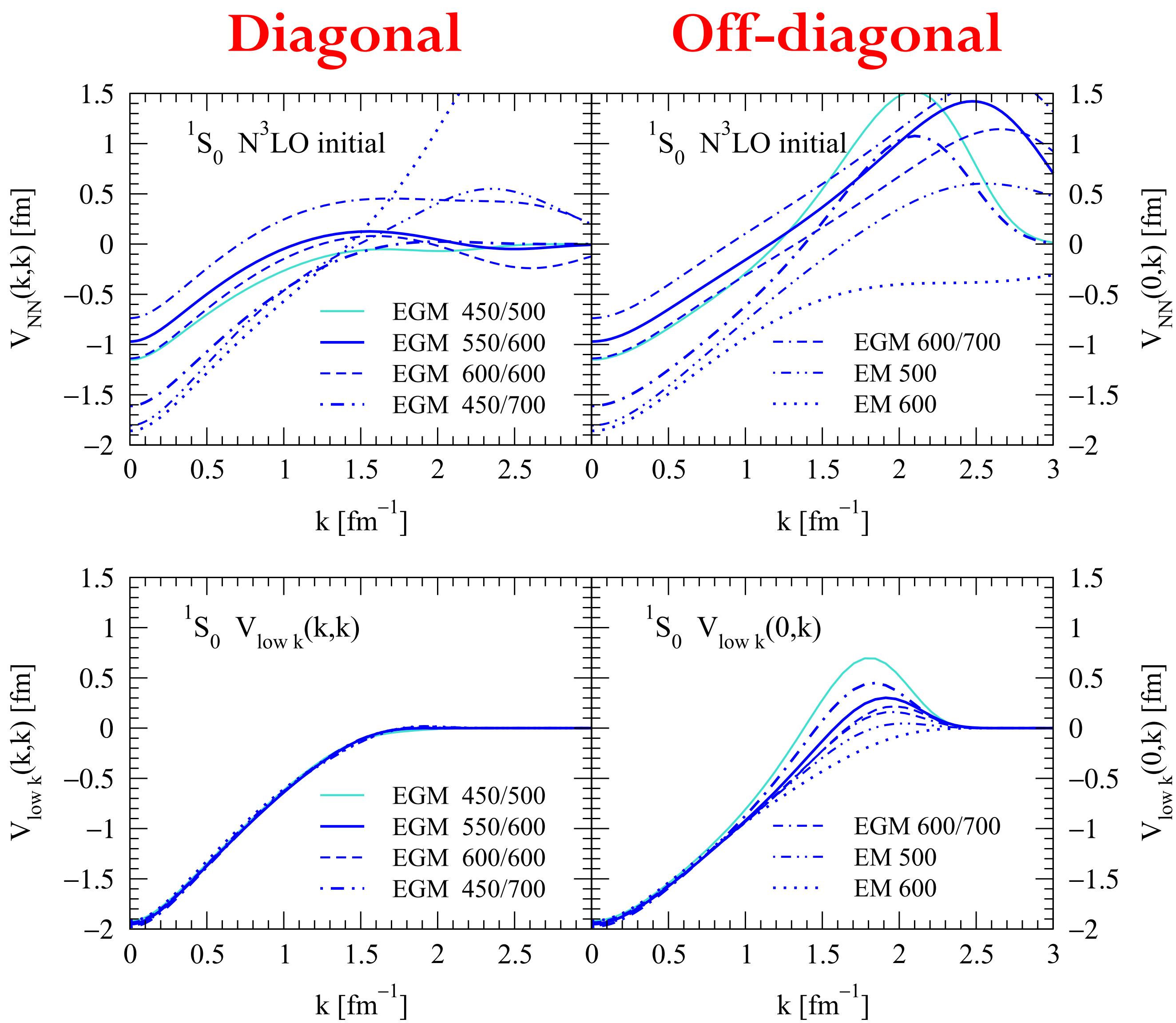


These are all our favorite OBE NN potentials…

These are all our favorite OBE NN potentials… **at low momentum**

Universal collapse in both diagonal/off-diagonal components, most partial waves

Renormalization of Chiral EFT Potentials

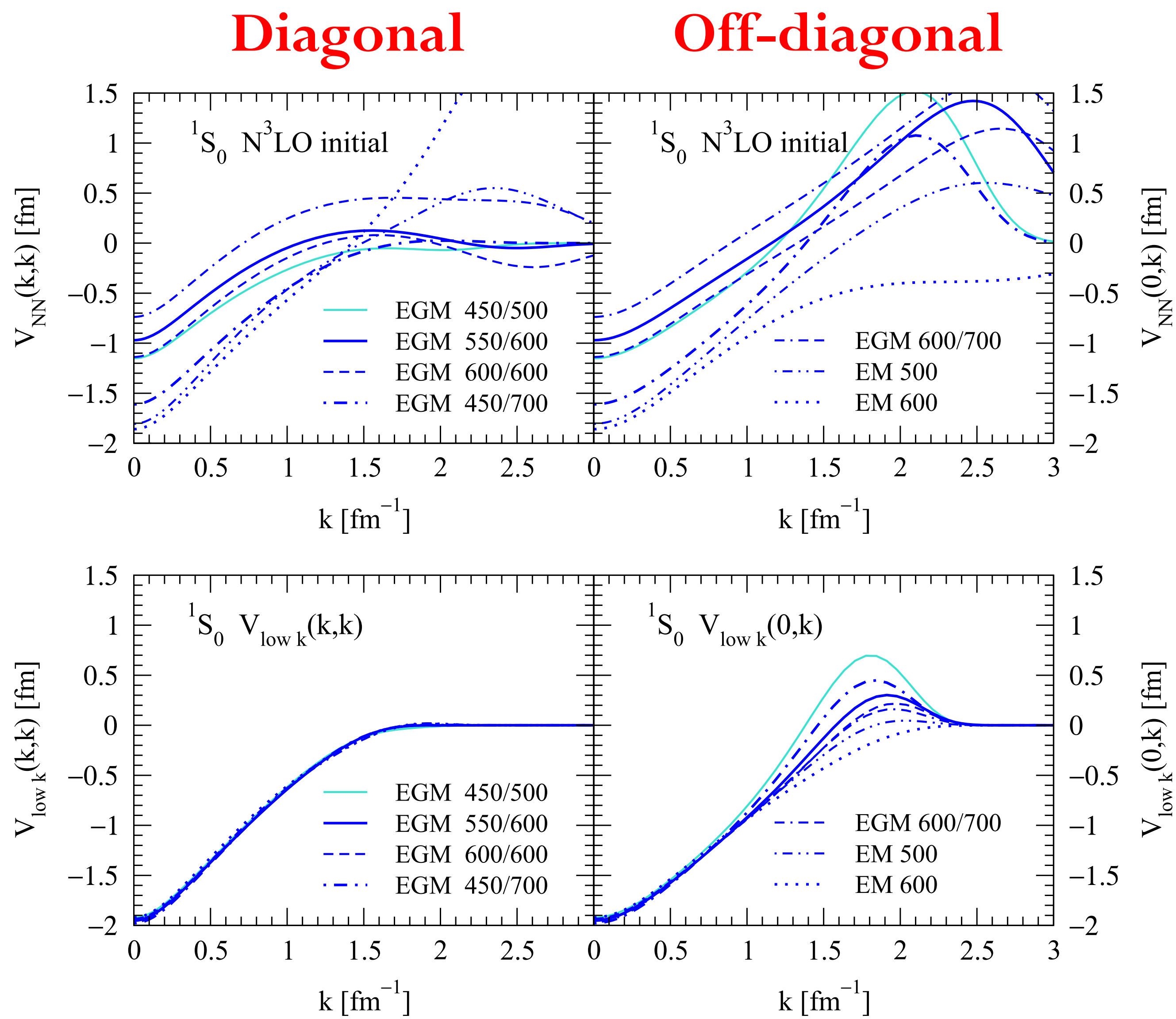


These are all our favorite Chiral EFT NN potentials…

These are all our favorite Chiral EFT NN potentials… **at low momentum**

Differences remain in off-diagonal matrix elements. Why?

Renormalization of Chiral EFT Potentials



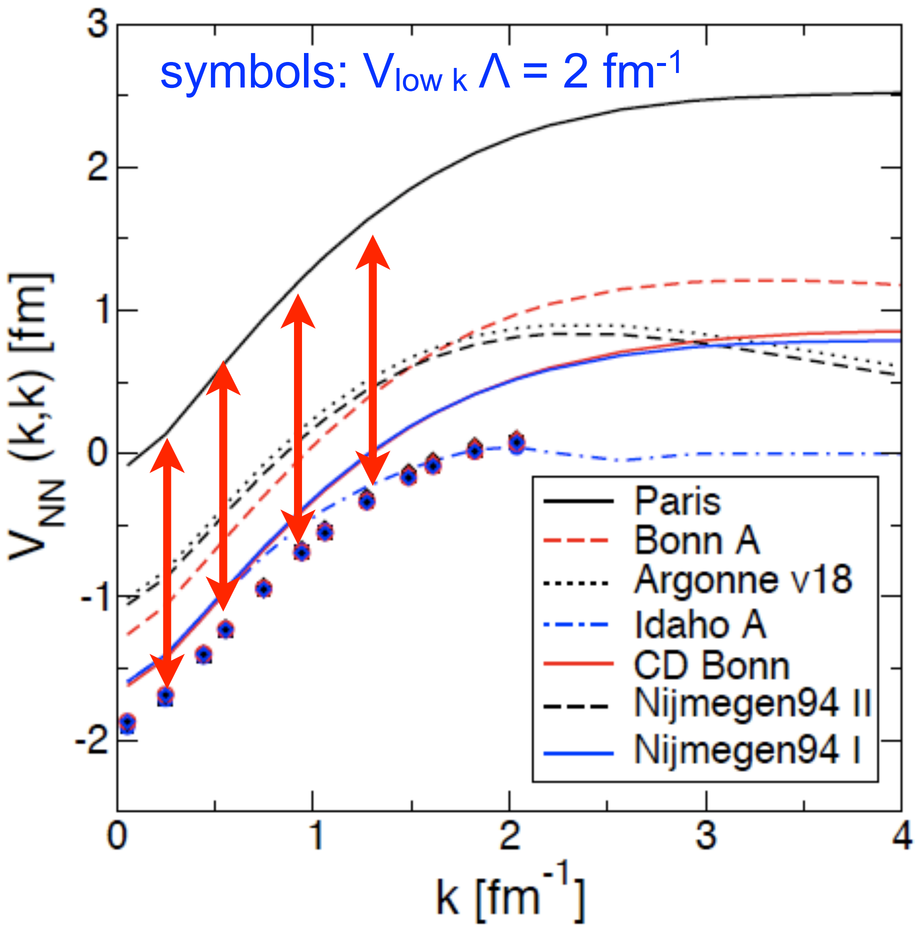
These are all our favorite Chiral EFT NN potentials…

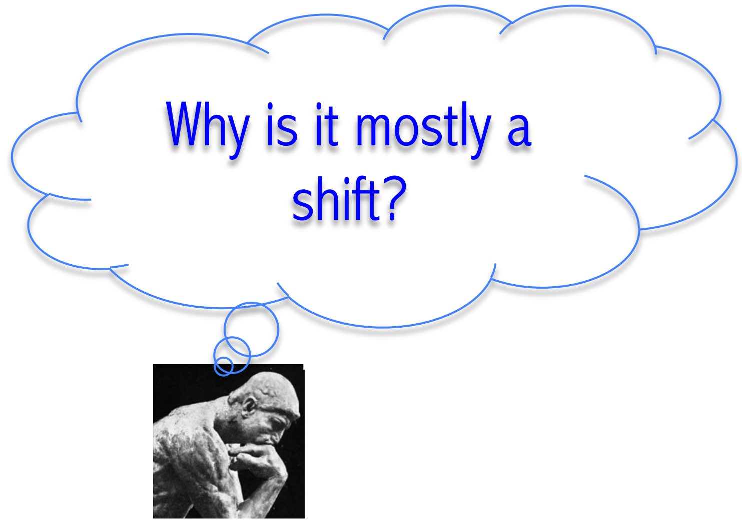
These are all our favorite Chiral EFT NN potentials…
at low momentum

Differences remain in off-diagonal matrix elements

Sensitive to agreement for phase shifts (not all fit perfectly)

Renormalization of NN Potentials



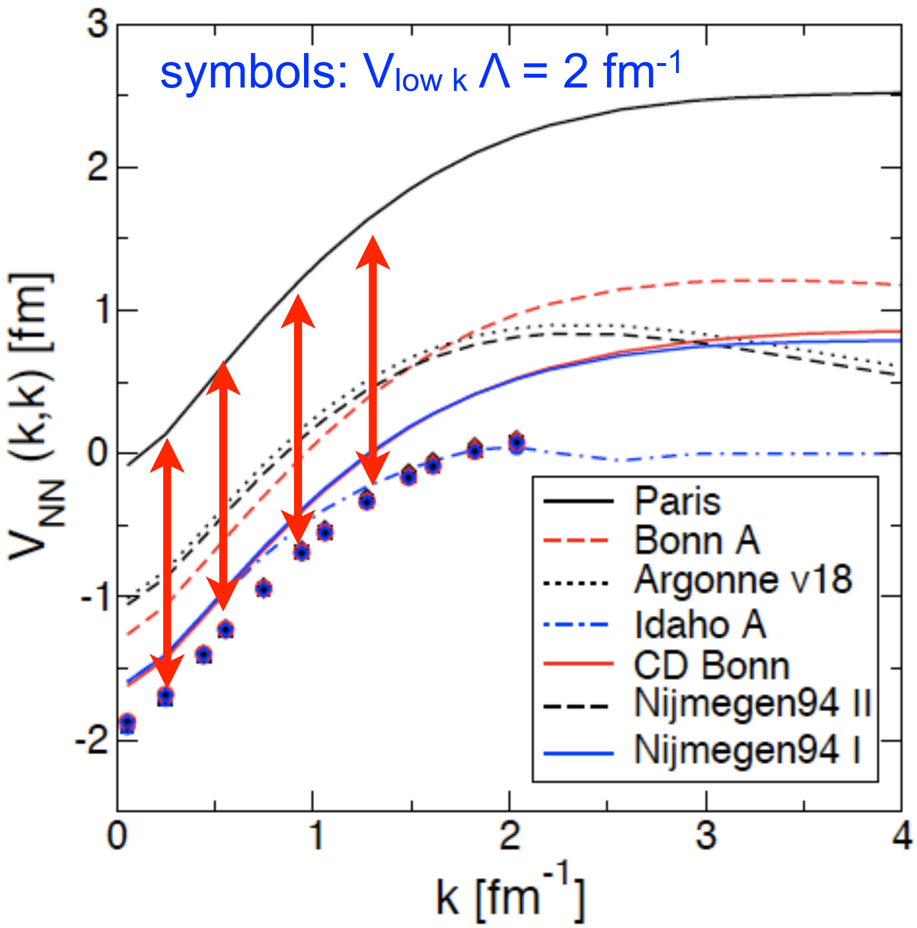


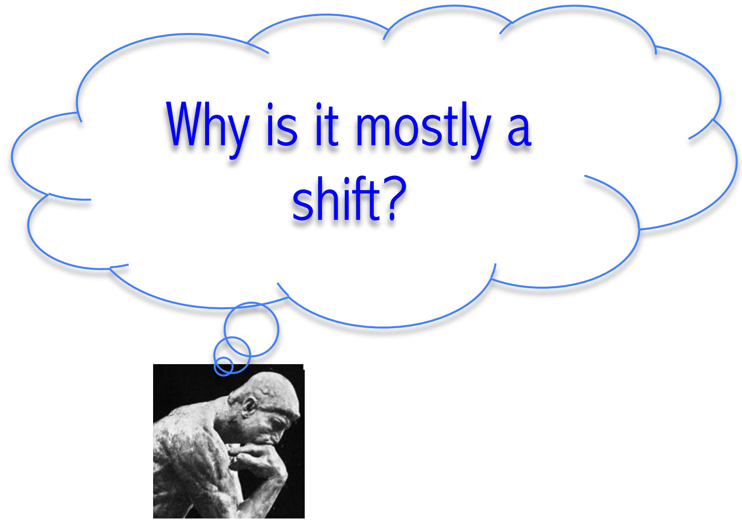
$$V_{\mathrm{eff}} = V_{\mathrm{L}} + \delta V_{\mathrm{c.t.}}(\Lambda)$$

Overall effect of evolving to low momentum

Main effect is shift in momentum space

Renormalization of NN Potentials





$$V_{\text{eff}} = V_{\text{L}} + \delta V_{\text{c.t.}}(\Lambda)$$

Overall effect of evolving to low momentum

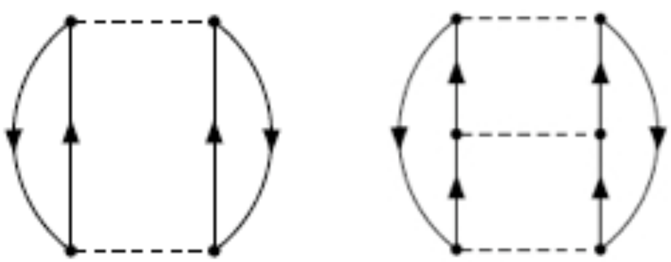
Main effect is shift in momentum space – delta function

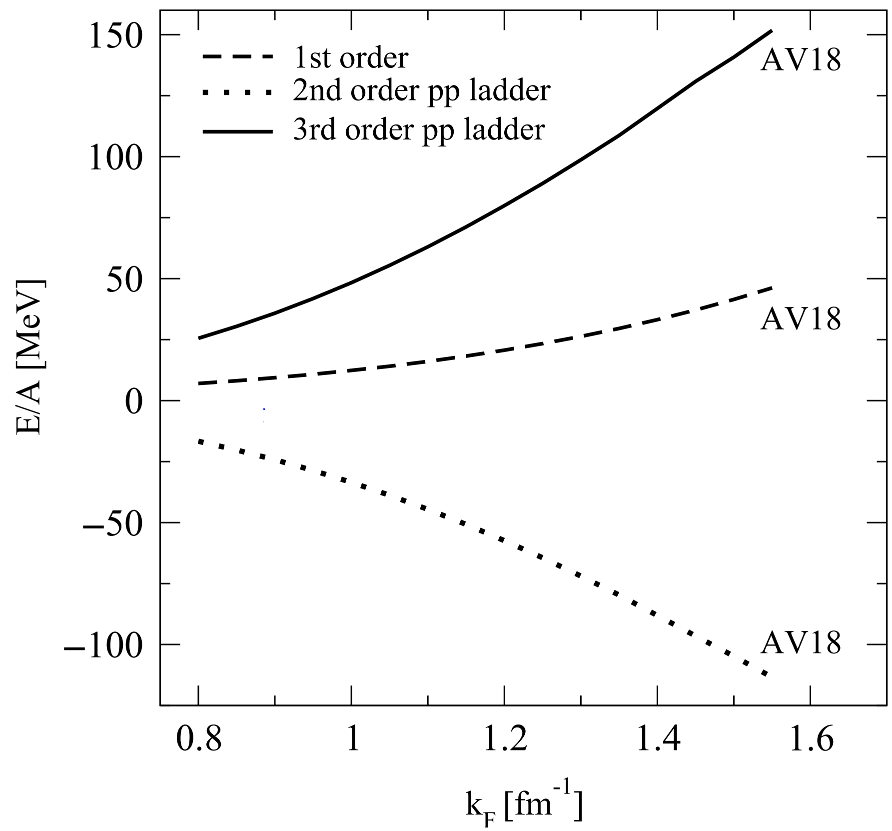
Removes hard core (unconstrained short-range physics)!

Improvements in Perturbation Theory

Explore improvements in symmetric infinite matter calculations

Order by order in **many-body perturbation theory (MBPT)**





No clear convergence with increasing order in bare potential

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Explore improvements in symmetric infinite matter calculations

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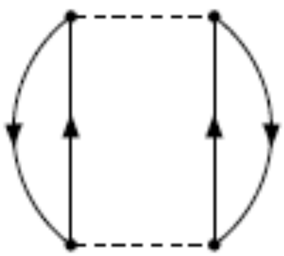

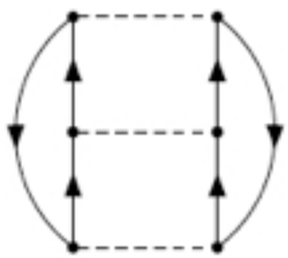

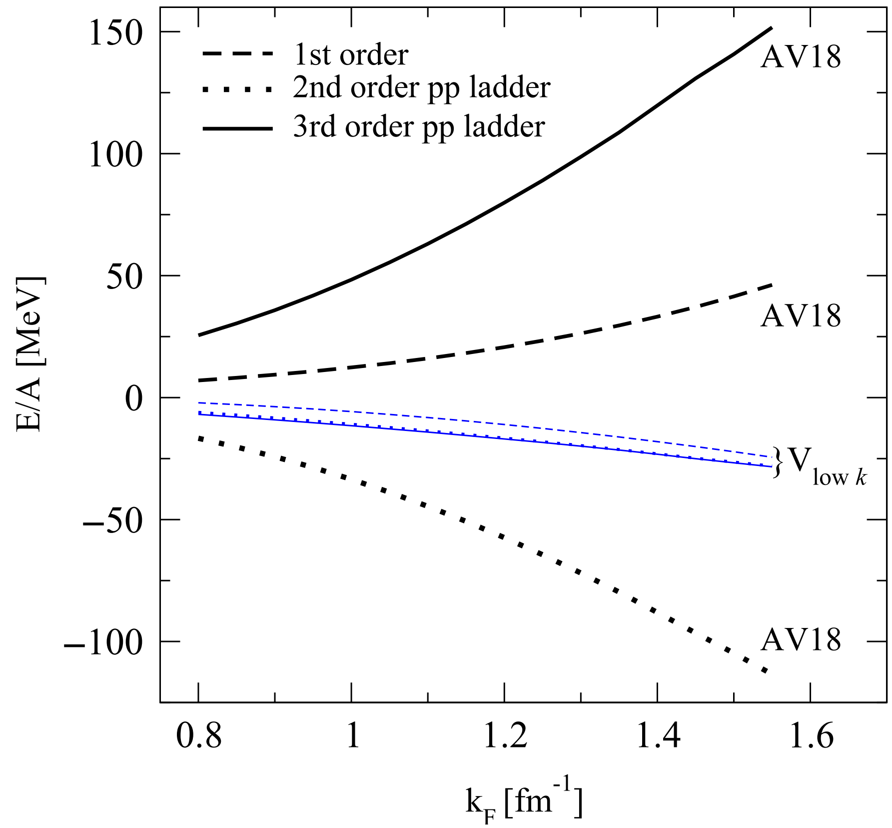

No clear convergence with increasing order in bare potential

Significant improvement with low-momentum interactions!

Improvements in Perturbation Theory

Explore improvements in symmetric infinite matter calculations

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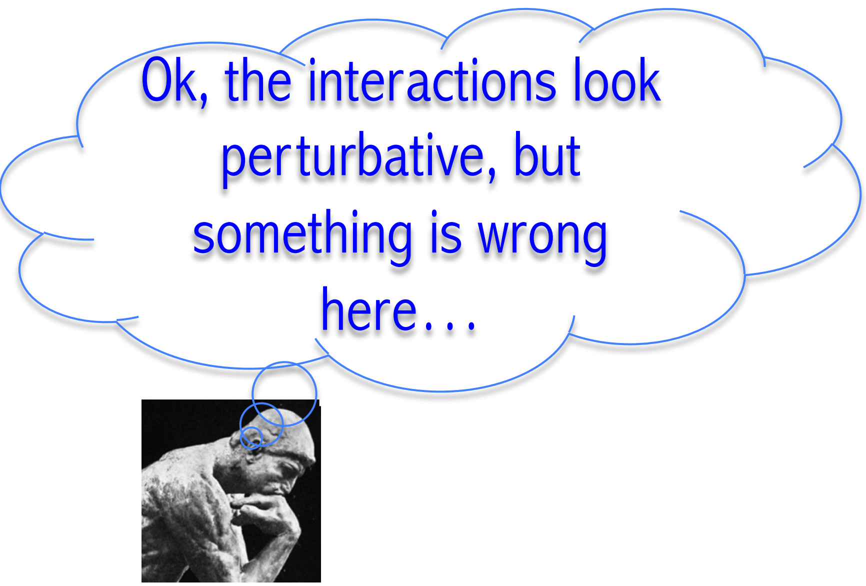

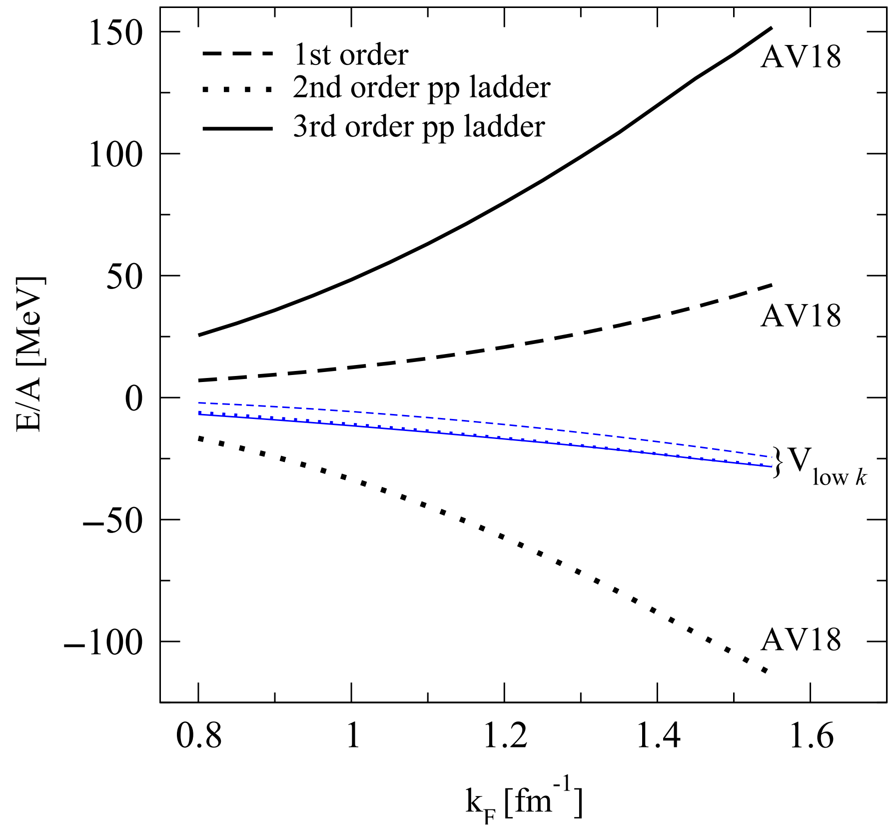

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Explore improvements in symmetric infinite matter calculations

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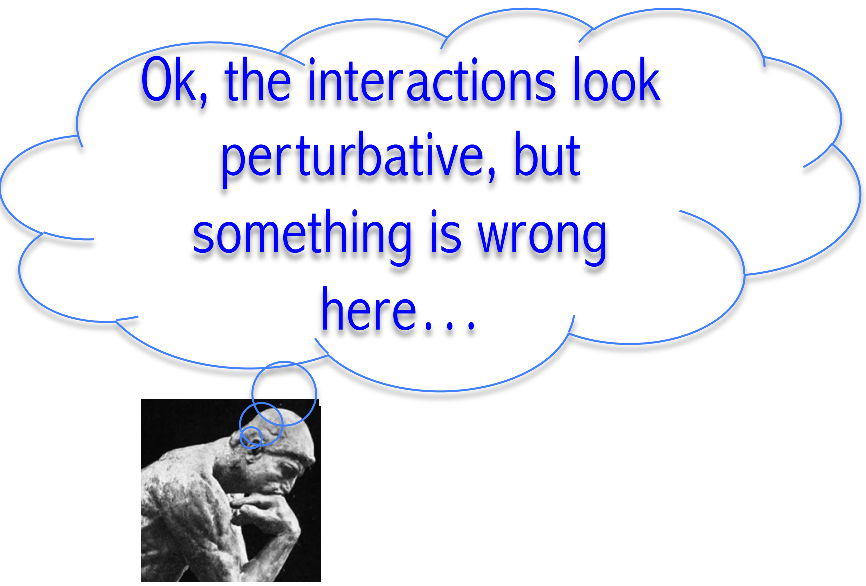
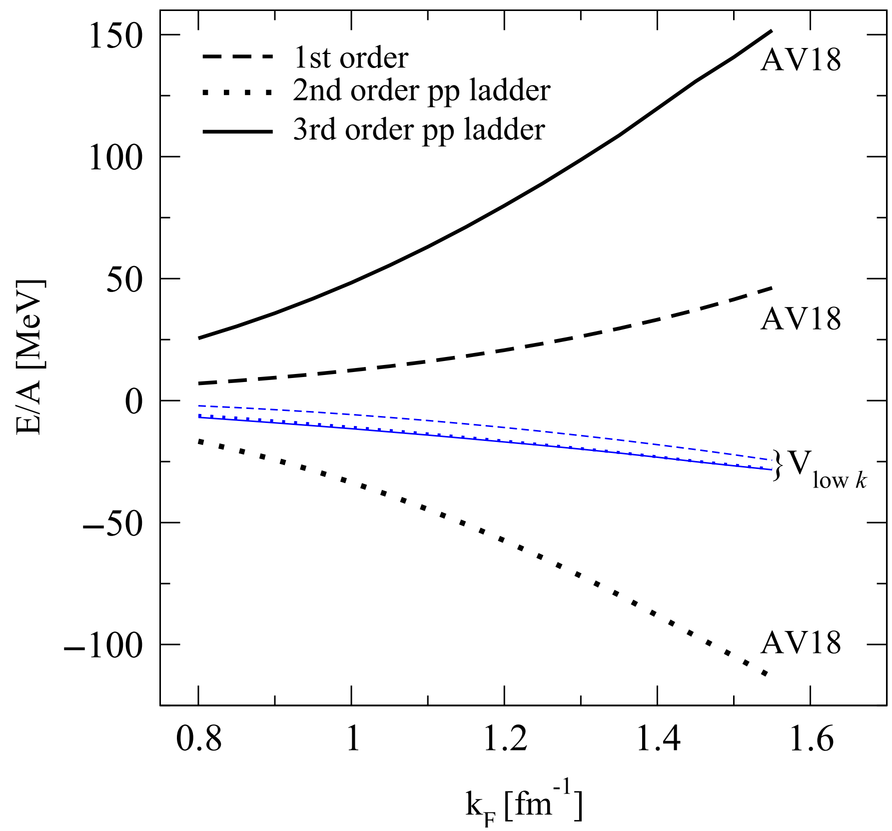




No clear convergence with increasing order in bare potential

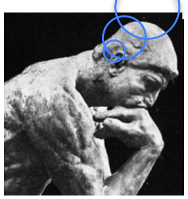
Significant improvement with low-momentum interactions!

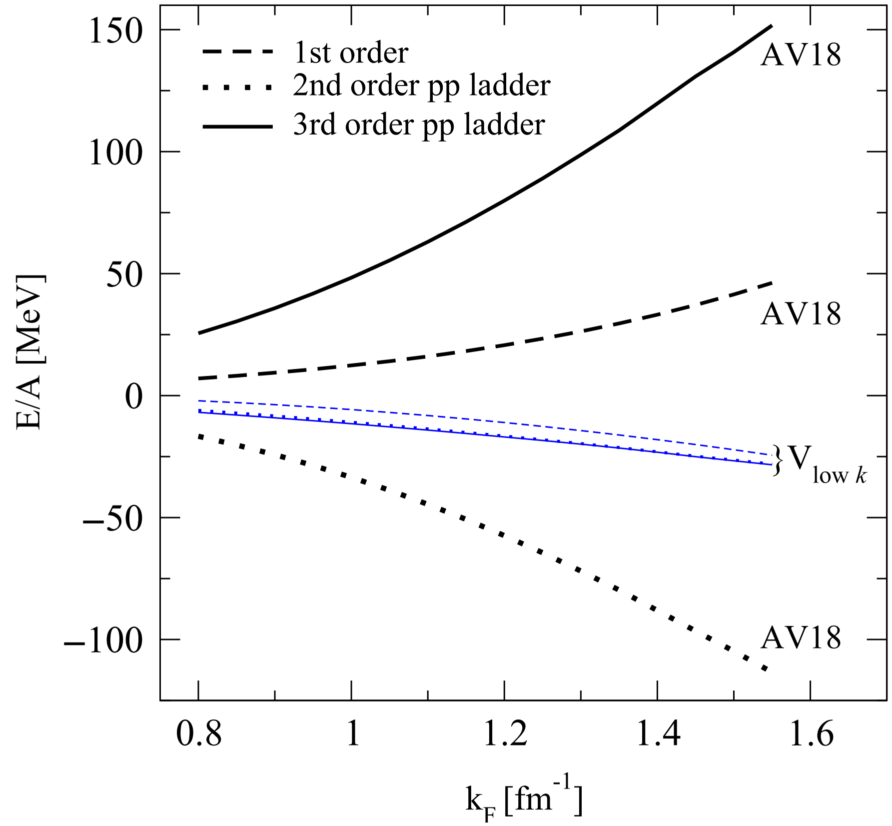
Does not saturate – what might be missing?

Improvements in Perturbation Theory

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{3\mathrm{N}}(\Lambda) + V_{4\mathrm{N}}(\Lambda) + \cdots$$

Ok, the interactions look perturbative, but something is wrong here…





No clear convergence with increasing order in bare potential

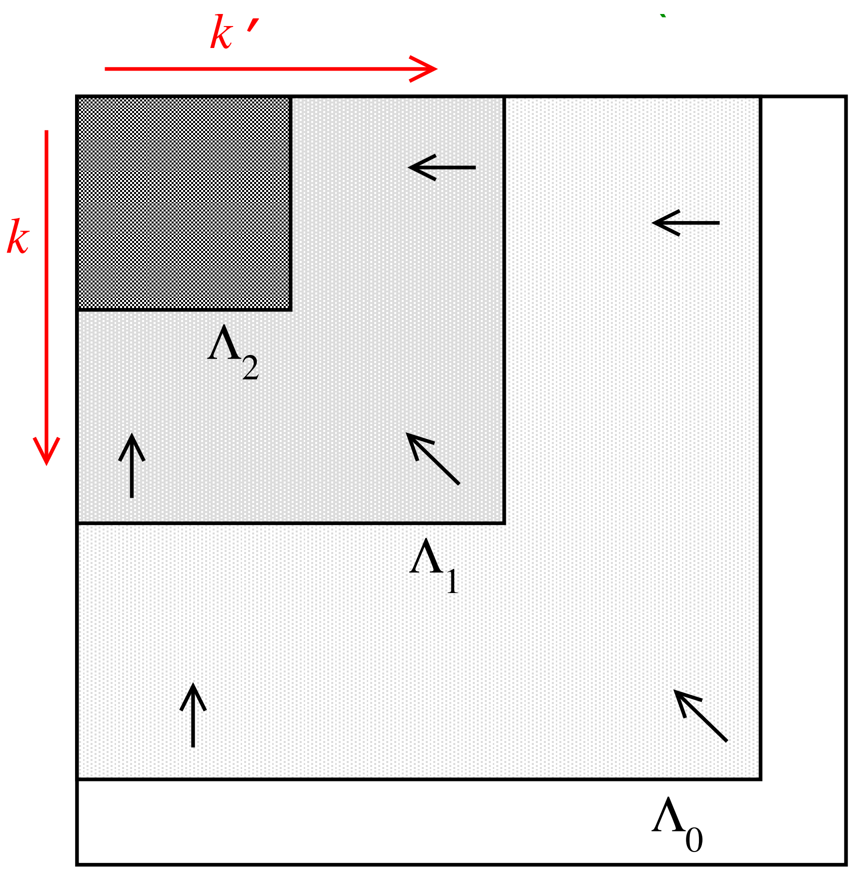
Significant improvement with low-momentum interactions!

Does not saturate – what might be missing?

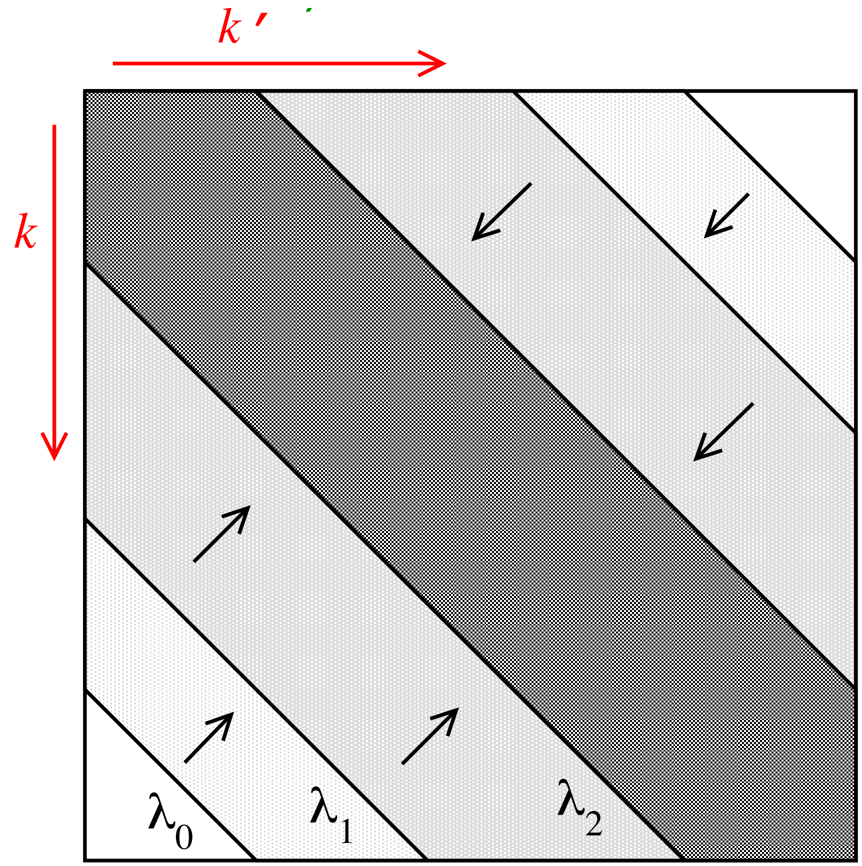
Similarity Renormalization Group

Wegner, Glazek/Wilson (1990s)

Complementary method to decouple low from high momenta



Decouples high-momentum



Similarity Renormalization Group

Drives Hamiltonian to band-diagonal

Similarity Renormalization Group

Wegner, Glazek/Wilson (1990s)

Apply a continuous unitary transformation, parameterized by s:

$$H = T + V \to H(s) = U(s) H U^{\dagger}(s) \equiv T + V(s)$$

where differentiating (exercise) yields:

$$\frac{\mathrm{d}H(s)}{\mathrm{d}s} = [\eta(s), H(s)] \quad \text{where} \quad \eta(s) \equiv \frac{\mathrm{d}U(s)}{\mathrm{d}s} U^{\dagger}(s)$$

Never explicitly construct unitary transformation

Instead **choose generator to obtain desired behavior**:

$$\eta(s) = [G(s), H(s)]$$

Many options, e.g.,

$$\eta(s) = [T, H(s)] \quad \text{Drives H(s) to band-diagonal form}$$

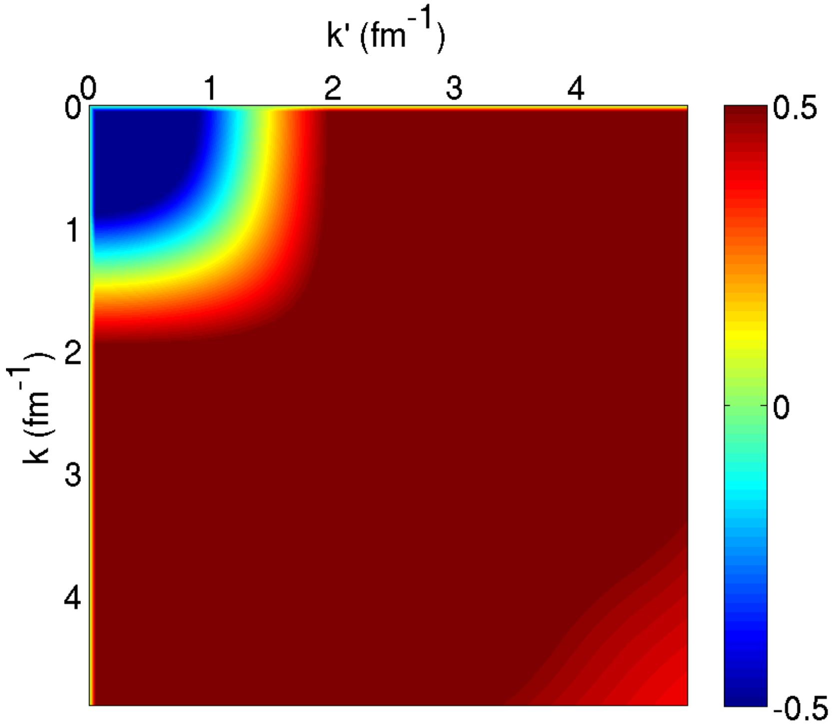
Illustration of SRG Flow

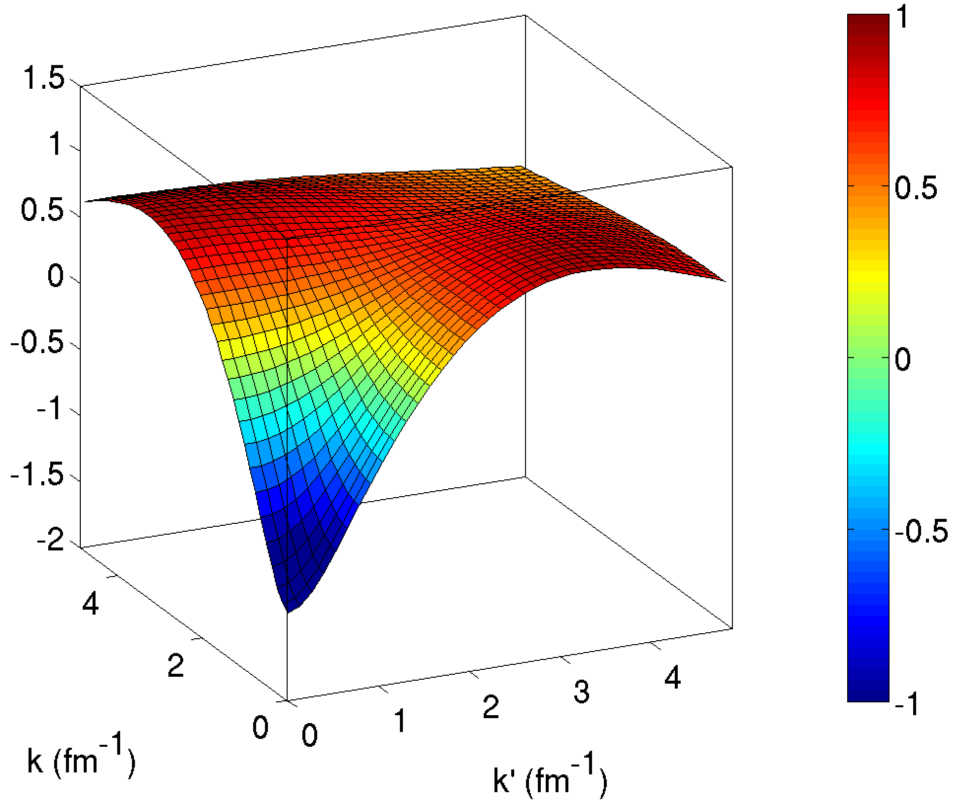
Drive H to band-diagonal form with kinetic-energy generator:

$$\eta(s) = [T, H(s)]$$

With alternate definition of flow parameter: $\lambda^2 = \frac{1}{\sqrt{s}}$

Argonne V_{18} $^1S_0$





$\lambda = 8.0\,\mathrm{fm}^{-1}$

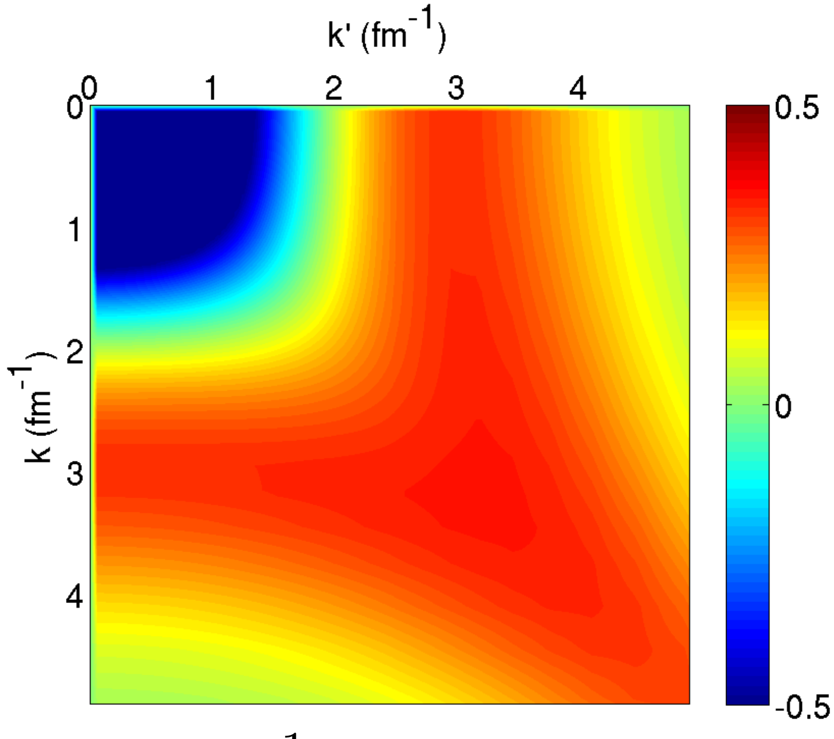
Illustration of SRG Flow

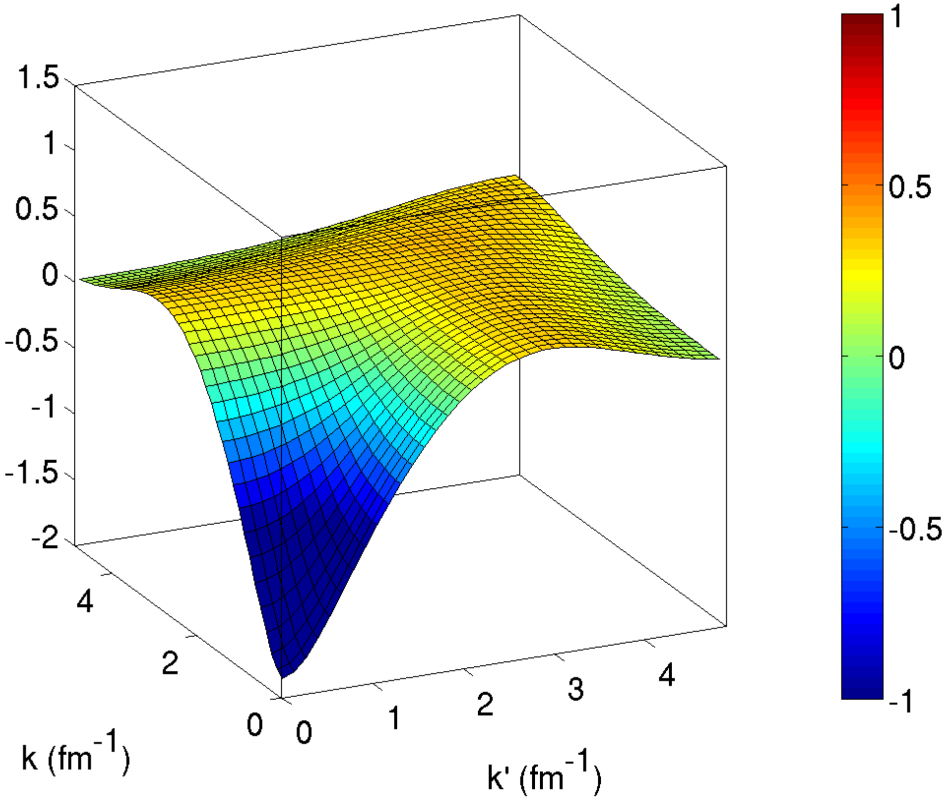
Drive H to band-diagonal form with standard choice:

$$\eta(s) = [T, H(s)]$$

With alternate definition of flow parameter: $\lambda^2 = \frac{1}{\sqrt{s}}$

Argonne V_{18} $^1S_0$





$\lambda = 4.0\,\mathrm{fm}^{-1}$

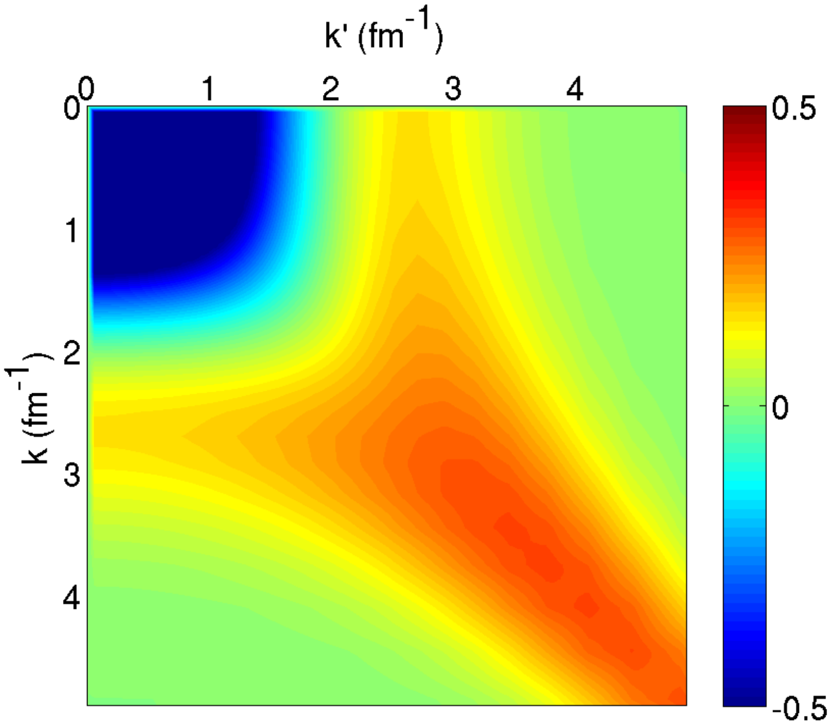
Illustration of SRG Flow

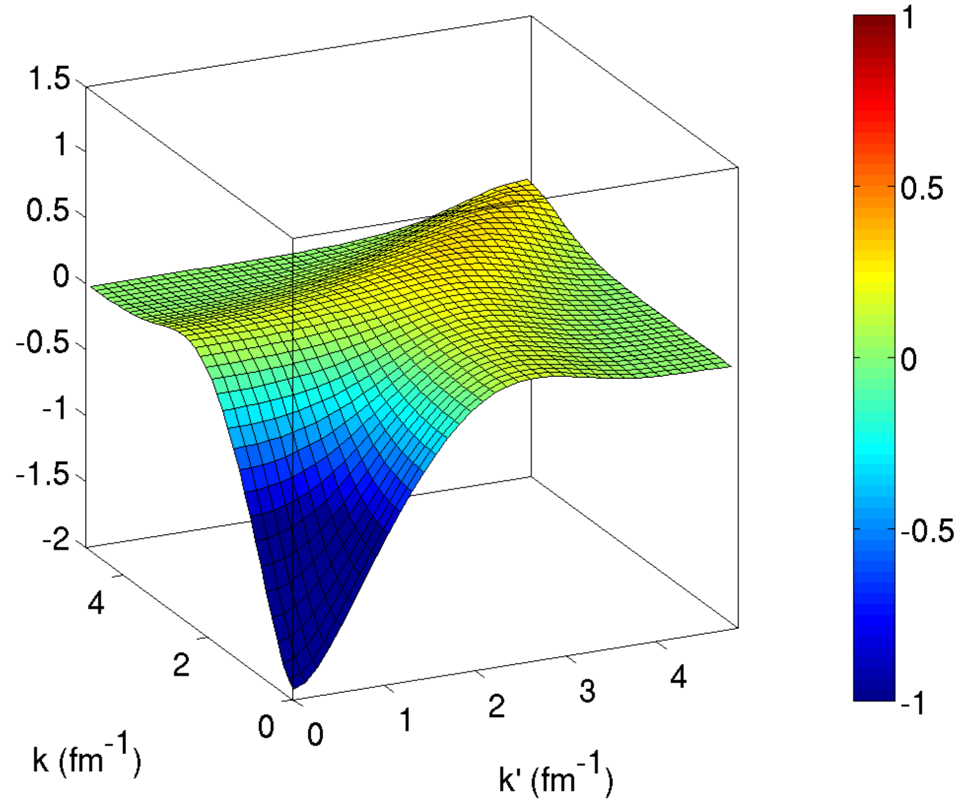
Drive H to band-diagonal form with standard choice:

$$\eta(s) = [T, H(s)]$$

With alternate definition of flow parameter: $\lambda^2 = \frac{1}{\sqrt{s}}$

Argonne V_{18} $\,^1S_0$





$\lambda = 3.0\,\mathrm{fm}^{-1}$

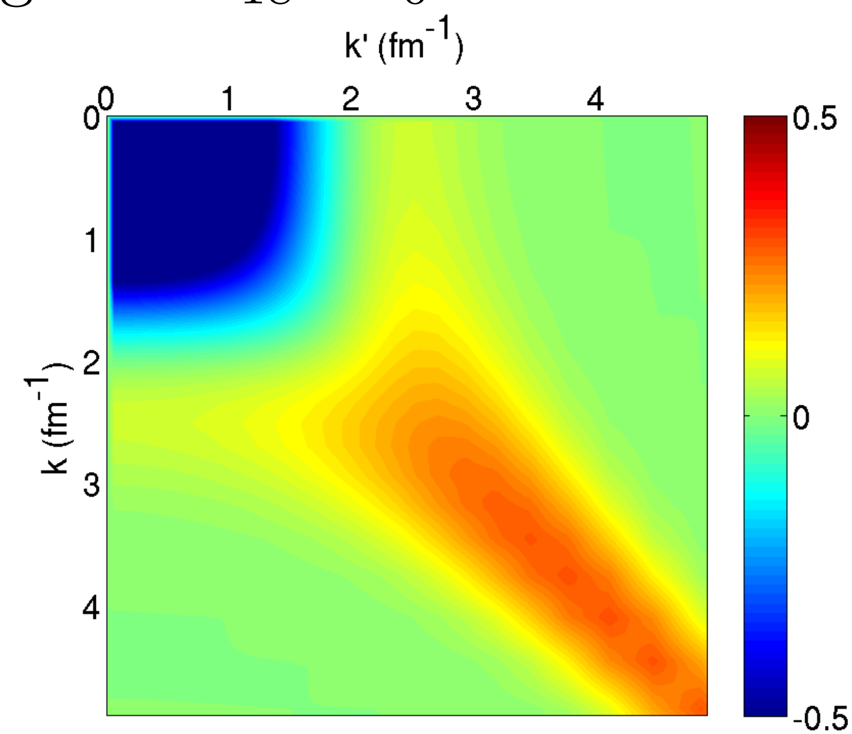
Illustration of SRG Flow

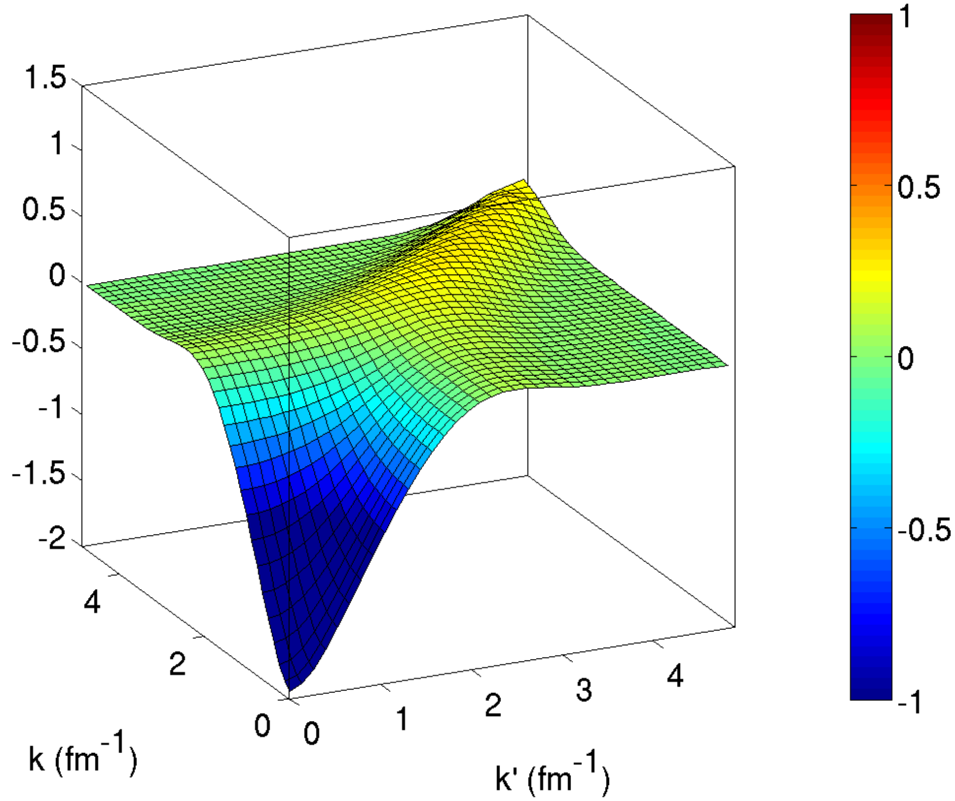
Drive H to band-diagonal form with standard choice:

$$\eta(s) = [T, H(s)]$$

With alternate definition of flow parameter: $\lambda^2 = \frac{1}{\sqrt{s}}$

Argonne V_{18} $^1S_0$





$\lambda = 2.5\,\mathrm{fm}^{-1}$

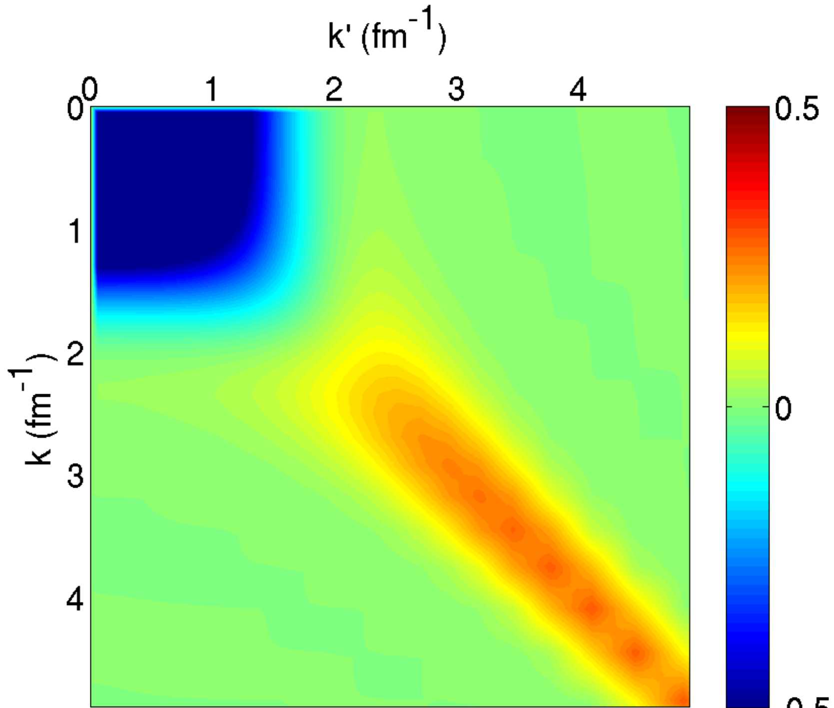
Illustration of SRG Flow

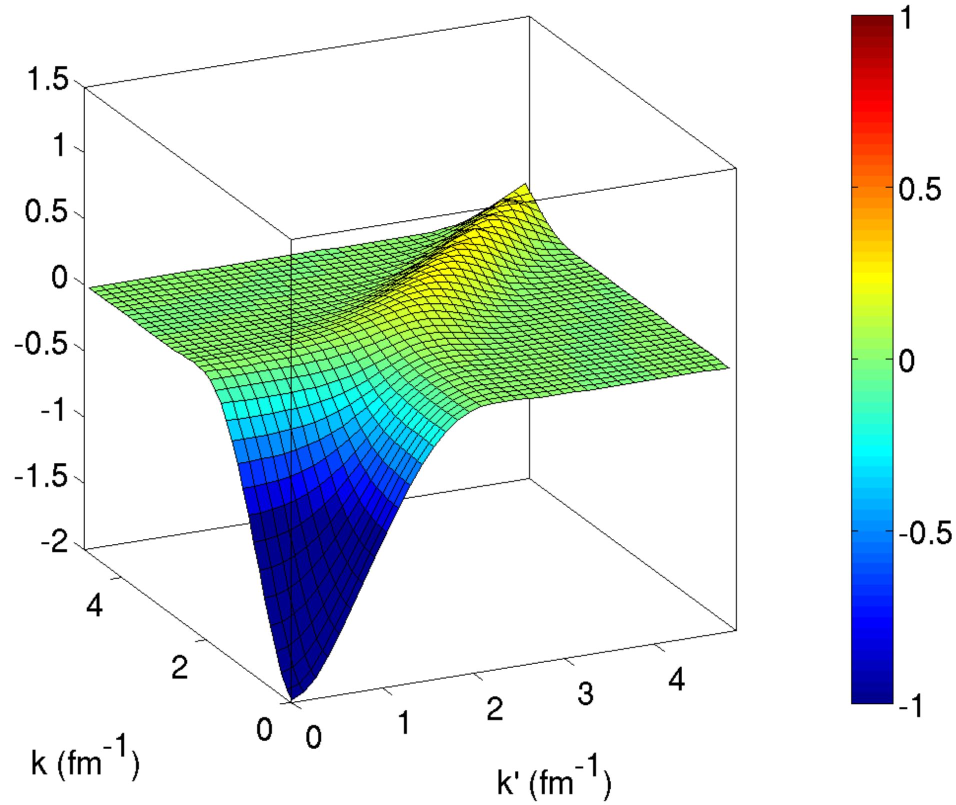
Drive H to band-diagonal form with standard choice:

$$\eta(s) = [T, H(s)]$$

With alternate definition of flow parameter: $\lambda^2 = \frac{1}{\sqrt{s}}$

Argonne V_{18} $^1S_0$





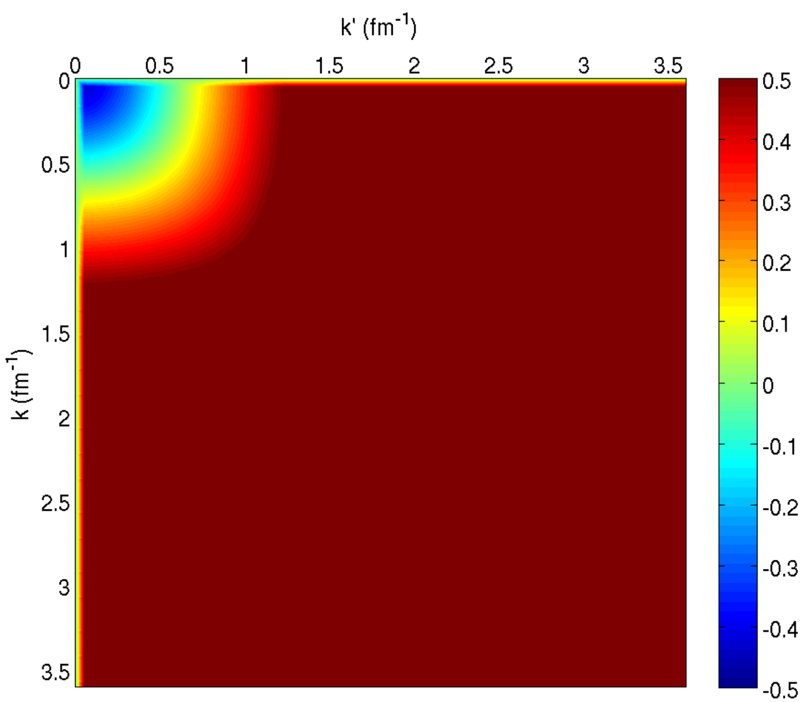
$\lambda = 2.0\,\mathrm{fm}^{-1}$

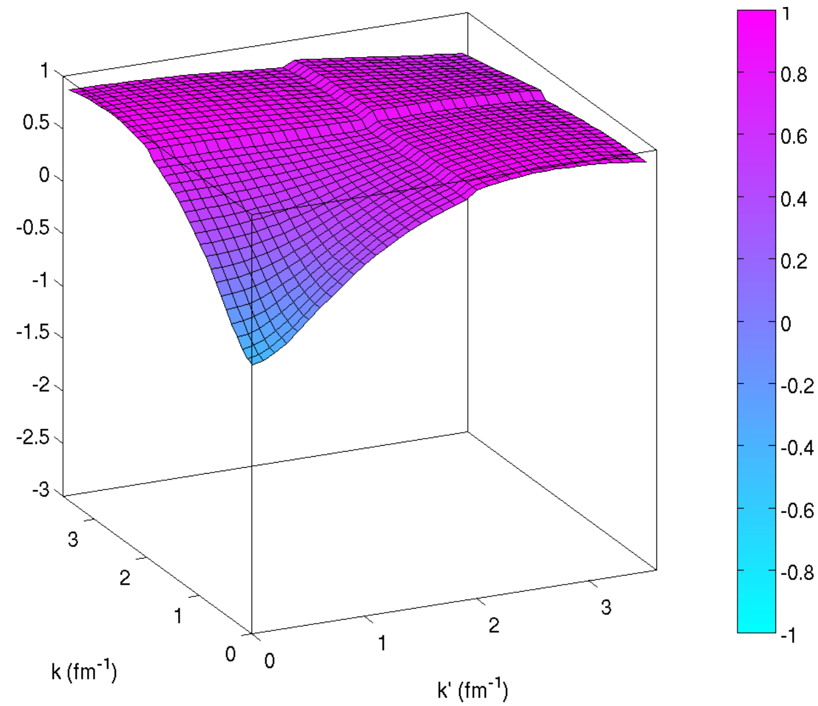
Other Generator Choices: Block Diagonal

Create block diagonal form like V_{lowk}?

$$G(s) = H_{\rm BD} = \begin{pmatrix} PH(s)P & 0 \\ 0 & QH(s)Q \end{pmatrix}$$

With alternate definition of flow parameter: $\lambda^2 = \frac{1}{\sqrt{s}}$





Argonne V_{18} $^3S_1$

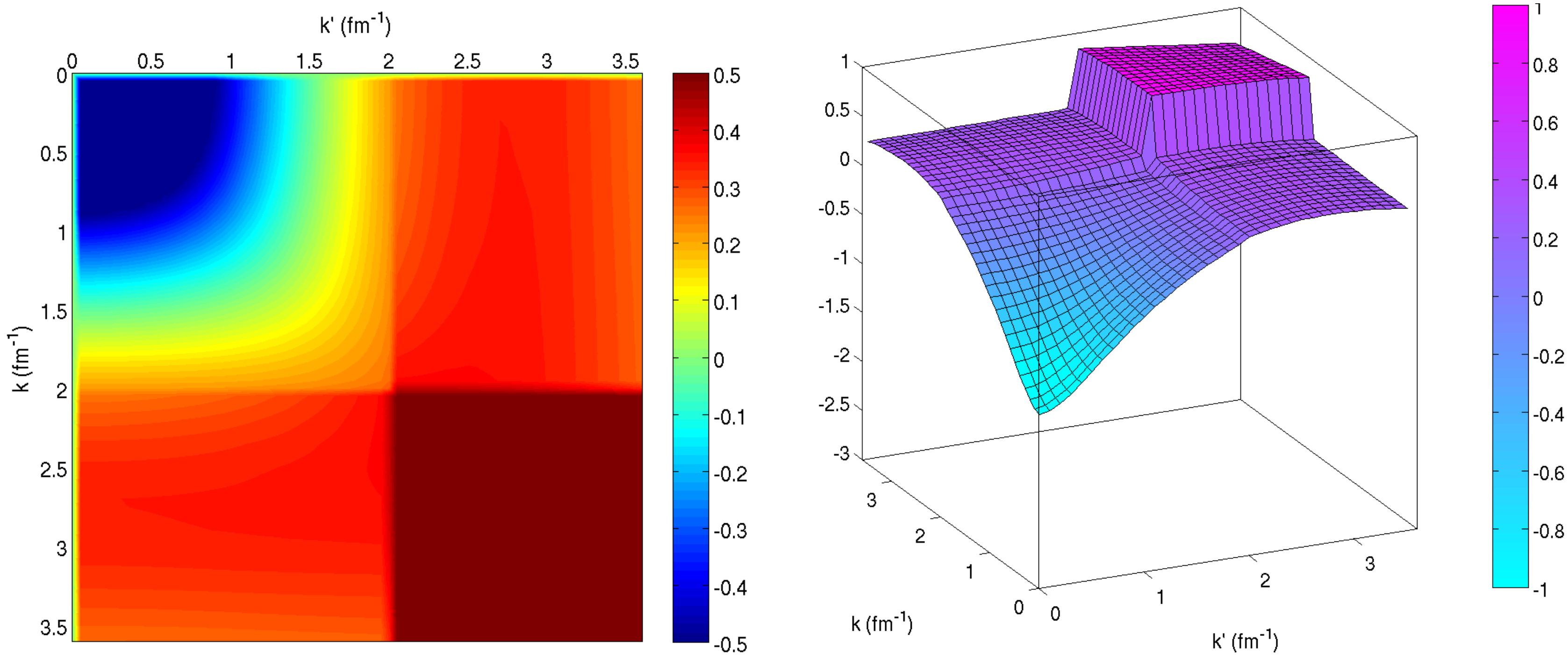
$\lambda = 10.0\,\mathrm{fm}^{-1}$

Other Generator Choices: Block Diagonal

Create block diagonal form like V_{lowk}?

$$G(s) = H_{\rm BD} = \begin{pmatrix} PH(s)P & 0 \\ 0 & QH(s)Q \end{pmatrix}$$

With alternate definition of flow parameter: $\lambda^2 = \frac{1}{\sqrt{s}}$



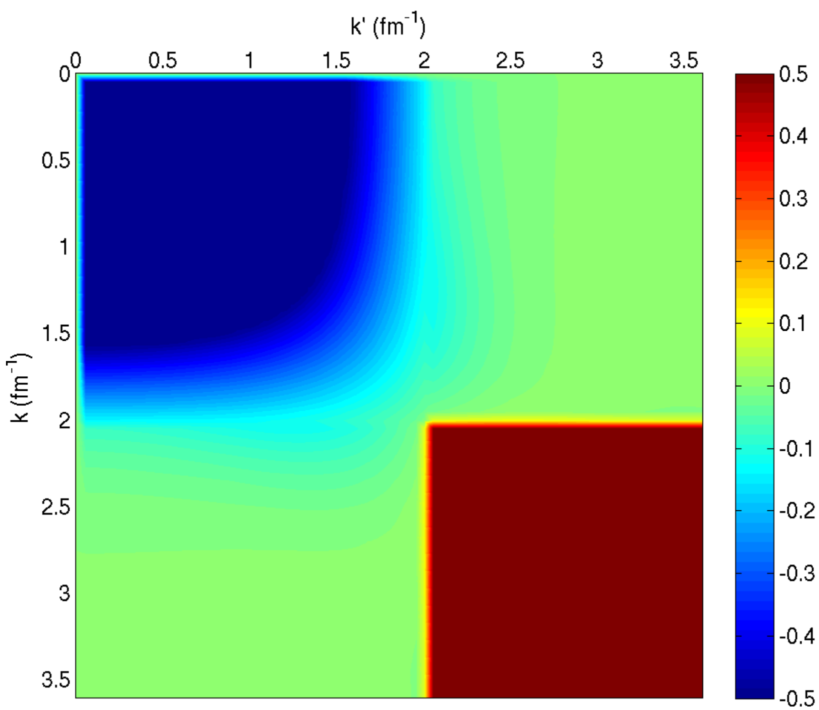
Argonne V_{18} $^3S_1$ $\lambda = 5.0\,\mathrm{fm}^{-1}$

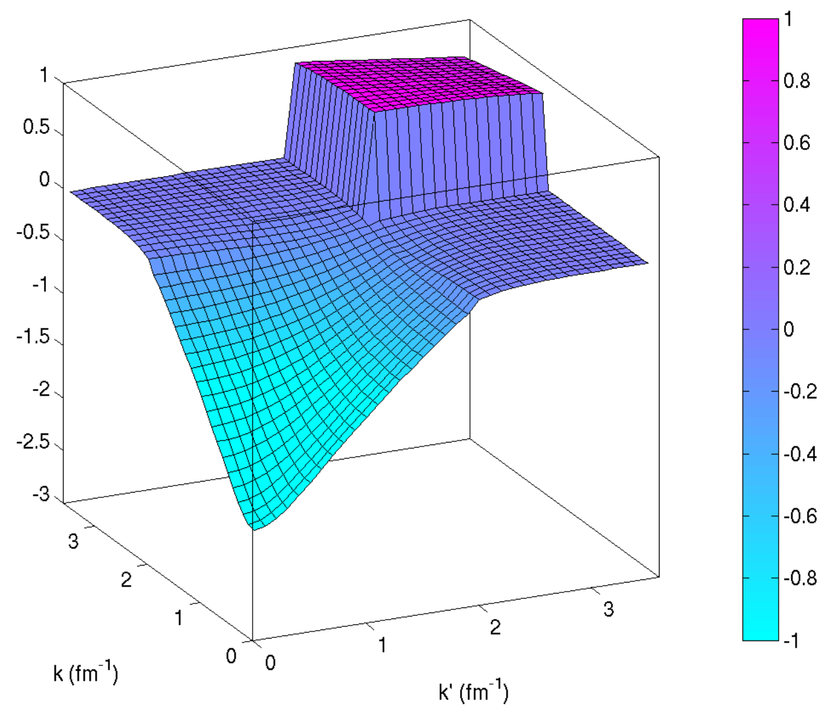
Other Generator Choices: Block Diagonal

Create block diagonal form like V_{lowk}?

$$G(s) = H_{\rm BD} = \begin{pmatrix} PH(s)P & 0 \\ 0 & QH(s)Q \end{pmatrix}$$

With alternate definition of flow parameter: $\lambda^2 = \dfrac{1}{\sqrt{s}}$

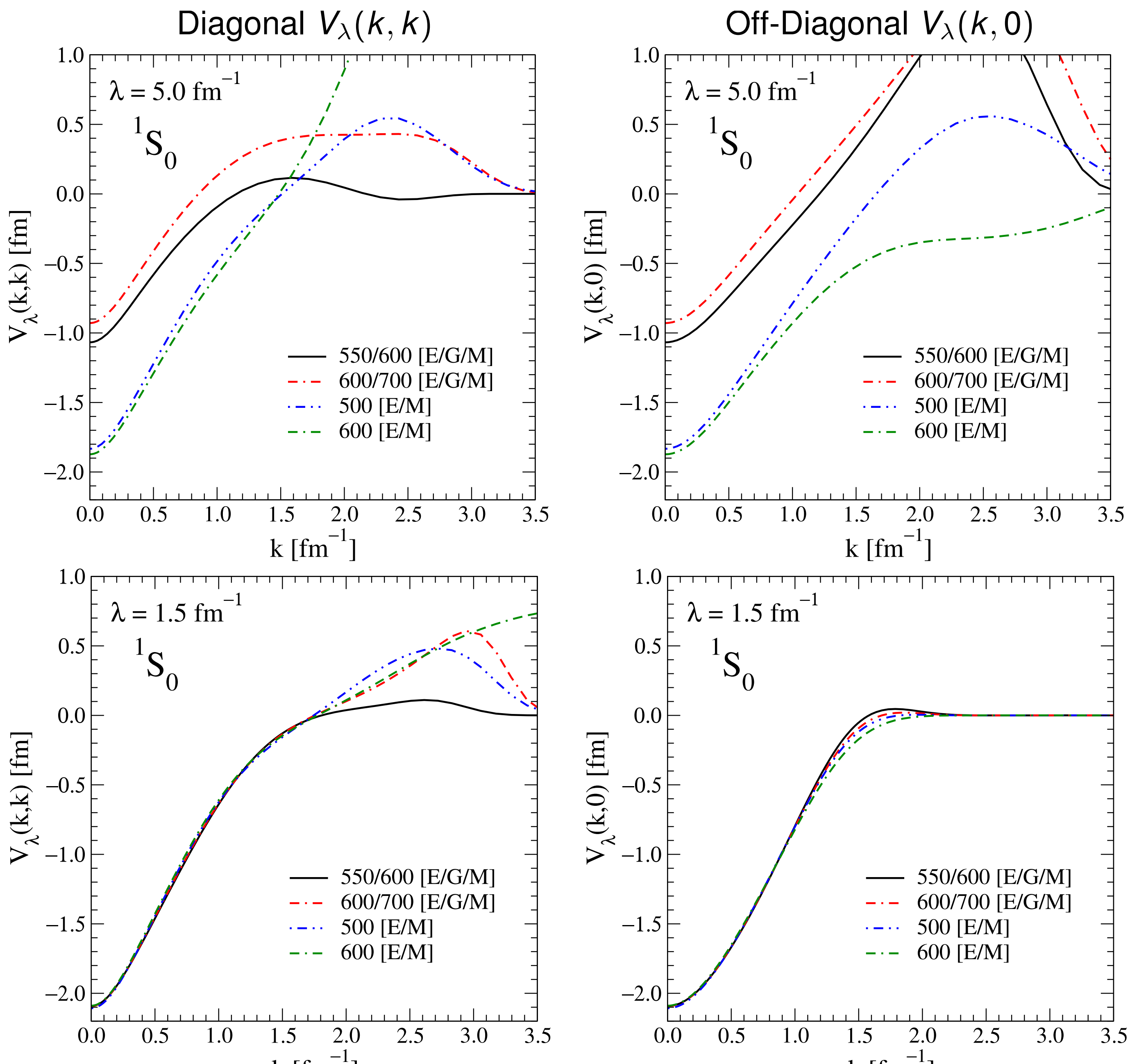




Argonne V_{18} $^3S_1$

$\lambda = 2.0\,\mathrm{fm}^{-1}$

SRG Renormalization of Chiral EFT Potentials



These are all our favorite Chiral EFT NN potentials…

These are all our favorite Chiral EFT NN potentials… **SRG evolved**

Exhibit similar “universal” behavior as low-momentum interactions!

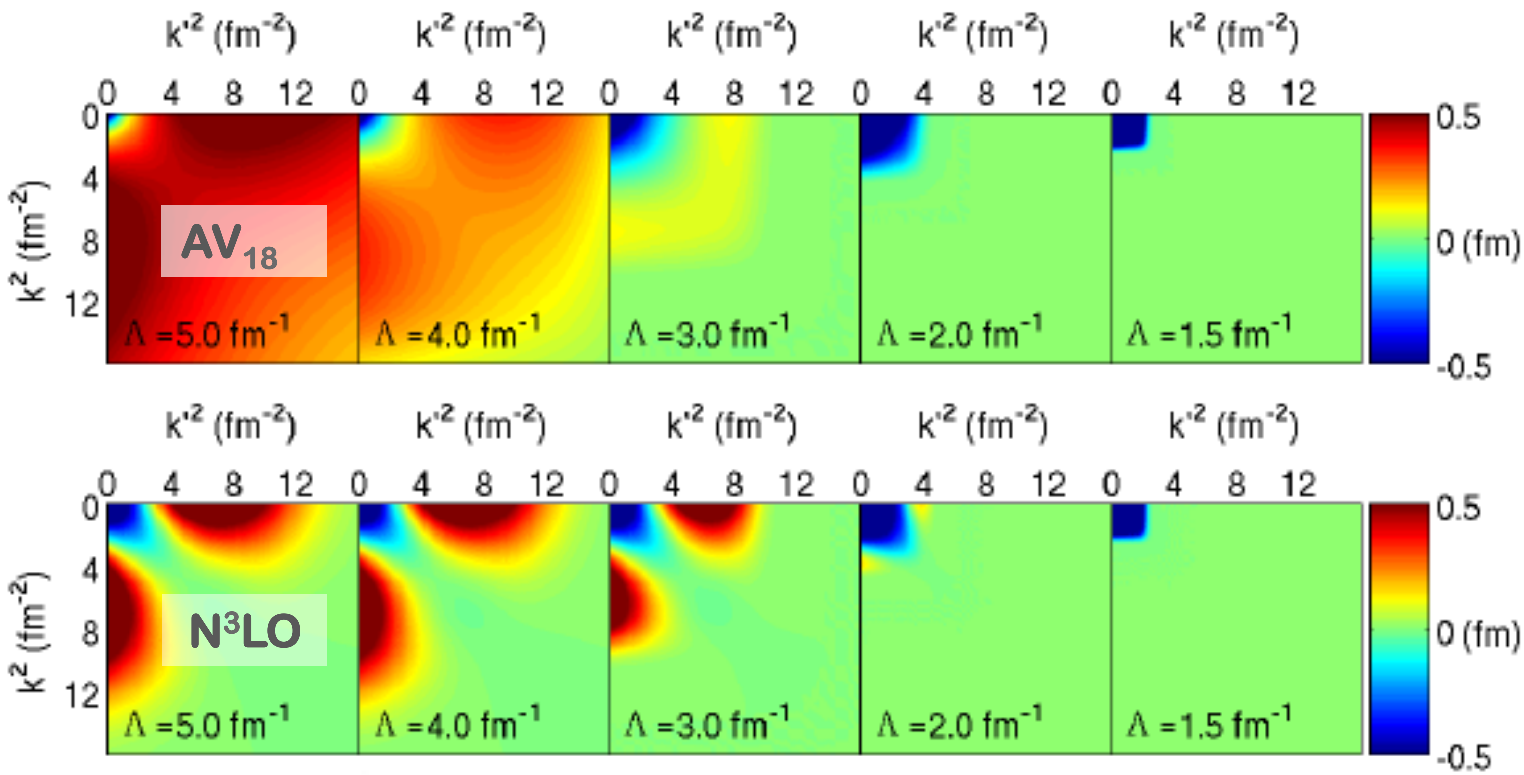
Renormalization of Nuclear Interactions

$$H(\Lambda) = T + V_{\rm NN}(\Lambda) + V_{\rm 3N}(\Lambda) + V_{\rm 4N}(\Lambda) + \cdots$$

Evolve momentum resolution scale of chiral interactions from initial Λ_χ

Remove coupling to high momenta, low-energy physics unchanged

Bogner, Kuo, Schwenk, Furnstahl



Universal at low-momentum

$V_{{\rm low}\,k}(\Lambda)$: lower cutoffs advantageous for nuclear structure calculations

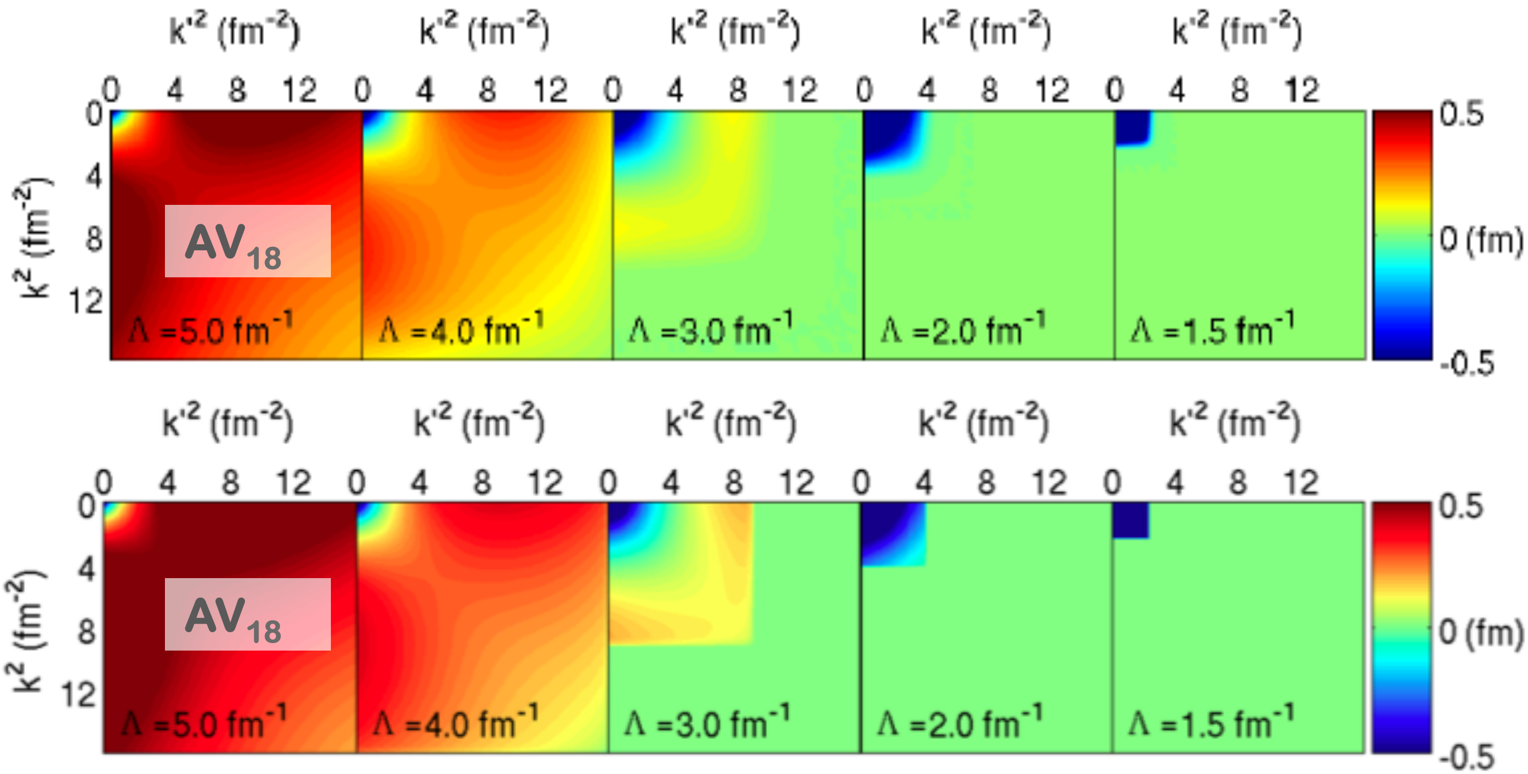
Smooth vs. Sharp Cutoffs

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{3\mathrm{N}}(\Lambda) + V_{4\mathrm{N}}(\Lambda) + \cdots$$

Can have sharp as well as smooth cutoffs

Remove coupling to high momenta, low-energy physics unchanged

Bogner, Kuo, Schwenk, Furnstahl

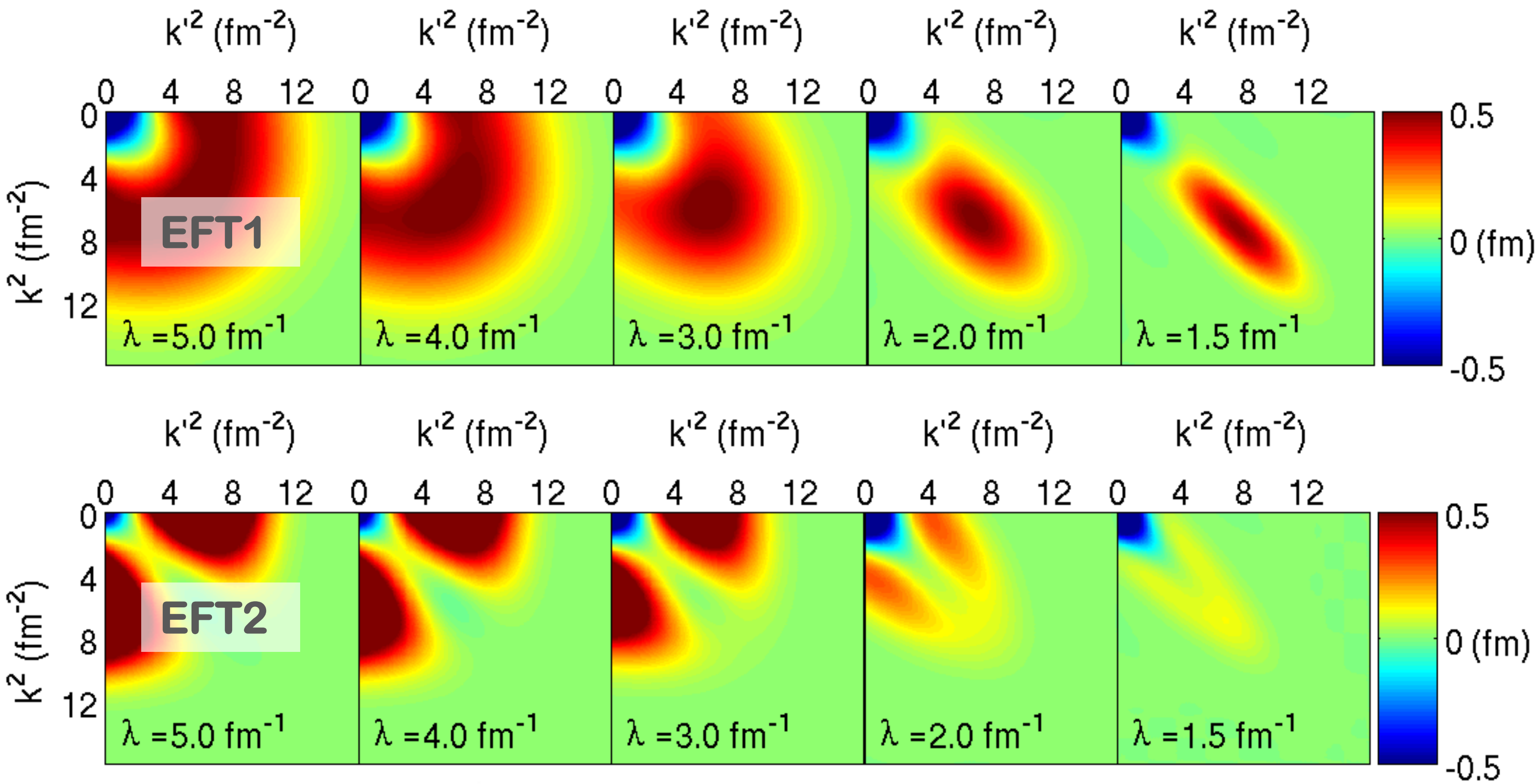


Similar but not exact same results – will be differences in calculations

SRG-Evolution of Different Initial Potentials

$$H(\Lambda) = T + V_{\rm NN}(\Lambda) + V_{\rm 3N}(\Lambda) + V_{\rm 4N}(\Lambda) + \cdots$$

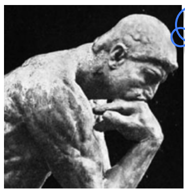
SRG evolution of two different chiral EFT potentials



Lots of pretty pictures, but how does it actually help?

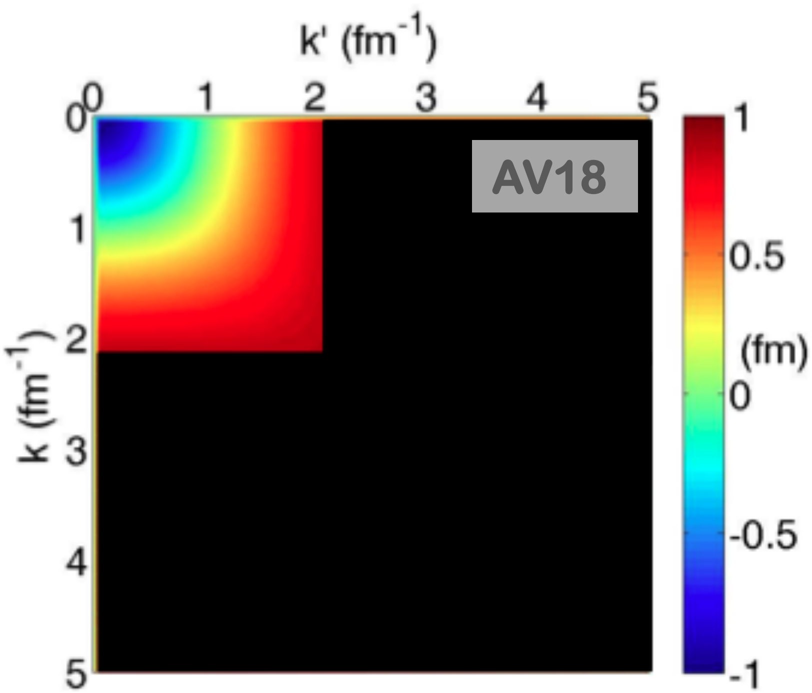
Revisit Low-Pass Filter Idea

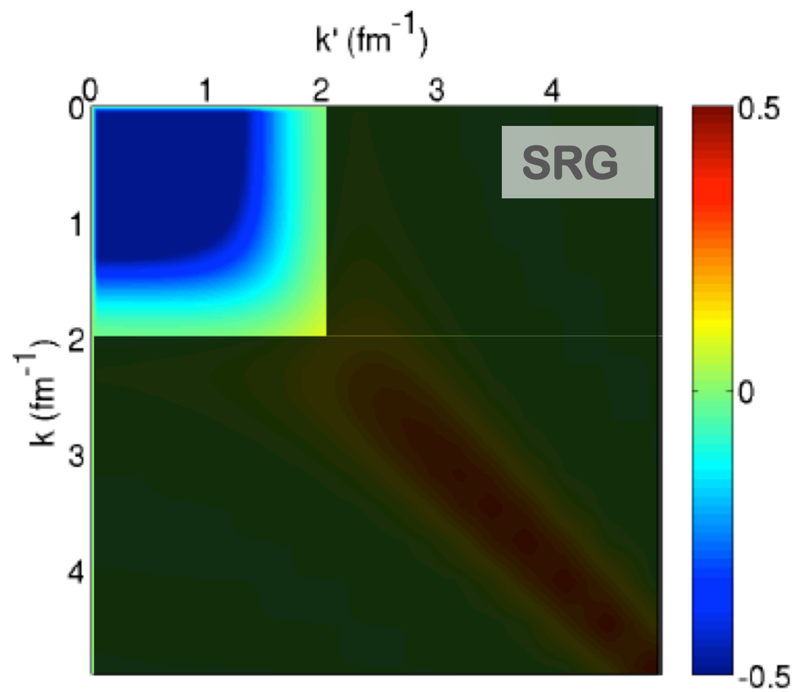
Ok, high momentum is a pain. I wonder what would happen to low-energy observables…



Low-to-high momentum makes life difficult for low-energy nuclear theorists

What's the difference now?





$$V_{\text{filter}}(k', k) \equiv 0; \;\; k, k' > 2.2\,\text{MeV}$$

Revisit Low-Pass Filter Idea

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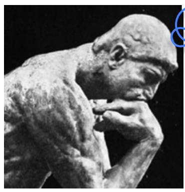
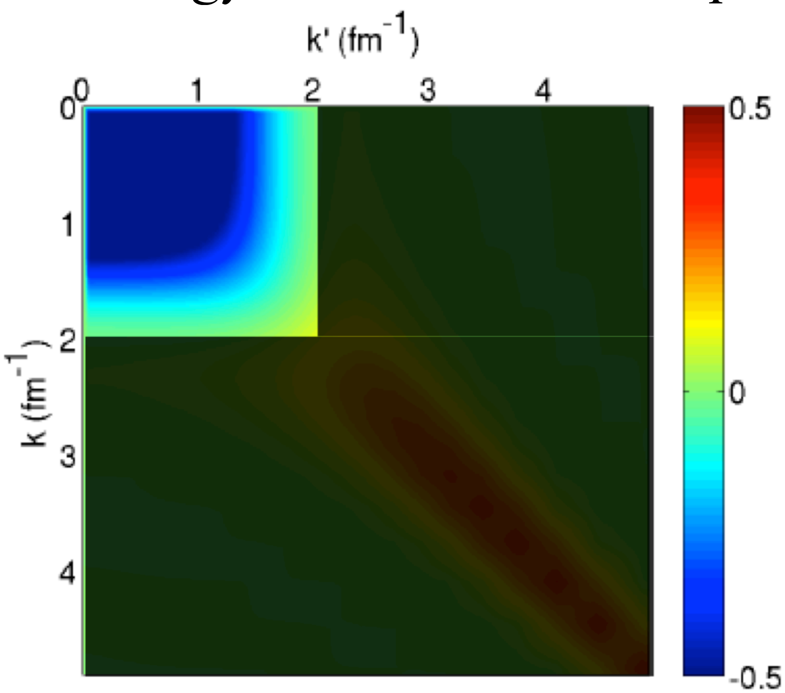
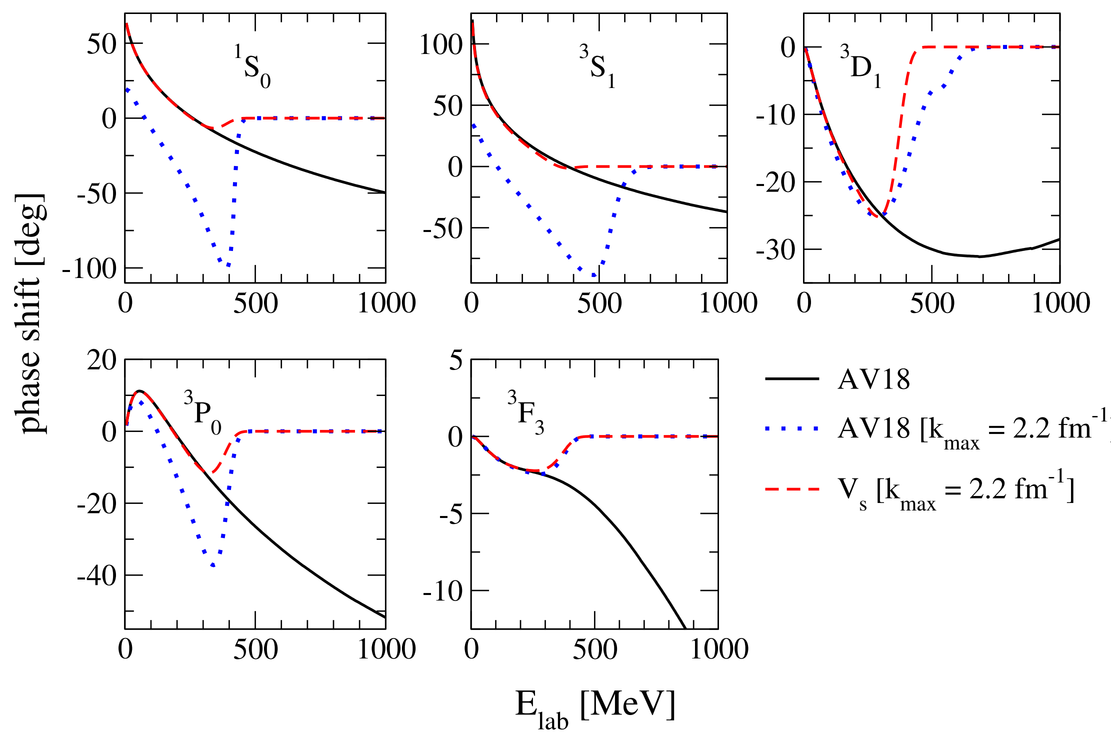


Low-to-high momentum makes life difficult for low-energy nuclear theorists

Low-energy observables were preserved – now sharp cut makes sense!





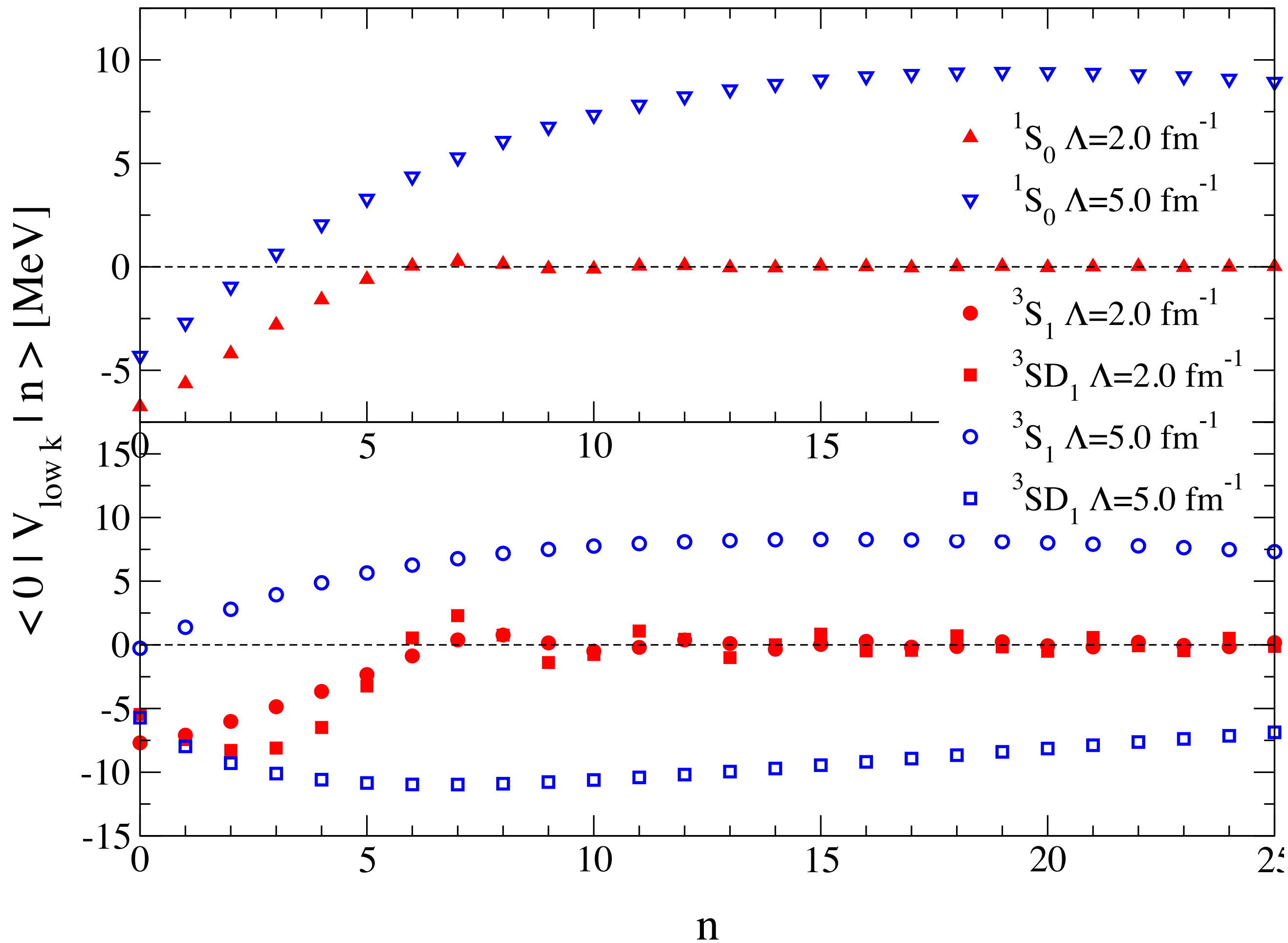
$$V_{\rm filter}(k', k) \equiv 0;\ \ k, k' > 2.2\,{\rm MeV}$$

Benefits of Lower Cutoffs

Often work in HO basis – does this make a difference there?

Removes coupling from low-to-high harmonic oscillator states

Expect to speed convergence in HO basis



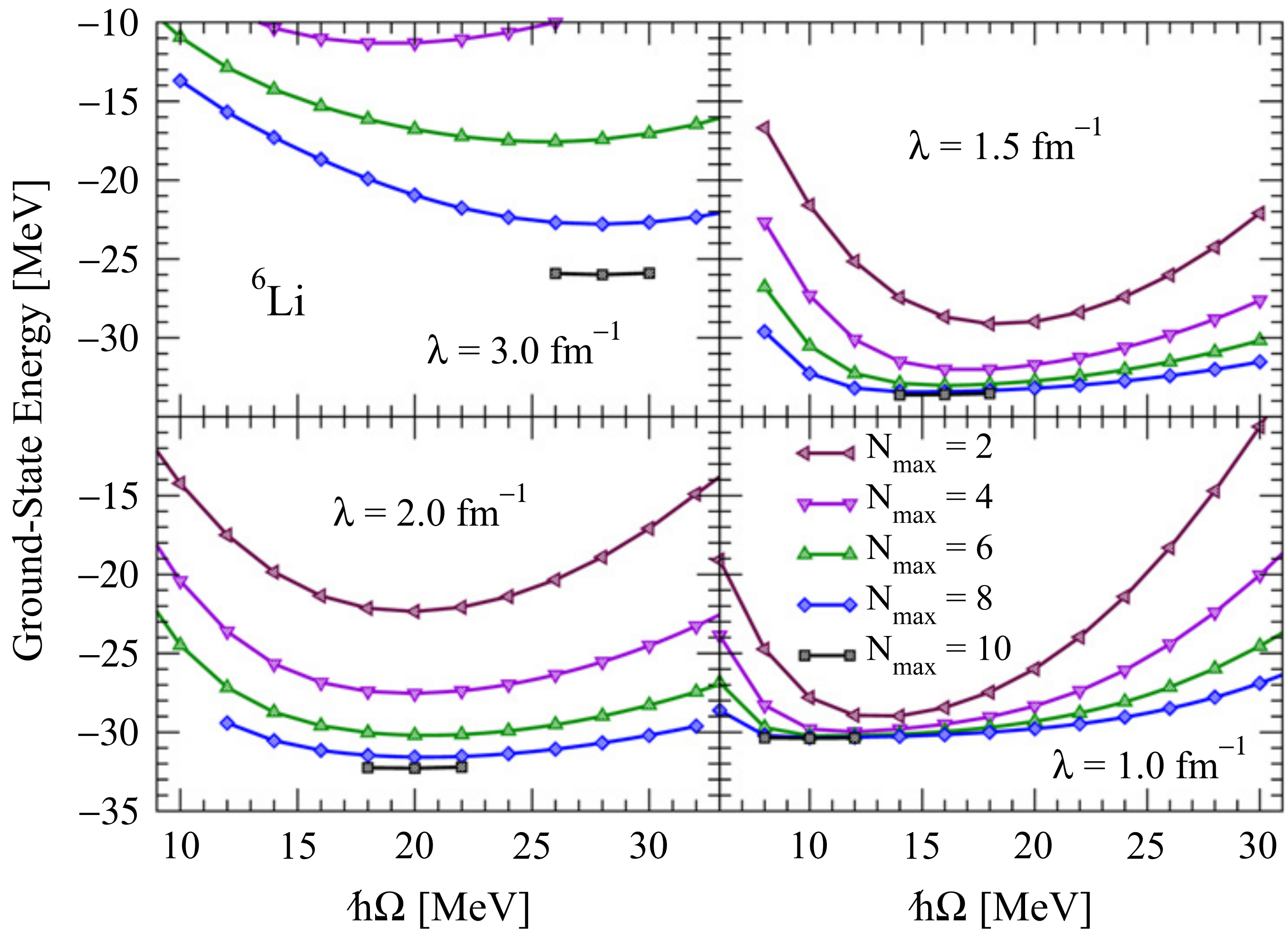
Explicitly see why this causes problems later!

Benefits of Lower Cutoffs

Exactly what happens in **no-core shell model calculations**

Probably equally helpful in normal shell-model calculations?

Come back to this later…



Benefits of Lower Cutoffs

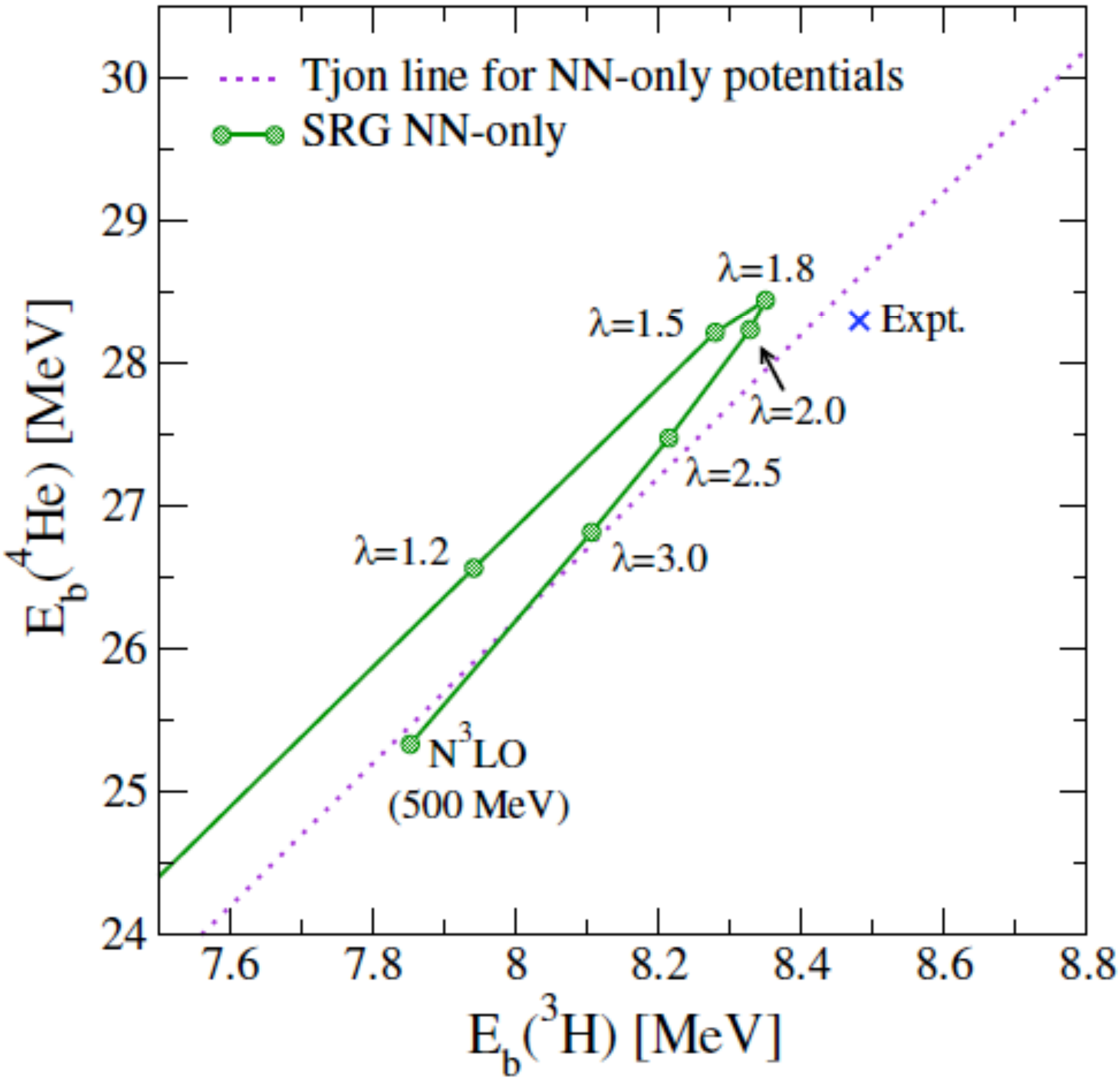
Use cutoff dependence to assess missing physics: return to Tjon line

Varying cutoff moves along line

Still never reaches experiment

Lesson: Variation in physical observables with cutoff indicates missing physics

Tool, not a parameter!

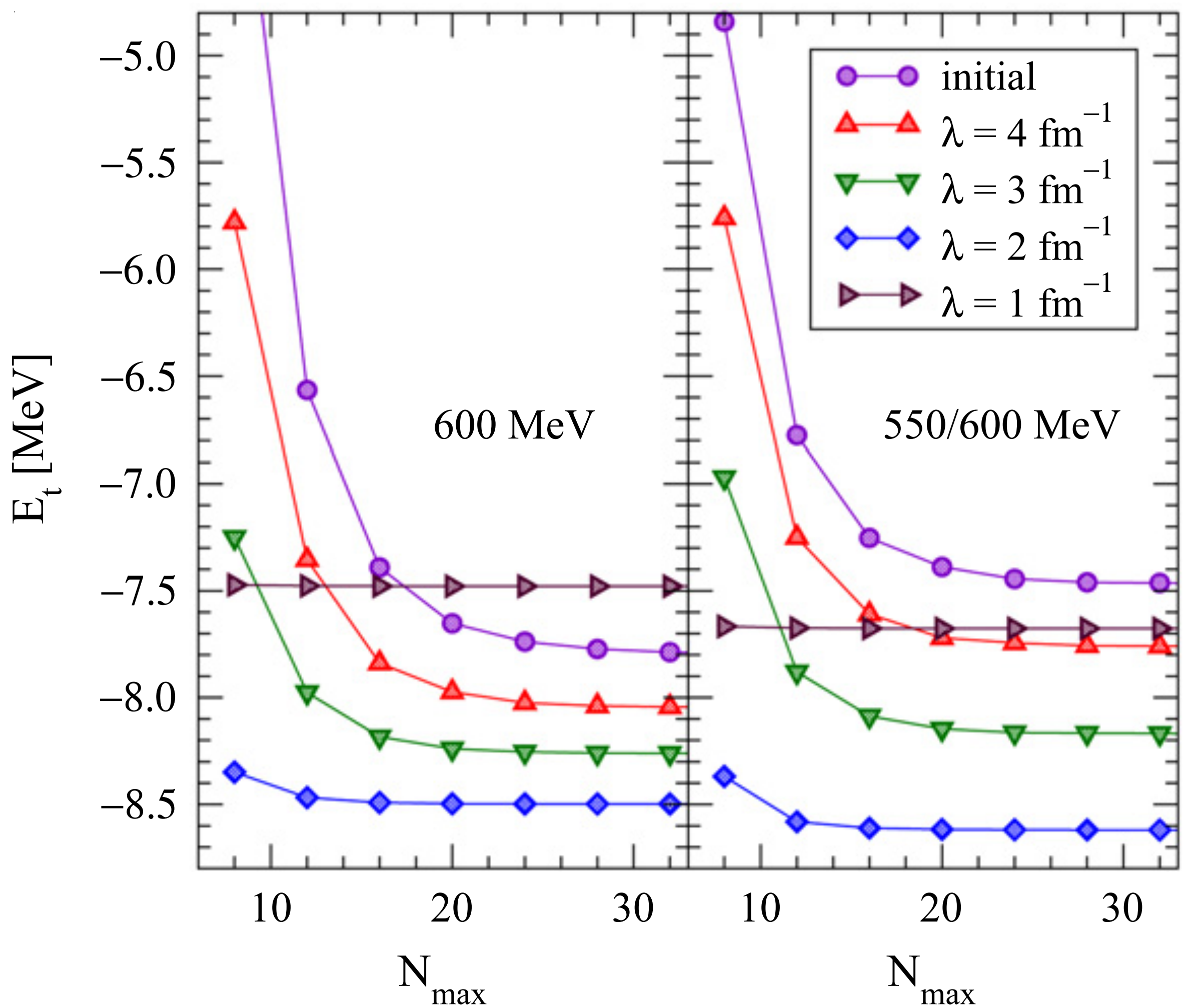


Benefits of Lower Cutoffs

Triton binding energy - again clearly improved convergence behavior

Clear dependence on cutoff – more than one, look closely…

What is the source(s)?



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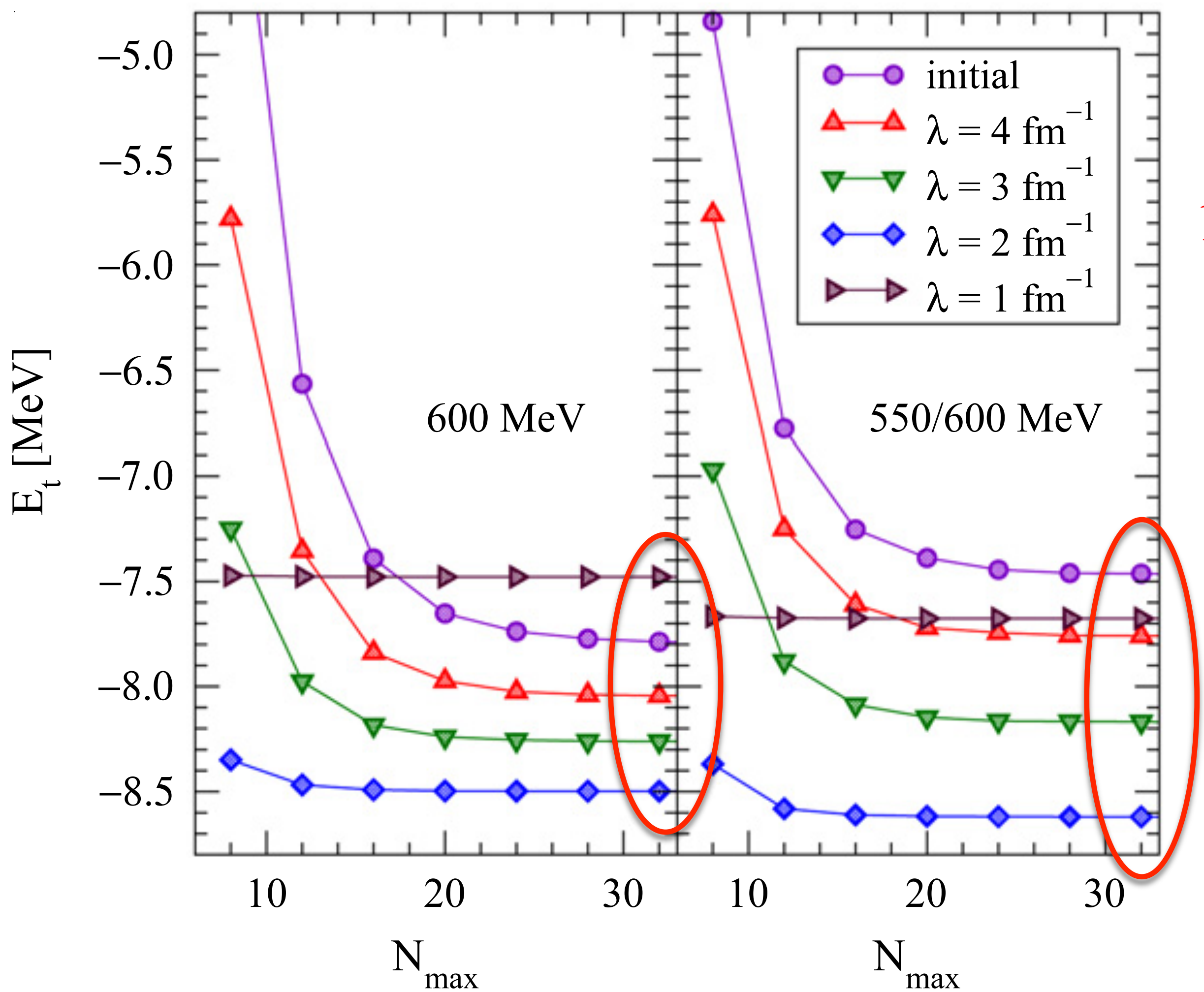

1) SRG cutoff dependence

Benefits of Lower Cutoffs

Triton binding energy - again clearly improved convergence behavior

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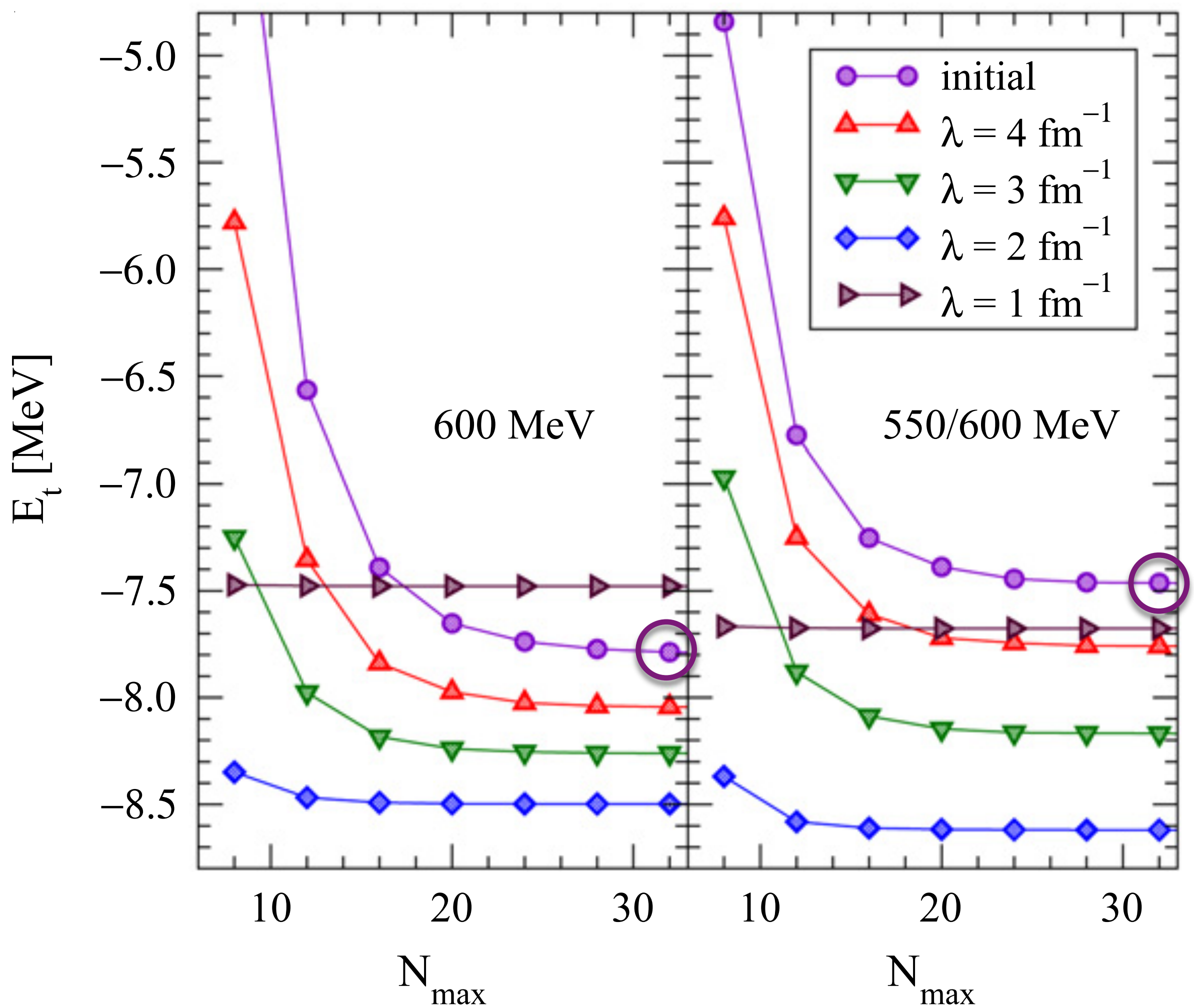

1) SRG cutoff dependence

2) Initial cutoff dependence

Something missing in each case!

Case 1: Price of Low Cutoffs = Induced Forces

Life Lesson: no free lunch – not even at Summer Schools, apparently ☹

Consider Hamiltonian with only two-body forces:

$$H = T + V_{\mathrm{NN}}$$

And $\eta(s) = [T, H(s)]$

$$\frac{\mathrm{d}H(s)}{\mathrm{d}s} = [\eta(s), H(s)] = \left[[T, T + V(s)], T + V(s)\right]$$

Simply expand with creation/annihilation operators:

Case 1: Price of Low Cutoffs = Induced Forces

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Simply expand with creation/annihilation operators:

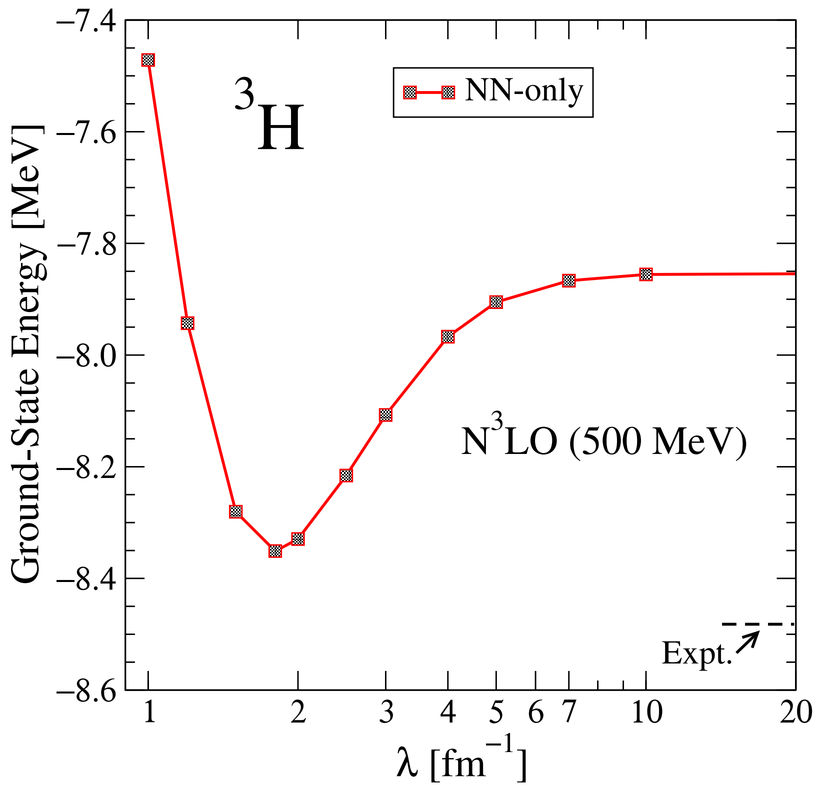
$$\frac{\mathrm{d}V(s)}{\mathrm{d}s} = \left[\left[\sum a^\dagger a, \sum a^\dagger a^\dagger a a\right], \sum a^\dagger a^\dagger a a\right] = \cdots + \sum a^\dagger a^\dagger a^\dagger a a a + \cdots$$

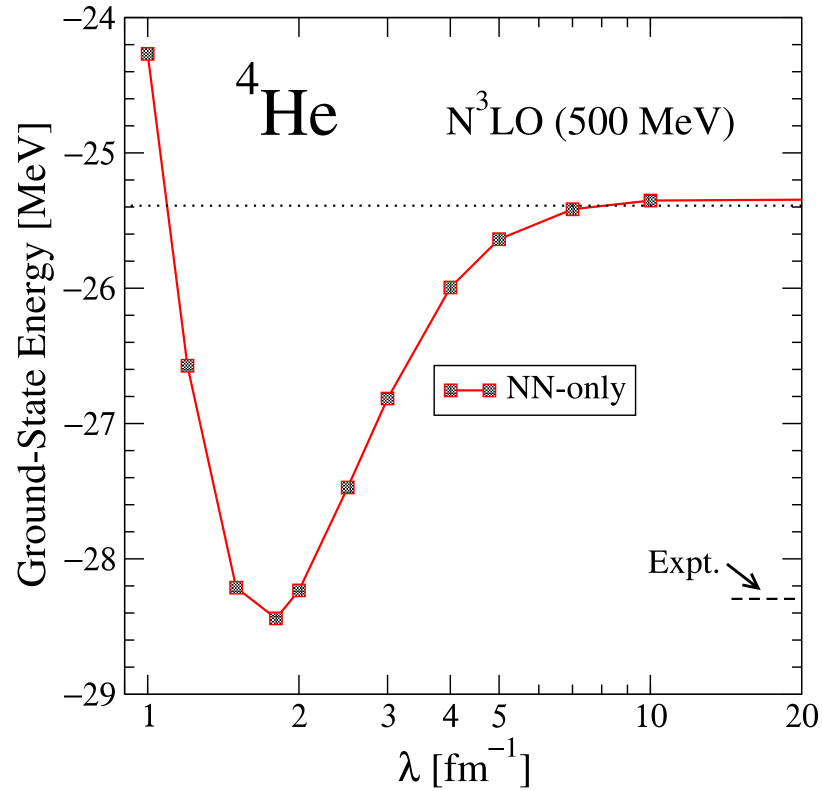
Three-body terms will appear even when initial 3-body forces absent

Call these **induced 3N forces (3N-ind)**

Induced 3N Forces

Effect of including 3N-ind? Exactly initial $V_{\rm NN}$ **up to neglected 4N-ind**

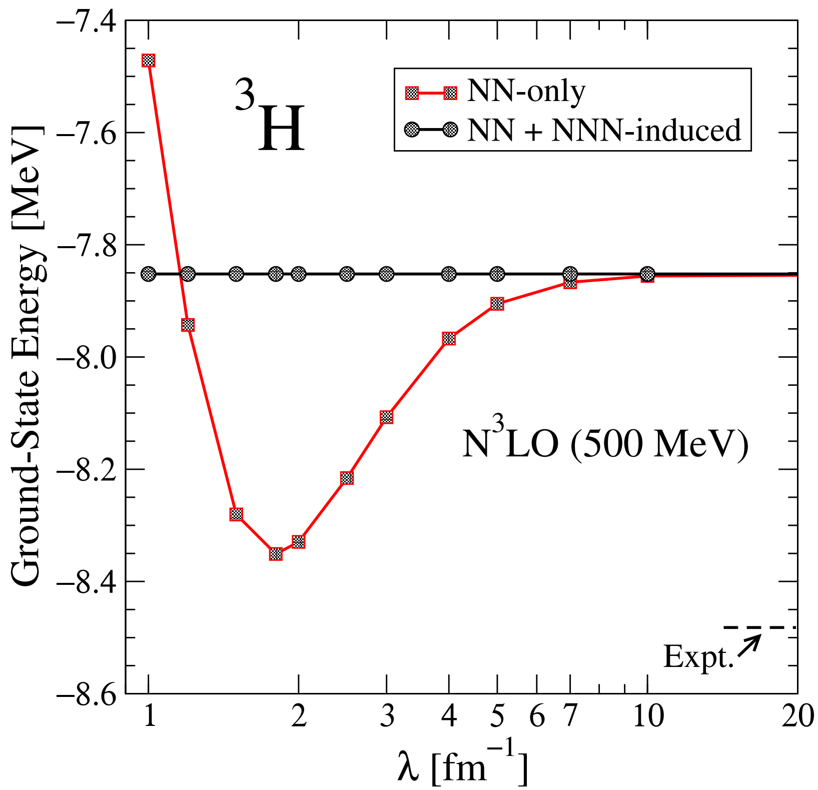


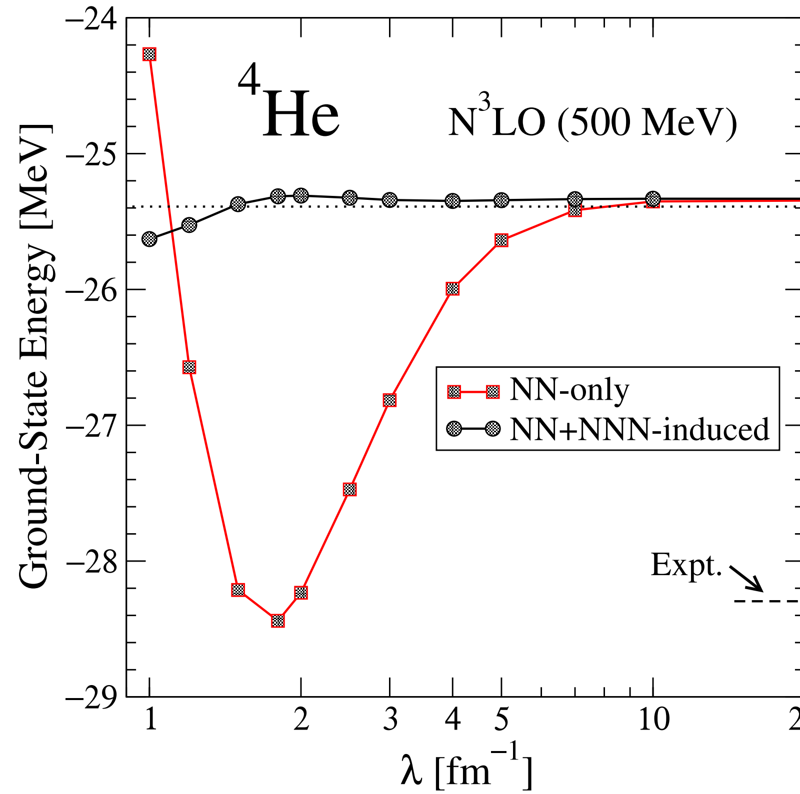


NN-only clear cutoff dependencs

Induced 3N Forces

Effect of including 3N-ind? Exactly initial $V_{\rm NN}$ **up to neglected 4N-ind**





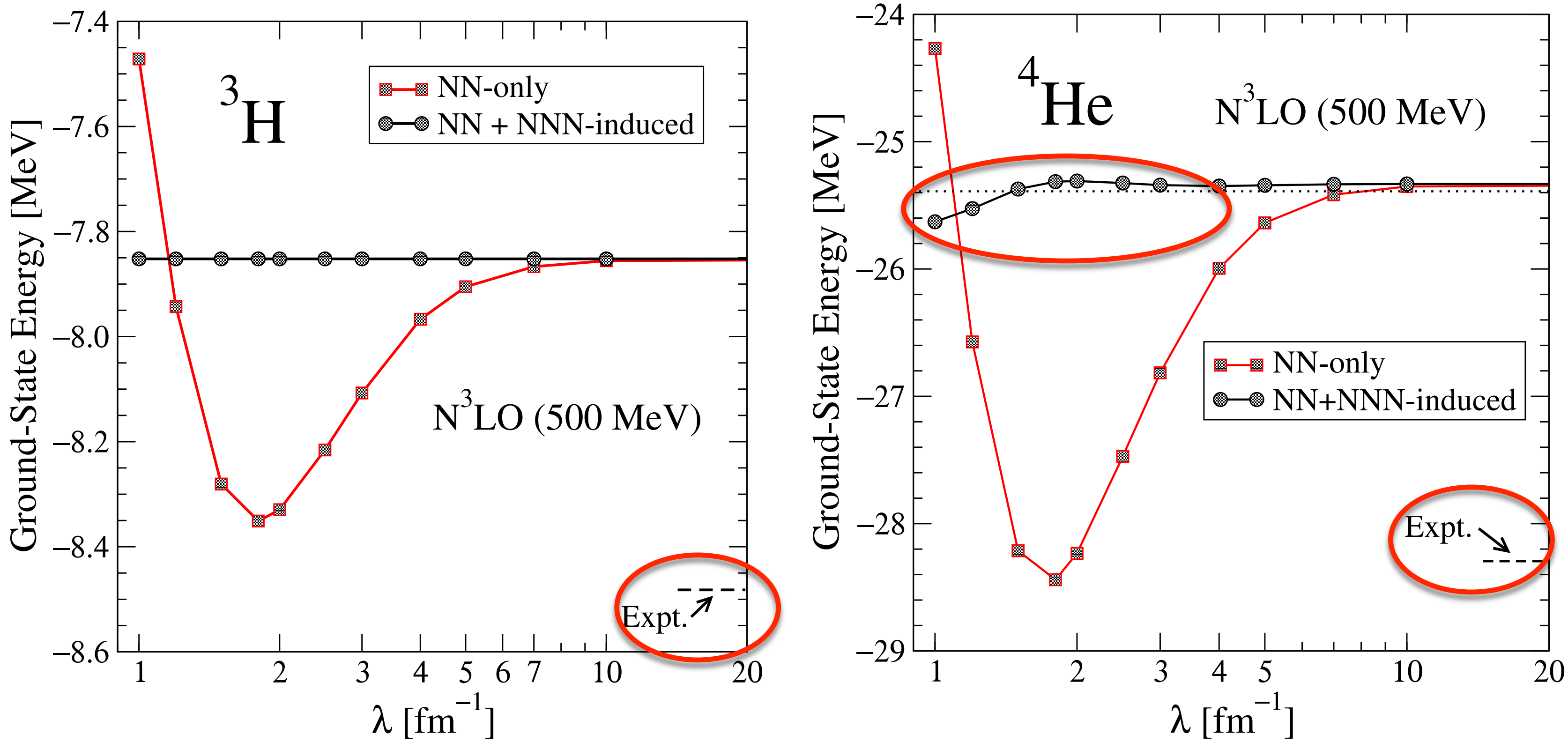
NN-only clear cutoff dependencs

3N-induced – dramatic reduction in cutoff dependence!

Lesson: SRG cutoff variation a sign of neglected induced forces

Induced 3N Forces

Effect of including 3N-ind? Exactly initial V_{NN} **up to neglected 4N-ind**



NN-only clear cutoff dependencs

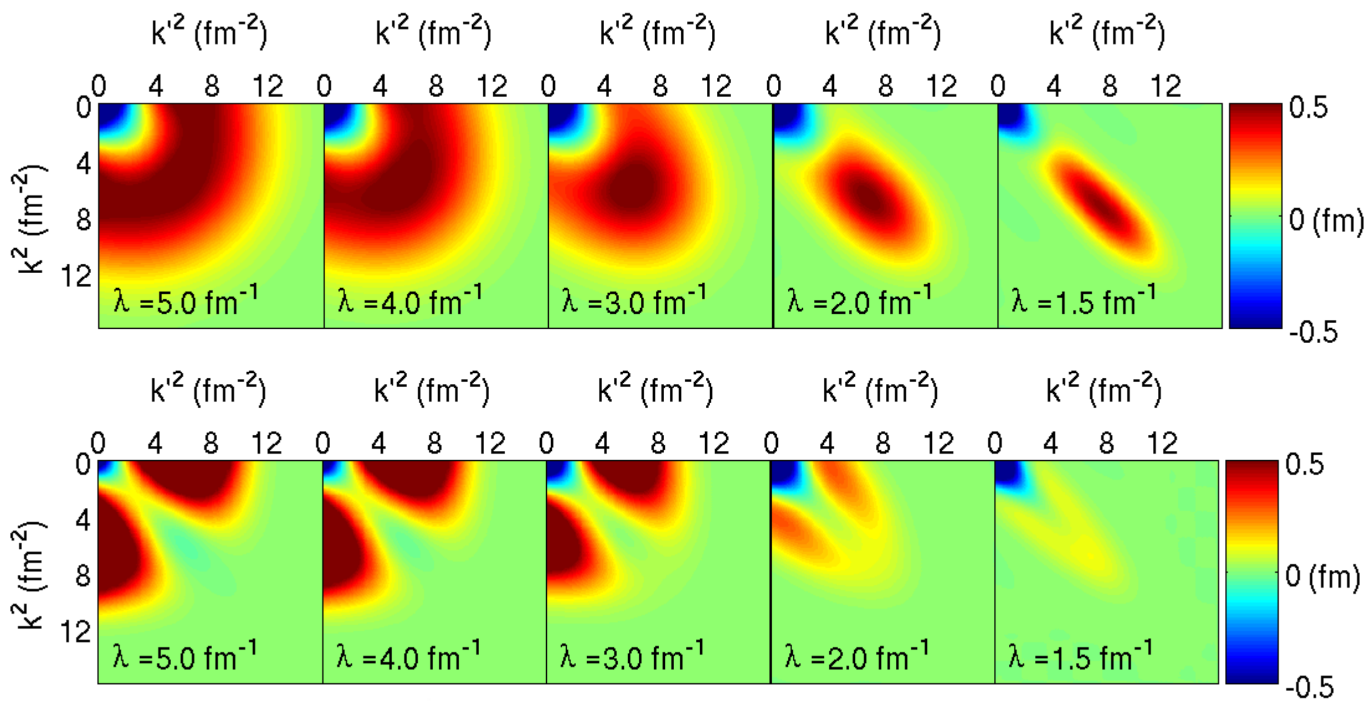
3N-induced – dramatic reduction in cutoff dependence!

Lesson: SRG cutoff variation a sign of neglected induced forces

Still far from experiment and remaining (minor) cutoff dependence!

Summary

Low-momentum interactions can be constructed from any V_{NN} via RG



Low-to-high momentum coupling not desirable in low-energy nuclear physics

Evolve to low-momentum while preserving low-energy physics

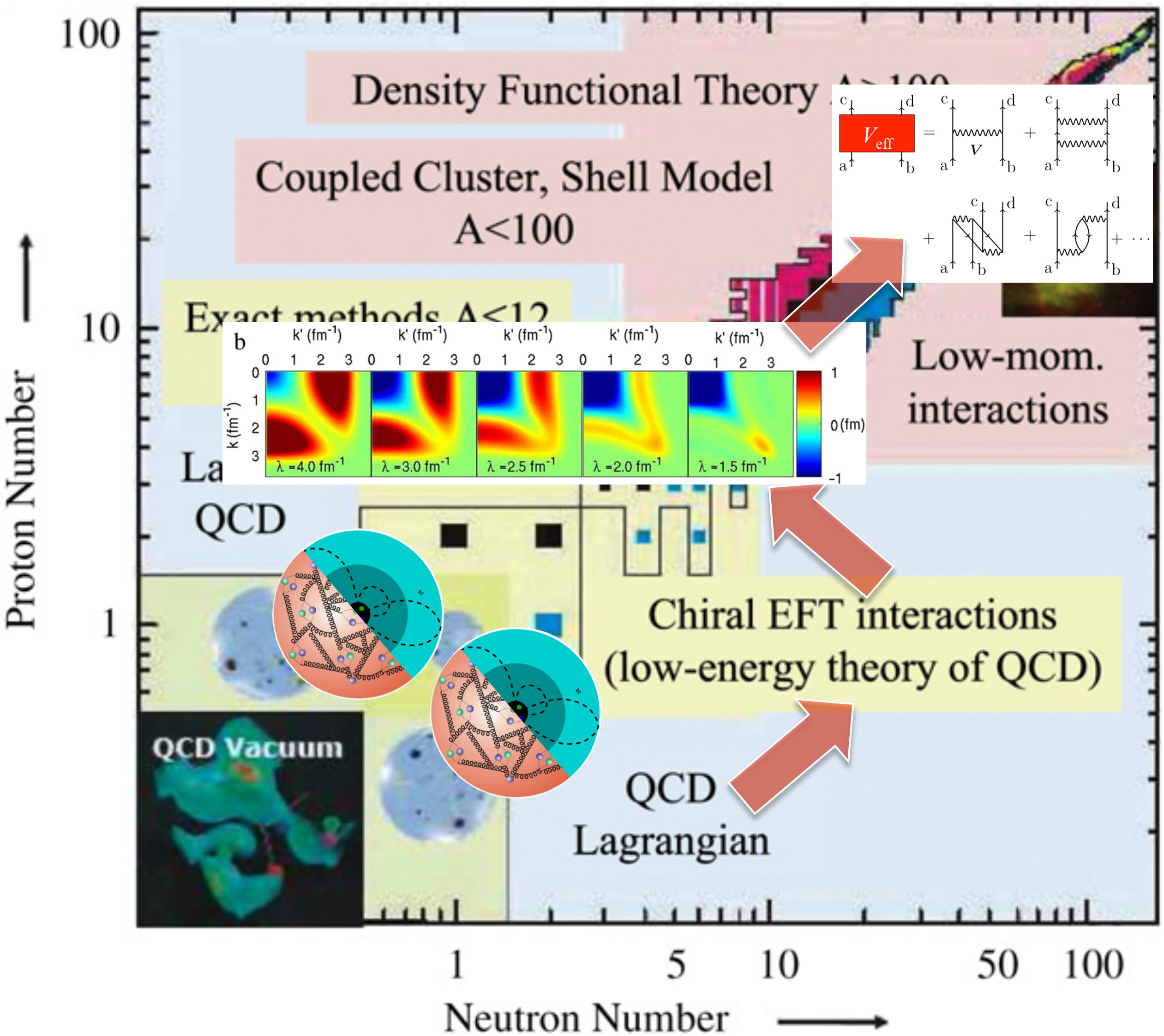
Universality attained near cutoff of data

Low-momentum cutoffs remove low-to-high harmonic oscillator couplings

Cutoff variation assesses missing physics interaction level: tool not a parameter

Part III: The Nuclear Many-Body Problem

To understand the properties of complex nuclei from first principles



Microscopic Valence-Space Interactions

Model spaces

Many-body perturbation theory (MBPT)

Calculating effective interaction

In-medium Similarity RG

Monopole part of interaction

Deficiencies of this approach

How will we approach this problem:

QCD → NN (3N) forces → Renormalize → "Solve" many-body problem → Predictions

The Nuclear Many-Body Problem

Nucleus strongly interacting many-body system – how to solve A-body problem?

$$H\psi_n = E_n\psi_n$$

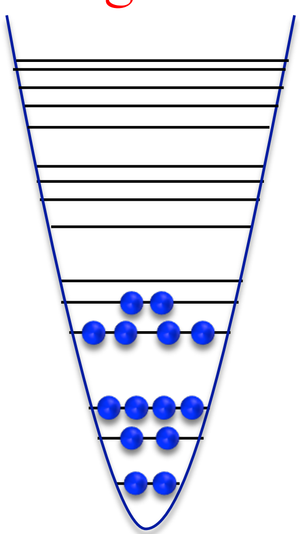
Quasi-exact solutions only in light nuclei (GFMC, NCSM…)

Large scale: controlled approximations to full Schrödinger Equation

Valence space: diagonalize exactly with reduced number of degrees of freedom

Medium-mass

Large scale



Limited range:

Closed shell ± 1

Even-even

Limited properties:

Ground states only

Some excited state

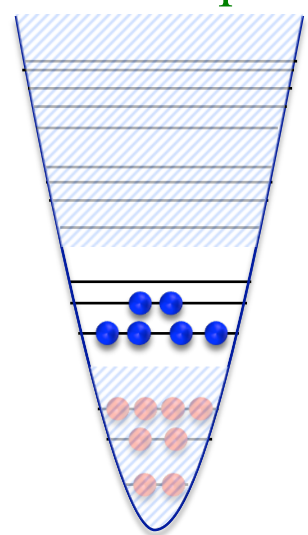
Coupled Cluster

In-Medium SRG

Green's Function

Medium-mass

Valence space



All nuclei near

closed-shell cores

All properties:

Ground states

Excited states

EW transitions

Coupled Cluster

In-Medium SRG

Perturbation Theory

From Momentum Space to HO Basis

To this point interaction matrix elements in momentum space, **partial waves**

$$\langle kK, lL|V|k'K, l'L\rangle_\alpha$$

To go to finite nuclei begin from Hamiltonian

$$H\psi_n = (T+V)\psi_n = E_n\psi_n$$

Assume many particles in the nucleus generate a **mean field** U:

U a one-body potential simple to solve (typically **Harmonic Oscillator**)

$$H = H_0 + H_1; \quad H_0 = T + U; \quad H_1 = V - U$$

So transform from momentum space to **Harmonic Oscillator Basis**

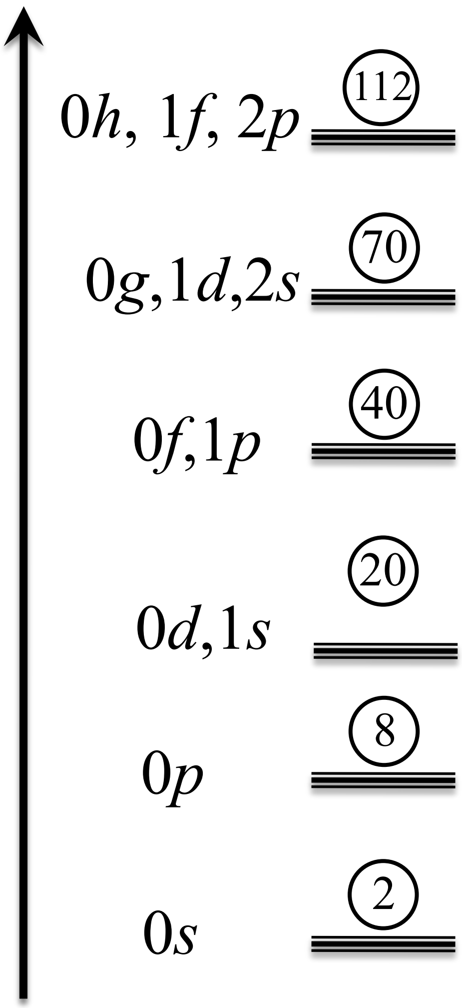
$$|nl, NL; \alpha\rangle = \int k^2 \mathrm{d}k\, K^2 \mathrm{d}K\, R_{nl}\left(\sqrt{2}\alpha k\right) R_{NL}\left(\sqrt{1/2}\alpha K\right) |kl, KL; \alpha\rangle$$

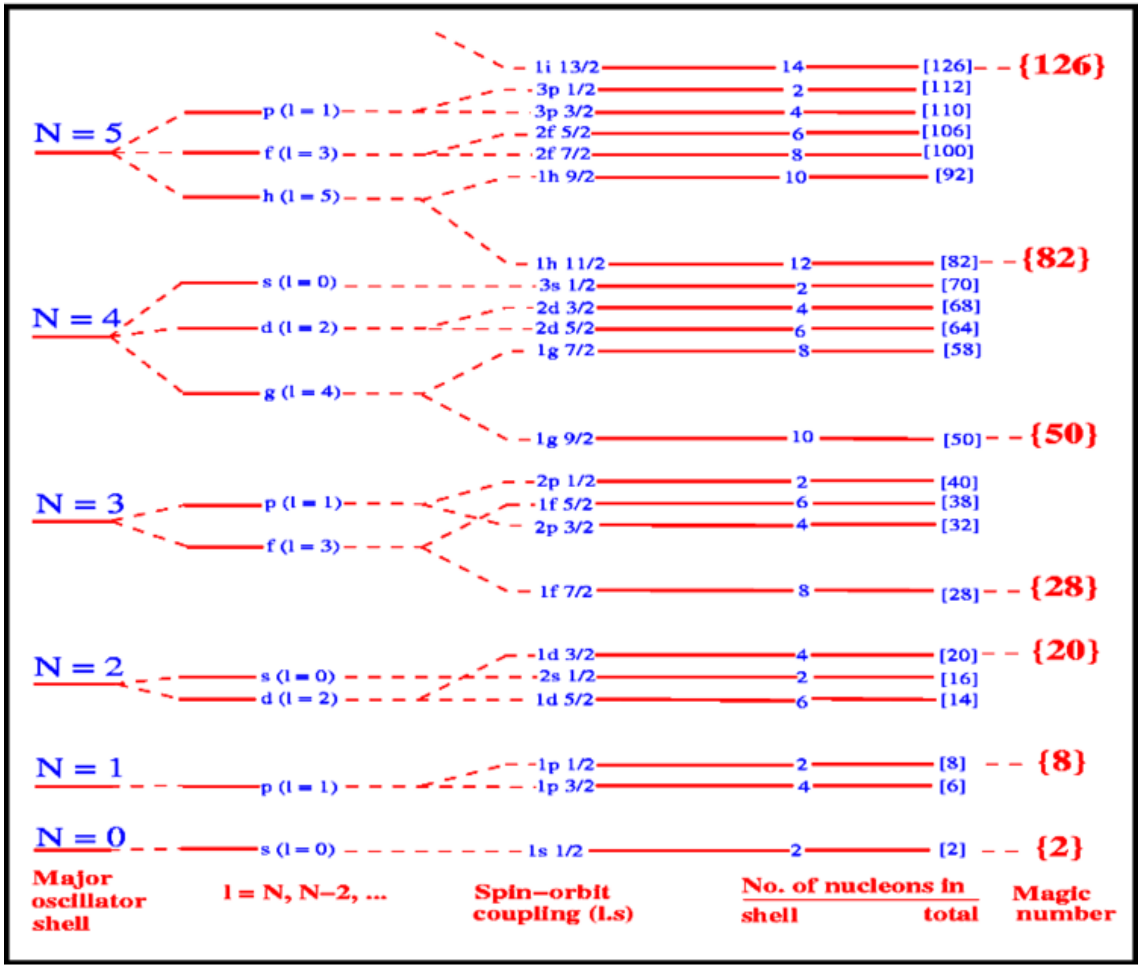
One more (ugly) transformation from center-of-mass to lab frame:

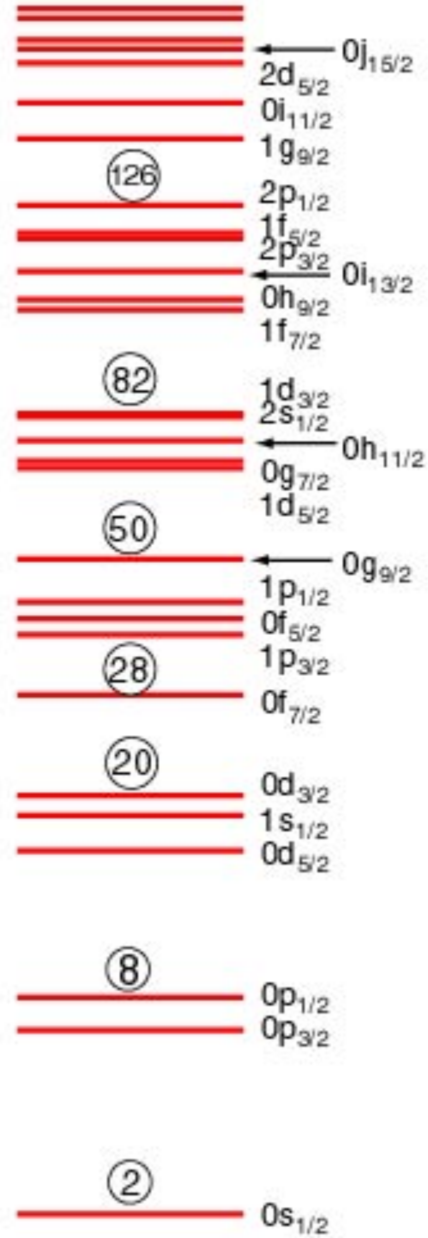
$$\rightarrow \langle ab; JT|V|cd; JT\rangle$$

Valence-Space Ideas

Begin with degenerate HO levels







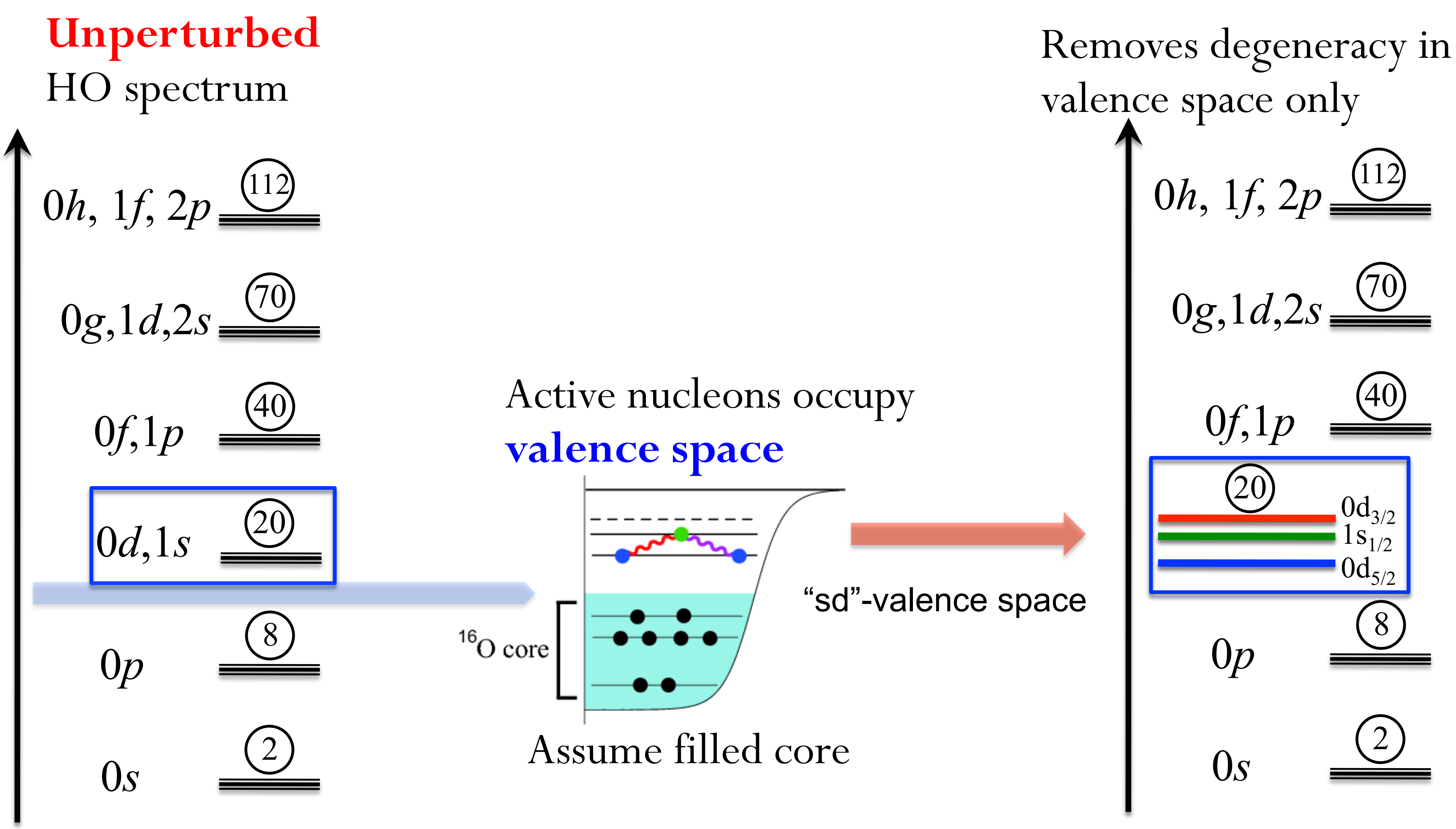
$$\langle ab;\, JT|V|cd;\, JT\rangle$$

Physics of V breaks HO degeneracy

Problem: Can't solve Schrodinger equation in full Hilbert space

Valence-Space Ideas

Nuclei understood as many-body system starting from closed shell, add nucleons



Valence-Space Ideas

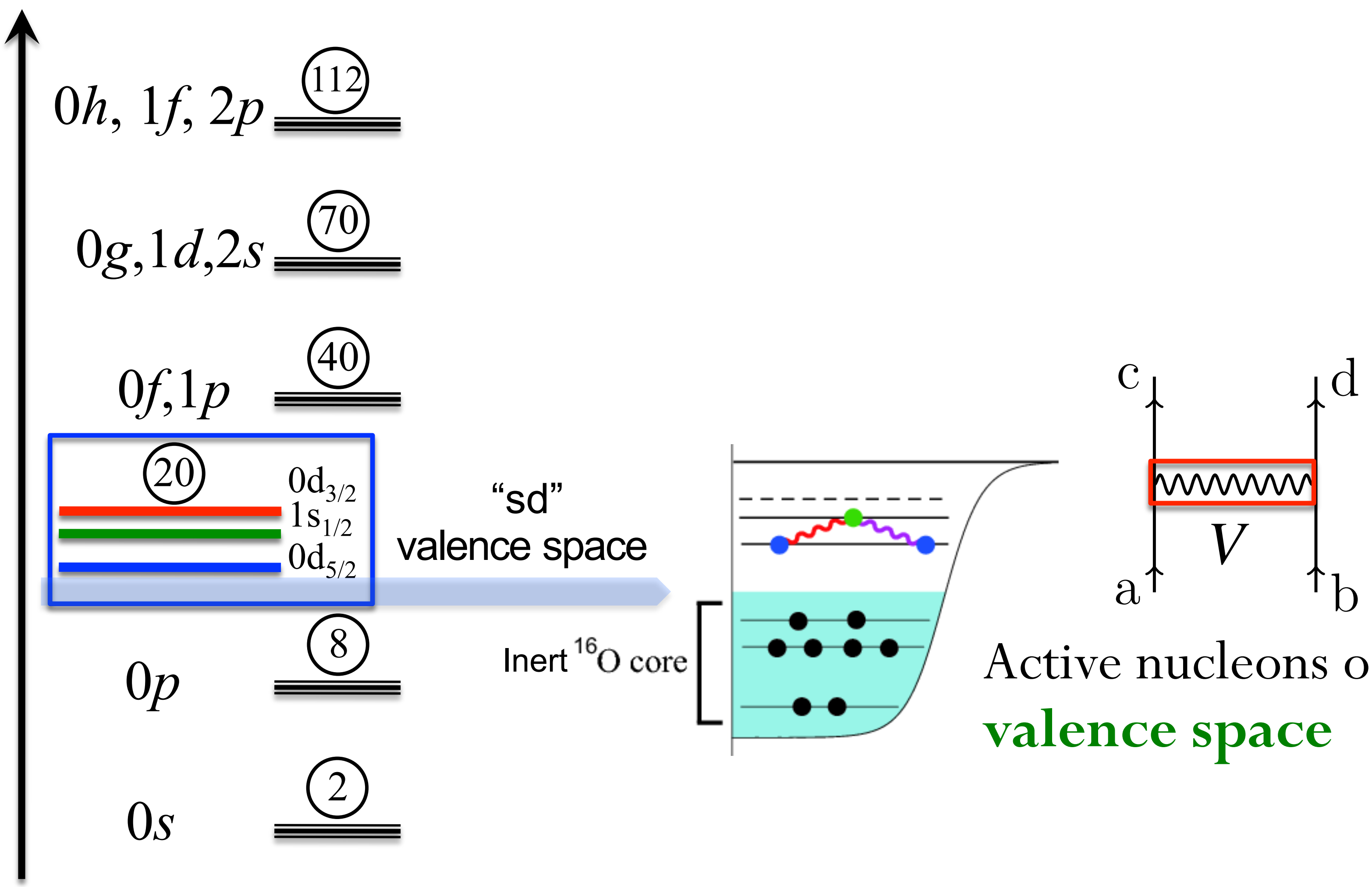
Nuclei understood as many-body system starting from closed shell, add nucleons

Valence-space Hamiltonian derived from nuclear forces:

Single-particle energies

Interaction matrix elements

$$H_{\text{v.s.}} = \sum_i \varepsilon_i a_i^\dagger a_i + V_{\text{v.s.}}$$



Active nucleons occupy **valence space**

Valence-Space Philosophy

Nuclei understood as many-body system starting from closed shell, add nucleons

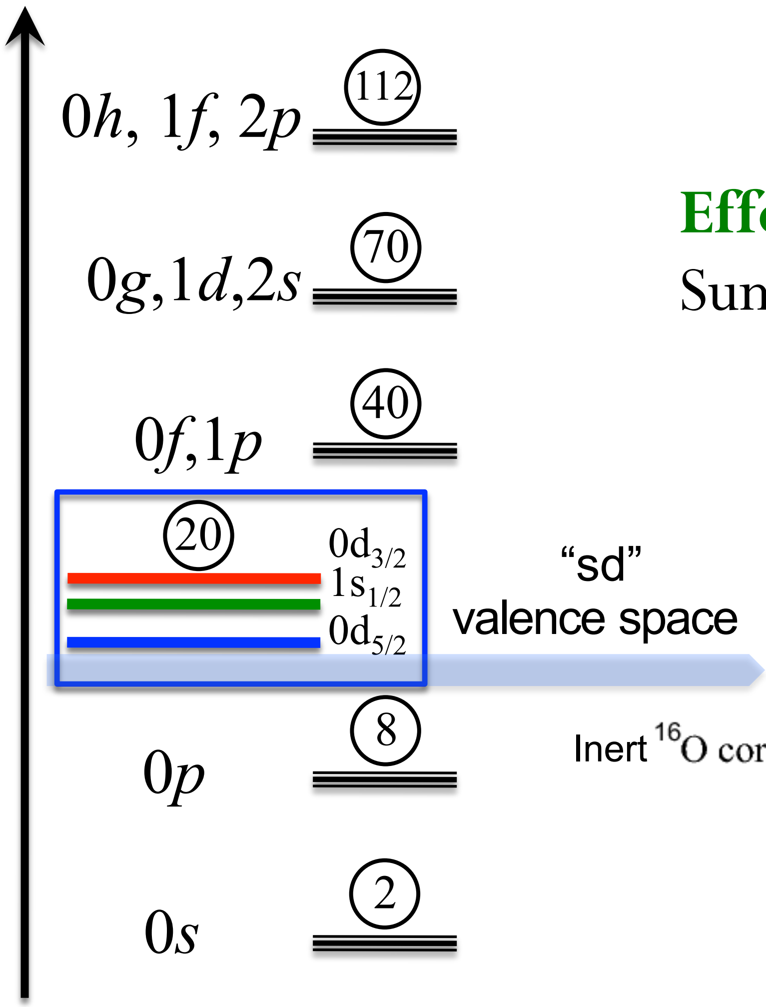
Valence-space Hamiltonian derived from nuclear forces:

Single-particle energies

Interaction matrix elements

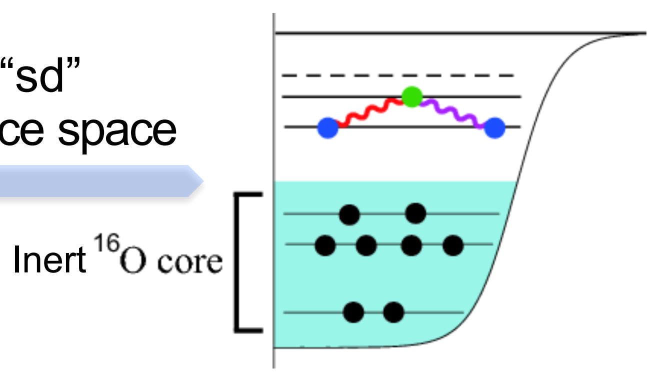
$$H_{\text{eff}} = \sum_i \varepsilon_{i_{\text{eff}}} a_i^\dagger a_i + V_{\text{eff}}$$

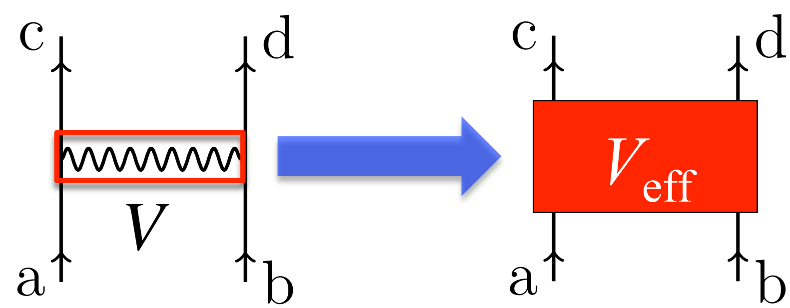
$$H\psi_n = E_n\psi_n \rightarrow PH_{\text{eff}}P\psi_i = E_iP\psi_i$$



Effective valence space Hamiltonian:

Sum all excitations outside valence space



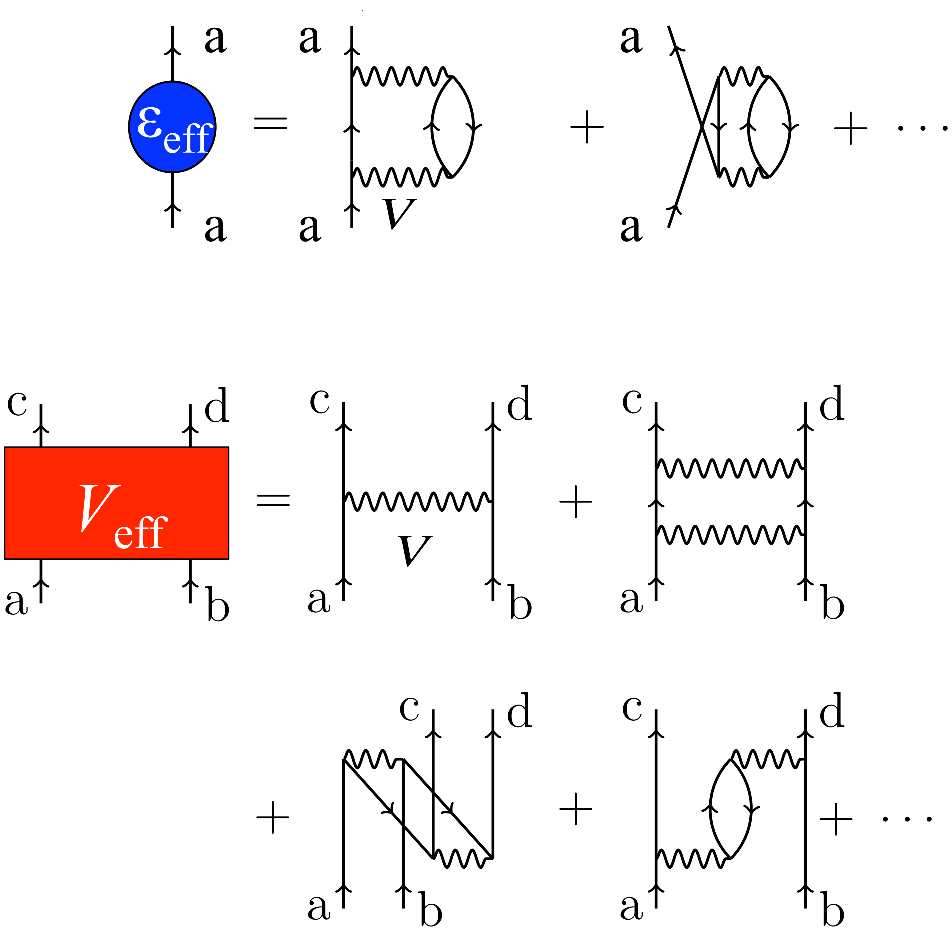


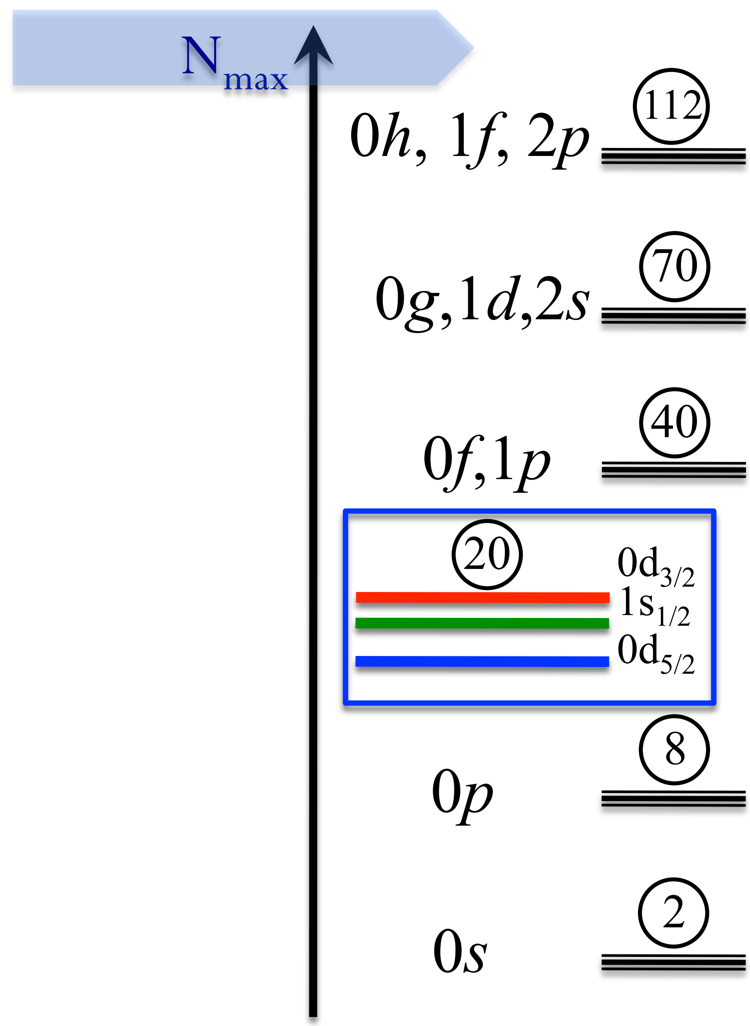
Decouple valence space from excitations

Perturbative Approach

1) Effective Hamiltonian: sum excitations outside valence space

2) Self-consistent single-particle energies

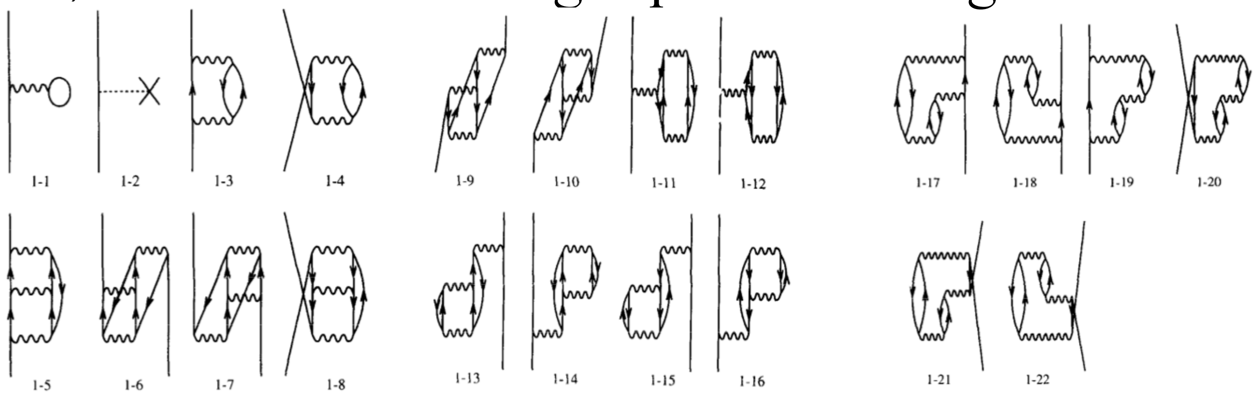


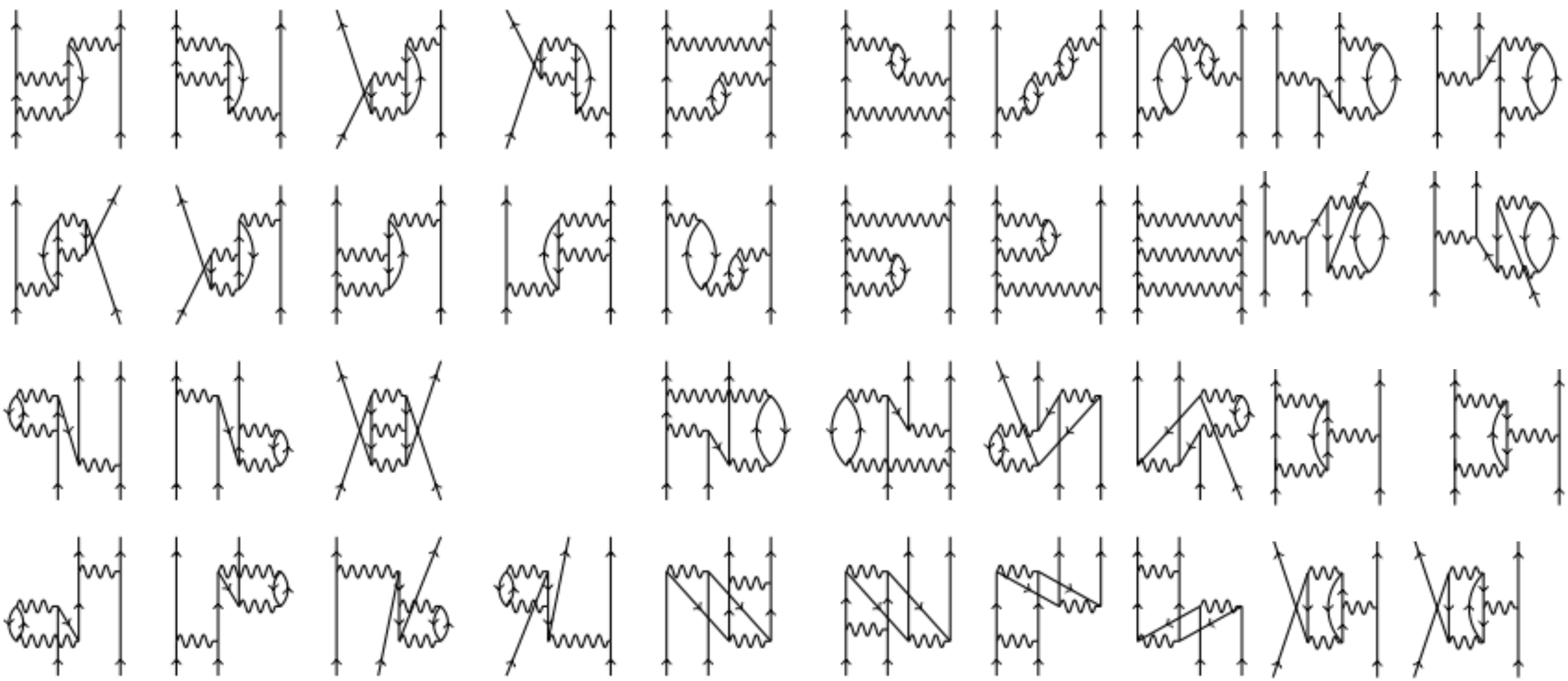


Perturbative Approach

1) Effective Hamiltonian: sum excitations outside valence space to **MBPT(3)**

2) Self-consistent single-particle energies



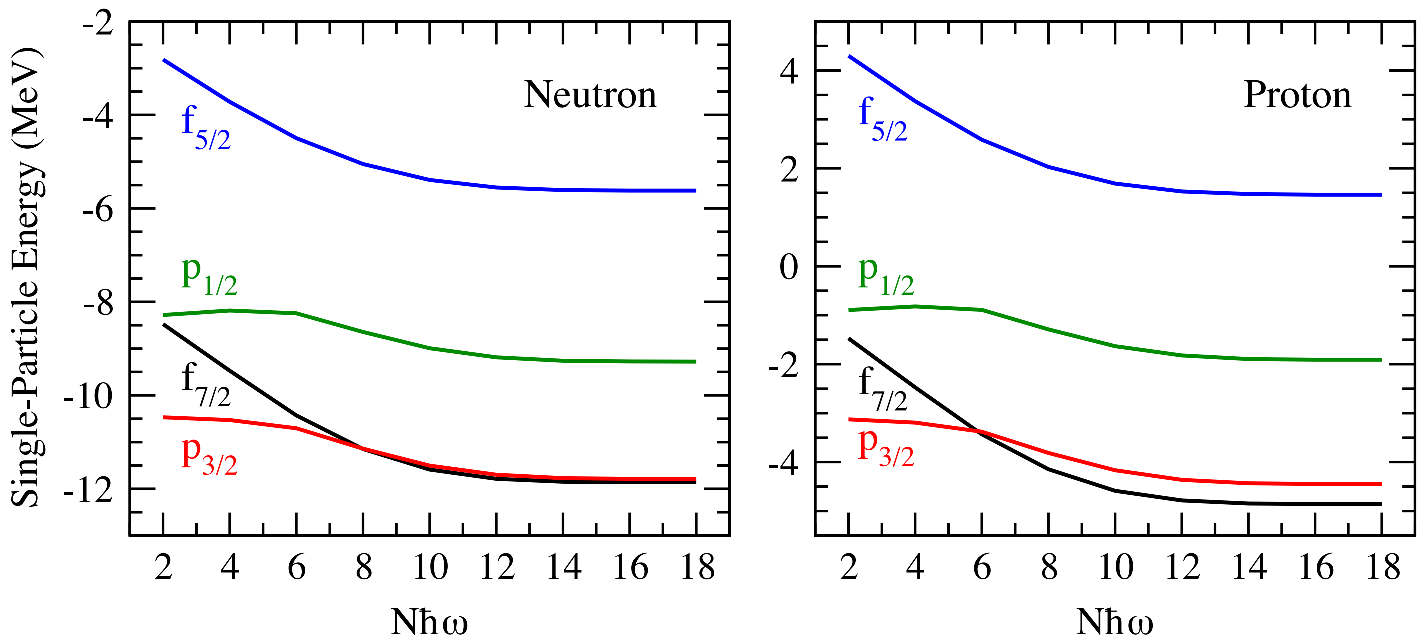


Perturbative Approach

1) Effective Hamiltonian: sum excitations outside valence space to MBPT(3)

2) Self-consistent single-particle energies

3) **Harmonic-oscillator basis** of 13-15 major shells: **converged!**



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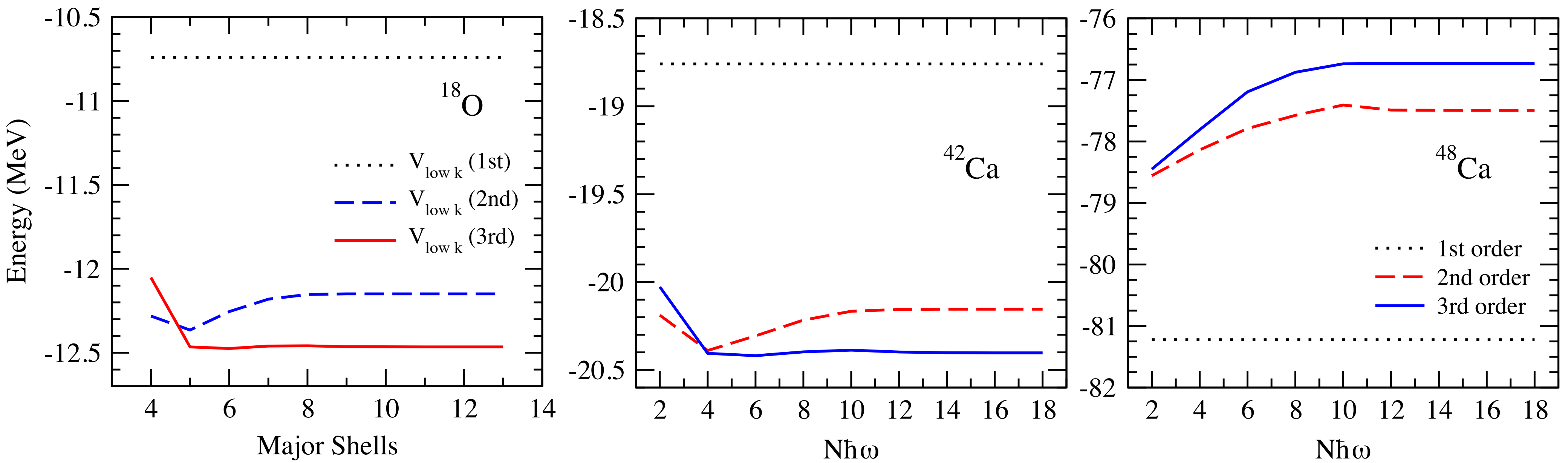
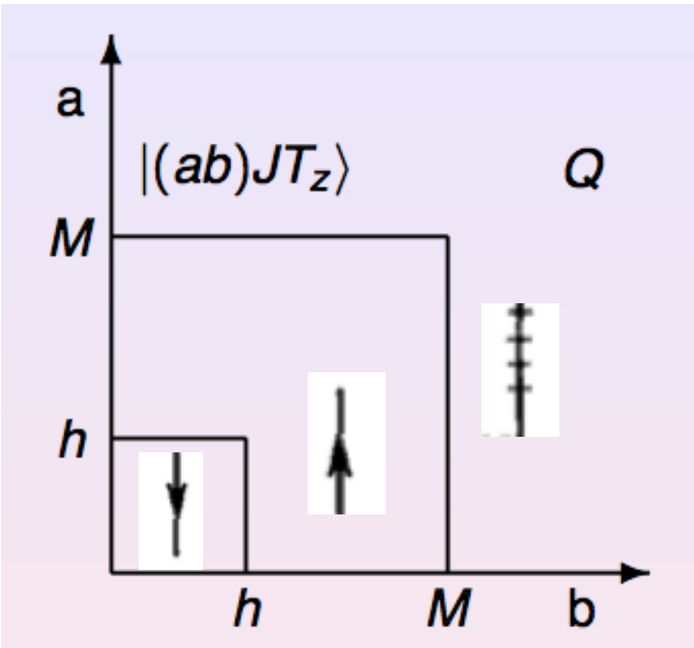
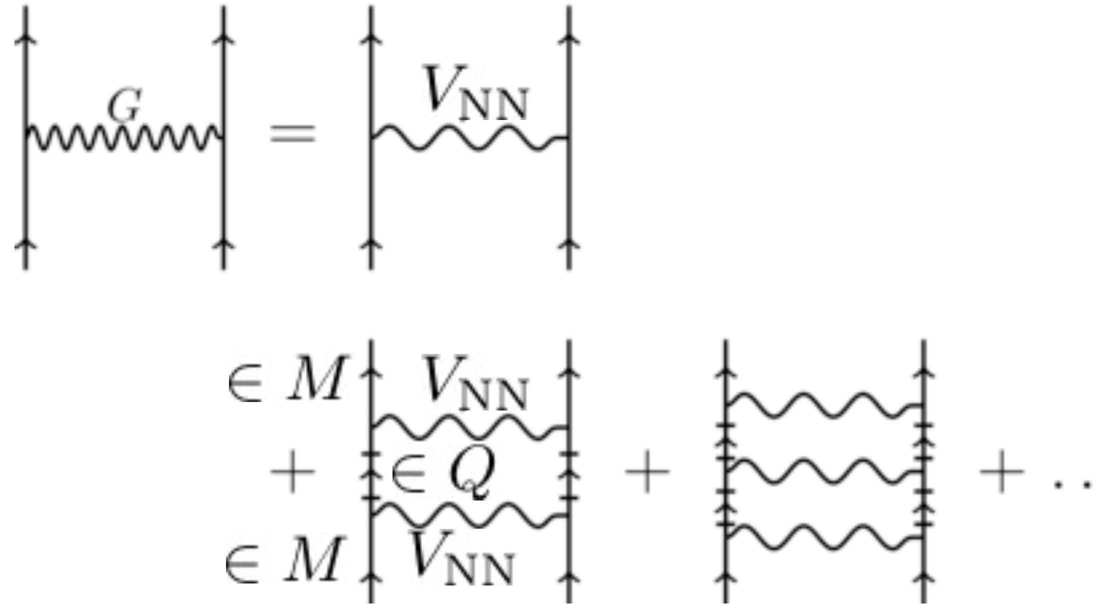


Aside: G-matrix Renormalization

Standard method for softening interaction in nuclear structure for decades:





Infinite summation of ladder diagrams

Need two model spaces:

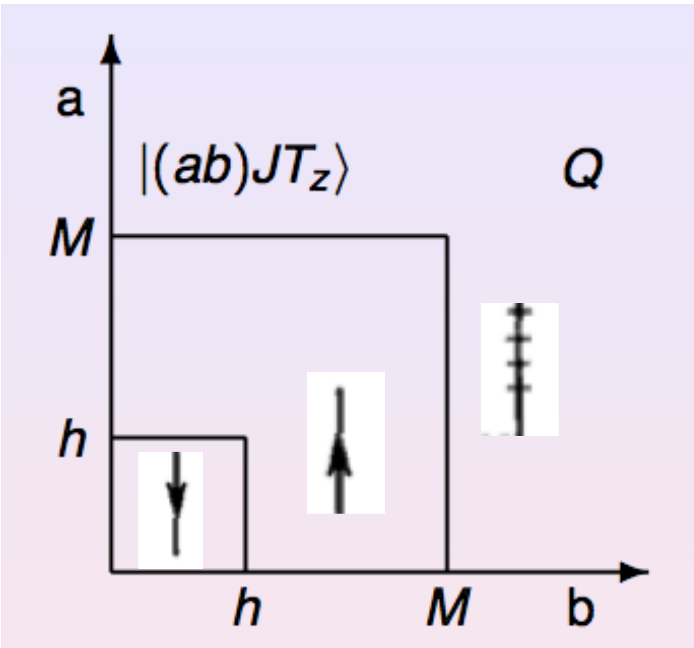
1) **M** space in which we will want to calculate (excitations allowed in M)

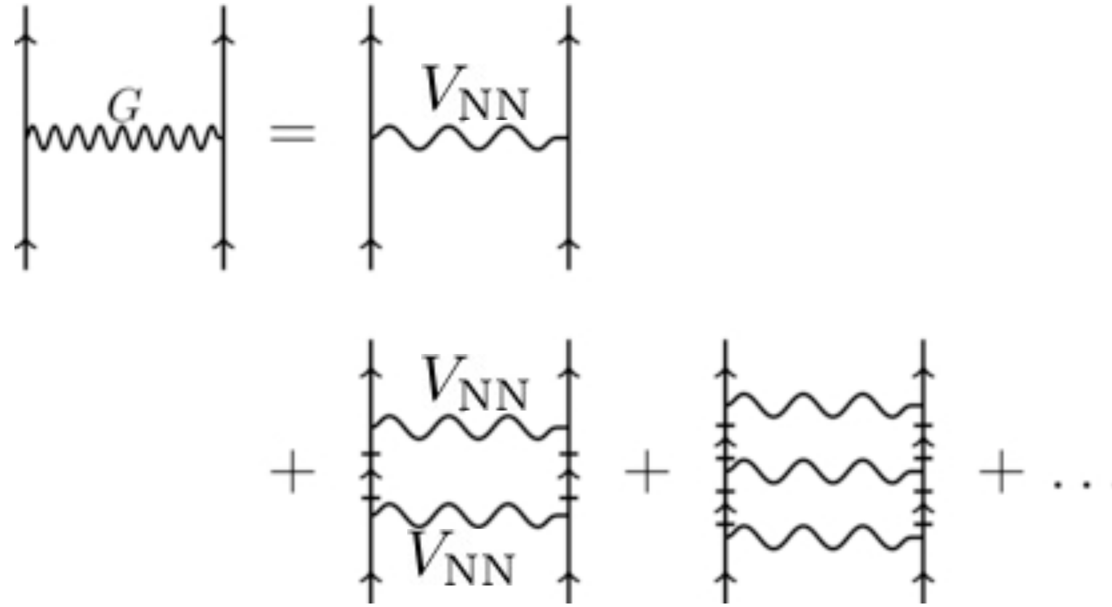
2) Large space **Q** in which particle excitations are allowed

To avoid double counting, can't overlap – **matrix elements depend on M**

Aside: G-matrix Renormalization

Standard method for softening interaction in nuclear structure for decades:





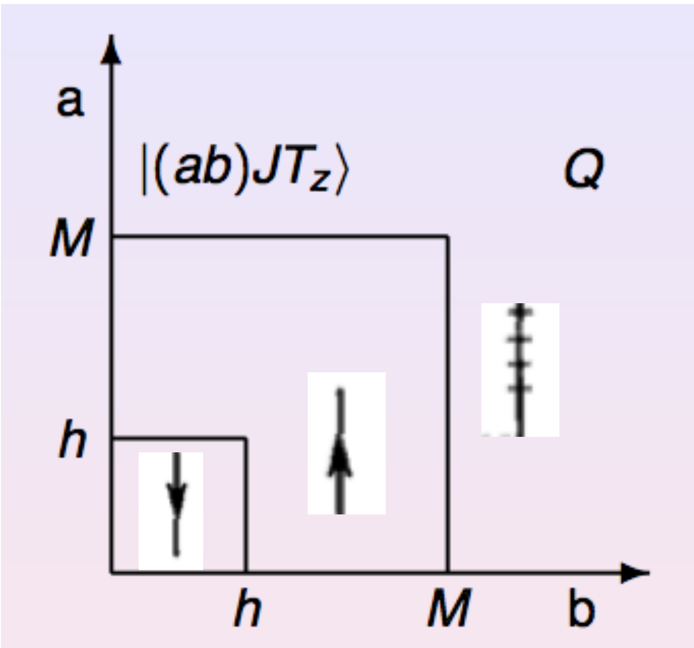
$$G_{ijkl}(\omega) = V_{ijkl} + \sum_{mn \in Q} V_{ijmn} \frac{Q}{\omega - \varepsilon_m - \varepsilon_n} G_{mnkl}(\omega)$$

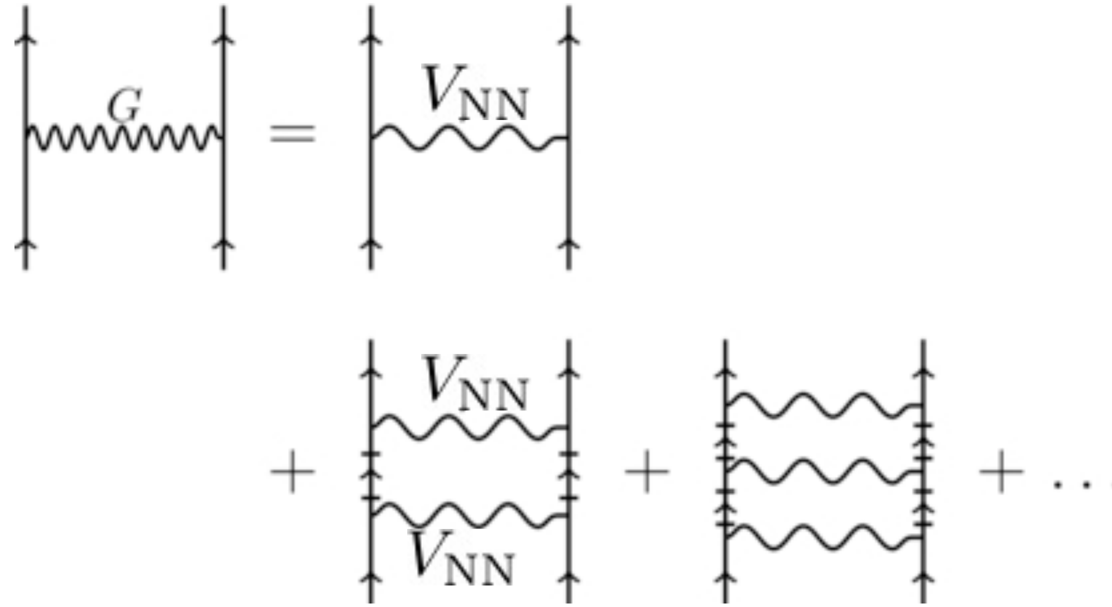
Iterative procedure

Dependence on arbitrary starting energy!

G-matrix Renormalization

Standard method for softening interaction in nuclear structure for decades:





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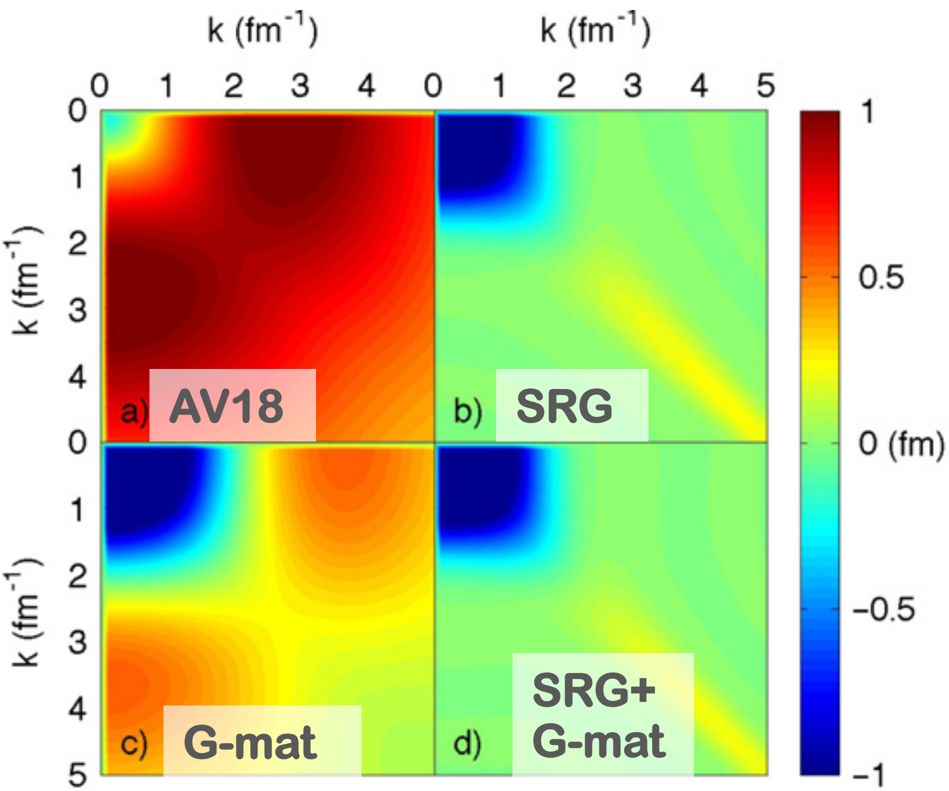
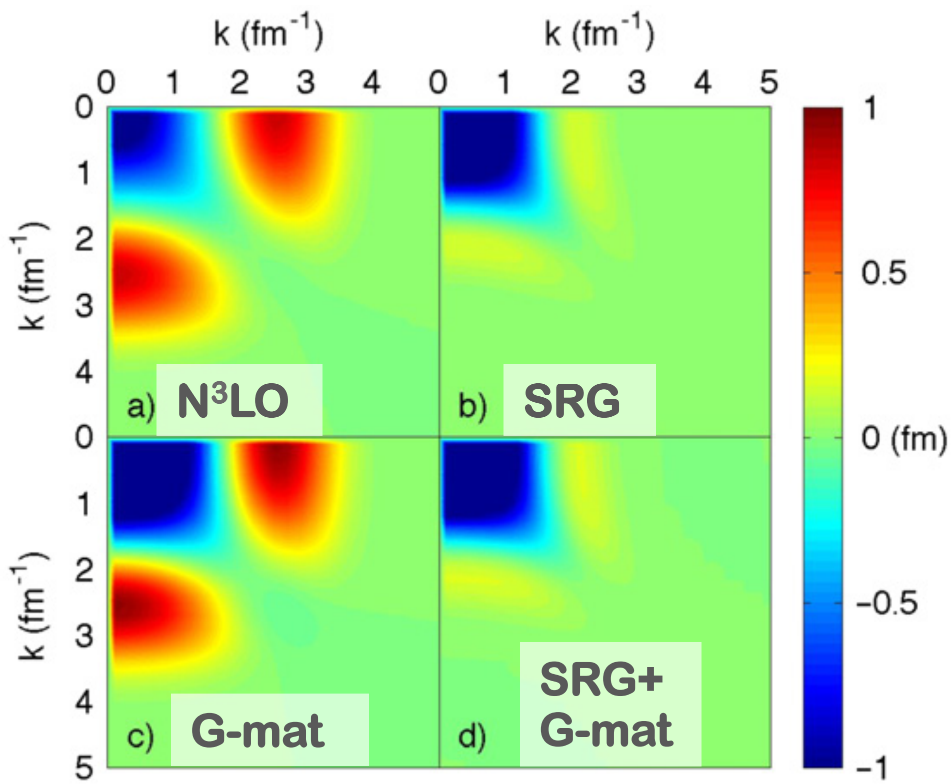
G-matrix Renormalization

Results of **G-matrix** renormalization vs. SRG





Removes some diagonal high-momentum components

Still large low-to-high coupling in both interactions

No indication of universality

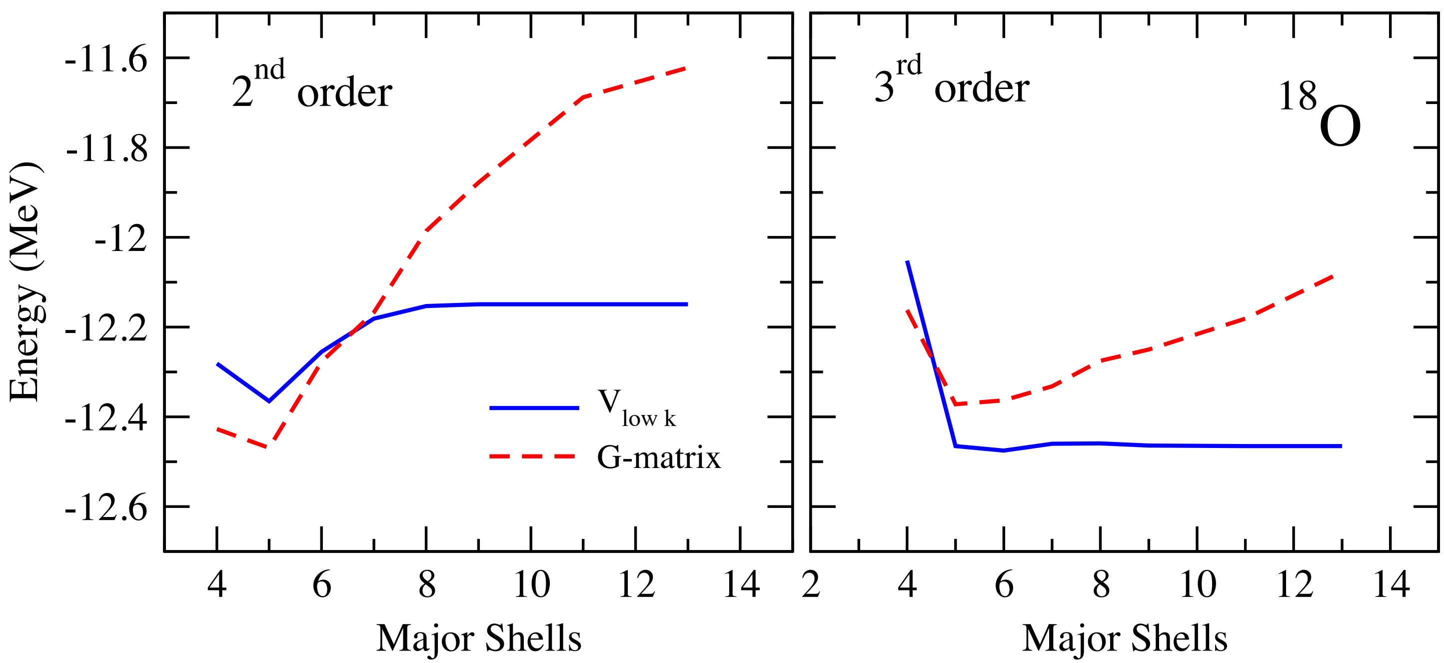
Clear difference compared with SRG-evolved interactions!

Perturbative Approach

1) Effective Hamiltonian: sum excitations outside valence space to MBPT(3)

2) Self-consistent single-particle energies

3) **Harmonic-oscillator basis** of 13-15 major shells: **converged!**



Compare vs G-matrix (no sign of convergence)

Clear benefit of low-momentum interactions!

Perturbative Approach

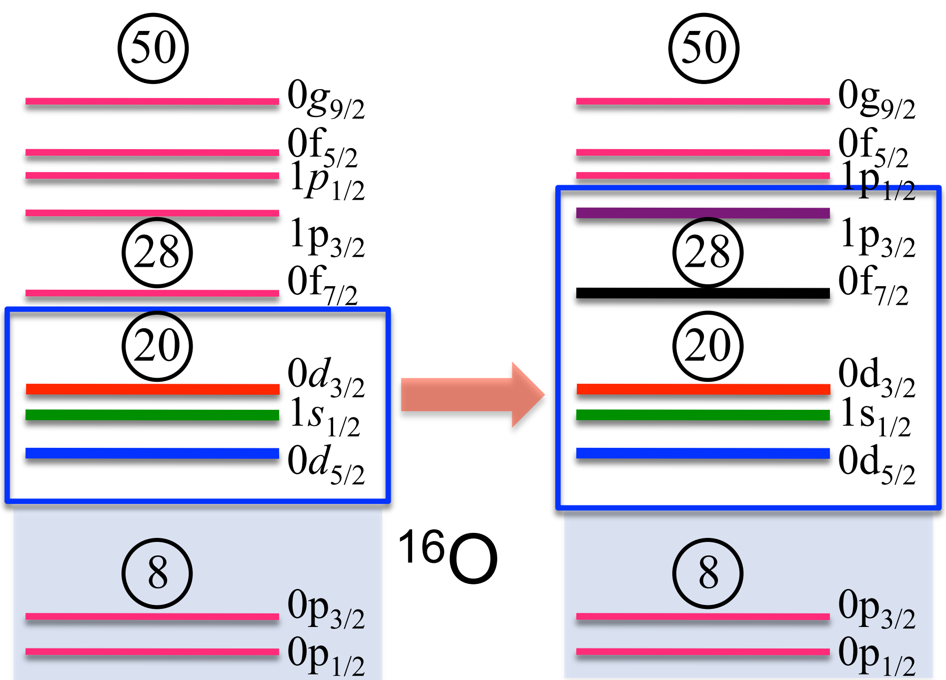
1) Effective Hamiltonian: sum excitations outside valence space to MBPT(3)

2) Self-consistent single-particle energies

3) Harmonic-oscillator basis of 13-15 major shells

4) Nuclear forces from chiral EFT

5) Requires **extended valence spaces**



Treat higher orbits nonperturbatively

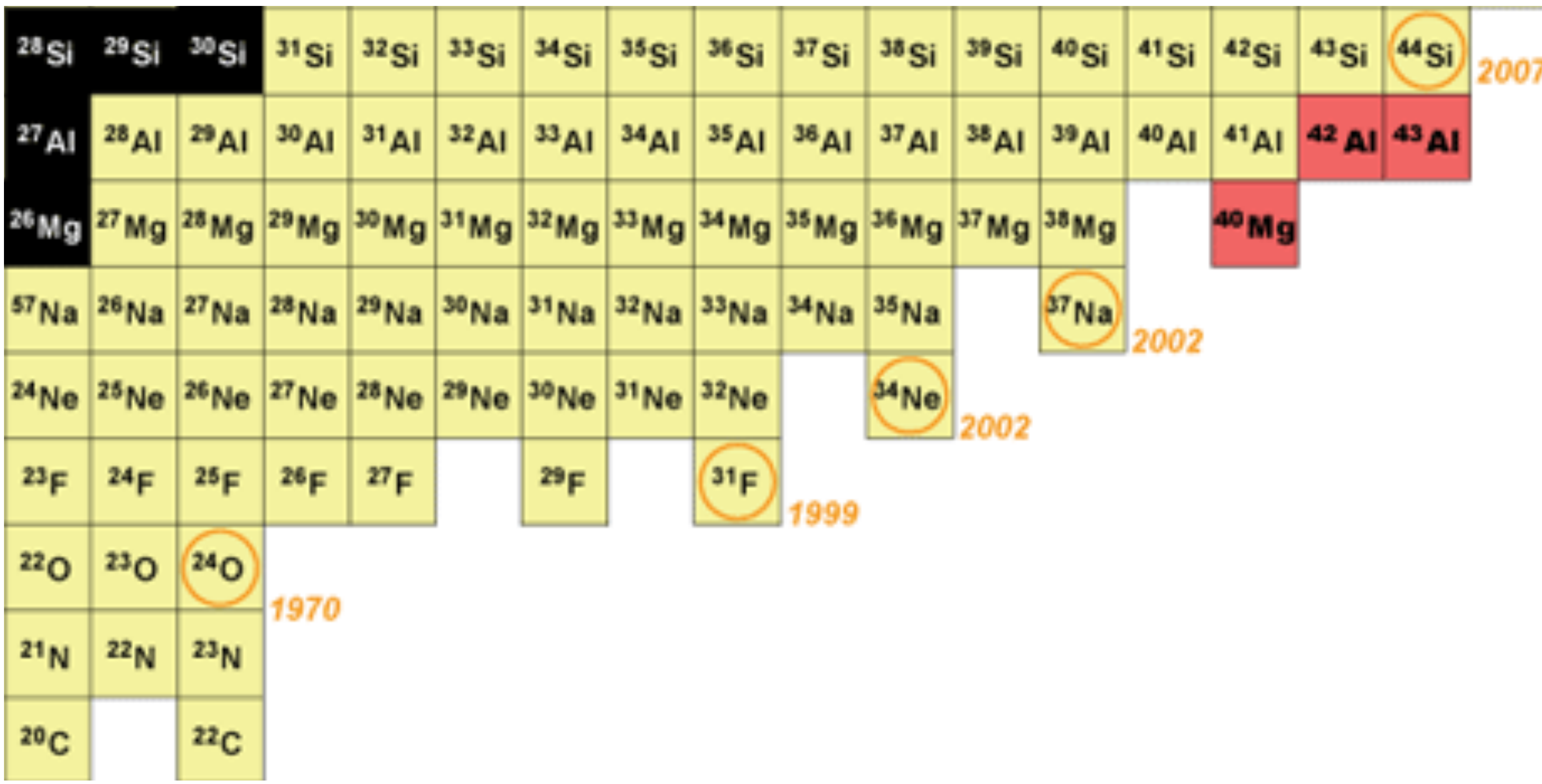
Limits of Nuclear Existence: Oxygen Anomaly

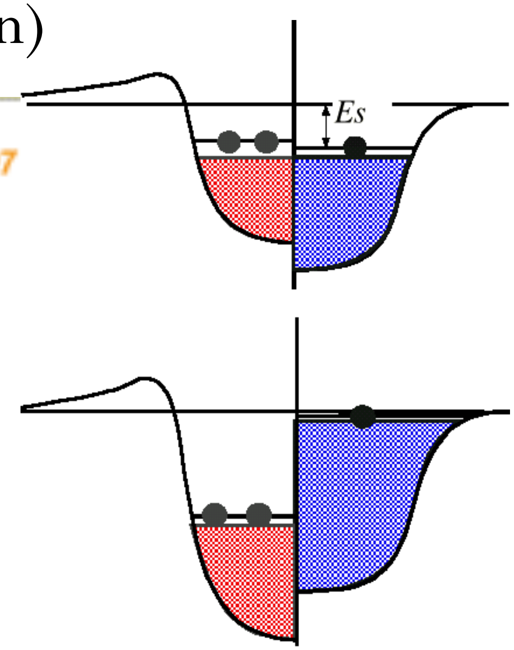
Where is the nuclear dripline?

Limits defined as last isotope with positive neutron separation energy

- Nucleons "drip" out of nucleus

Neutron dripline experimentally established to Z=8 (Oxygen)





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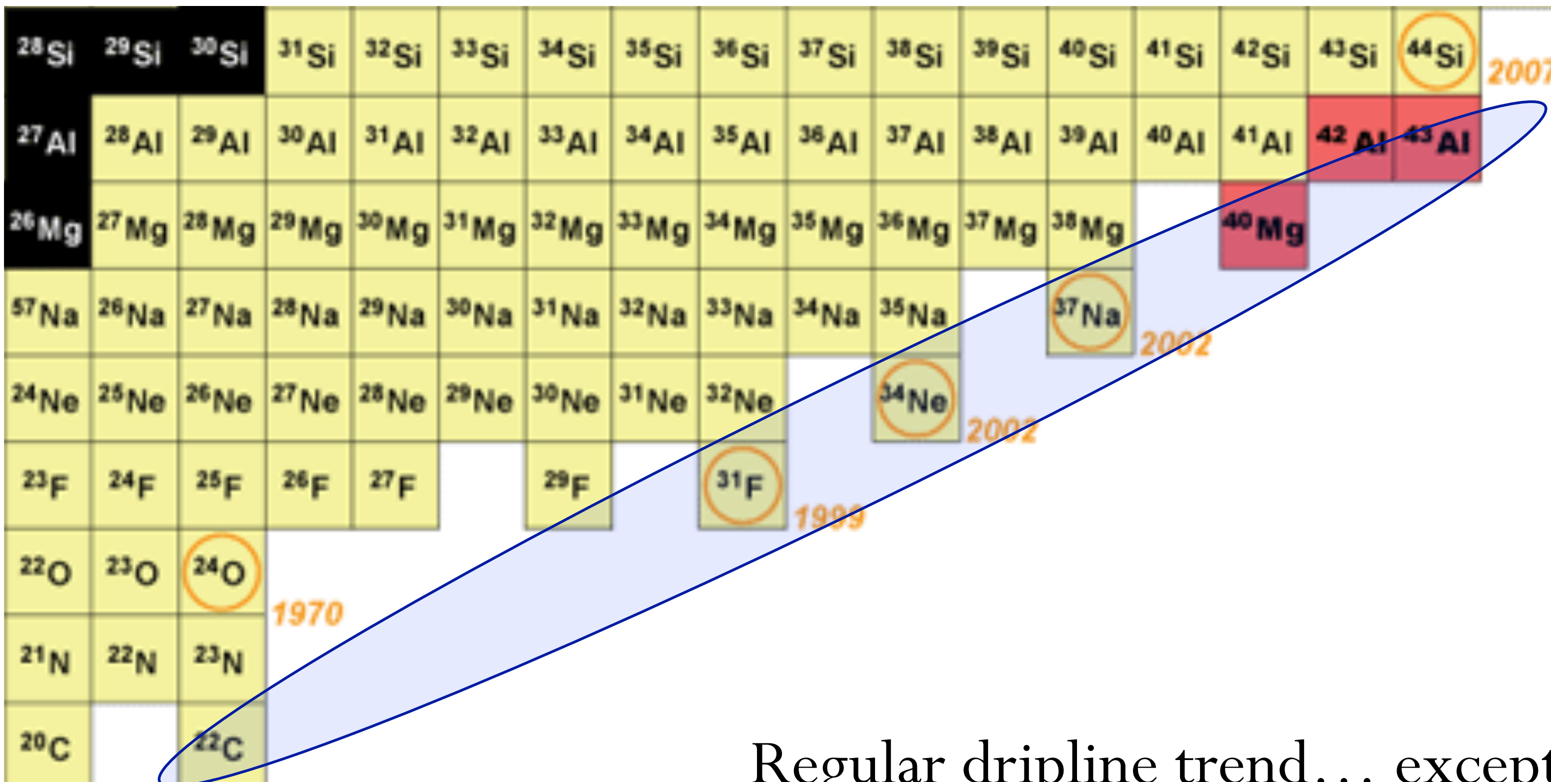

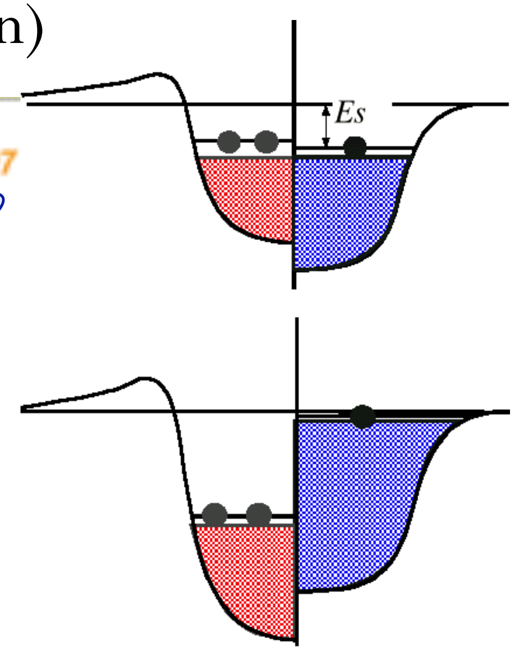

Regular dripline trend… except oxygen

Adding one proton binds 6 additional neutrons

Limits of Nuclear Existence: Oxygen Anomaly

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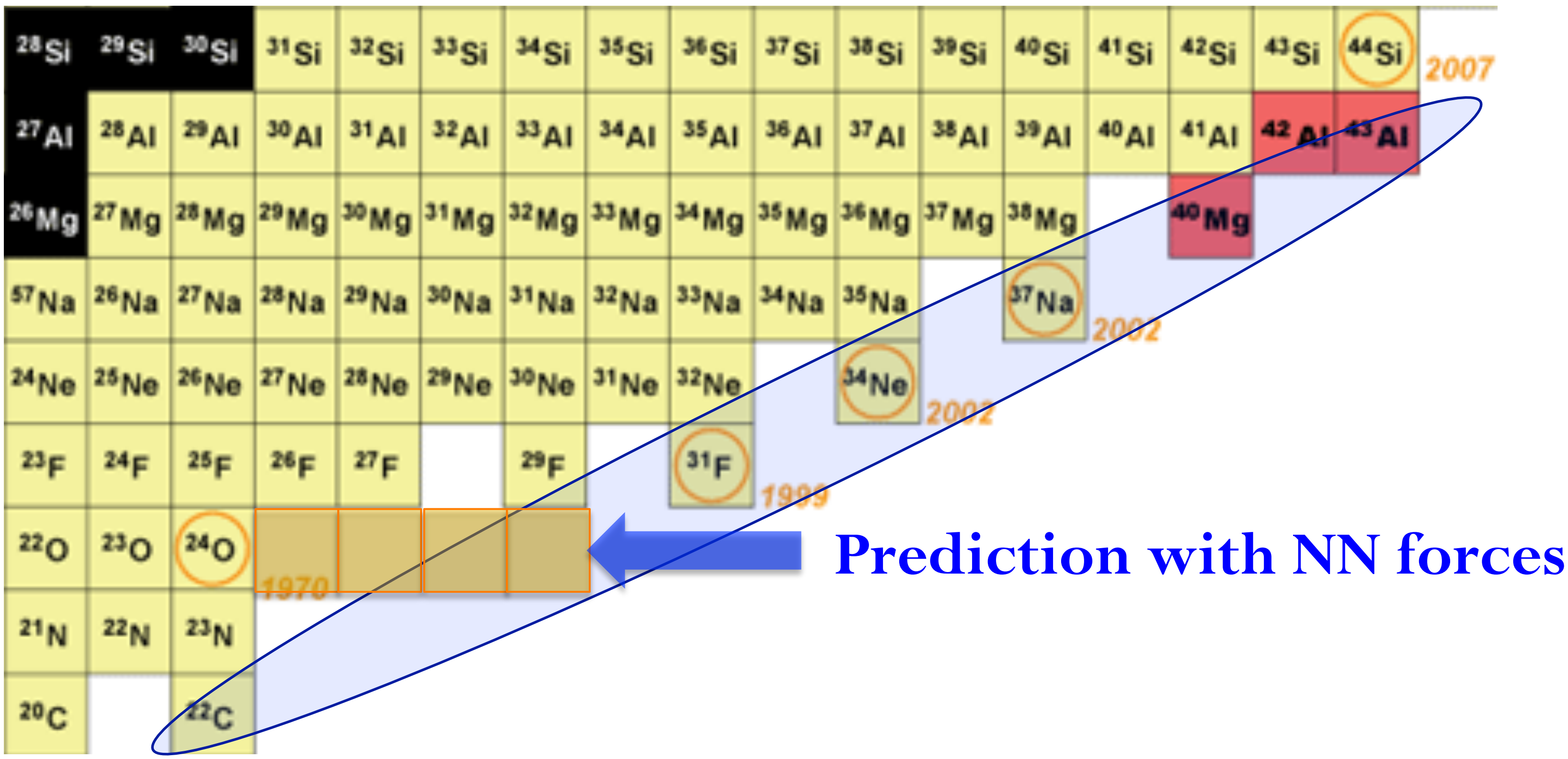


Microscopic picture: **NN-forces too attractive**

Incorrect prediction of dripline

Monopole Part of Valence-Space Interactions

Microscopic MBPT – effective interaction in chosen model space

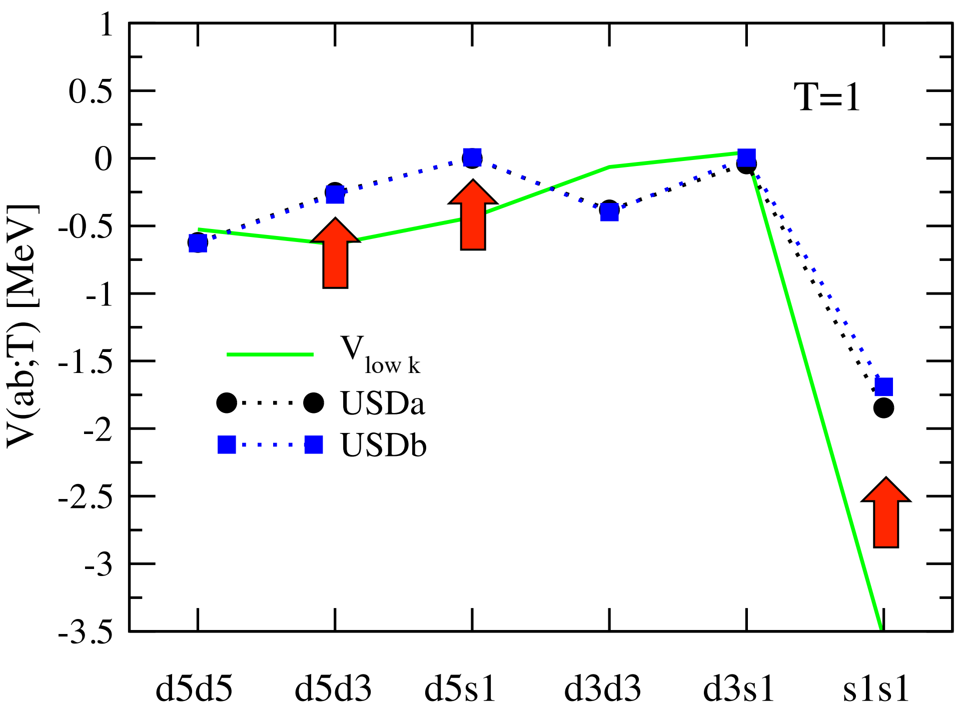
Works near closed shells: deteriorates beyond this

Deficiencies improved adjusting particular two-body matrix elements

Monopoles:
Angular average of interaction

$$V_{ab}^{T} = \frac{\sum_J (2J+1) V_{abab}^{JT}}{\sum_J (2J+1)}$$

Determines interaction of orbit *a* with *b*: *evolution of orbital energies*



$$\Delta\varepsilon_a = V_{ab} n_b$$

Microscopic **low-momentum** interactions

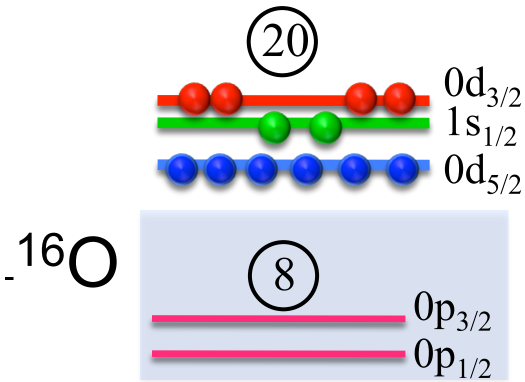
Phenomenological **USD** interactions

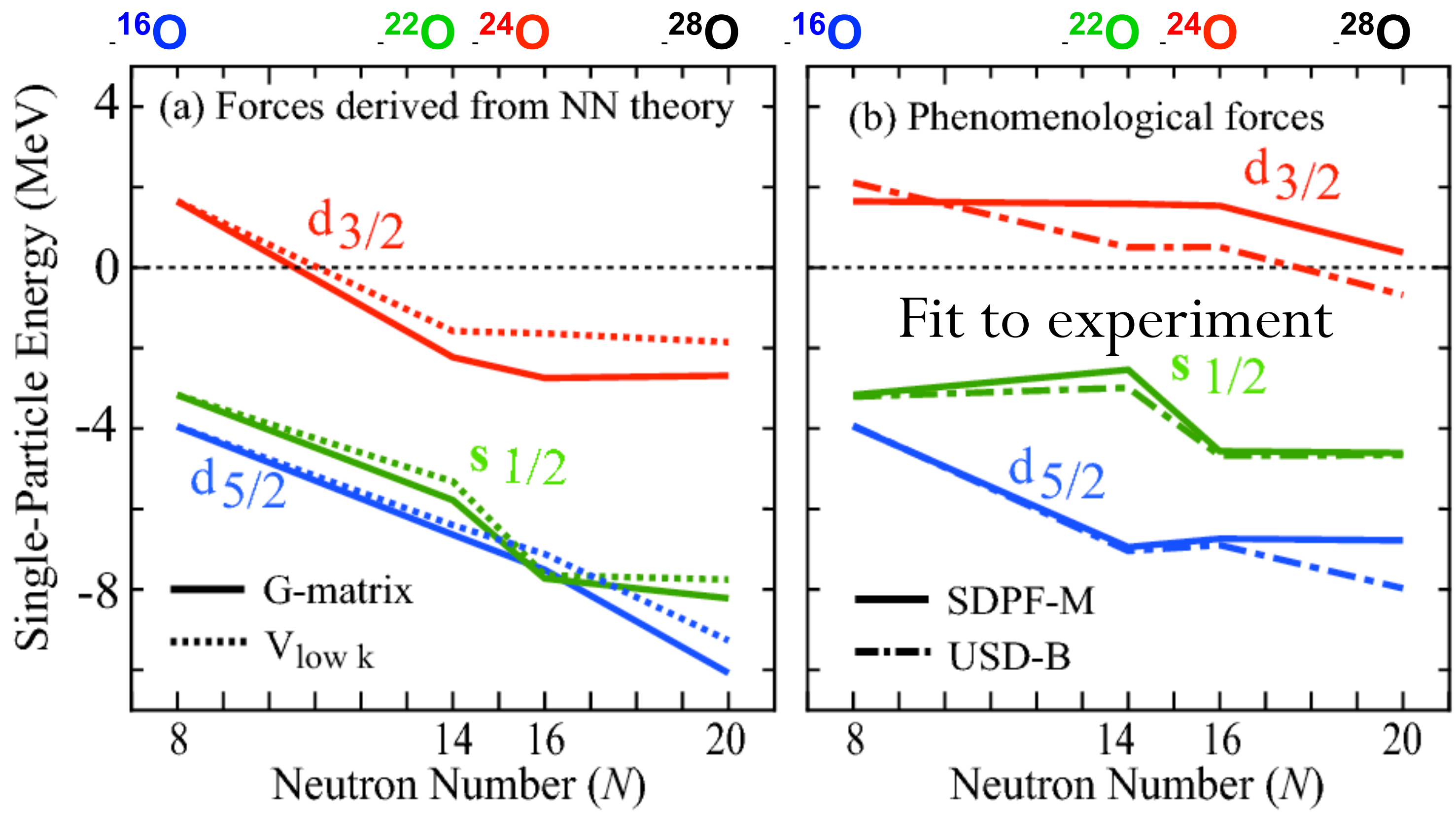
Clear shifts in **low-lying orbitals**:

- T=1 repulsive shift

Physics in Oxygen Isotopes

Calculate evolution of *sd*-orbital energies from interactions





Microscopic NN Theories

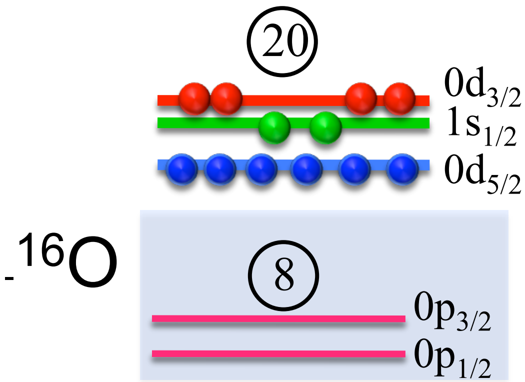
$d_{3/2}$ orbit bound to ^{28}O

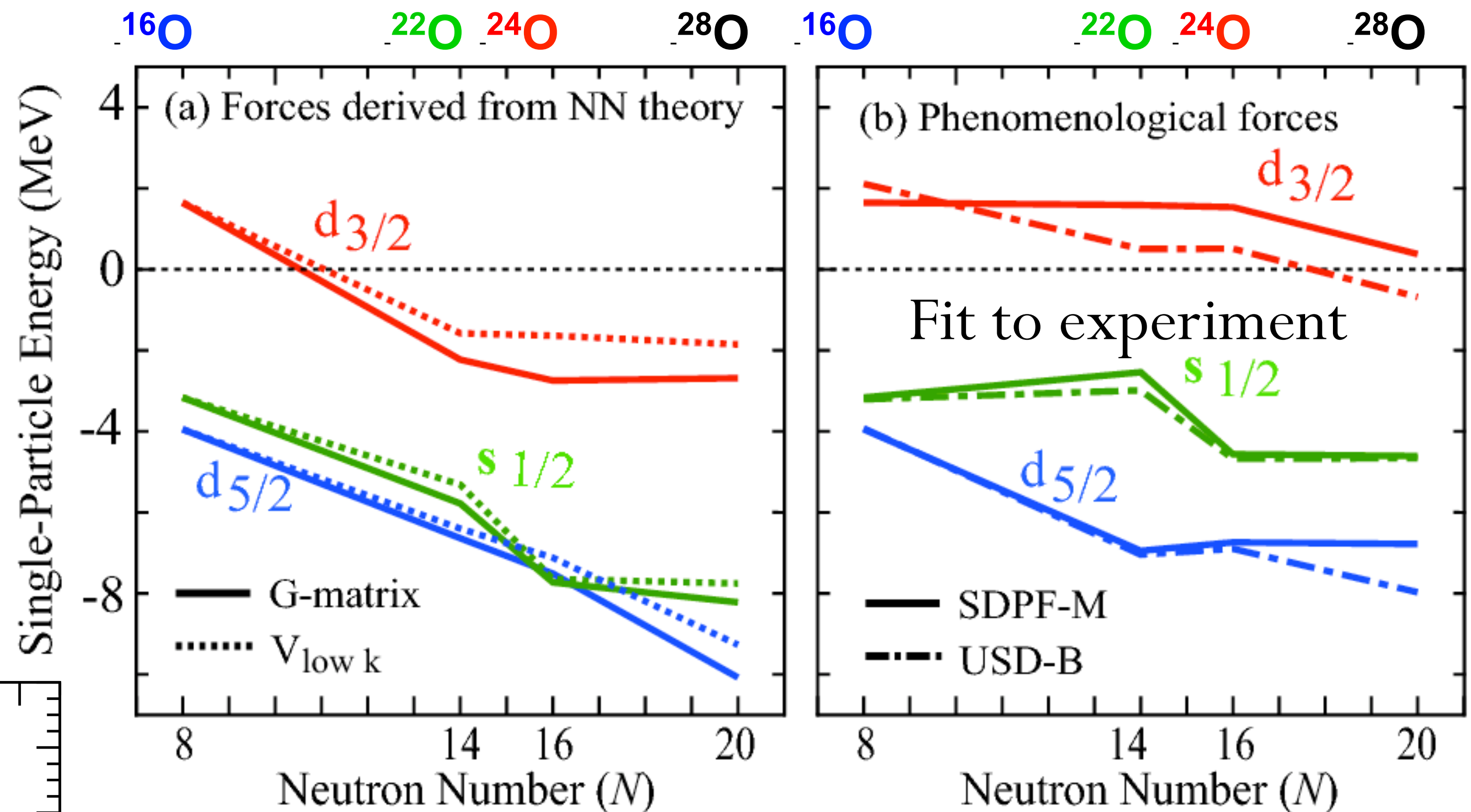
Phenomenological Models

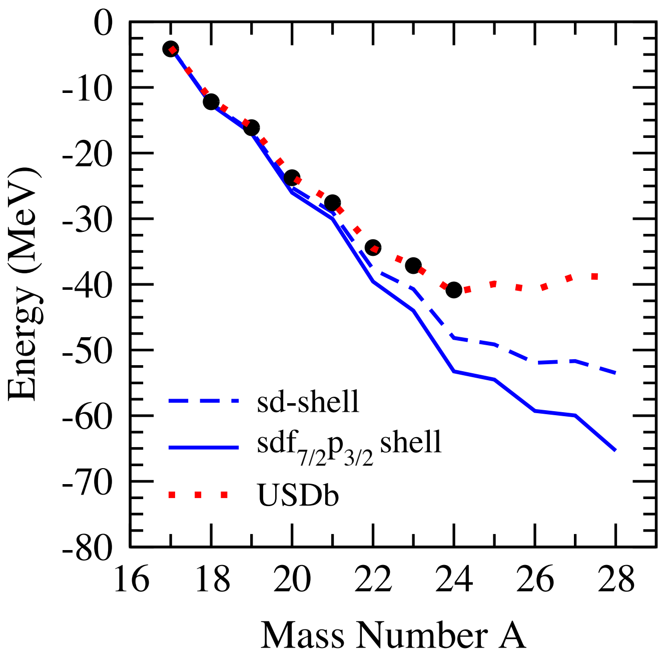
$d_{3/2}$ orbit unbound

Physics in Oxygen Isotopes

Calculate evolution of *sd*-orbital energies from interactions







Microscopic NN Theories	*Phenomenological Models*
$d_{3/2}$ orbit bound to ^{28}O	$d_{3/2}$ orbit unbound
Dripline at ^{28}O	Dripline at ^{24}O

Oxygen anomaly unexplained with NN forces

Origin of monopole shifts: Neglected 3N forces

-- **See lecture of A. Poves**

Perturbative Approach

1) Effective Hamiltonian: sum excitations outside valence space to MBPT(3)

2) Self-consistent single-particle energies

3) Harmonic-oscillator basis of 13-15 major shells

4) Nuclear forces from chiral EFT

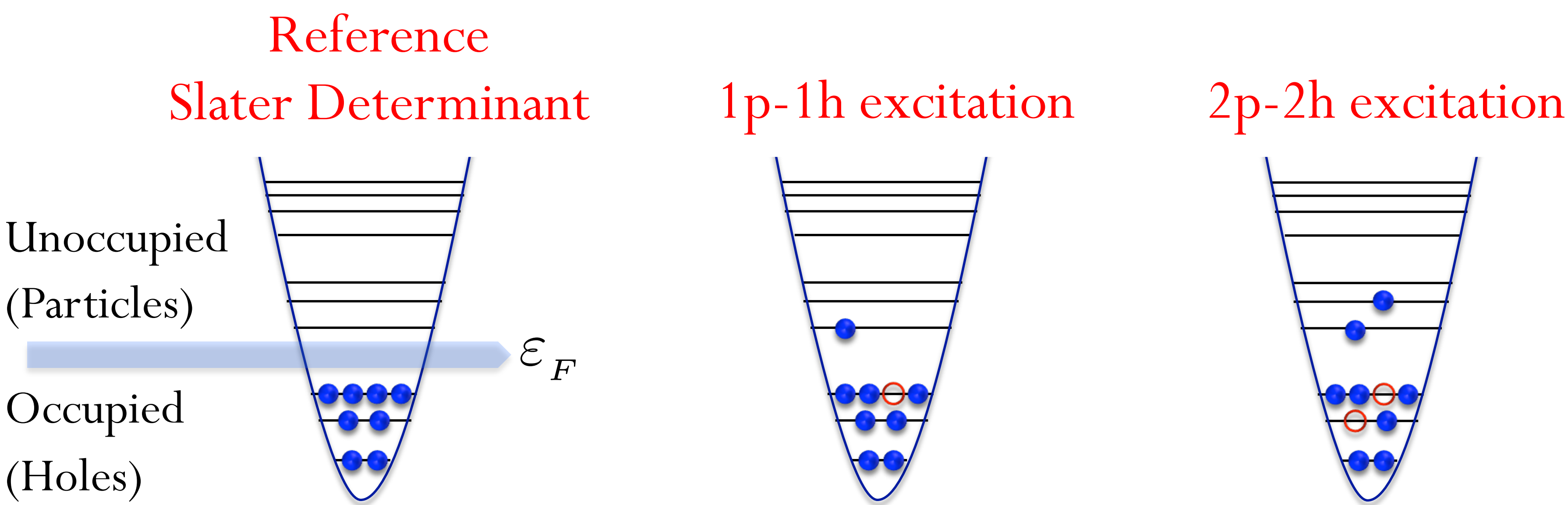
5) Requires extended valence spaces

<u>Limitations</u>

- Uncertain perturbative convergence
- Core physics inconsistent or absent
- Degenerate valence space requires HO basis (HF requires nontrivial extension)
- Must treat additional orbitals nonperturbatively (extend valence space)

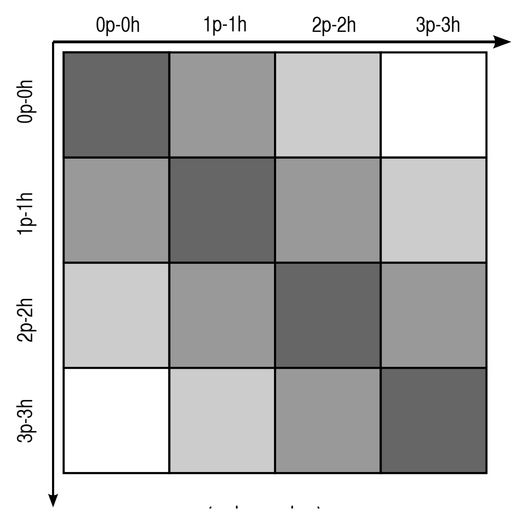
Particle/Hole Excitations

Consider basis states as excitations from some reference state:



$$|\Phi\rangle = \prod_{i=1}^{N} a_i^\dagger |0\rangle \qquad |\Phi_i^a\rangle = a_a^\dagger a_i |\Phi\rangle \qquad |\Phi_{ij}^{ab}\rangle = a_a^\dagger a_i a_b^\dagger a_j |\Phi\rangle$$

Hamiltonian schematically given in terms of ph excitations



$\langle i|H|j\rangle$

Normal-Ordered Hamiltonian

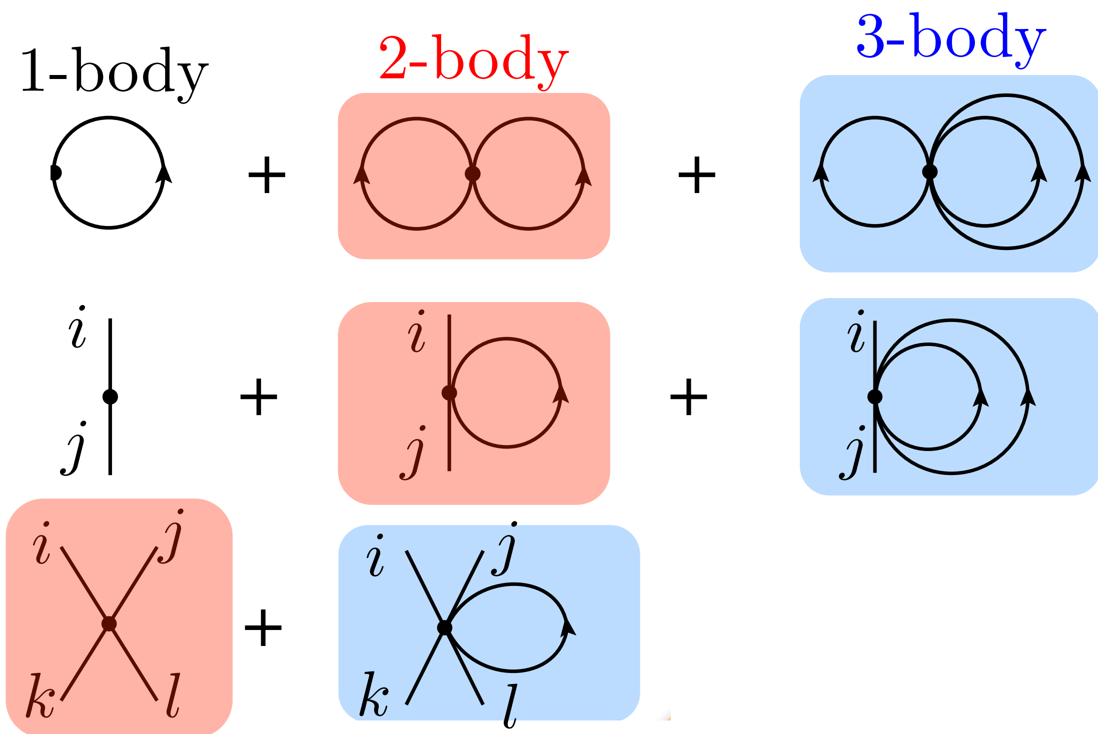
Now rewrite exactly the initial Hamiltonian in normal-ordered form

$$H_{\mathrm{N.O.}} = E_0 + \sum_{ij} f_{ij} \left\{a_i^\dagger a_j\right\} + \frac{1}{4}\sum_{jkl} \Gamma_{ijkl} \left\{a_i^\dagger a_j^\dagger a_l a_k\right\} + \frac{1}{36}\sum_{ijklmn} W_{ijklmn} \left\{a_i^\dagger a_j^\dagger a_k^\dagger a_l a_m a_n\right\}$$

N.O. 0-body $\rightarrow E_0 =$

N.O. 1-body $\rightarrow \; f =$

N.O. 2-body $\rightarrow \; \Gamma =$



Normal-ordered Hamiltonian w.r.t. reference state

Loop = **sum over occupied states**

Include dominant 1-,2-,3-body physics in NO

Nonperturbative In-Medium SRG

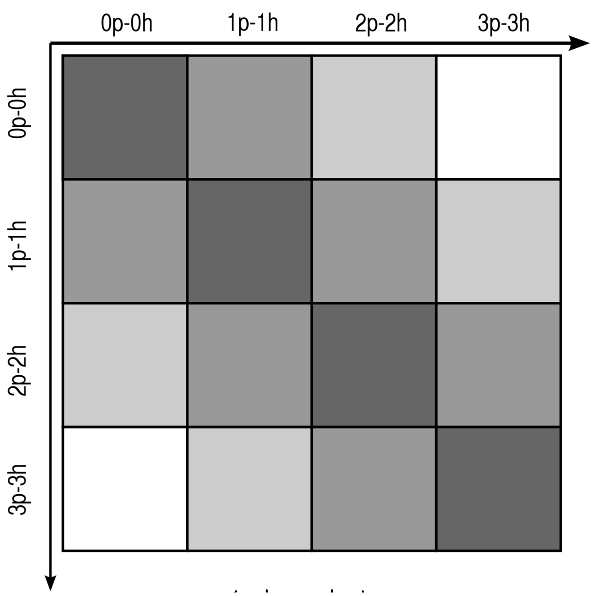
Tsukiyama, **Bogner**, Schwenk, PRL (2011)

In-Medium SRG continuous unitary trans. drives off-diagonal physics to zero

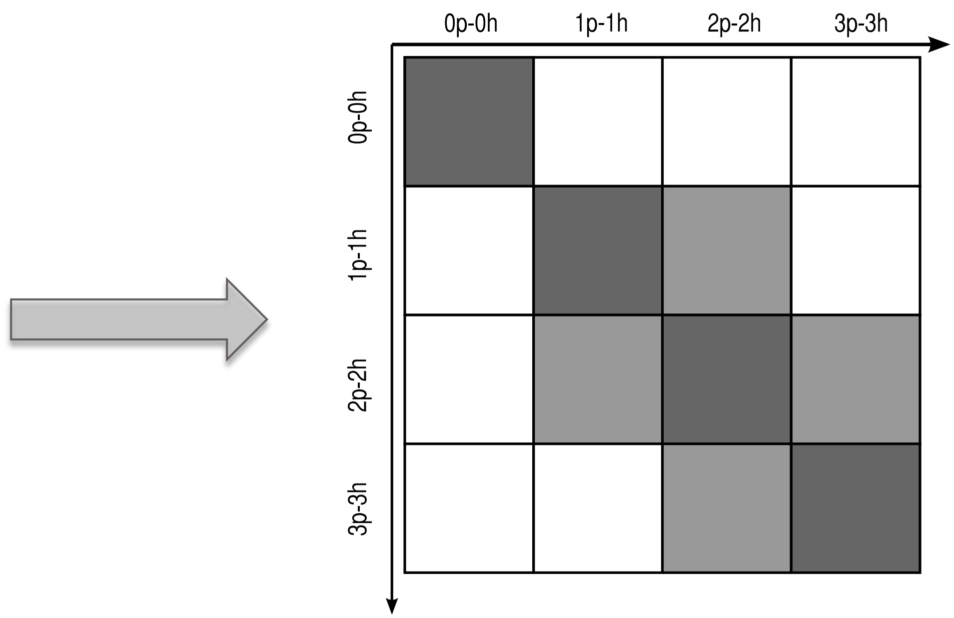
$$H(s) = U(s)HU^{\dagger}(s) \equiv H^{\mathrm{d}}(s) + H^{\mathrm{od}}(s) \rightarrow H^{\mathrm{d}}(\infty)$$

From uncorrelated Hartree-Fock reference state (e.g., ^{16}O) define:

$$H^{\mathrm{od}} = \langle p|H|h\rangle + \langle pp|H|hh\rangle + \cdots + \mathrm{h.c.}$$



$$\langle i|H|j\rangle$$



$$\langle \mathrm{np}\mathrm{n}h|H(\infty)|\Phi_{\mathrm{core}}\rangle = 0$$

Drives all n-particle n-hole couplings to 0 – **decouples** core from excitations

IM-SRG: Flow Equation Formulation

Define *U(s)* implicitly from particular choice of generator:

$$\eta(s) \equiv (\mathrm{d}U(s)/\mathrm{d}s)\, U^\dagger(s)$$

chosen for desired decoupling behavior – e.g.,

$$\eta_I(s) = \left[H^{\mathrm{d}}(s), H^{\mathrm{od}}(s)\right]$$ Wegner (1994)

Solve **flow equation** for Hamiltonian (coupled DEs for 0,1,2-body parts)

$$\frac{\mathrm{d}H(s)}{\mathrm{d}s} = [\eta(s), H(s)] \qquad H(s) = E_0(s) + f(s) + \Gamma(s) + \cdots$$

Hamiltonian and generator truncated at 2-body level: **IM-SRG(2)**

0-body flow drives uncorrelated ref. state to fully correlated ground state

$$E_0(\infty) \to \text{Core Energy}$$

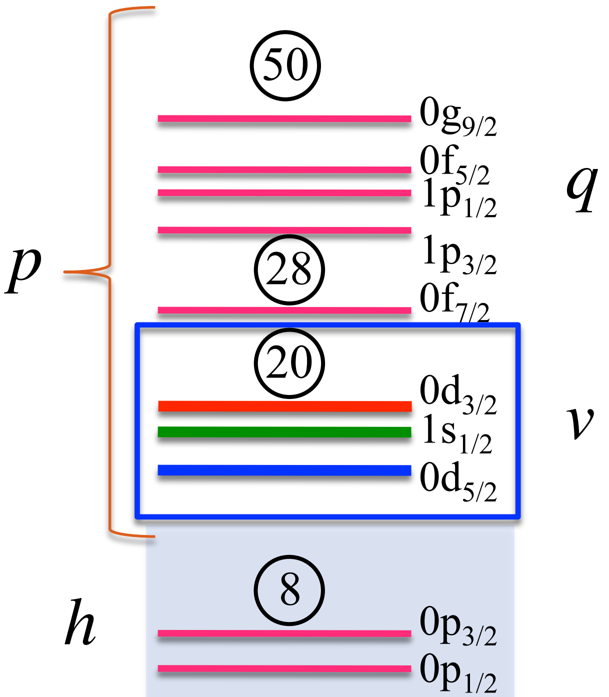
Ab initio method for energies of **closed-shell systems**

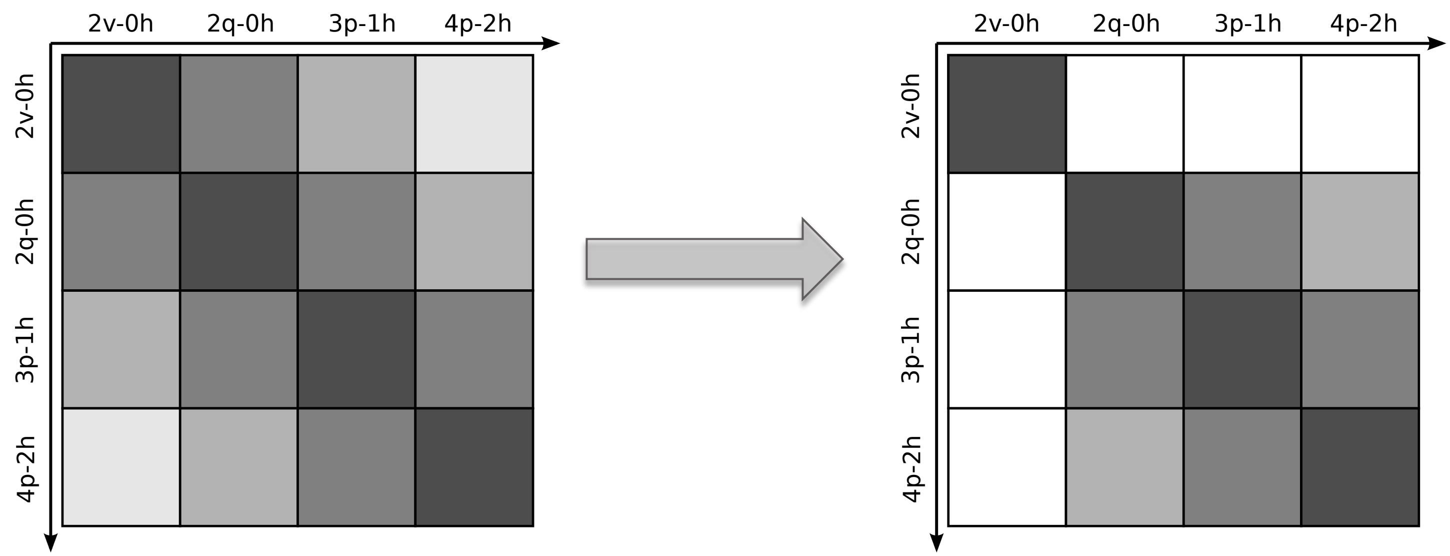
IM-SRG: Valence-Space Hamiltonians

Tsukiyama, **Bogner**, Schwenk, PRC (2012)

Open-shell systems

Separate *p* states into valence states (*v*) and those above valence space (*q*)





$$H(s=0) \rightarrow H(\infty)$$

Redefine H^{od} to **decouple valence space from excitations** outside *v*

$$H^{\mathrm{od}} = \langle p|H|h\rangle + \langle pp|H|hh\rangle + \langle v|H|q\rangle + \langle pq|H|vv\rangle + \langle pp|H|hv\rangle + \mathrm{h.c.}$$

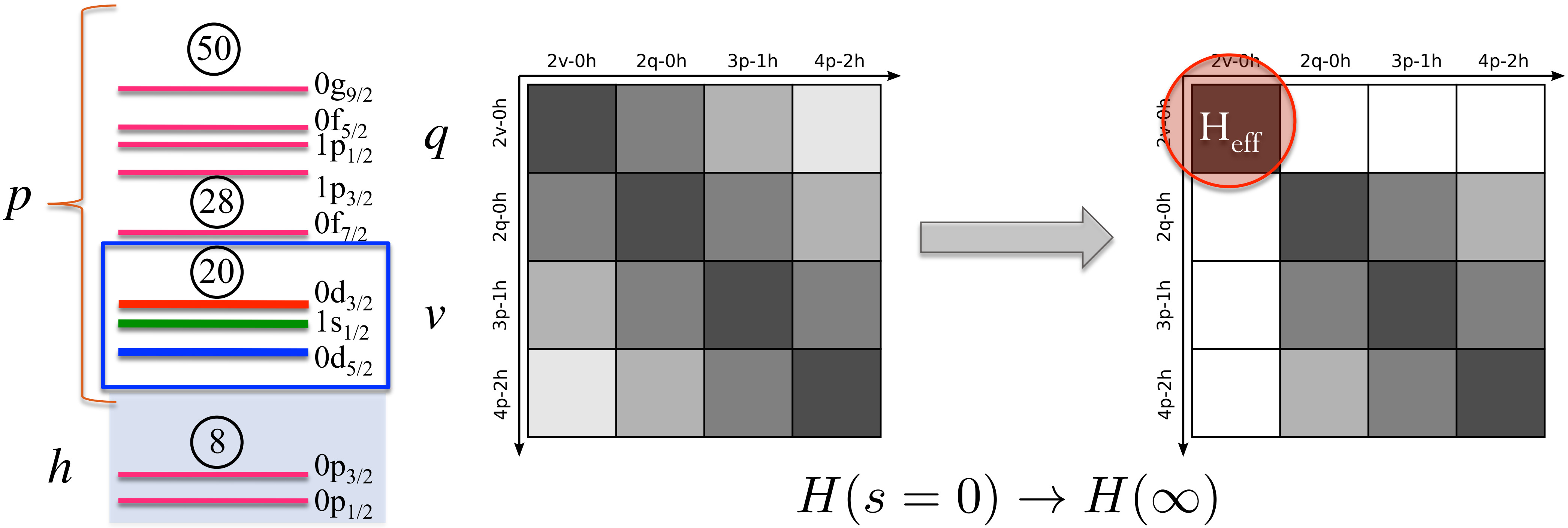
$$E_0(\infty) \rightarrow \text{Core Energy} \quad f(\infty) \rightarrow \text{SPEs} \quad \Gamma(\infty) \rightarrow V_{\mathrm{eff}}$$

IM-SRG: Valence-Space Hamiltonians

Tsukiyama, **Bogner**, Schwenk, PRC (2012)

Open-shell systems

Separate *p* states into valence states (*v*) and those above valence space (*q*)



Core physics included consistently (**absolute energies, radii…**)

Inherently nonperturbative – no need for extended valence space

Non-degenerate valence-space orbitals

NN-only IM-SRG Monopoles

Testing ab initio IM-SRG shell model monopoles

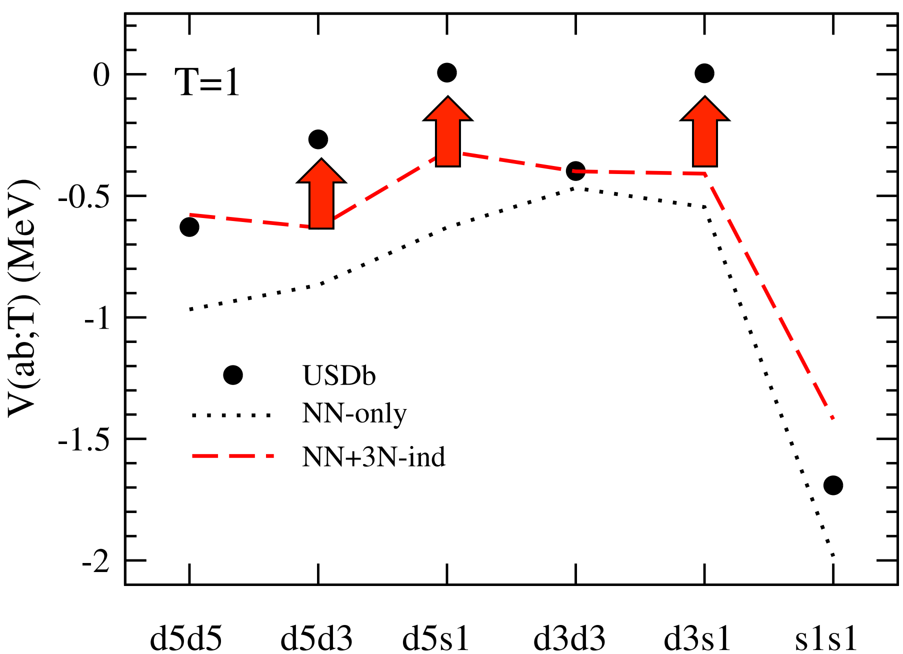
Monopoles:
Angular average of interaction

$$V_{ab}^{T} = \frac{\sum_J (2J+1) V_{abab}^{JT}}{\sum_J (2J+1)}$$

Determines interaction of orbit *a* with *b*: *evolution of orbital energies*

$$\Delta\varepsilon_a = V_{ab} n_b$$

Improvements over MBPT?

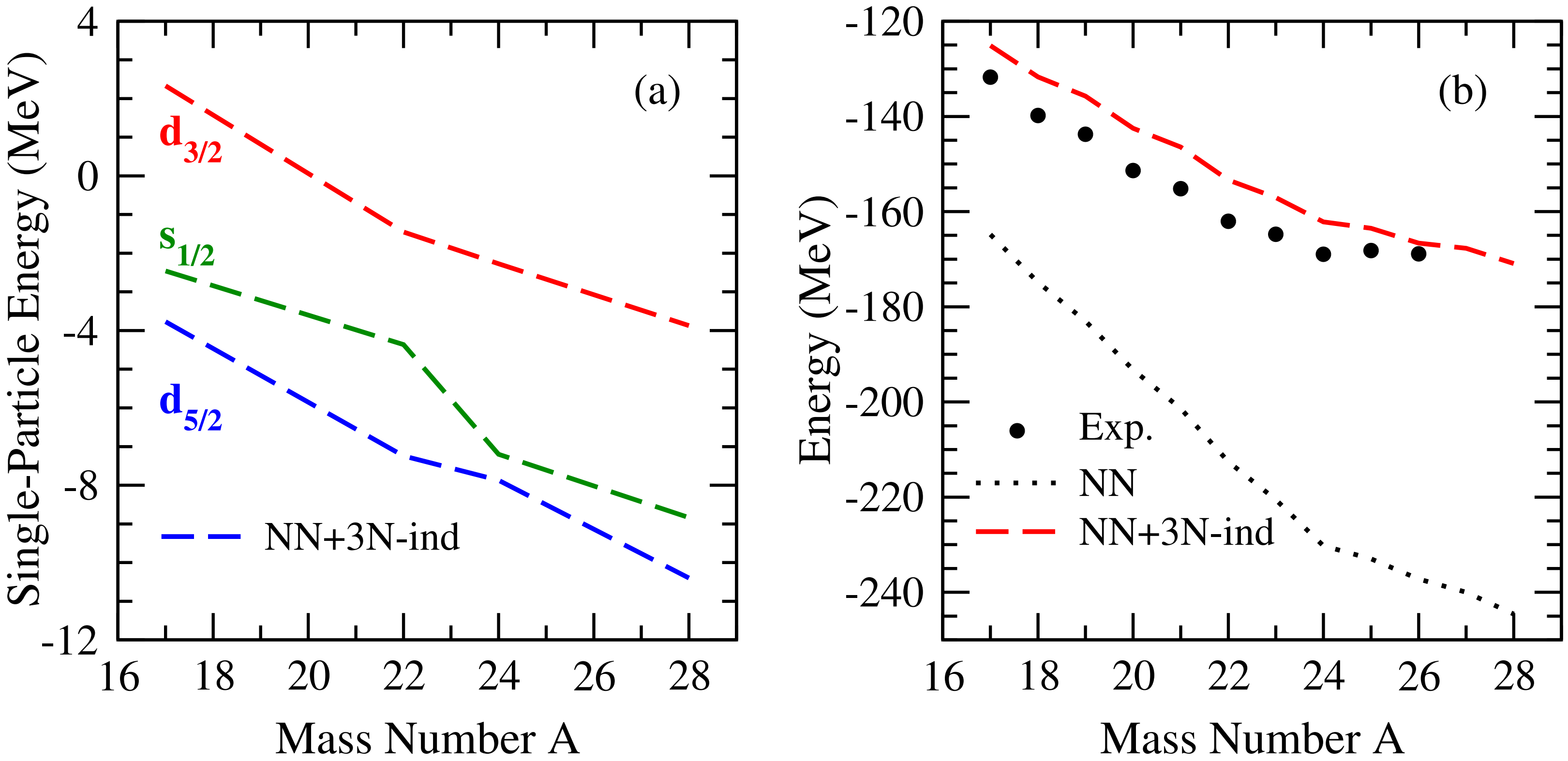


NN-only significantly too attractive

NN+3N-ind improved but $\boldsymbol{d_{3/2}}$ monopoles too attractive

Comparison with Large-Space Methods

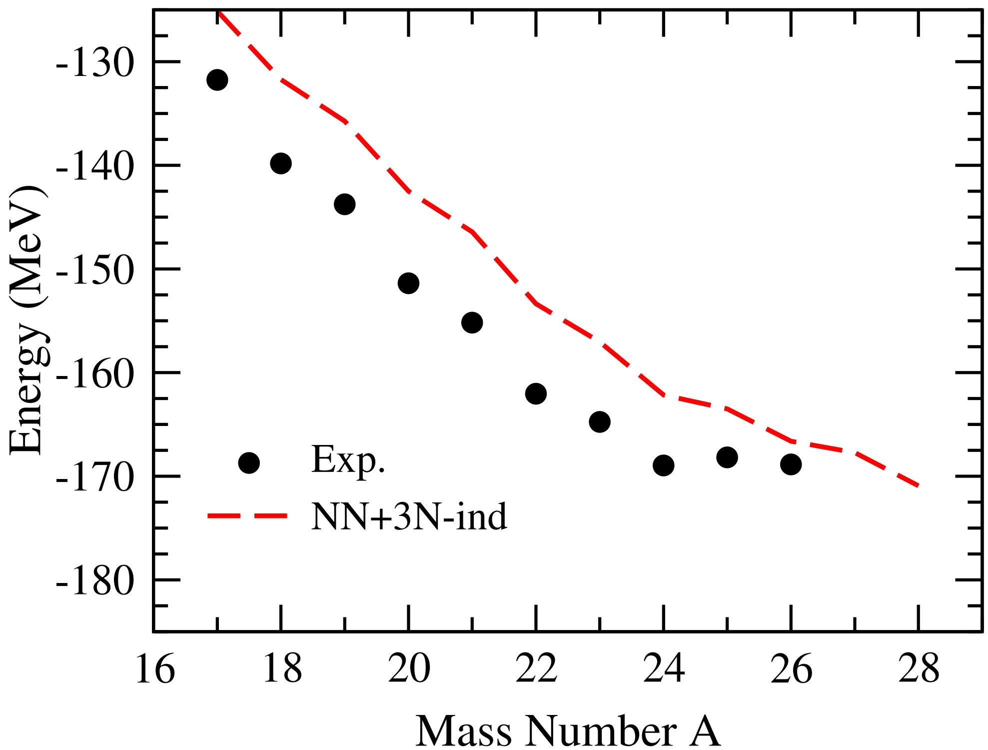
Results from SRG-evolved NN and NN+3N-ind forces

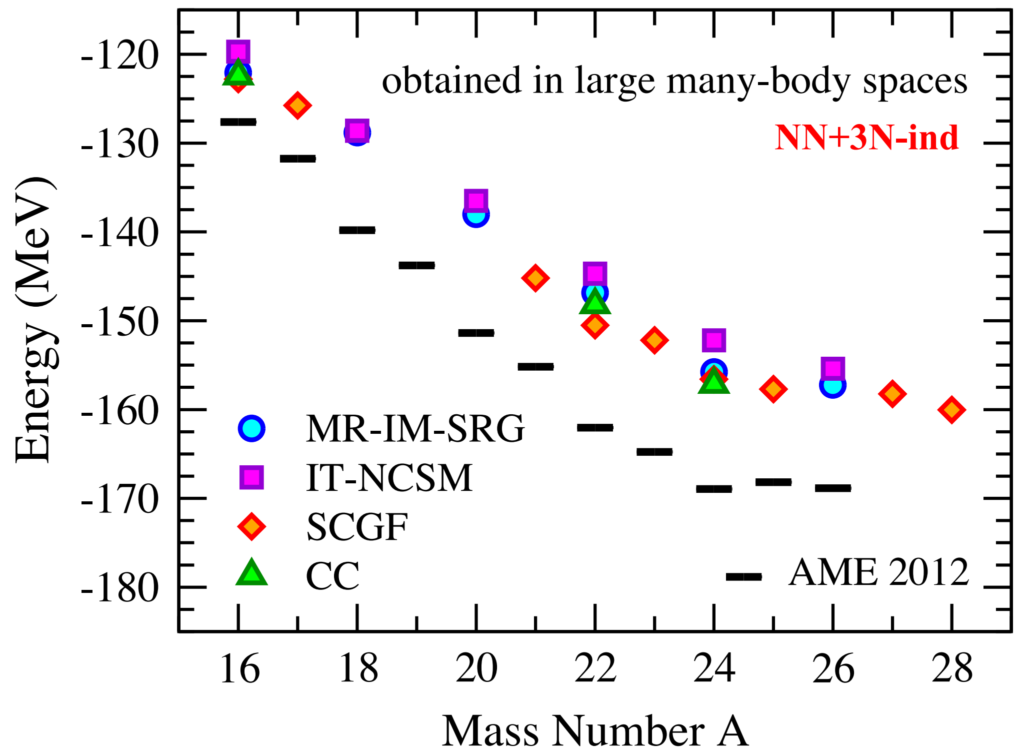


Dripline still not reproduced

Comparison with Large-Space Methods

Large-space methods with **same SRG-evolved NN+3N-ind forces**

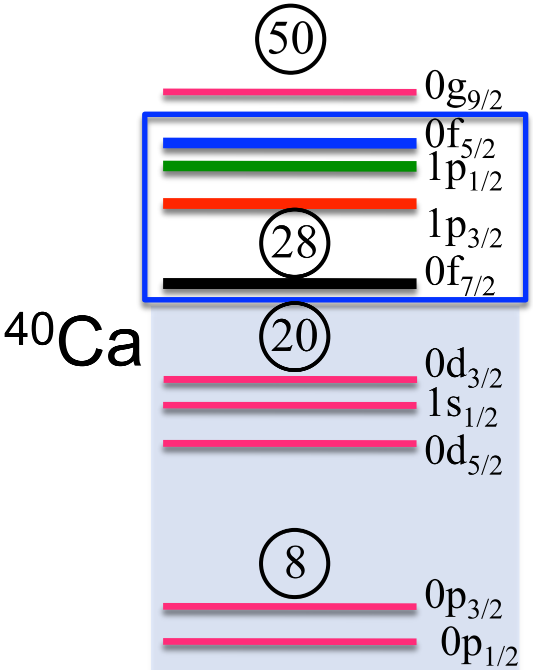


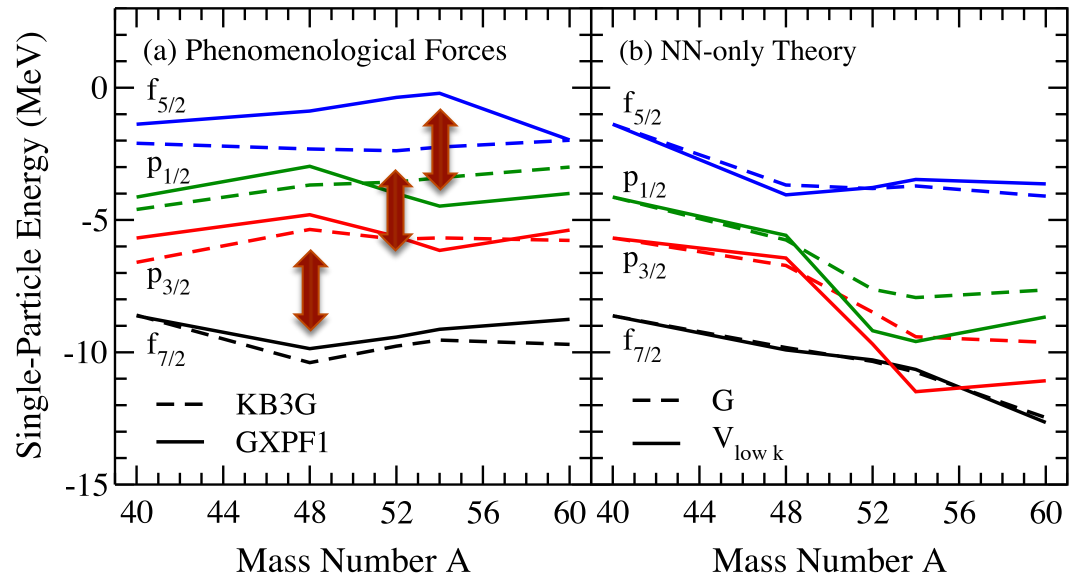


Agreement between all methods with same input forces

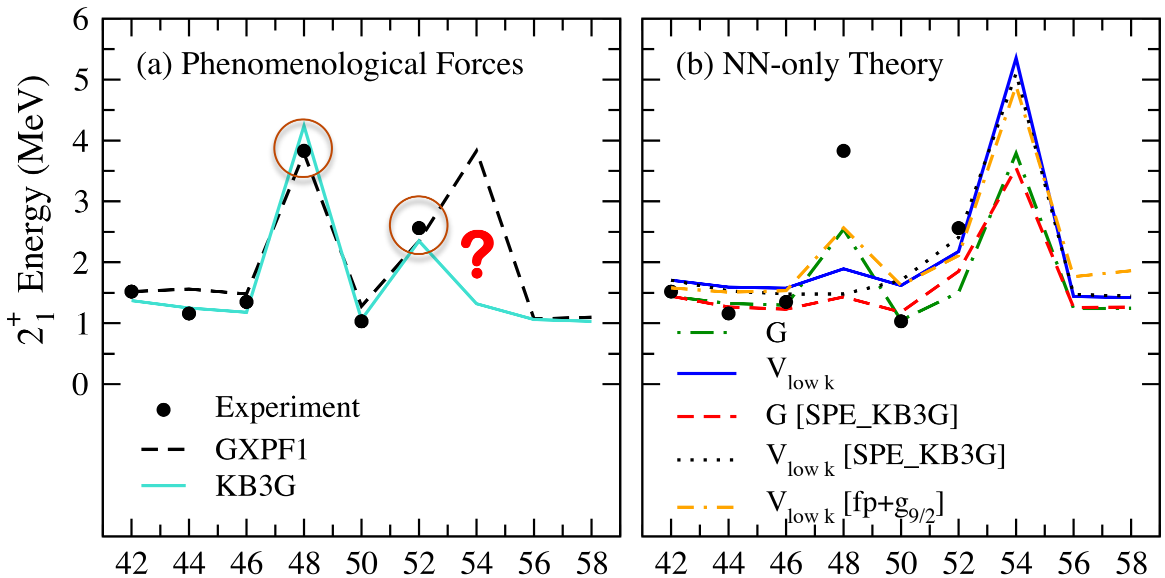
No reproduction of dripline in any case

Calcium Isotopes: Magic Numbers





GXPF1: Honma, Otsuka, Brown, Mizusaki (2004)
KB3G: Poves, Sanchez-Solano, Caurier, Nowacki (2001)



Phenomenological Forces

Large gap at ^{48}Ca

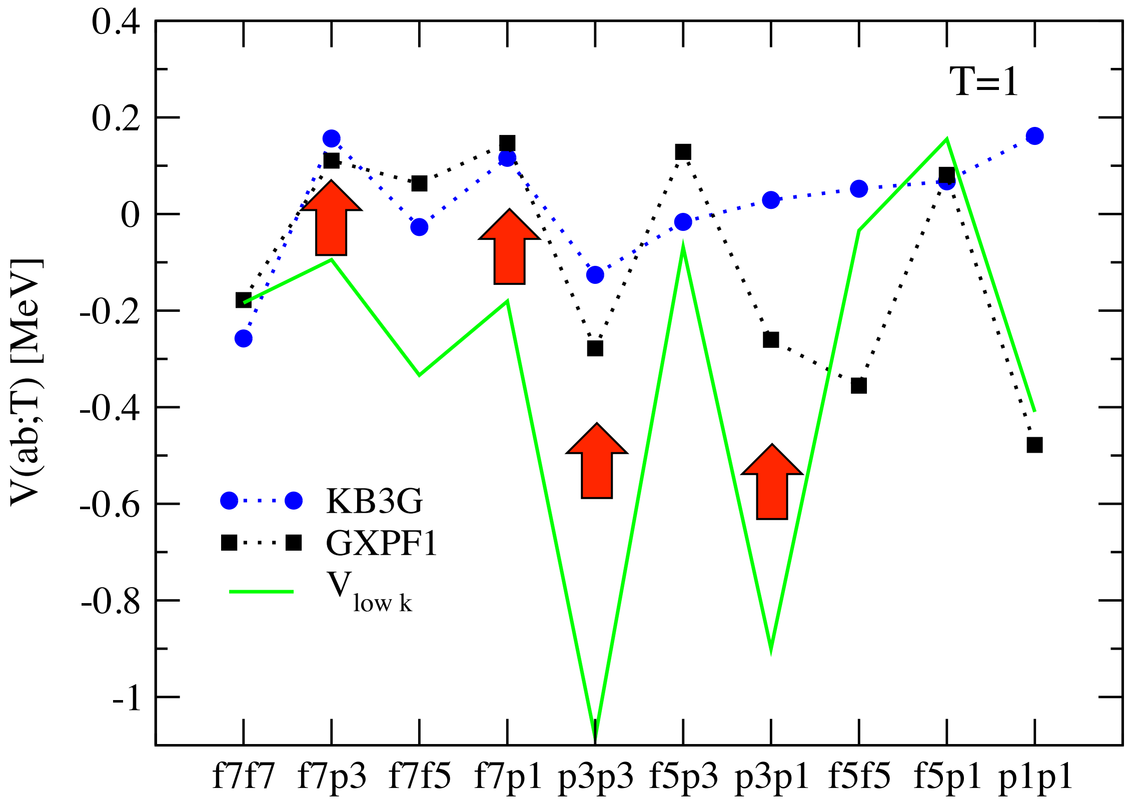
Discrepancy at *N=34*

Microscopic NN Theory

Small gap at ^{48}Ca

N=28: first standard magic number not reproduced in microscopic NN theories

Phenomenological vs. Microscopic



Compare monopoles from:

Microscopic **low-momentum** interactions

Phenomenological **KB3G**, **GXPF1** interactions

Shifts in **low-lying orbitals**:

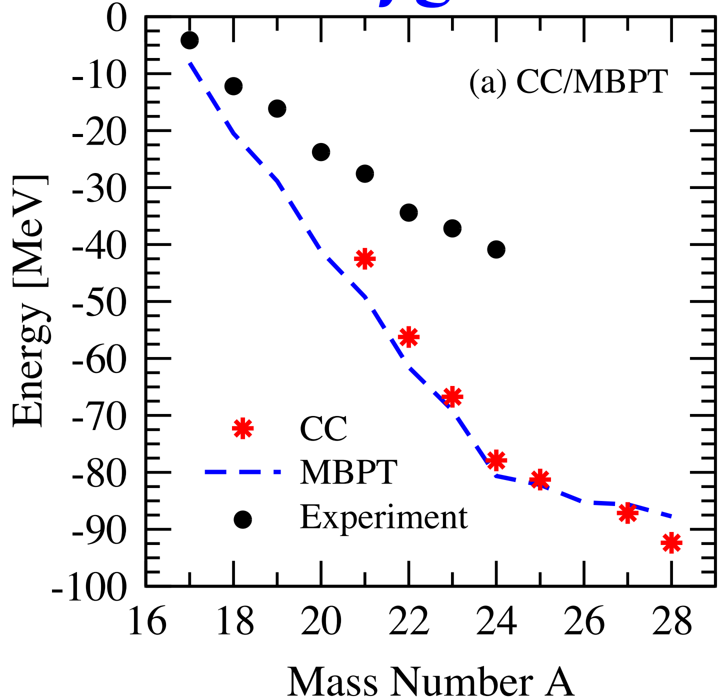
- T=1 repulsive shift

Comparison to Coupled Cluster

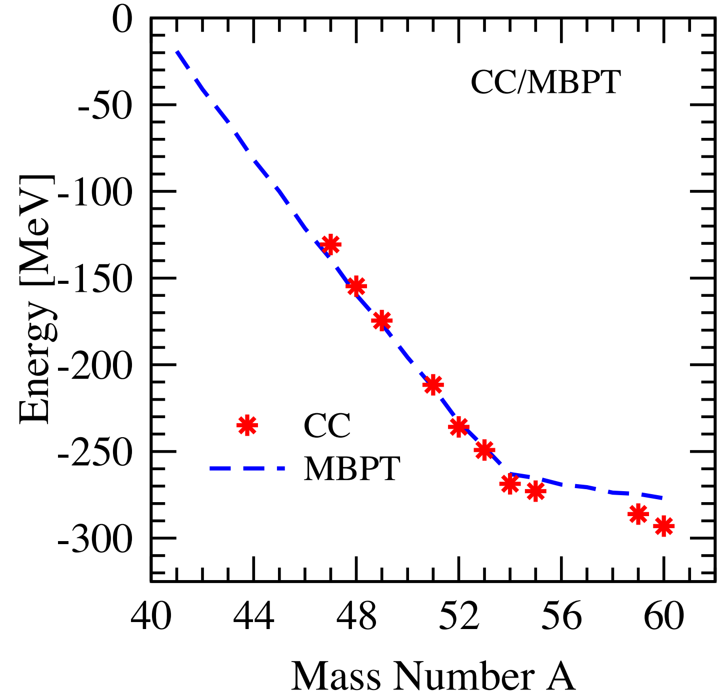
Many-body method insufficient?

Benchmark against *ab-initio* Coupled Cluster at NN-only level

Oxygen



Calcium



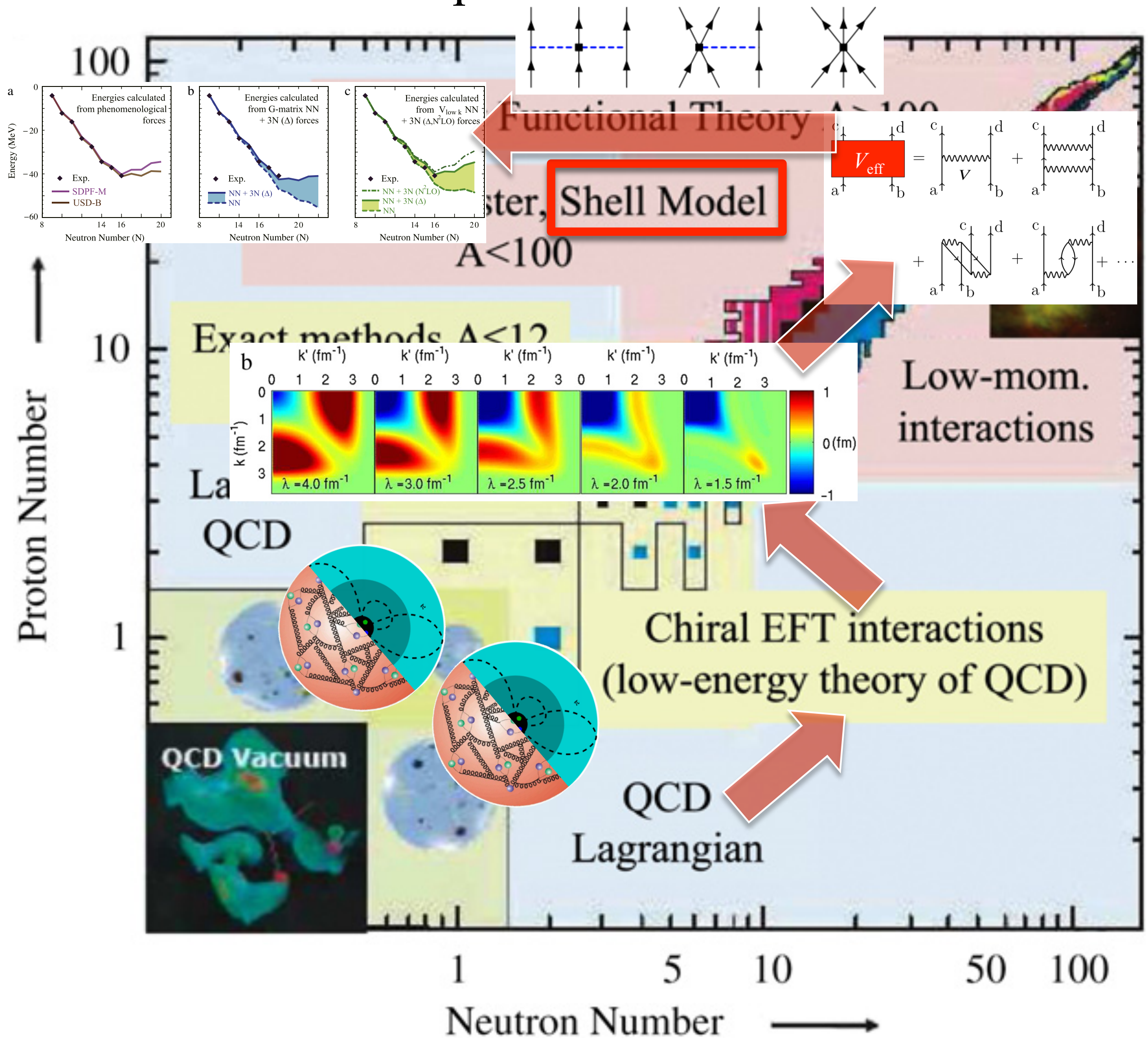
SPEs: one-particle attached CC energies in ^{17}O and ^{41}Ca

Small difference in many-body methods

Include **3N forces** to improve agreement with experiment

Part IV: Three-Nucleon Forces to Nuclei

To understand the properties of complex nuclei from first principles



Three-Nucleon Forces

Basic ideas – why needed?

3N from chiral EFT

Implementing in shell model

Relation to monopoles

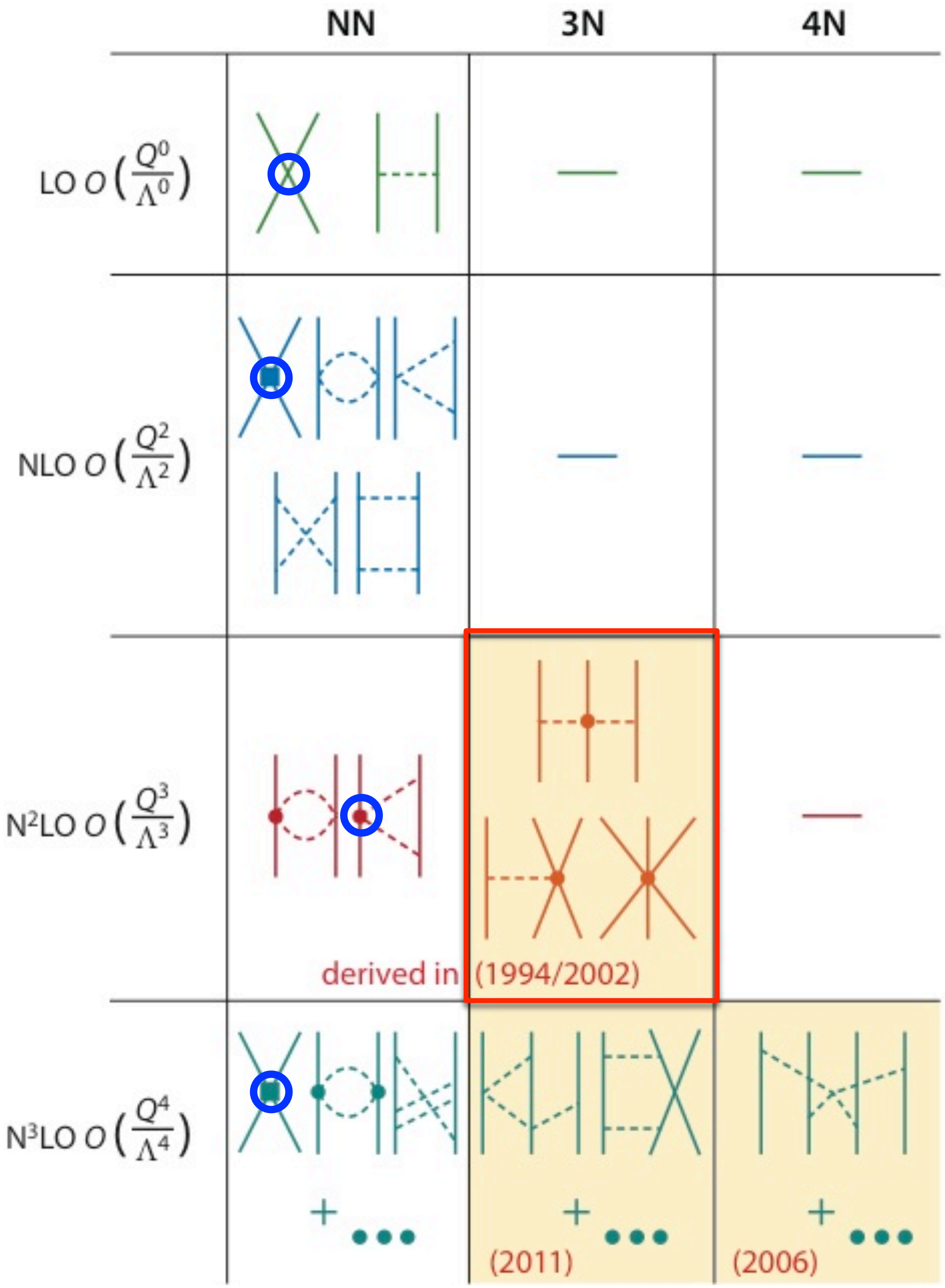
Predictions/new discoveries

Connections beyond structure

How will we approach this problem:

QCD → NN (3N) forces → Renormalize → "Solve" many-body problem → Predictions

Chiral Effective Field Theory: Nuclear Forces

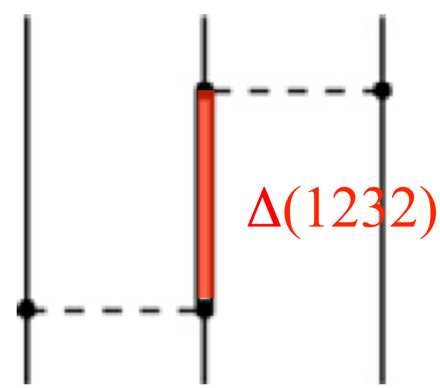


Weinberg, van Kolck, Kaplan, Savage, Wise

Nucleons interact via pion exchanges and contact interactions

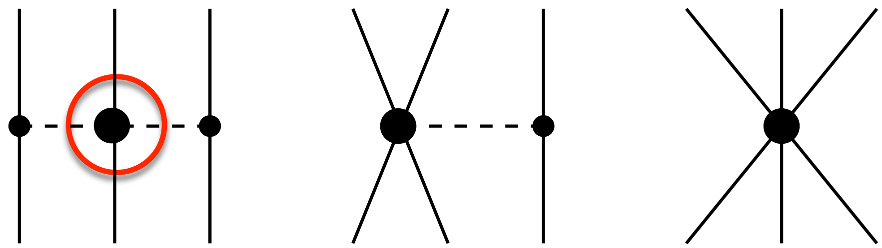
Consistent treatment of NN, 3N,…

NN couplings fit to scattering data



Chiral EFT: N^{2}LO 3N

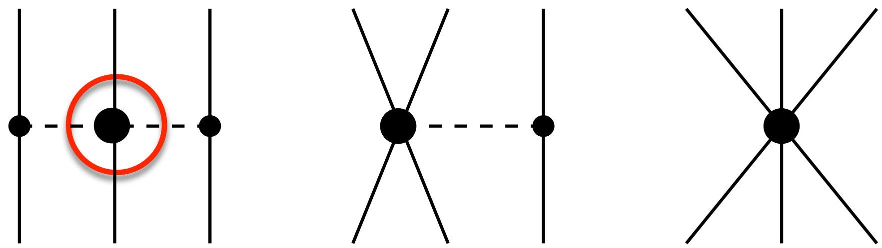
First non-vanishing 3N contributions: Next-to-next-to-leading order $\nu = 3$



$$V_{3N}^{(3)} = \frac{g_A^2}{8F_\pi^4} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2][q_3^2 + M_\pi^2]} [\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 (-4c_1 M_\pi^2 + 2c_3 \vec{q}_1 \cdot \vec{q}_3) + c_4 \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2] - \frac{g_A D}{8F_\pi^2} \frac{\vec{\sigma}_3 \cdot \vec{q}_3}{q_3^2 + M_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{q}_3 + \frac{1}{2} E \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3$$

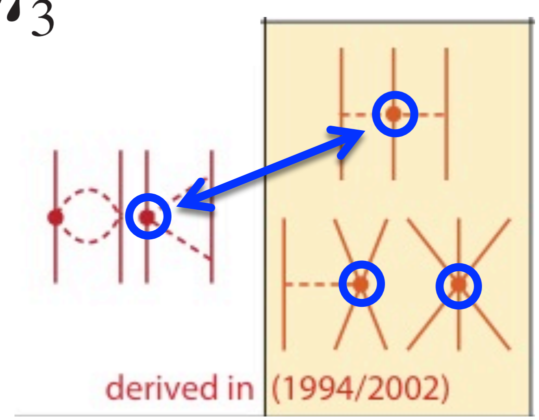
Chiral EFT: N²LO 3N

First non-vanishing 3N contributions: Next-to-next-to-leading order $\nu = 3$



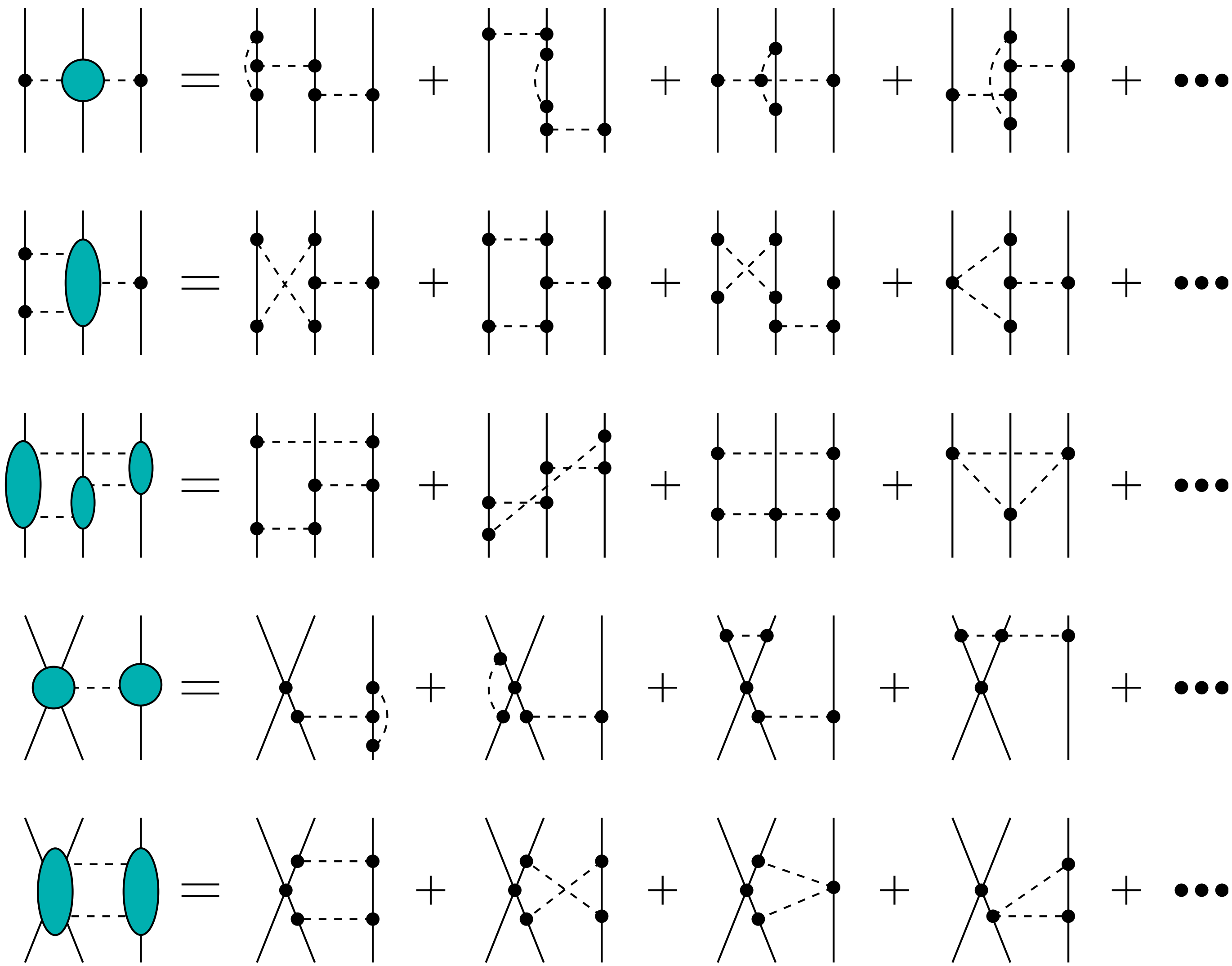
$$V_{3N}^{(3)} = \frac{g_A^2}{8F_\pi^4} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2][q_3^2 + M_\pi^2]} [\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 (-4c_1 M_\pi^2 + 2c_3 \vec{q}_1 \cdot \vec{q}_3) + c_4 \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2] - \frac{g_A D}{8F_\pi^2} \frac{\vec{\sigma}_3 \cdot \vec{q}_3}{q_3^2 + M_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{q}_3 + \frac{1}{2} E \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3$$

Three undetermined πN couplings from NN fit



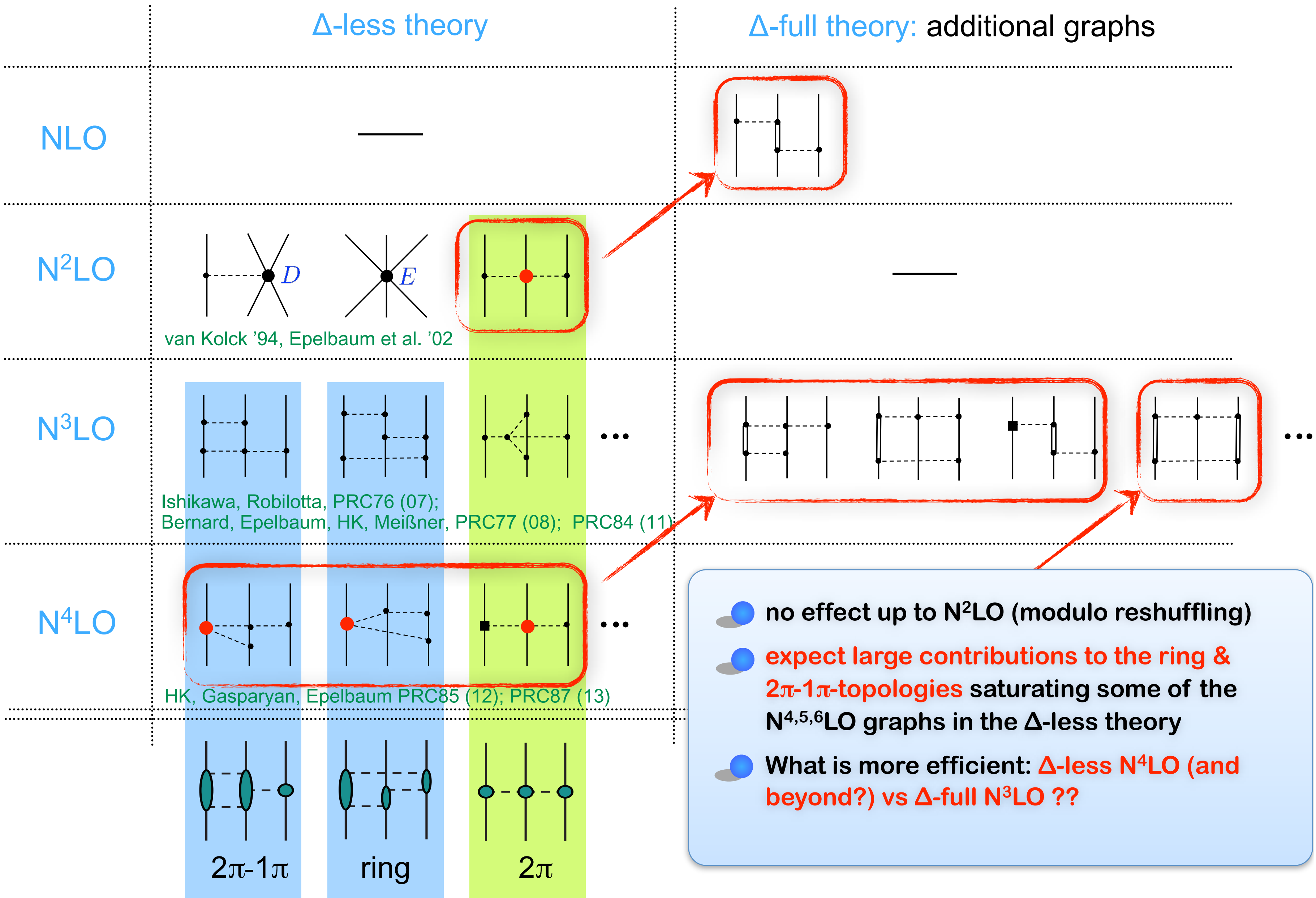
Chiral EFT: N^{3}LO 3N

Next-to-next-to-next-to-leading order $\nu = 4$



Good news: **no new constants**   Bad news: well, there's all this

Aside: Effects of Adding Explicit Deltas



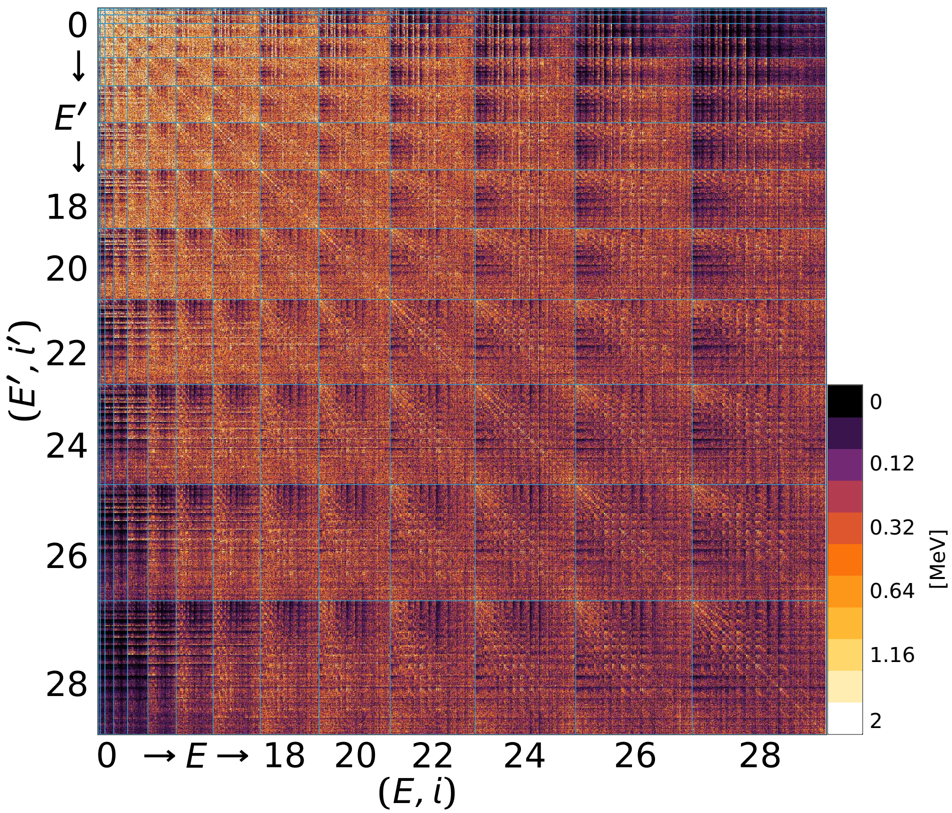
- no effect up to N^{2}LO (modulo reshuffling)
- expect large contributions to the ring & 2π-1π-topologies saturating some of the N^{4,5,6}LO graphs in the Δ-less theory
- What is more efficient: Δ-less N^{4}LO (and beyond?) vs Δ-full N^{3}LO ??

Reshuffles effects to different chiral orders

SRG Evolution in HO Basis

Most common to SRG evolve 3N in HO basis:

3B-Jacobi HO matrix elements



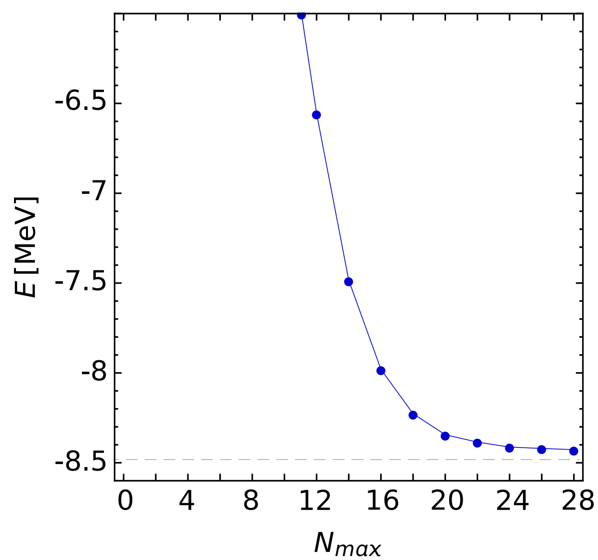
$$\alpha = 0.00\,\mathrm{fm}^4$$

$\lambda = \infty\,\mathrm{fm}^{-1}$

$$\langle E'i'JT|\,\tilde{\mathrm{H}}_\alpha - \mathrm{T}_{\mathrm{int}}\,|EiJT\rangle$$

$$J^\pi = \tfrac{1}{2}^+,\, T = \tfrac{1}{2},\, \hbar\Omega = 24\,\mathrm{MeV}$$

NCSM ground state ^{3}H



1) SRG-evolve both NN and 3N: NN+3N-full

2) NN Vlowk, refit 3N: NN+3N-fit

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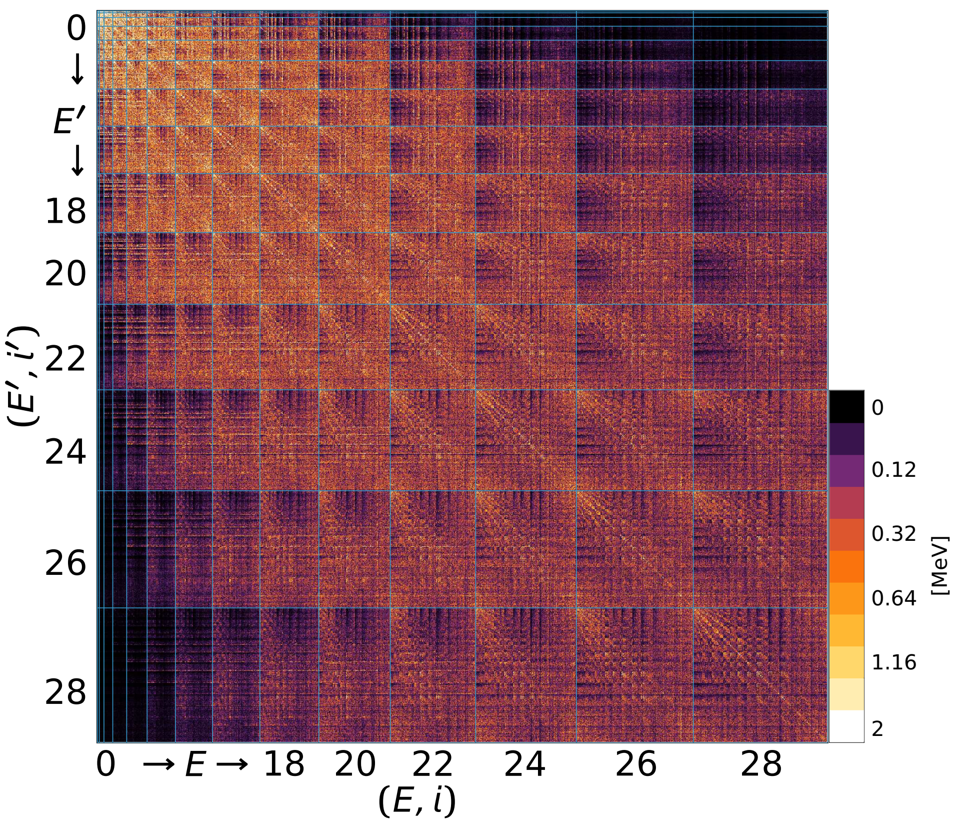


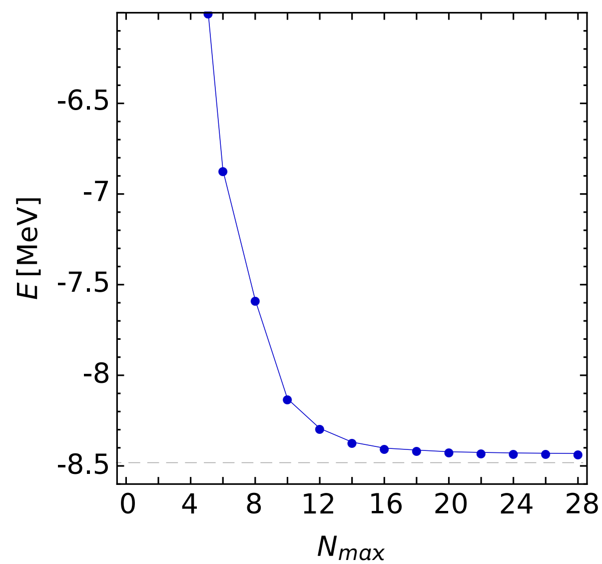
$\alpha = 0.02\,\mathrm{fm}^4$

$\lambda = 2.66\,\mathrm{fm}^{-1}$

$\langle E' i' J T | \widetilde{\mathrm{H}}_\alpha - \mathrm{T}_{\mathrm{int}} | E i J T \rangle$

$J^\pi = \frac{1}{2}^+, T = \frac{1}{2}, \hbar\Omega = 24\,\mathrm{MeV}$

NCSM ground state ^{3}H



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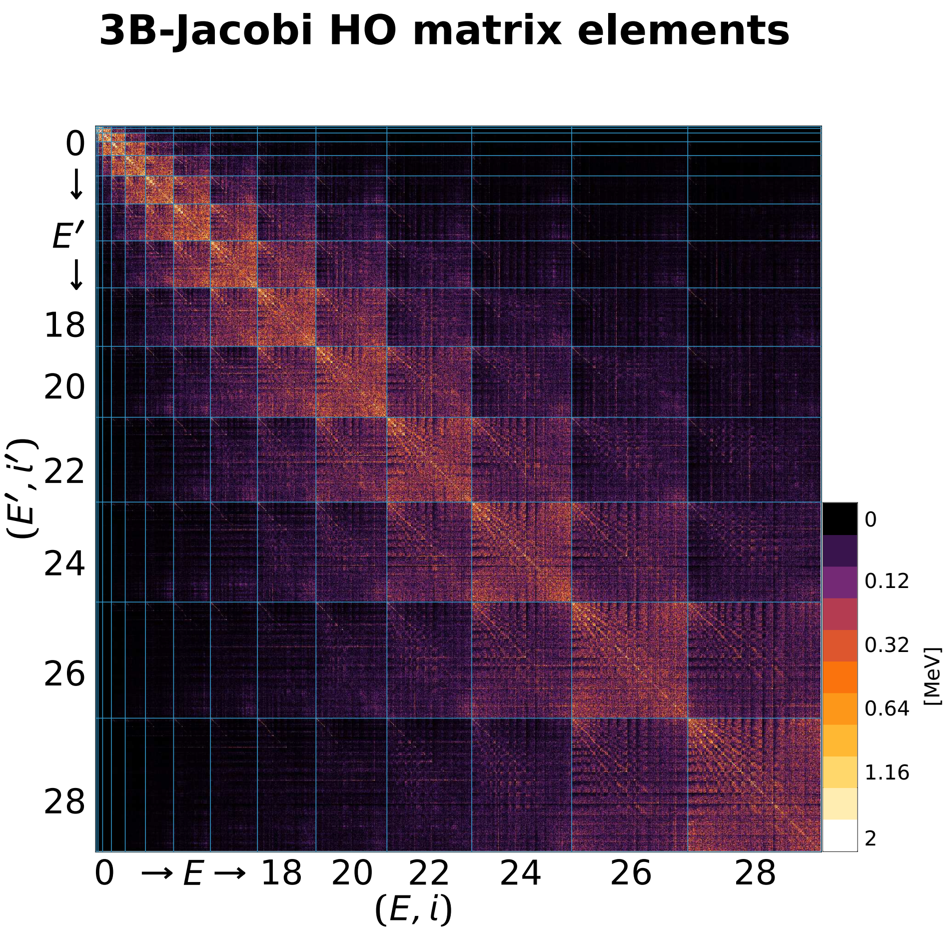
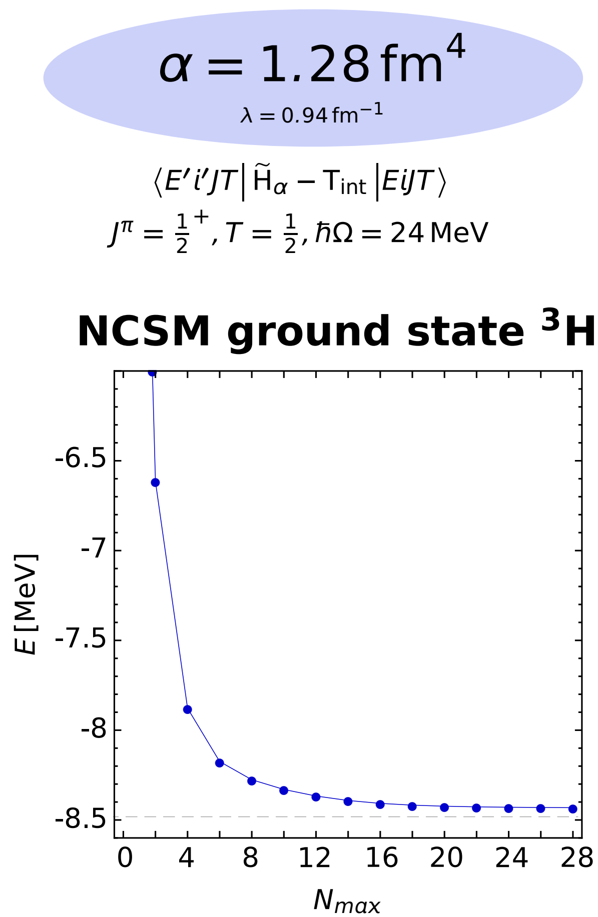


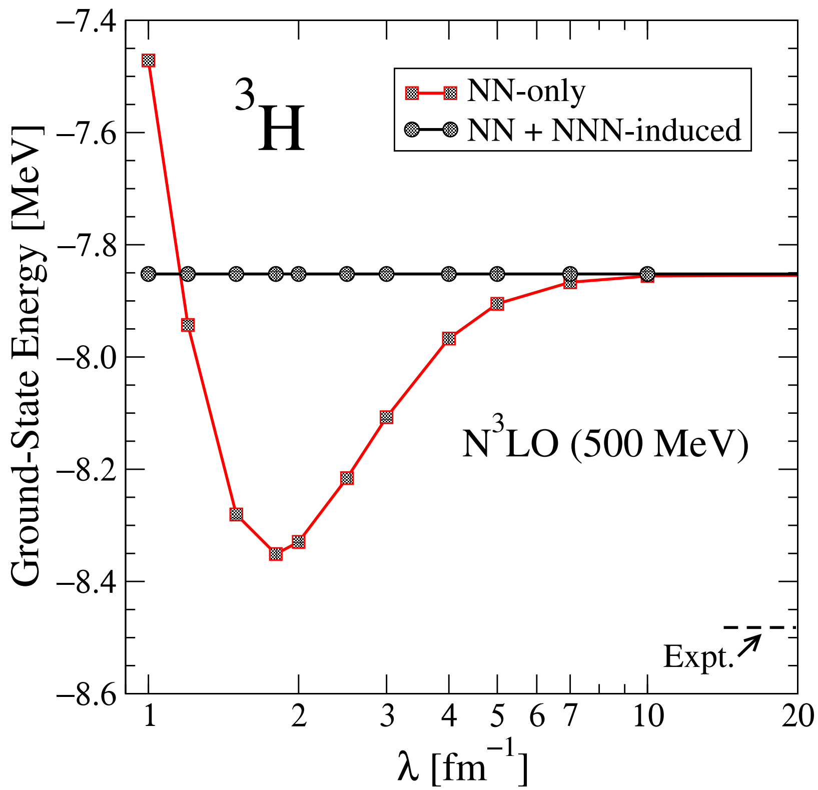


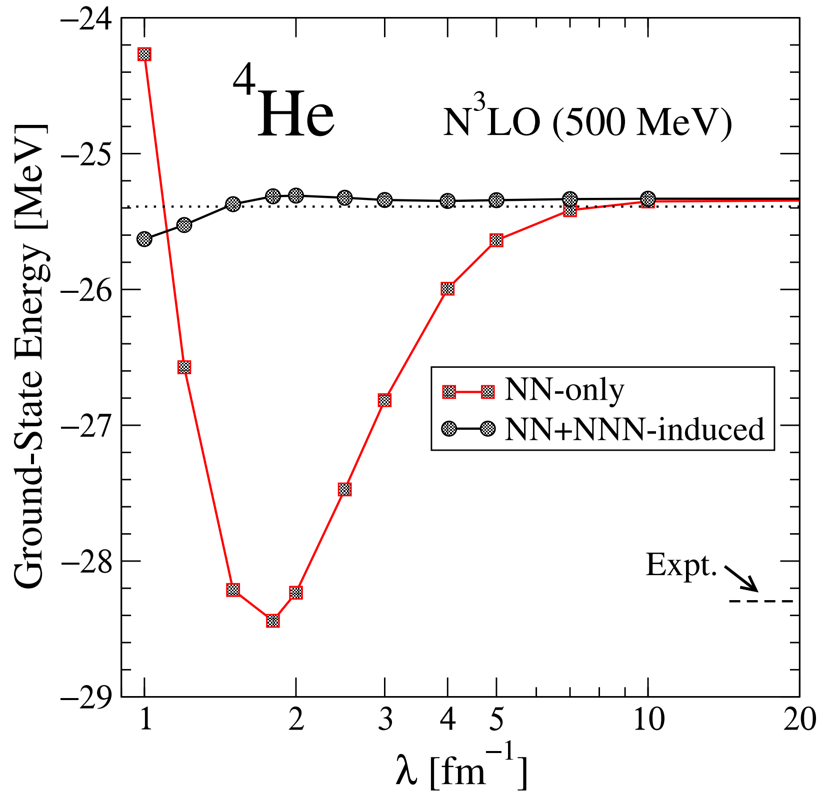
1) SRG-evolve both NN and 3N: NN+3N-full

2) NN Vlowk, refit 3N: NN+3N-fit

Induced 3N Forces

Effect of including 3N-ind? Exactly initial $V_{\rm NN}$ up to neglected 4N-ind



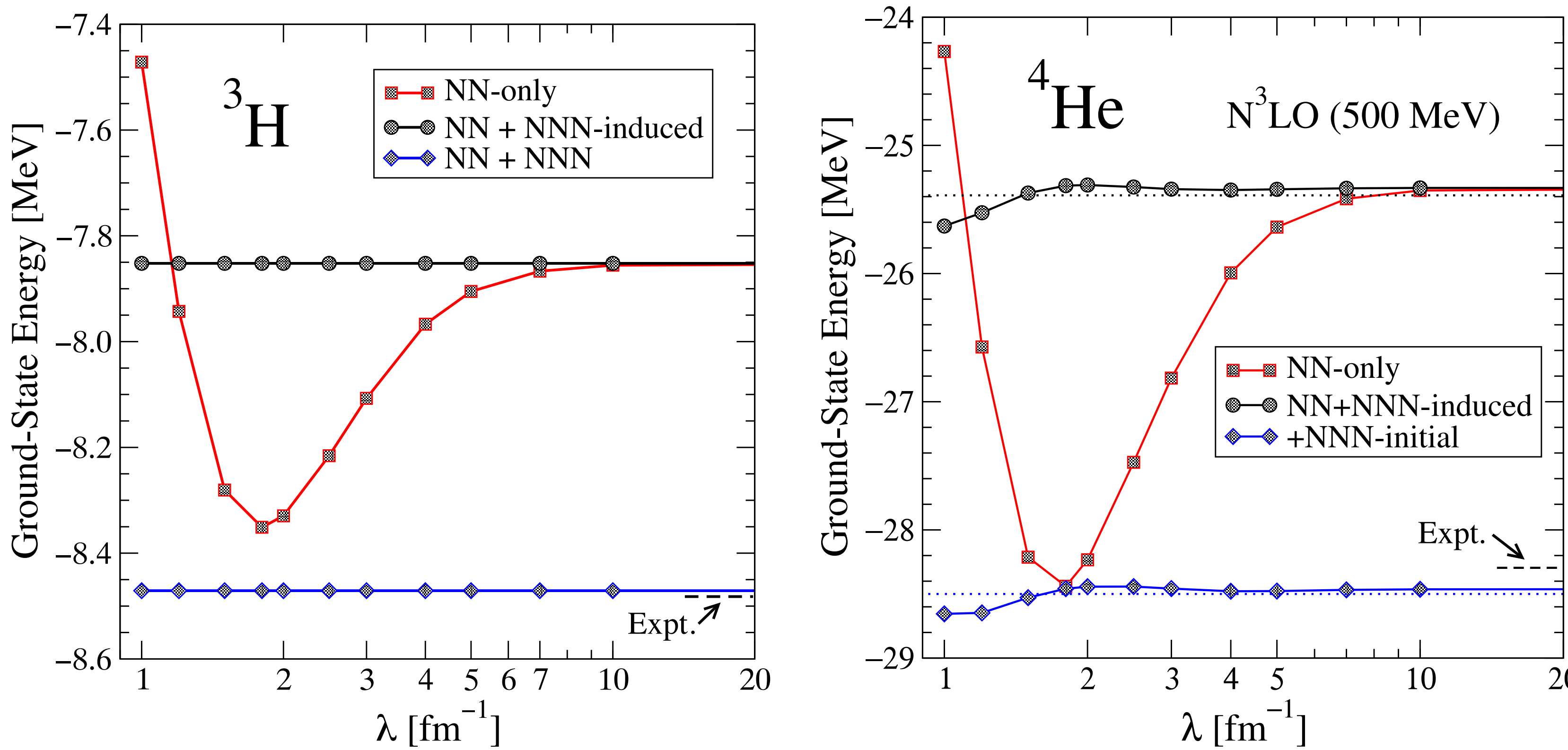


NN-only clear cutoff dependencs

3N-ind: dramatic reduction in cutoff dependence, no agreement with experiment

Induced 3N Forces

Effect of including 3N-ind? Exactly initial $V_{\rm NN}$ up to neglected 4N-ind



NN-only clear cutoff dependencs

3N-ind: dramatic reduction in cutoff dependence, no agreement with experiment

NN+3N-full retains cutoff independence, reproduces experiment!

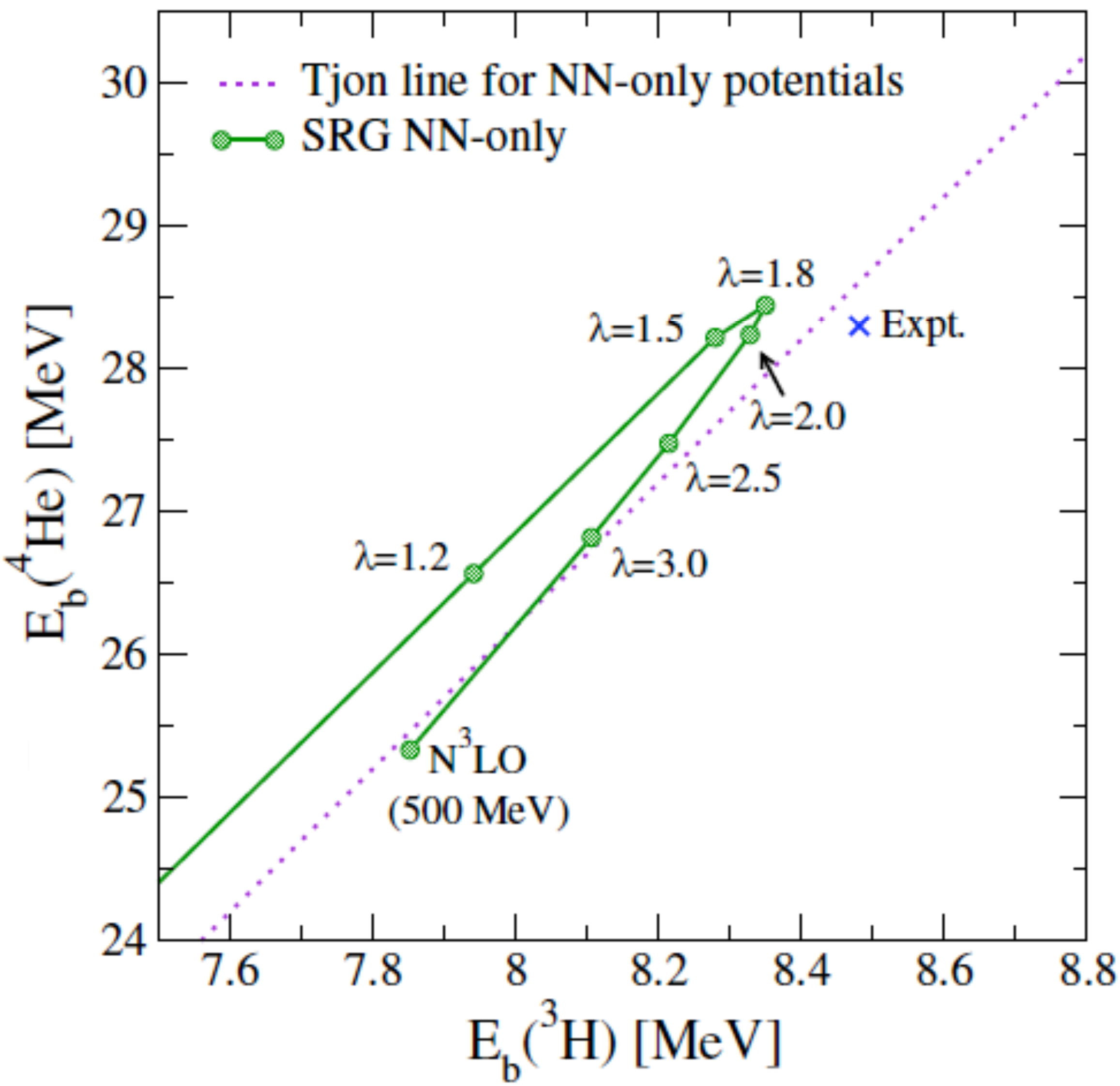
Benefits of Lower Cutoffs

Use cutoff dependence to assess missing physics: return to Tjon line

Varying cutoff moves along line

Still never reaches experiment

Tool, not a parameter!



Benefits of Lower Cutoffs

Use cutoff dependence to assess missing physics: return to Tjon line

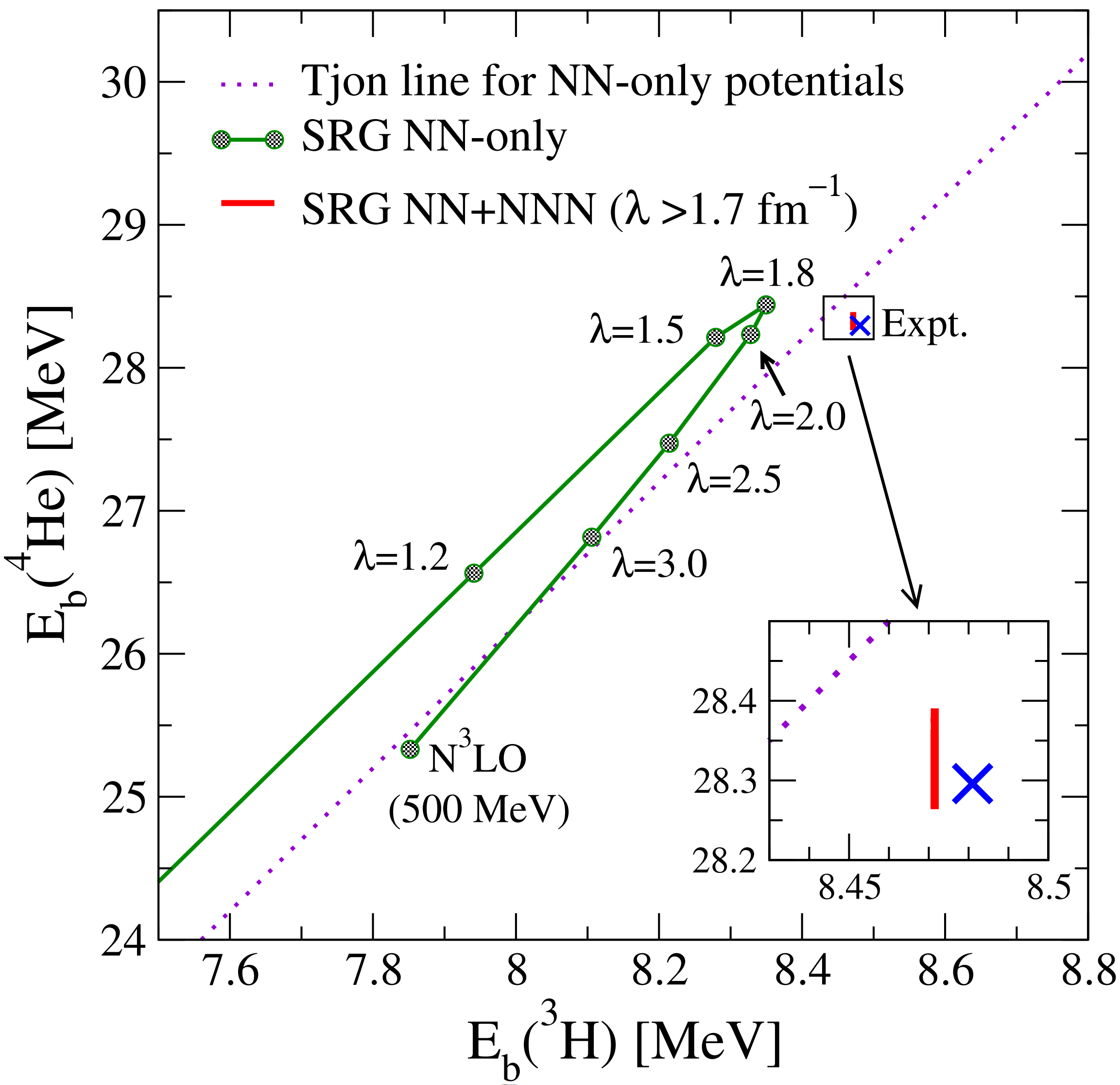
Varying cutoff moves along line

Still never reaches experiment

Tool, not a parameter!

Including 3N reaches expt.

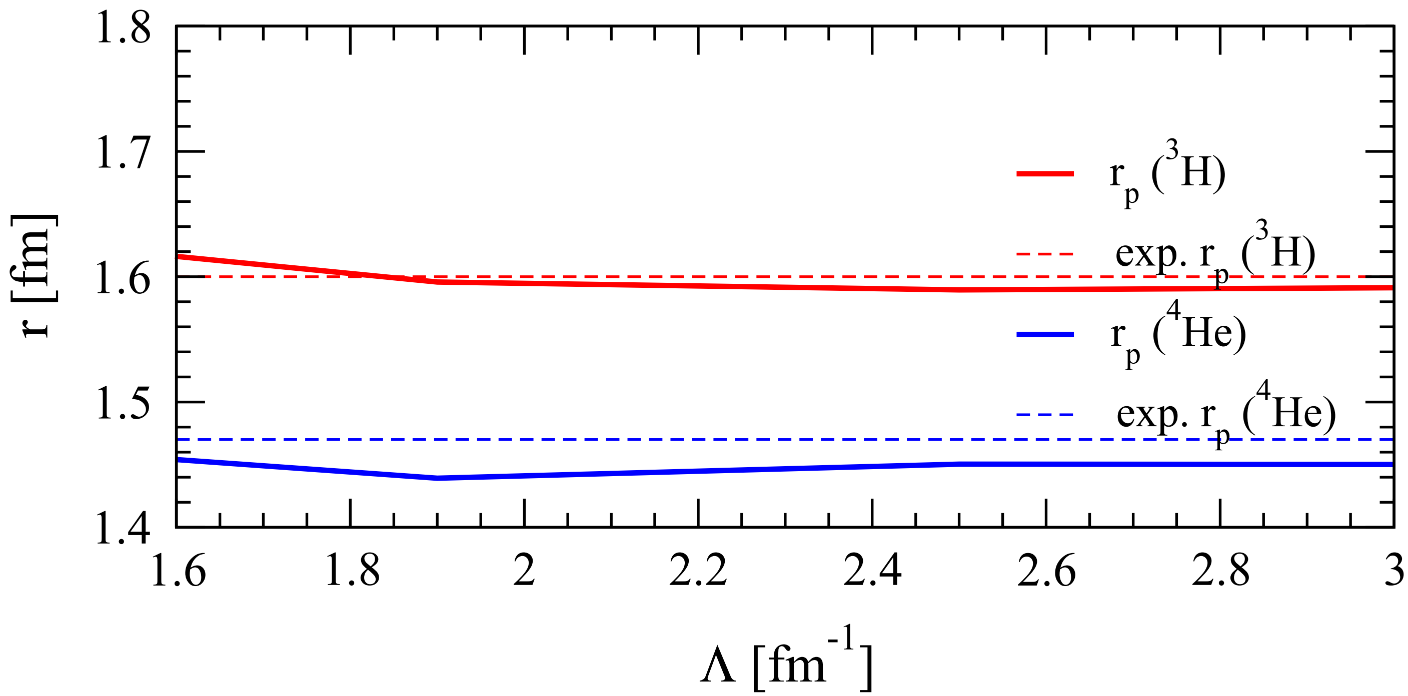
Why not perfect fit?



Cutoff Variation with 3N Forces

Use cutoff variation to assess missing physics in few body systems

Radii of triton and alpha particle calculated from NN+3N forces

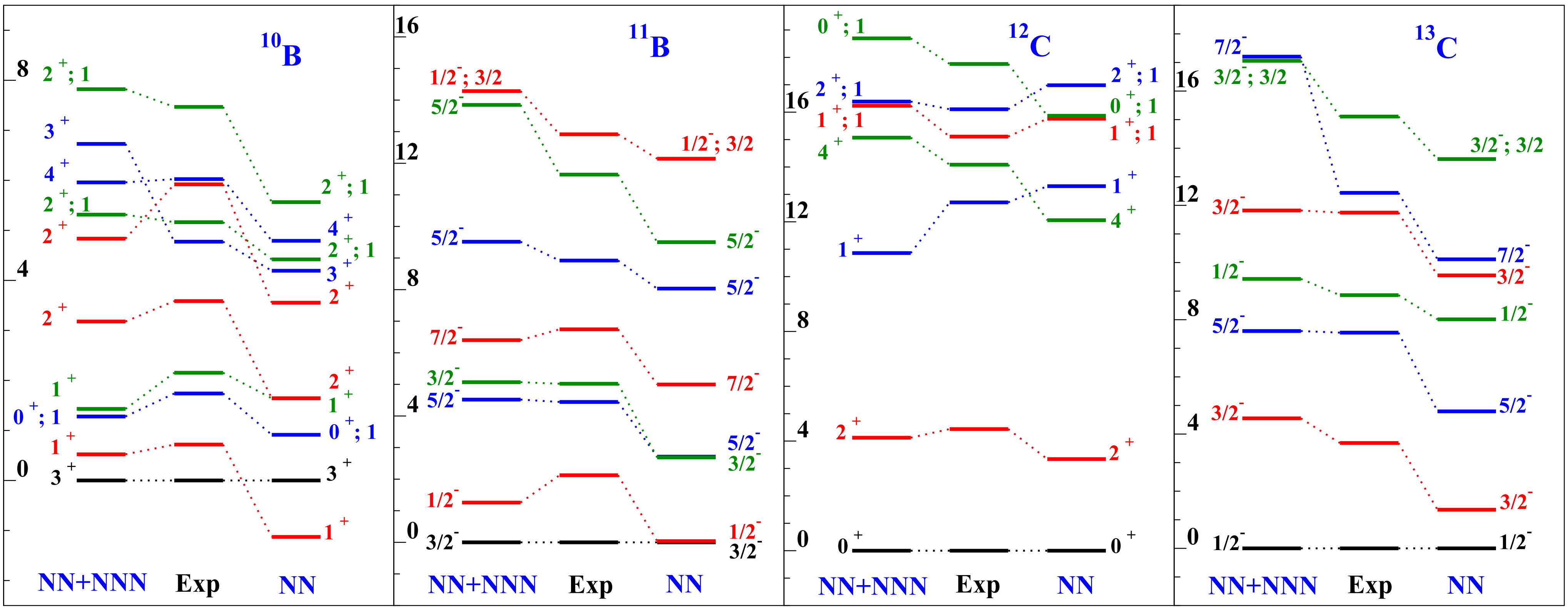


Minimal cutoff variation

Chiral Three-Body Forces in Light Nuclei

Importance of chiral 3N forces established in light nuclei

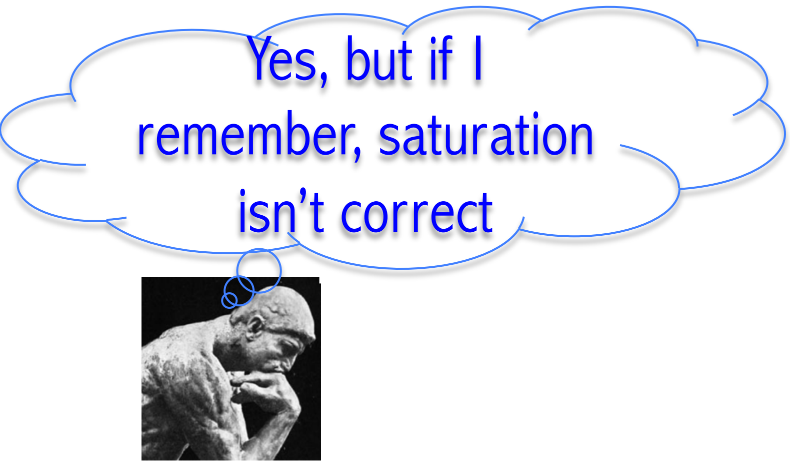
Converged NCSM (Navratil 2007)

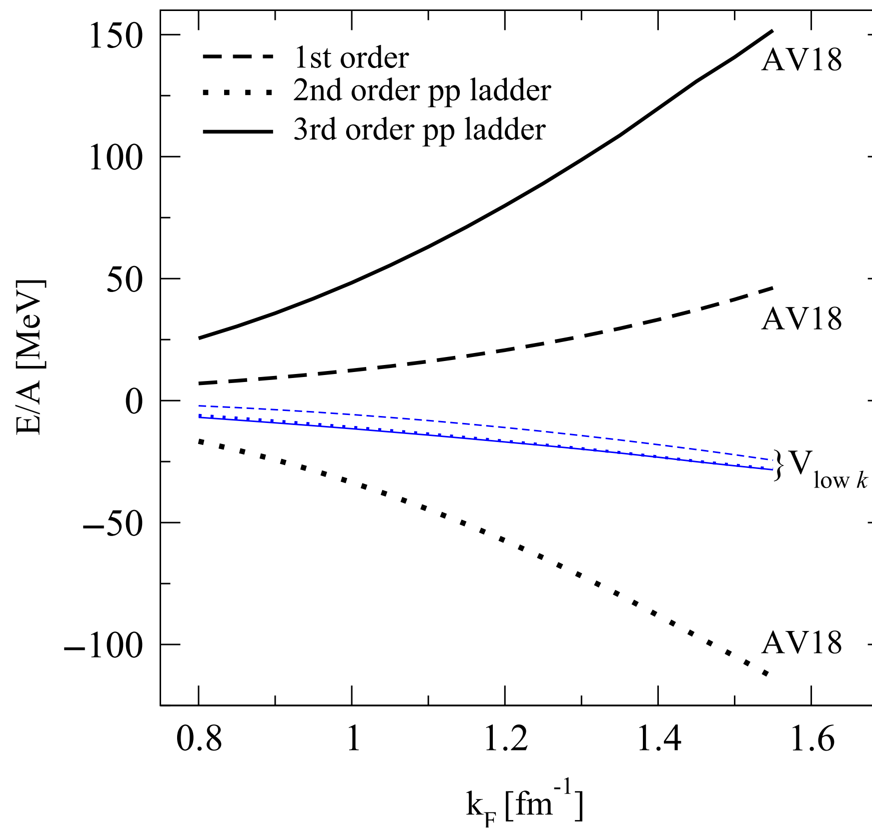


They work! What about nuclear matter?

Perturbative in Symmetric Nuclear Matter?

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{3\mathrm{N}}(\Lambda) + V_{4\mathrm{N}}(\Lambda) + \cdots$$

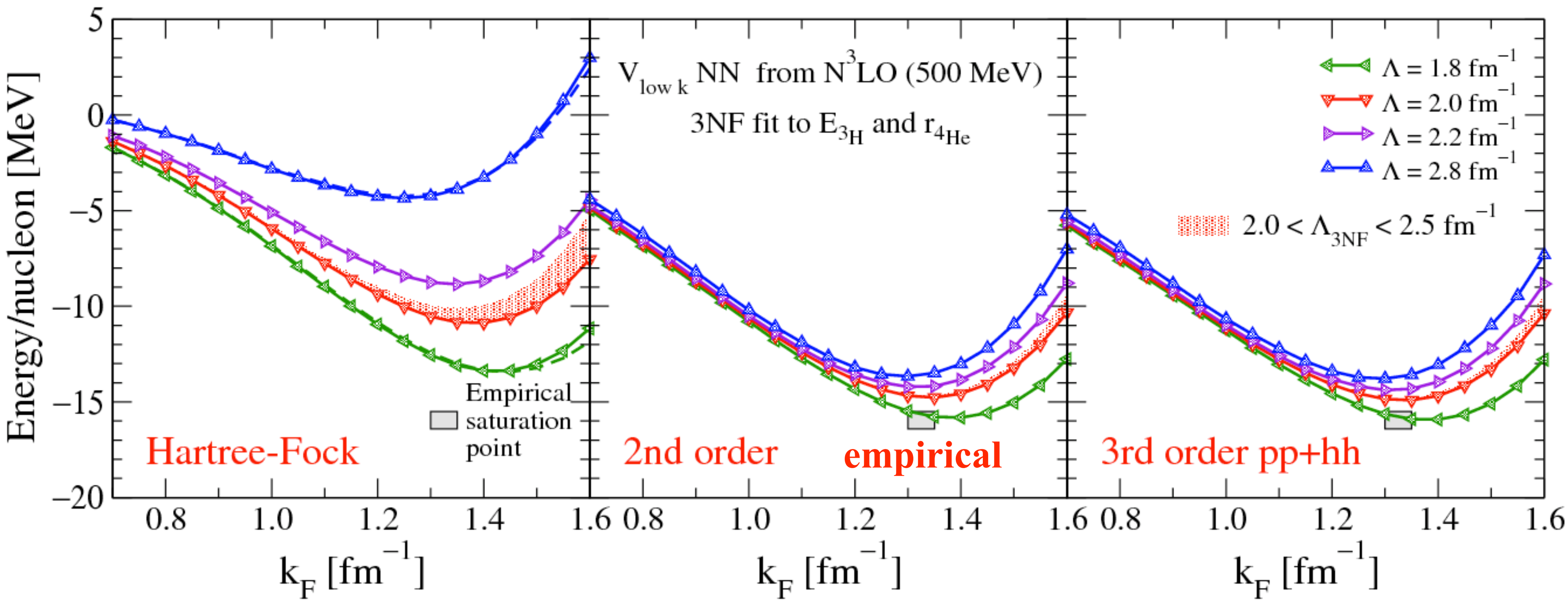




Significant improvement with low-momentum interactions!

Perturbative in Symmetric Nuclear Matter?

$$H(\Lambda) = T + V_{\rm NN}(\Lambda) + V_{\rm 3N}(\Lambda) + V_{\rm 4N}(\Lambda) + \cdots$$



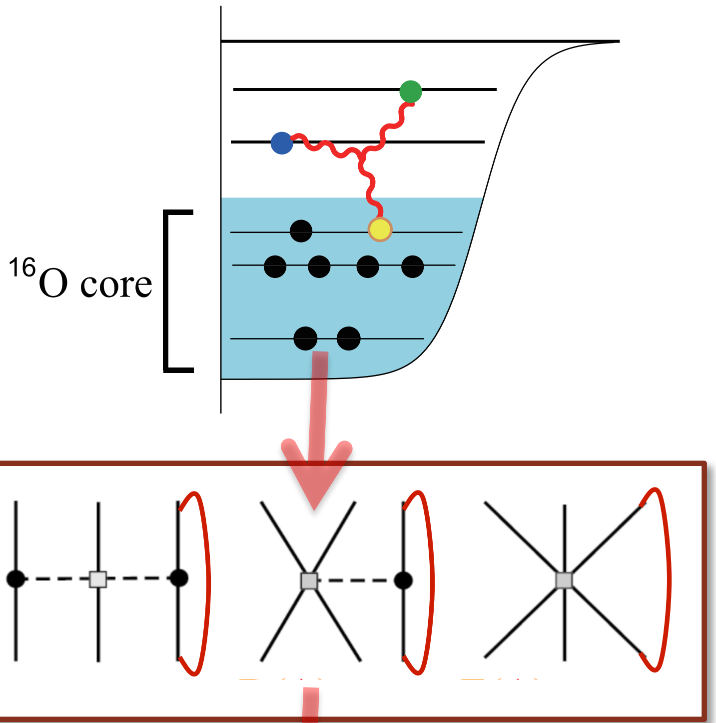
Now NN+3N-fit remain perturbative and reproduce saturation!

Minor but non-negligible cutoff variation

3N Forces for Valence-Shell Theories

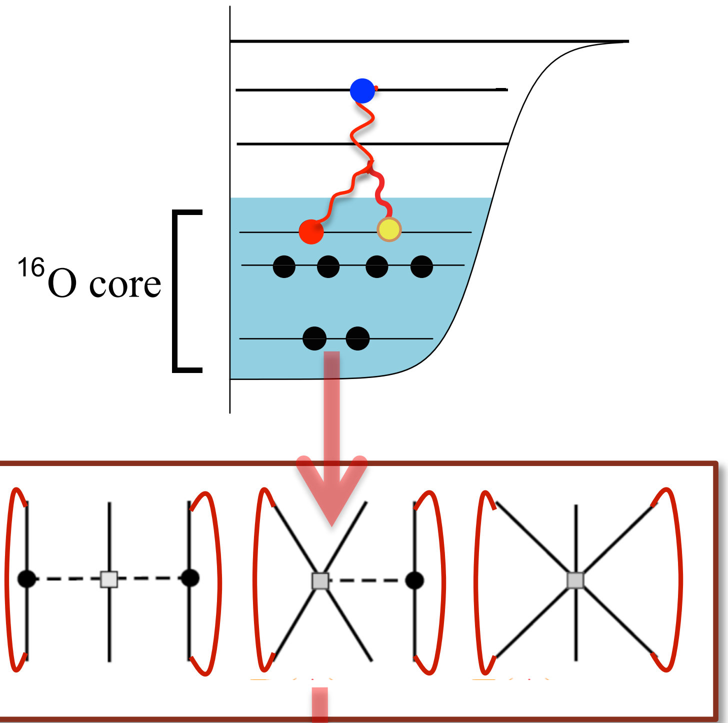
Normal-ordered 3N: contribution to valence neutron interactions

Effective two-body



$$\langle ab|\, V_{3N,\mathrm{eff}}\,|a'b'\rangle = \sum_{\alpha=core}\langle \alpha ab|\, V_{3N}\,|\alpha a'b'\rangle$$

Effective one-body



$$\langle a|\, V_{3N,\mathrm{eff}}\,|a'\rangle = \frac{1}{2}\sum_{\alpha\beta=core}\langle \alpha\beta a|\, V_{3N}\,|\alpha\beta a'\rangle$$

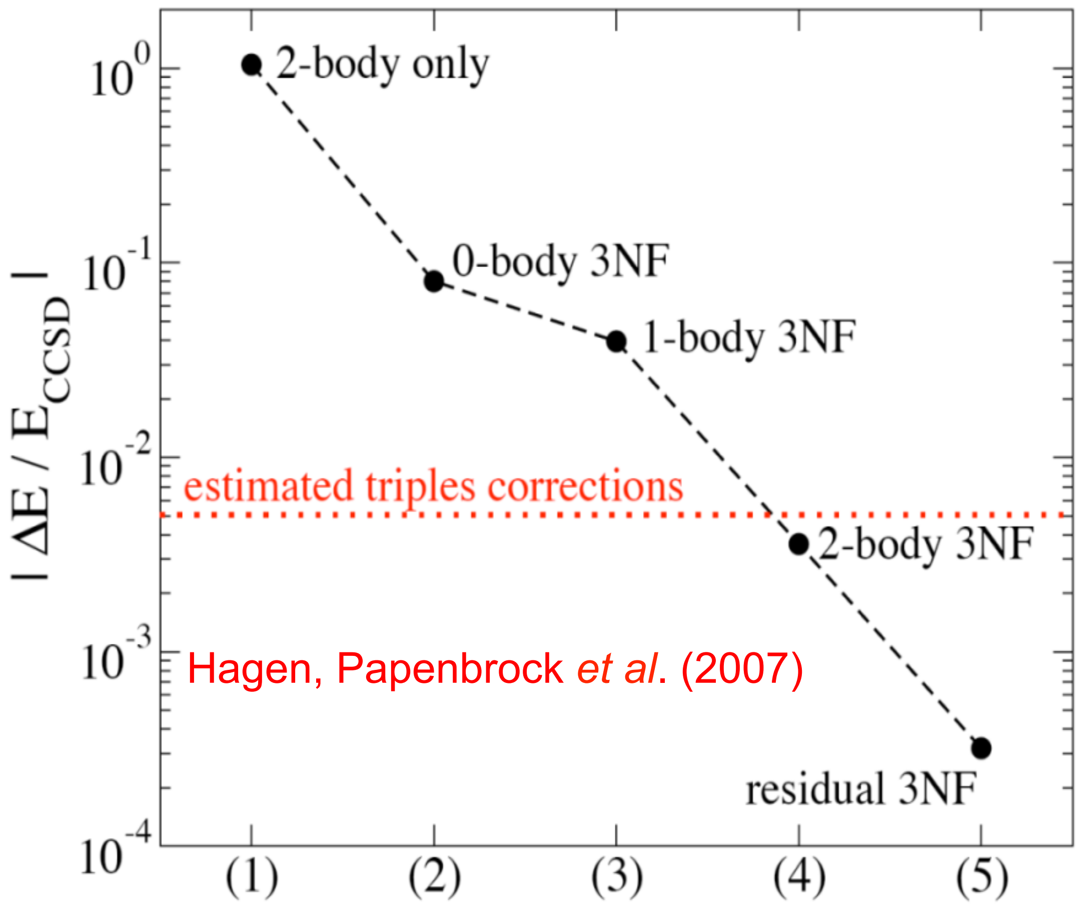
Combine with microscopic NN: eliminate empirical adjustments

3N Forces for Valence-Shell Theories

Effects of residual 3N between 3 valence nucleons?

Normal-ordered 3N: microscopic contributions to inputs for CI Hamiltonian

Effects of residual 3N between 3 valence nucleons?



Coupled-Cluster theory with 3N: benchmark of ^{4}He

0- 1- and 2-body of 3NF dominate

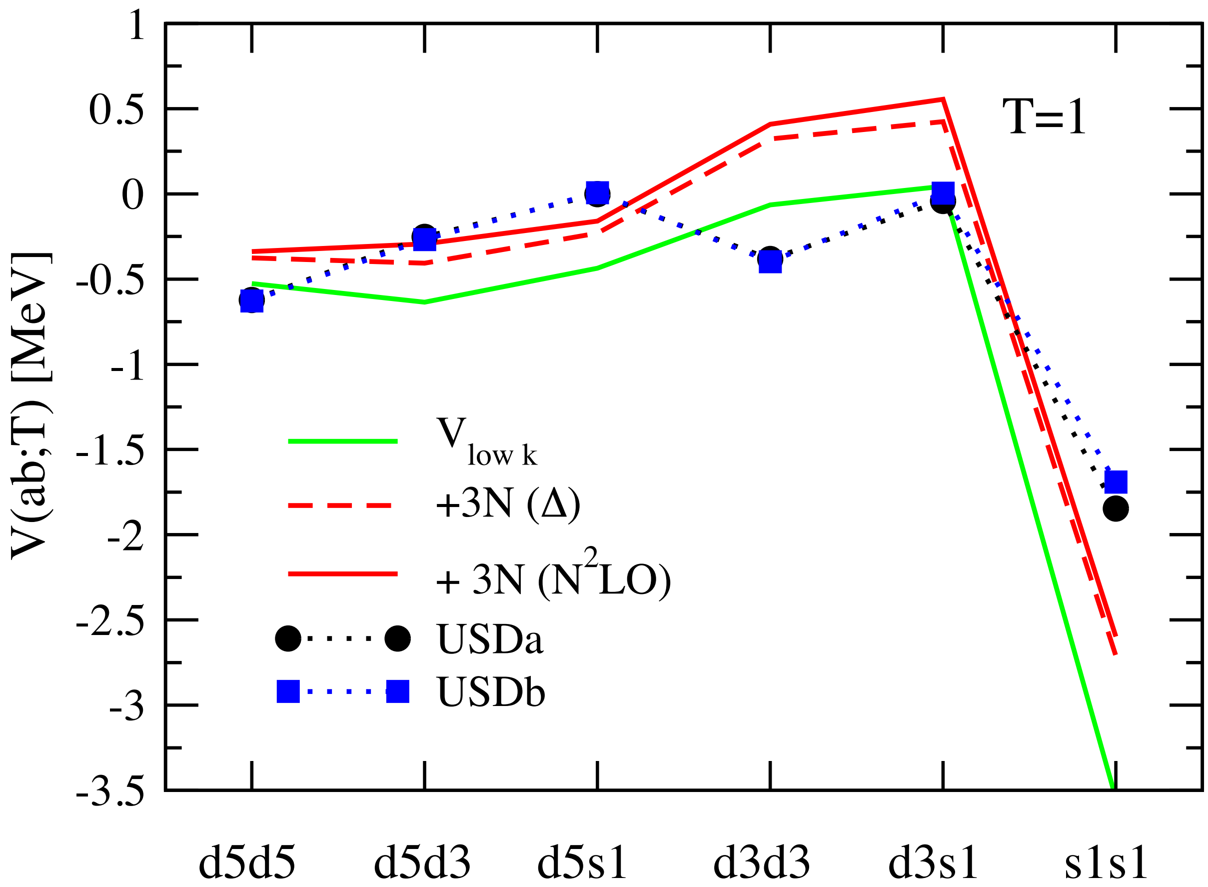
Residual 3N can be neglected

Work on ^{16}O in progress

Approximated residual 3N by summing over valence nucleon

– Nucleus-dependent: effect small, not negligible by ^{24}O

Two-body 3N: Monopoles in *sd*-shell



Dominant effect from **one-Δ** – as expected from cutoff variation

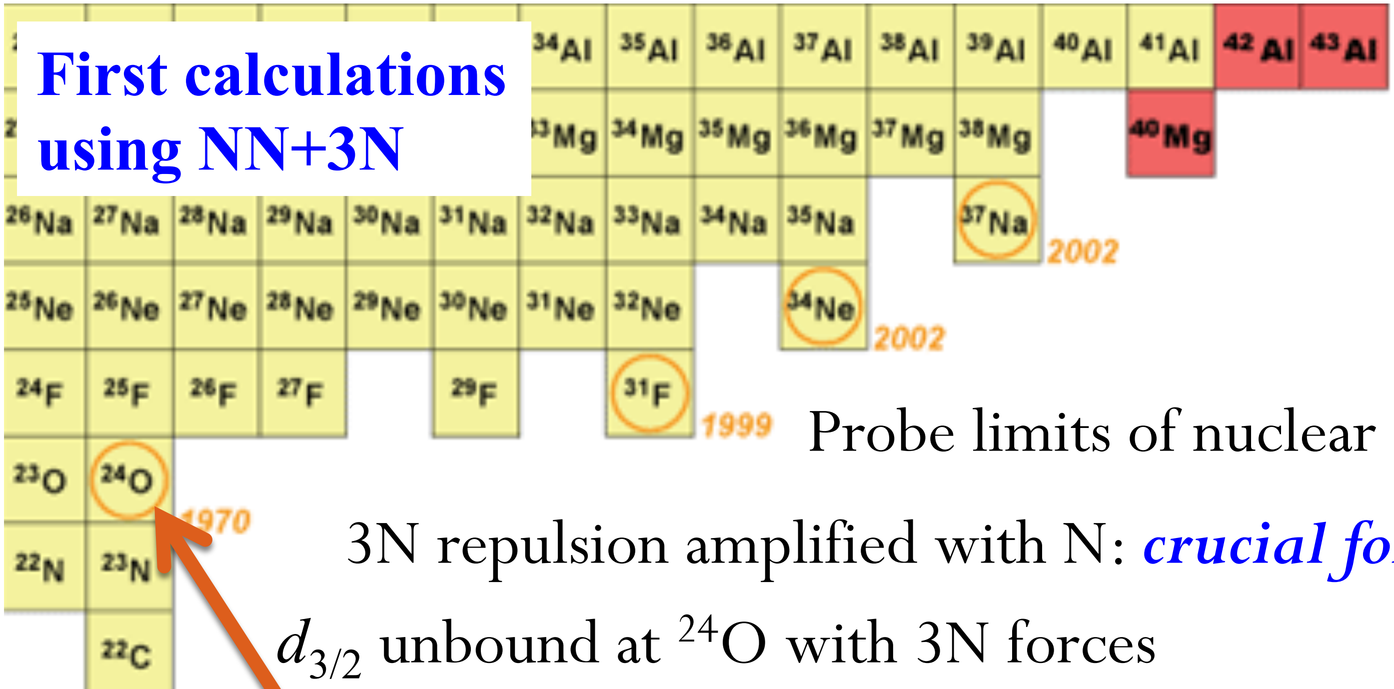
3N forces produce clear repulsive shift in monopoles

First calculations to show missing monopole strength due to neglected 3N

Future: Improved treatment of high-lying orbits

Oxygen Anomaly

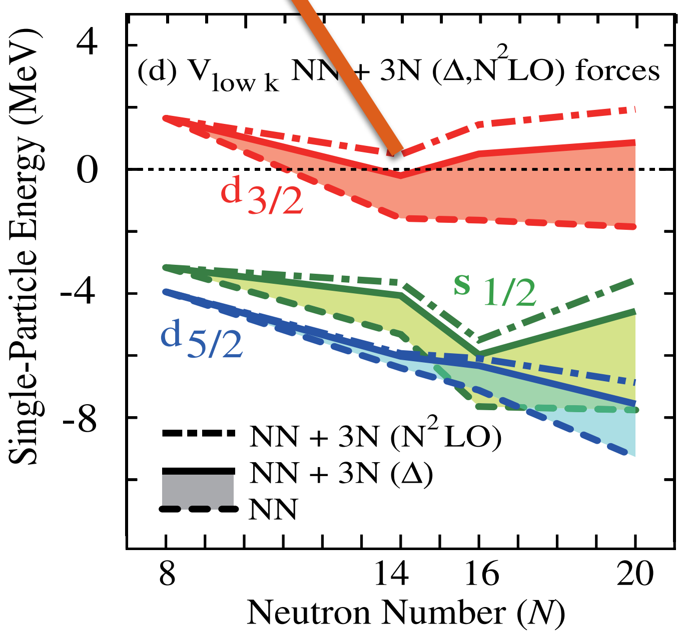
First calculations using NN+3N



Probe limits of nuclear existence with 3N forces

3N repulsion amplified with N: ***crucial for neutron-rich nuclei***

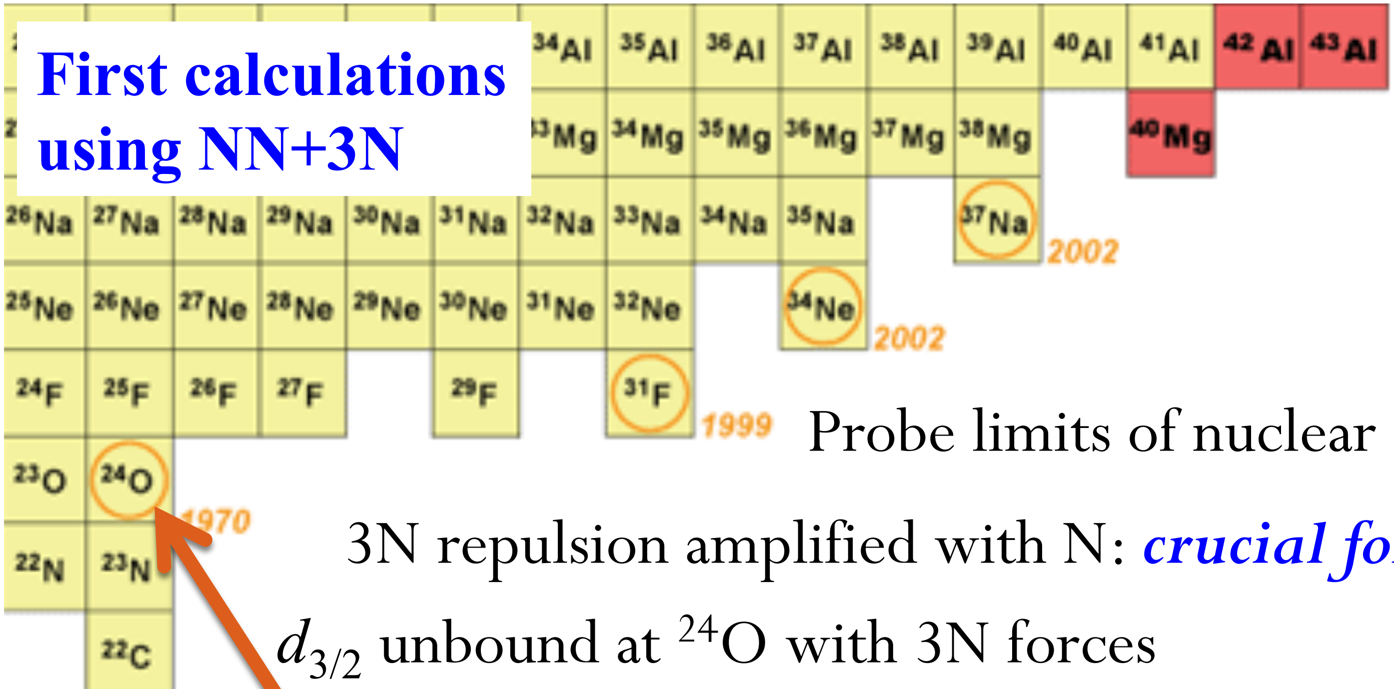
$d_{3/2}$ unbound at ^{24}O with 3N forces



Otsuka, Suzuki, JDH, Schwenk, Akaishi, PRL (2010)

Oxygen Anomaly

First calculations using NN+3N



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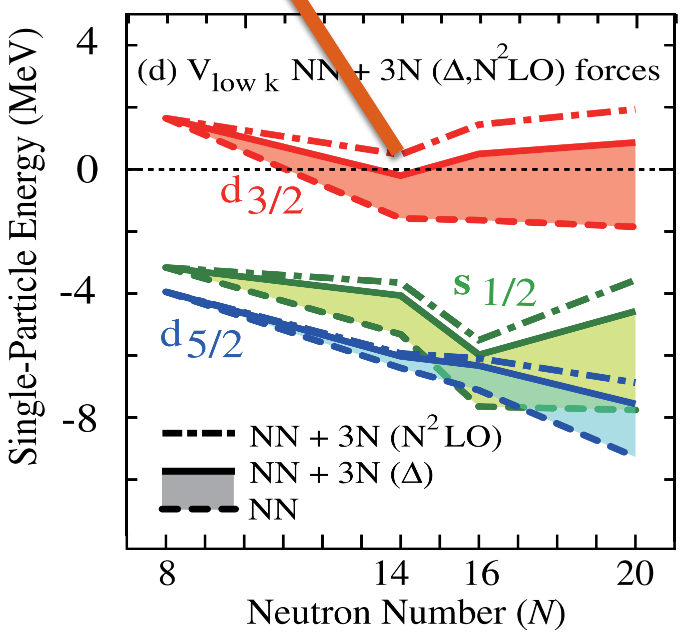
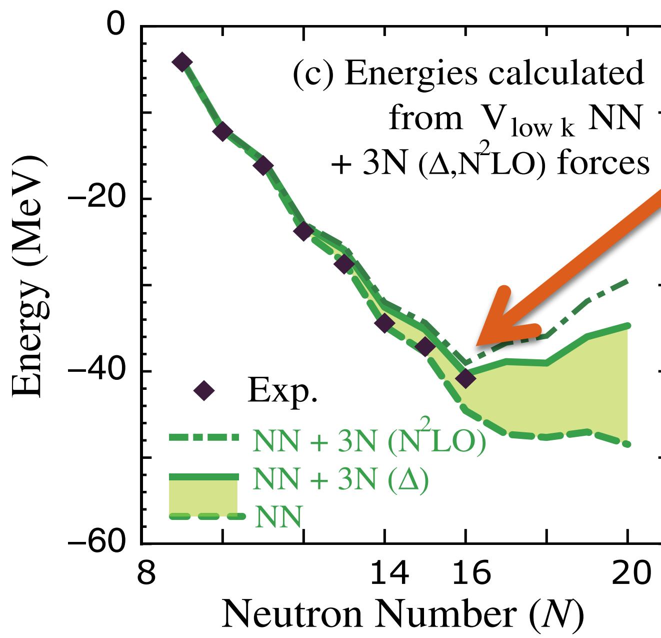




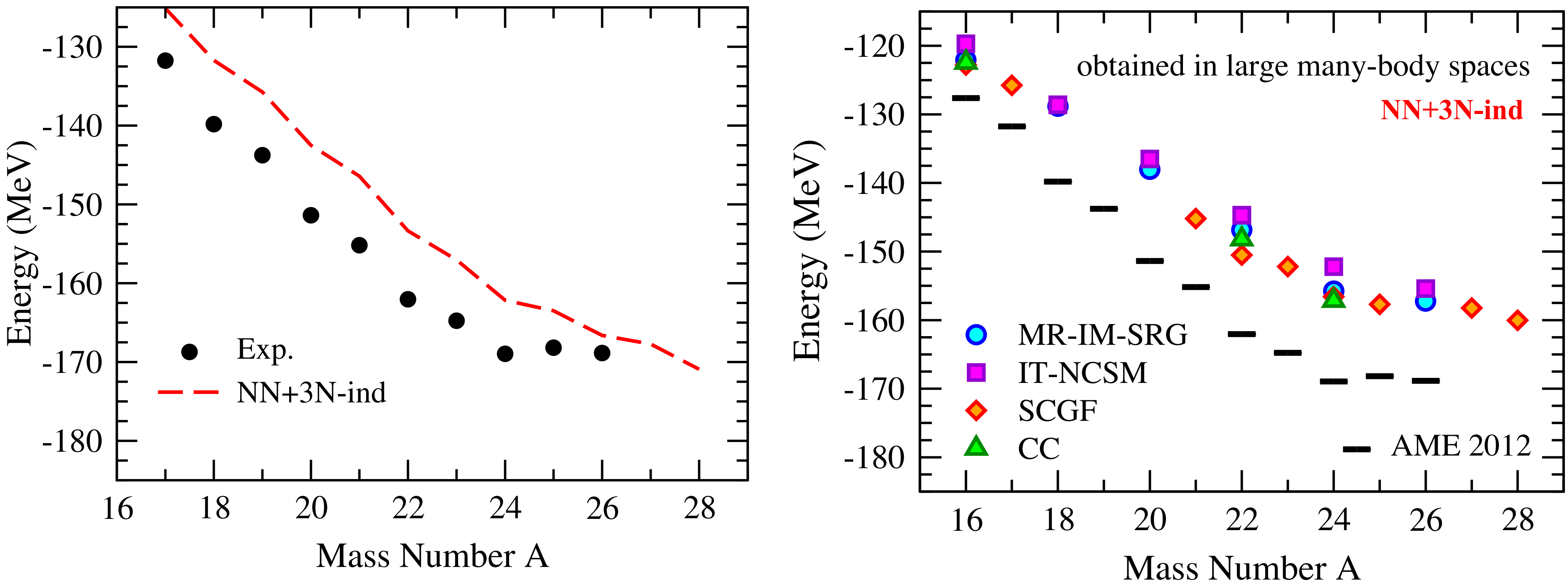
Isotopes unbound beyond ^{24}O

First microscopic explanation of oxygen anomaly

Otsuka, Suzuki, JDH, Schwenk, Akaishi, PRL (2010)

Comparison with Large-Space Methods

Large-space methods with **same SRG-evolved NN+3N-ind forces**



Agreement between all methods with same input forces

No reproduction of dripline in any case

Normal-Ordered Hamiltonian

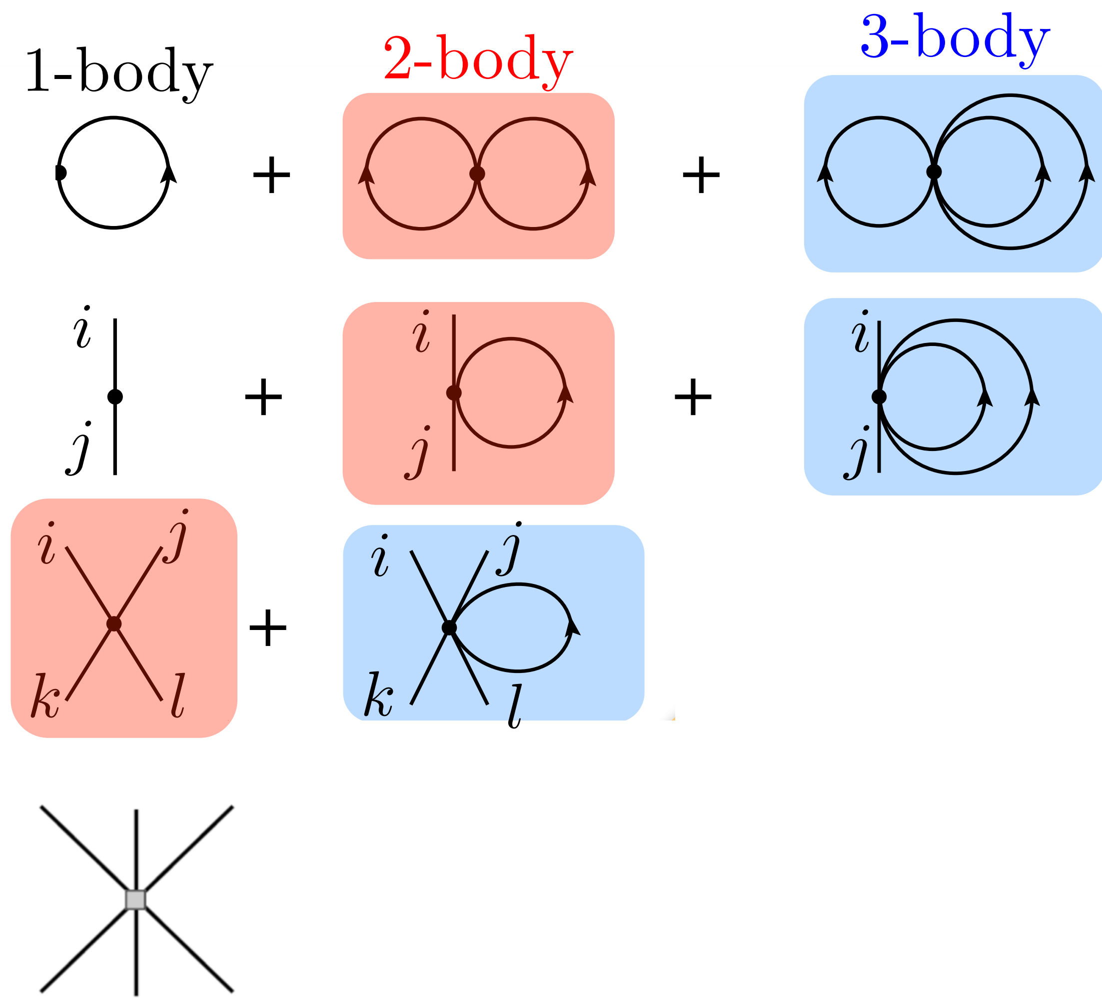
Now rewrite exactly the initial Hamiltonian in normal-ordered form

$$H_{\text{N.O.}} = E_0 + \sum_{ij} f_{ij} \left\{ a_i^\dagger a_j \right\} + \frac{1}{4} \sum_{jkl} \Gamma_{ijkl} \left\{ a_i^\dagger a_j^\dagger a_l a_k \right\} + \frac{1}{36} \sum_{ijklmn} W_{ijklmn} \left\{ a_i^\dagger a_j^\dagger a_k^\dagger a_l a_m a_n \right\}$$

N.O. 0-body $\rightarrow E_0 =$

N.O. 1-body $\rightarrow \quad f =$

N.O. 2-body $\rightarrow \quad \Gamma =$



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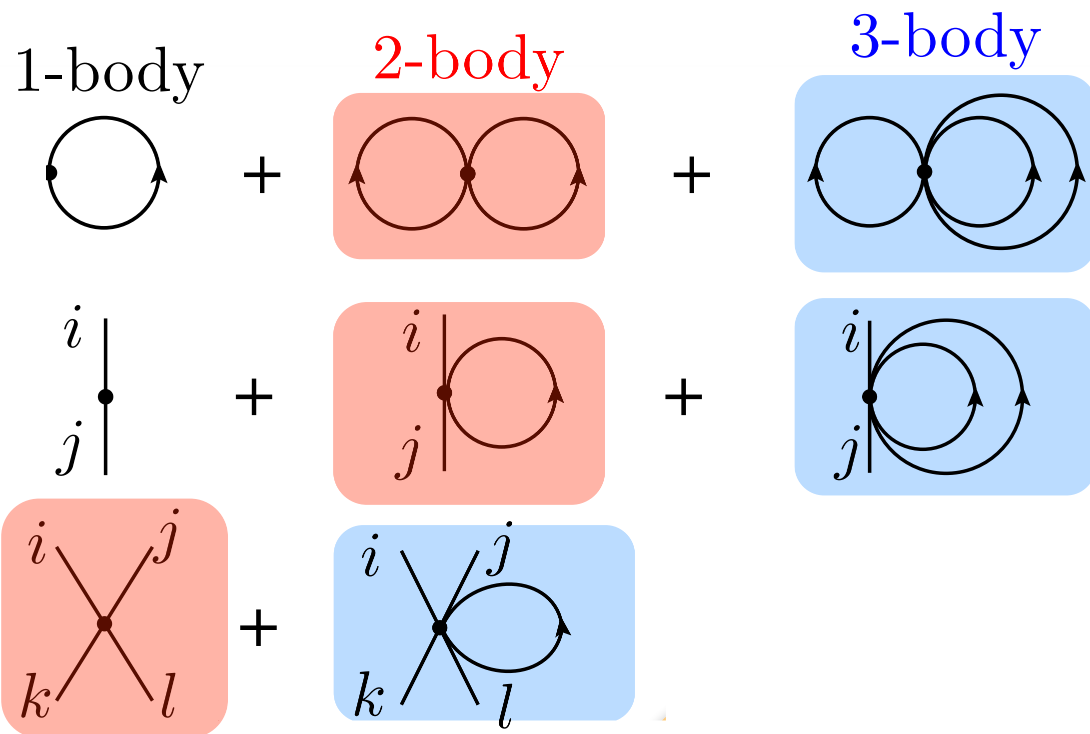
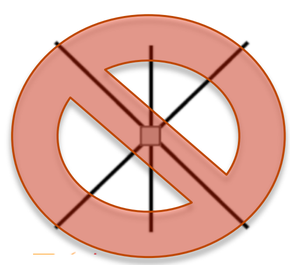
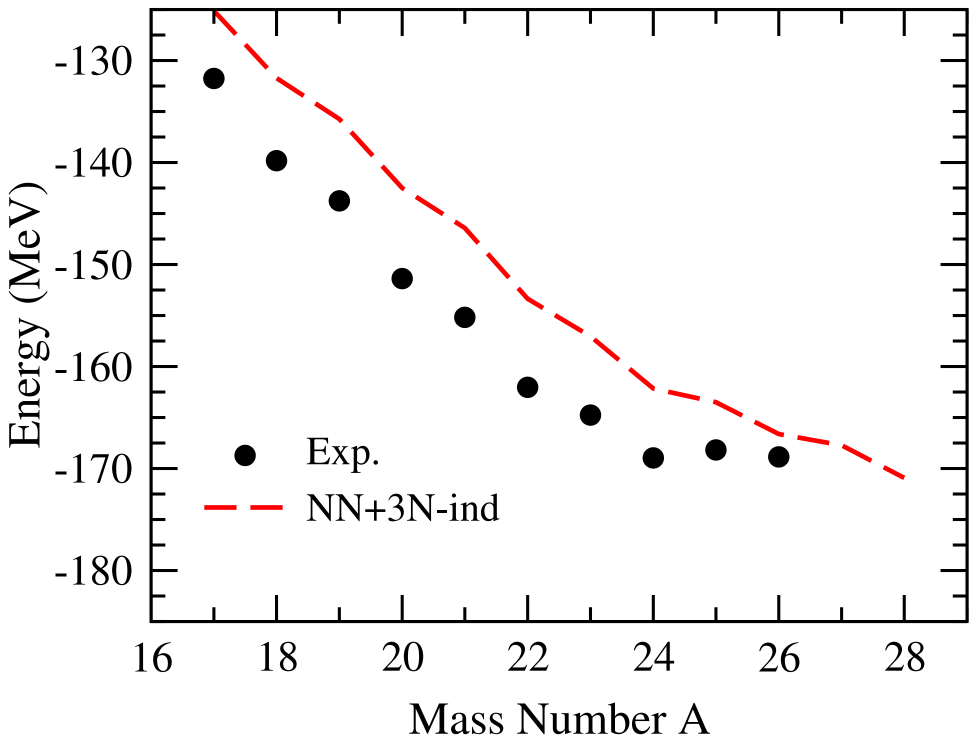


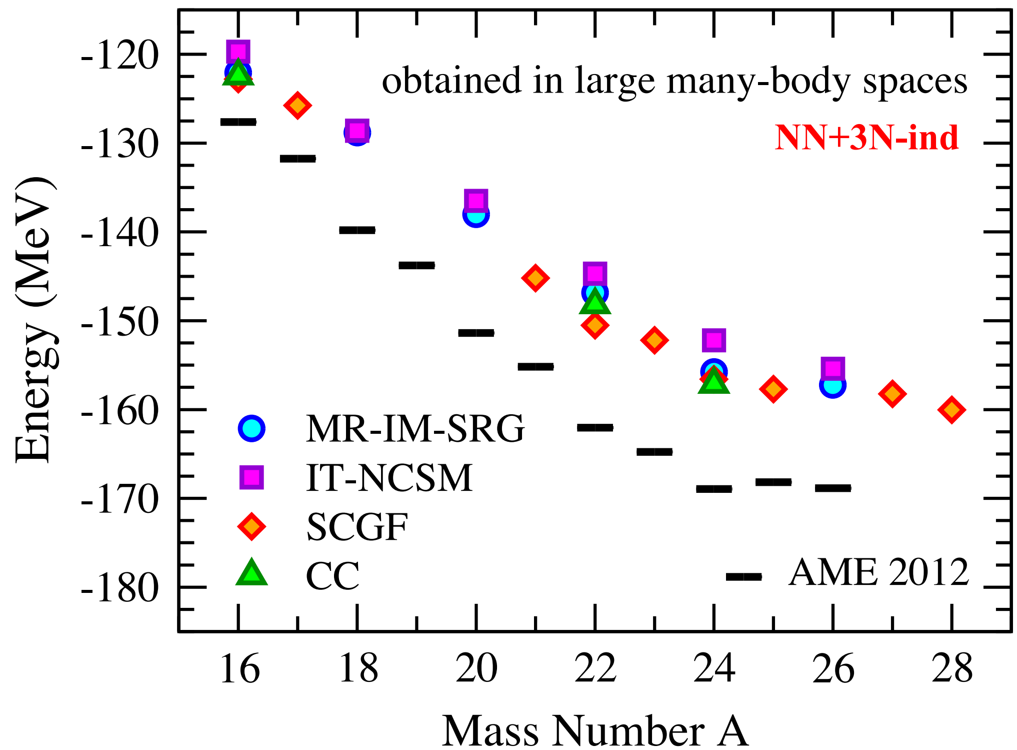
Neglect residual 3N



Comparison with Large-Space Methods

Large-space methods with **same SRG-evolved NN+3N-ind forces**



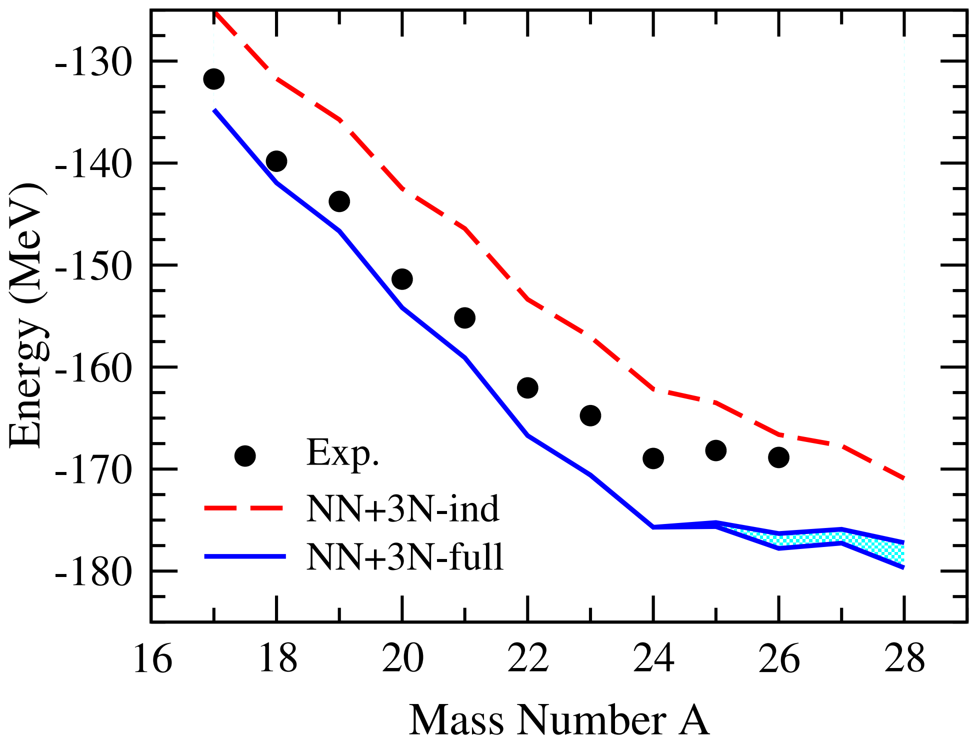


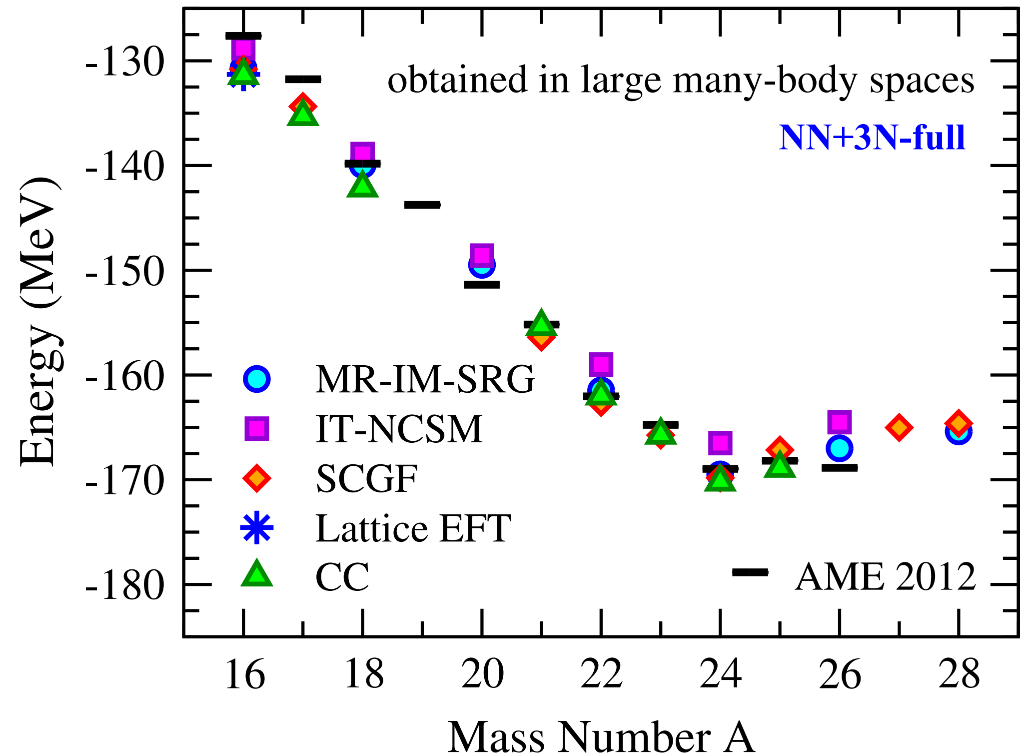
Agreement between all methods with same input forces

No reproduction of dripline in any case

Comparison with Large-Space Methods

Large-space methods with **same SRG-evolved NN+3N-full forces**





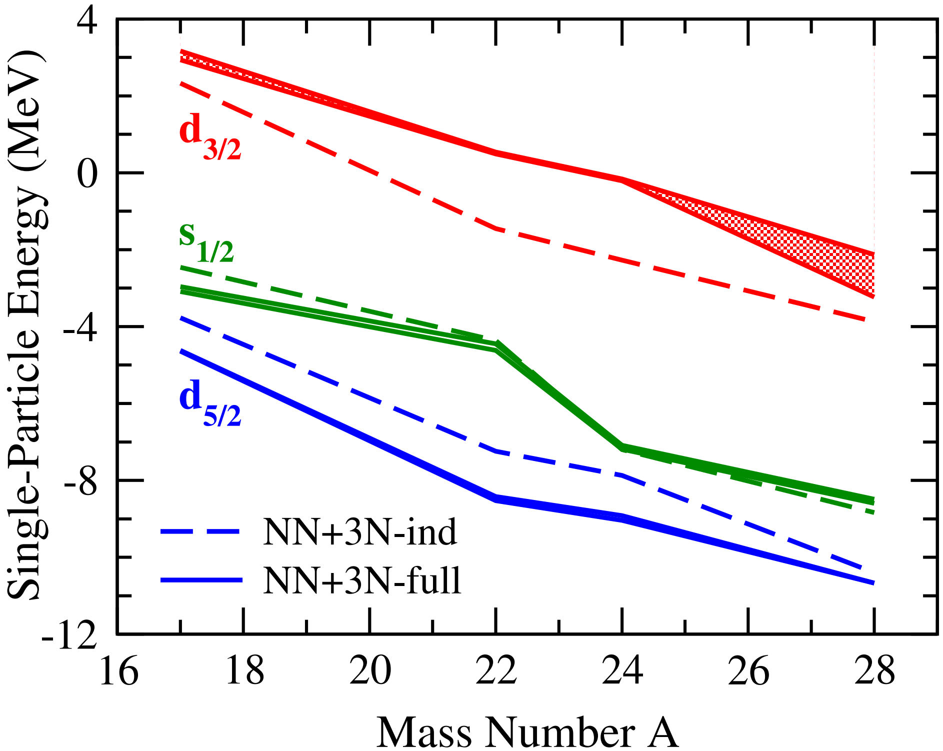
Agreement between all methods with same input forces

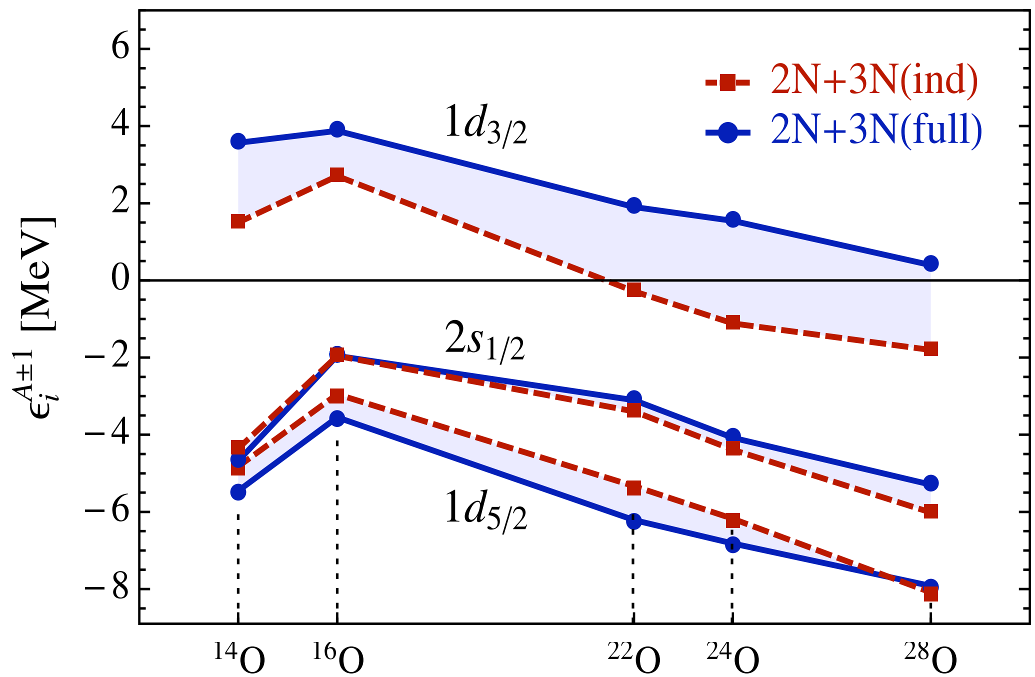
Clear improvement with NN+3N-full

Validates valence-space results

Oxygen Dripline Mechanism

Self-consistent Green's Function with **same SRG-evolved NN+3N forces**





Cipollone, Barbieri, Navrátil, PRL (2013)

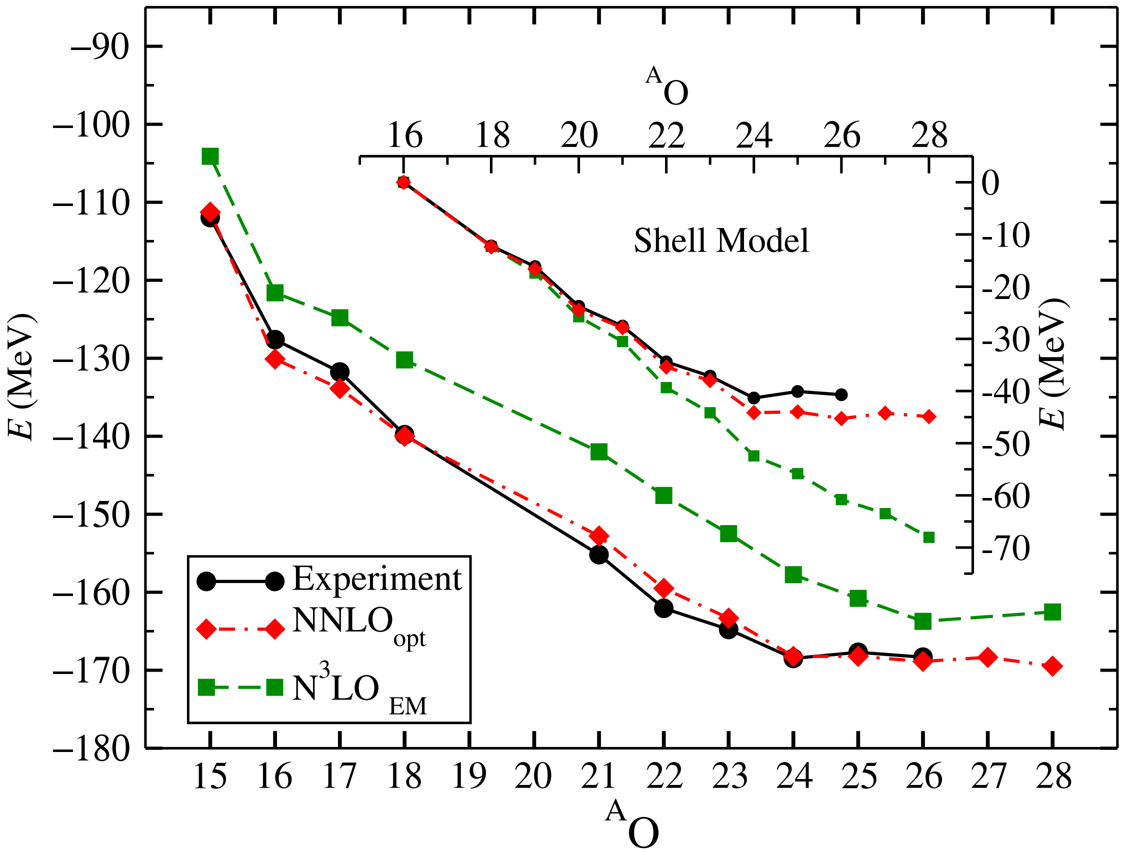
Robust mechanism driving dripline behavior

3N repulsion raises $d_{3/2}$, lessens decrease across shell

Similar to first MBPT NN+3N calculations in oxygen

Optimized Chiral Forces N^{2}LO NN-Only

Recent calculations at N^{2}LO without 3N forces found a remarkable result



Ekström et al (PRL 2013)

Oxygen dripline reproduced with NN forces only!

What does this mean about 3N?

Optimized Chiral Forces N^{2}LO NN-Only

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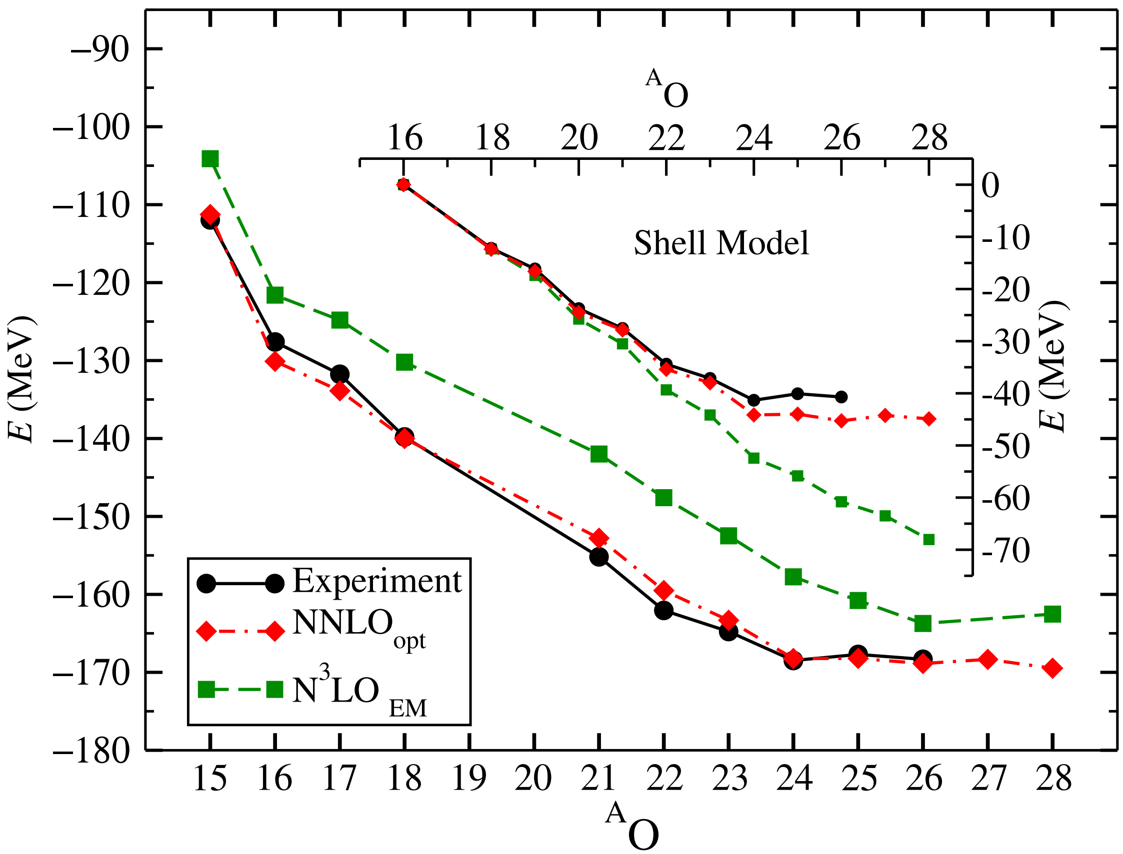
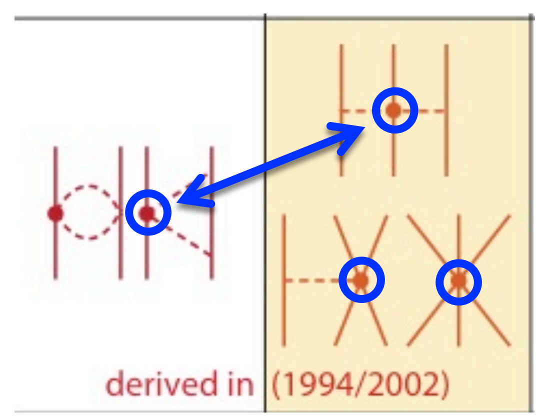


Ekström et al (PRL 2013)

Oxygen dripline reproduced with NN forces only!

Power counting dictates 3N forces be included



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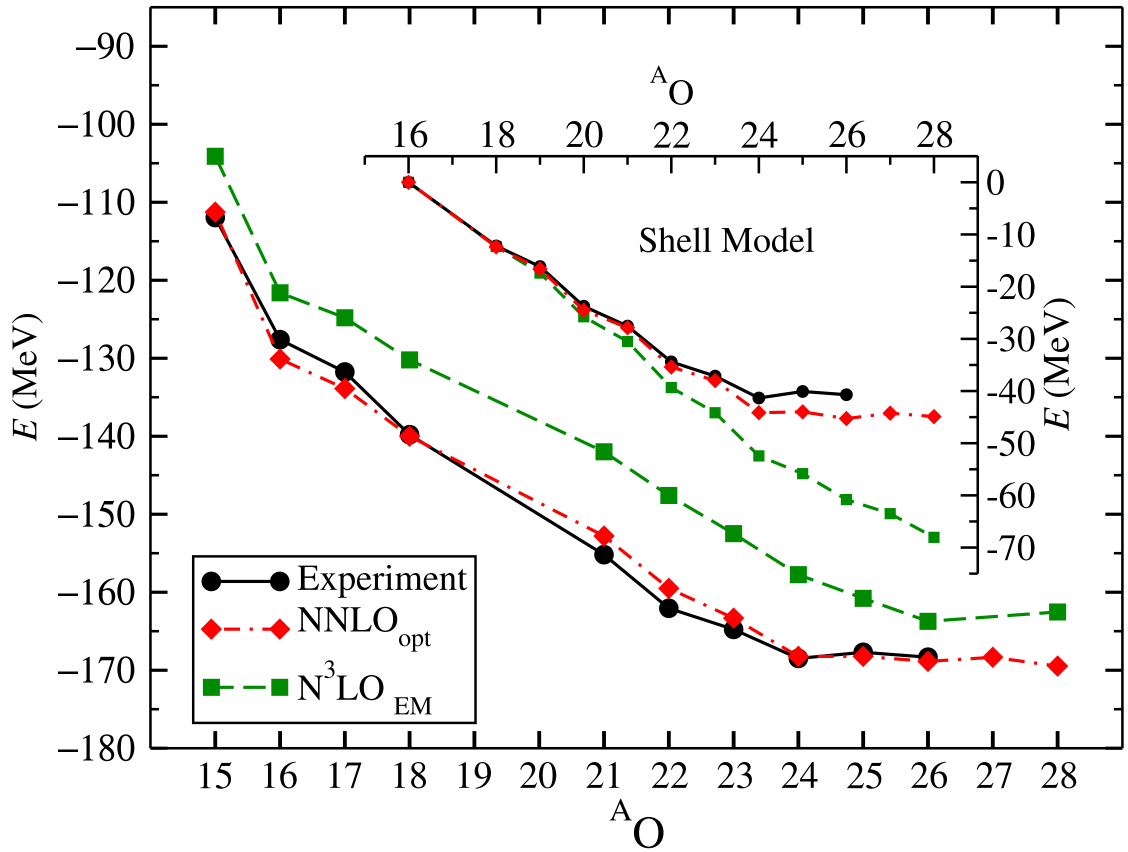


Ekström et al (PRL 2013)

Oxygen dripline reproduced with NN forces only

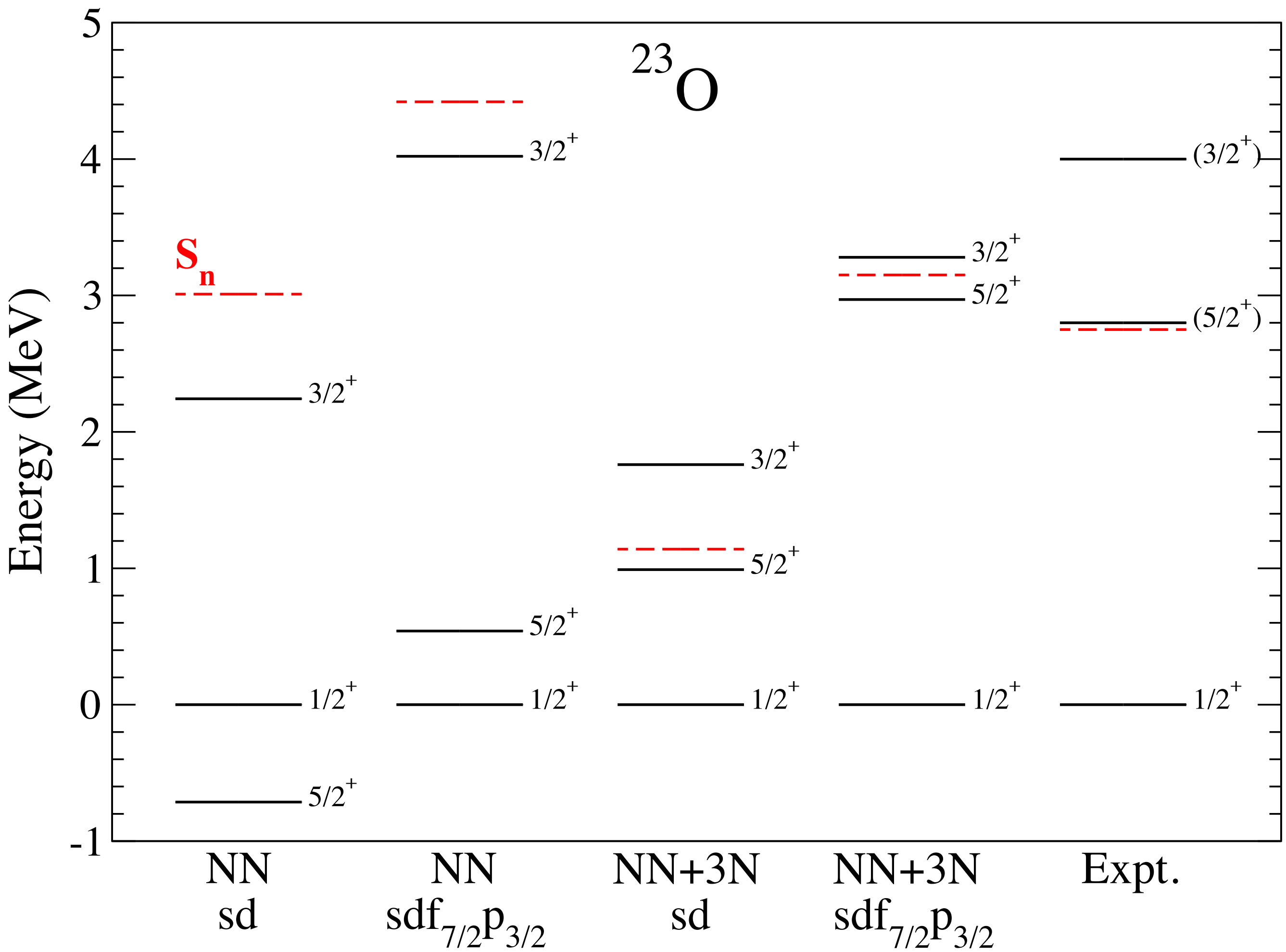
Unnaturally large couplings when 3N fit in ^{3}H(?) – results off the plot!

Lesson: 3N forces unavoidable part of theory – must investigate importance

Impact on Spectra: ^{23}O

Neutron-rich oxygen spectra with NN+3N

$5/2^+$, $3/2^+$ energies reflect ^{22,24}O shell closures



***sd*-shell NN only**

Wrong ground state

$5/2^+$ too low

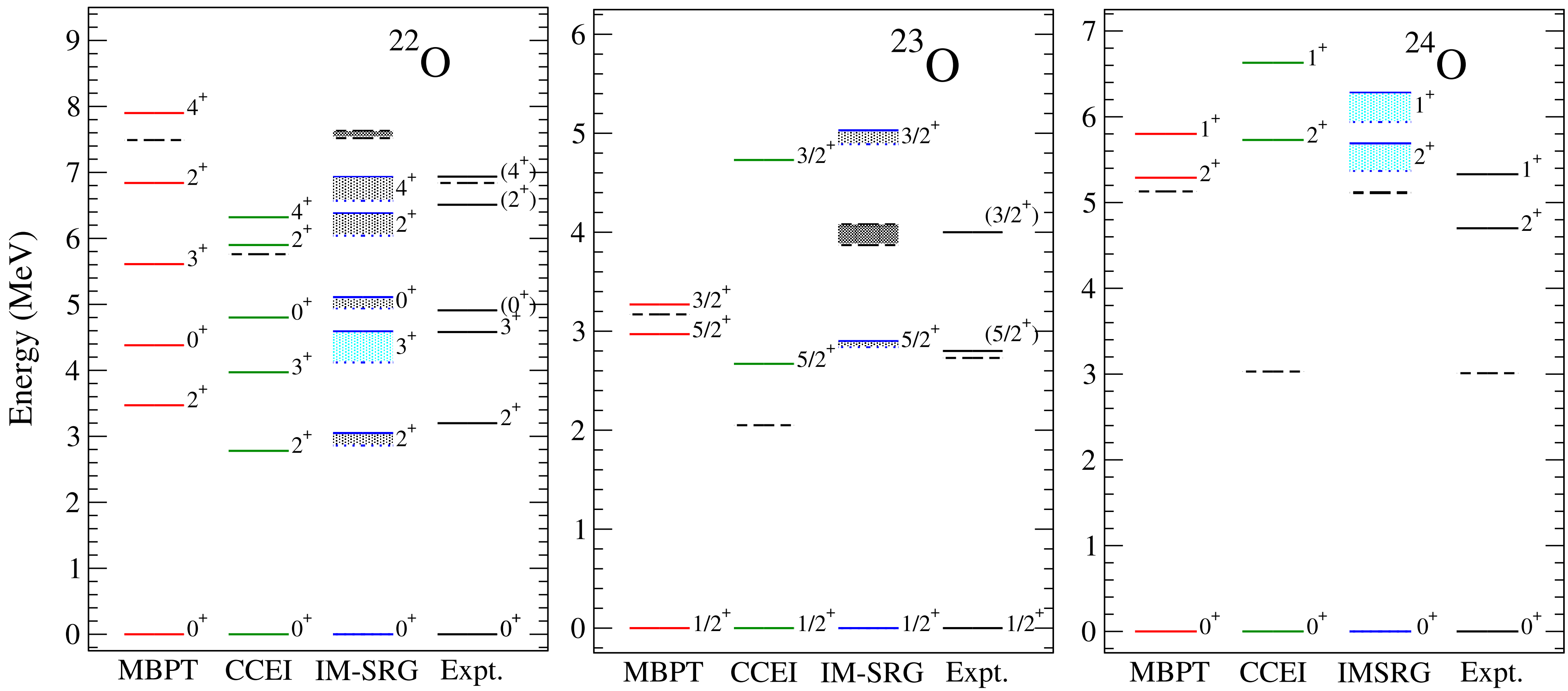
$3/2^+$ bound

NN+3N

Clear improvement in extended valence space

Comparison with MBPT/CCEI Oxygen Spectra

Oxygen spectra: Effective interactions from **Coupled-Cluster theory**

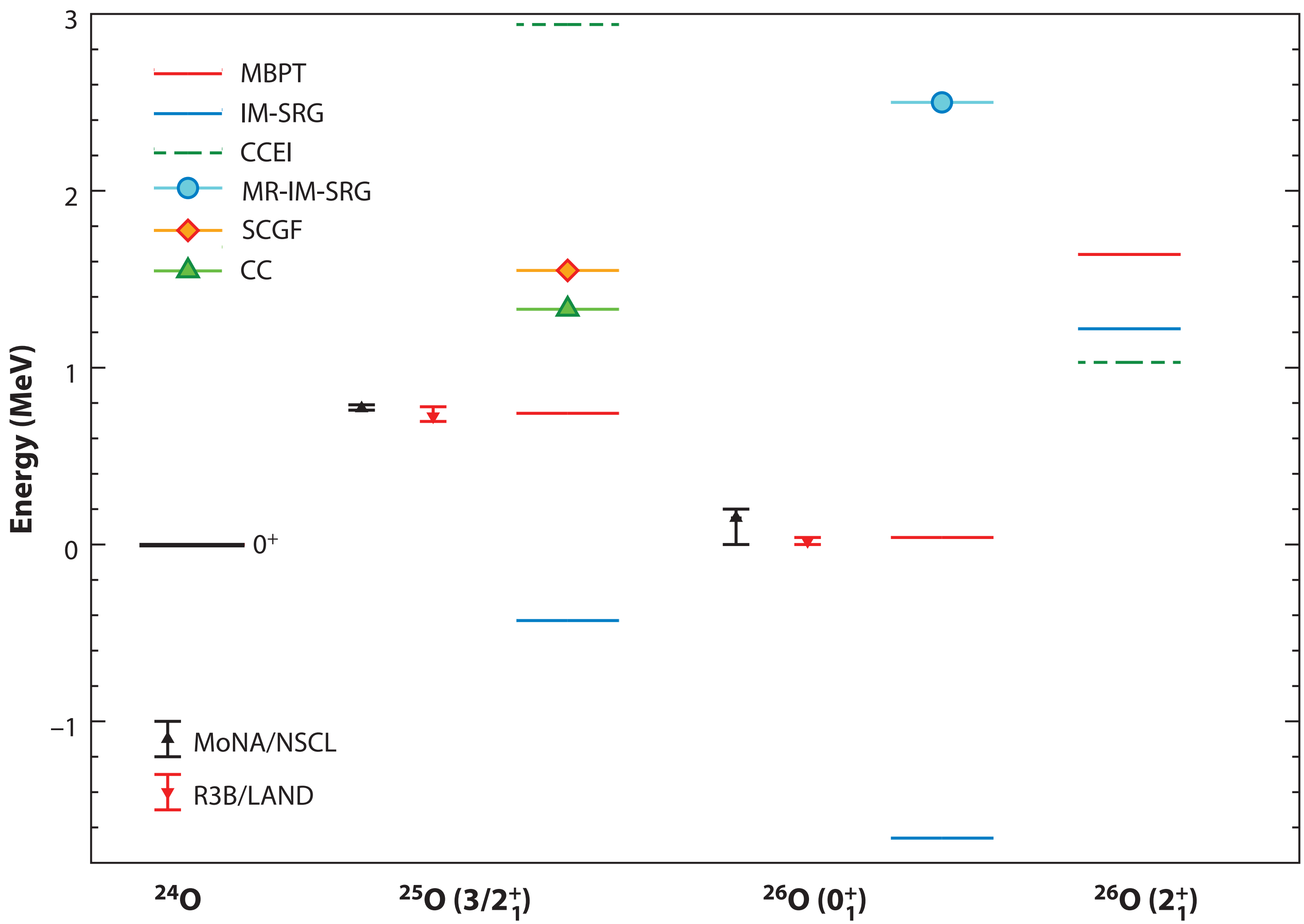


MBPT in extended valence space

IM-SRG/**CCEI** spectra agree within ~300 keV

Beyond the Oxygen Dripline

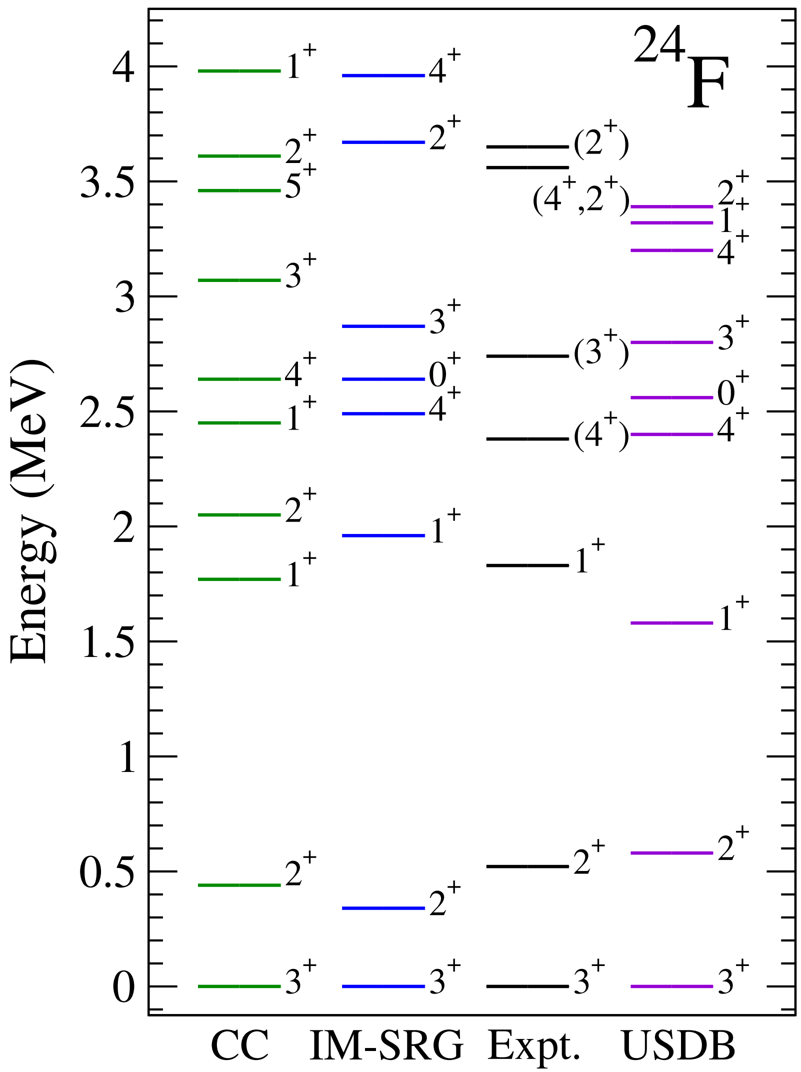
Physics beyond dripline highly sensitive to 3N and continuum effects



Prediction of low-lying 2^+ in ^{26}O (recently measured at RIKEN)

Experimental Connection: ^{24}F Spectrum

^{24}F spectrum: **IM-SRG** (*sd* shell), **full CC**, **USDB**



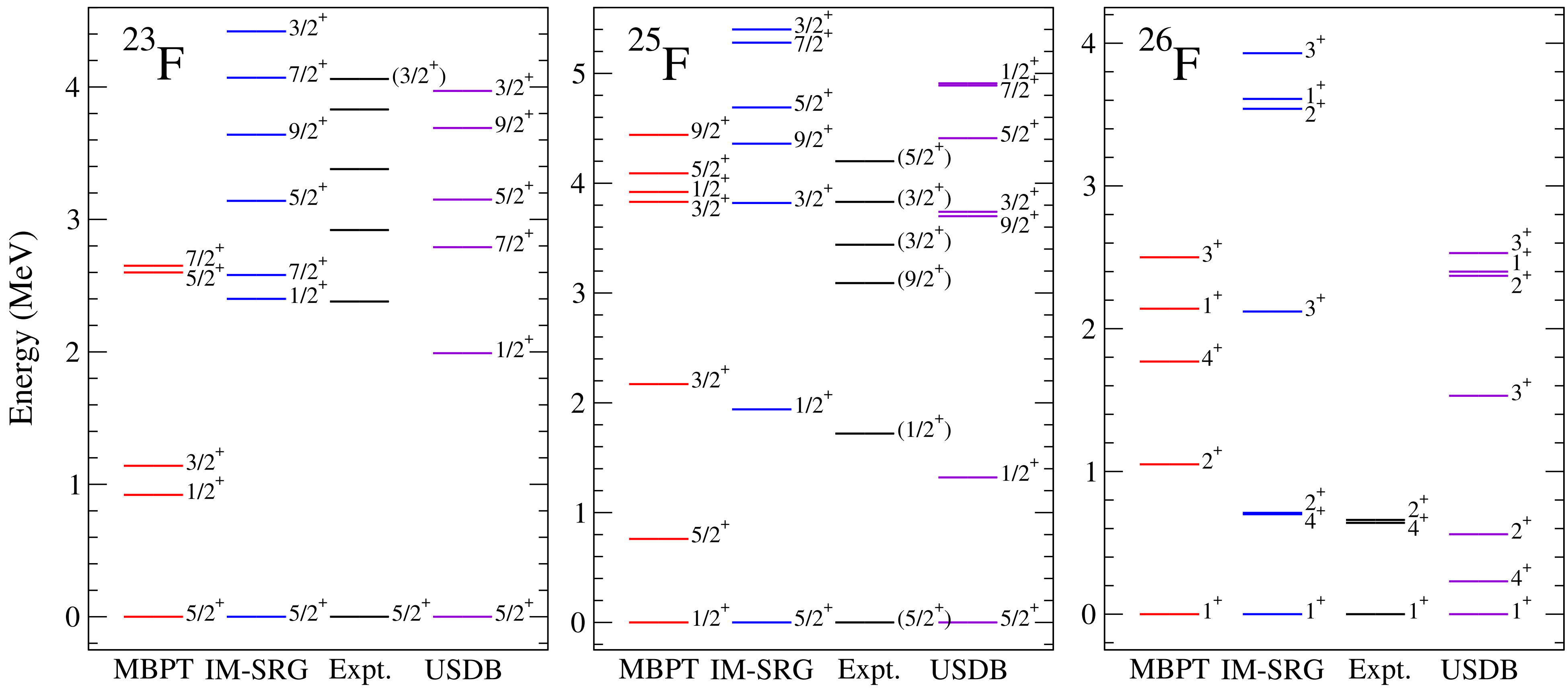
Ekström et al., PRL (2014)
Cáceres et al., arXiv:1501.01166
Hebeler, JDH, Menéndez, Schwenk, ARNPS (2015)

New measurements from GANIL

IM-SRG: comparable with phenomenology, good agreement with new data

Fully Open Shell: Neutron-Rich Fluorine Spectra

Fluorine spectroscopy: **MBPT** and **IM-SRG** (*sd* shell) from NN+3N forces



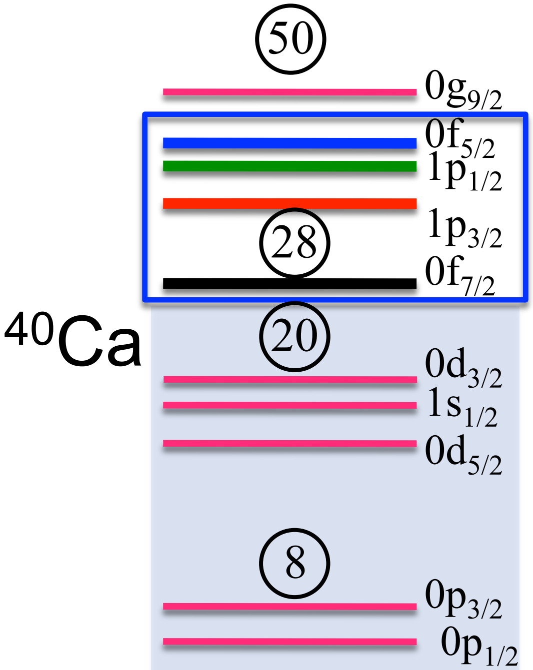
Bogner, Hergert, JDH, Schwenk, in prep.

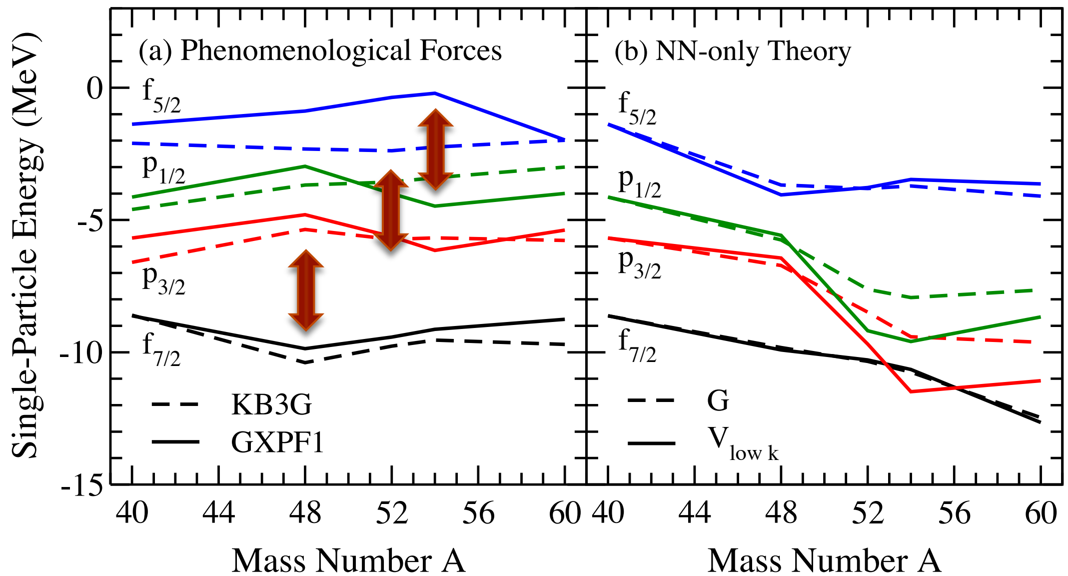
IM-SRG: **competitive with phenomenology**, good agreement with data

Preliminary results already for scalar operators: charge radii, E0 transitions

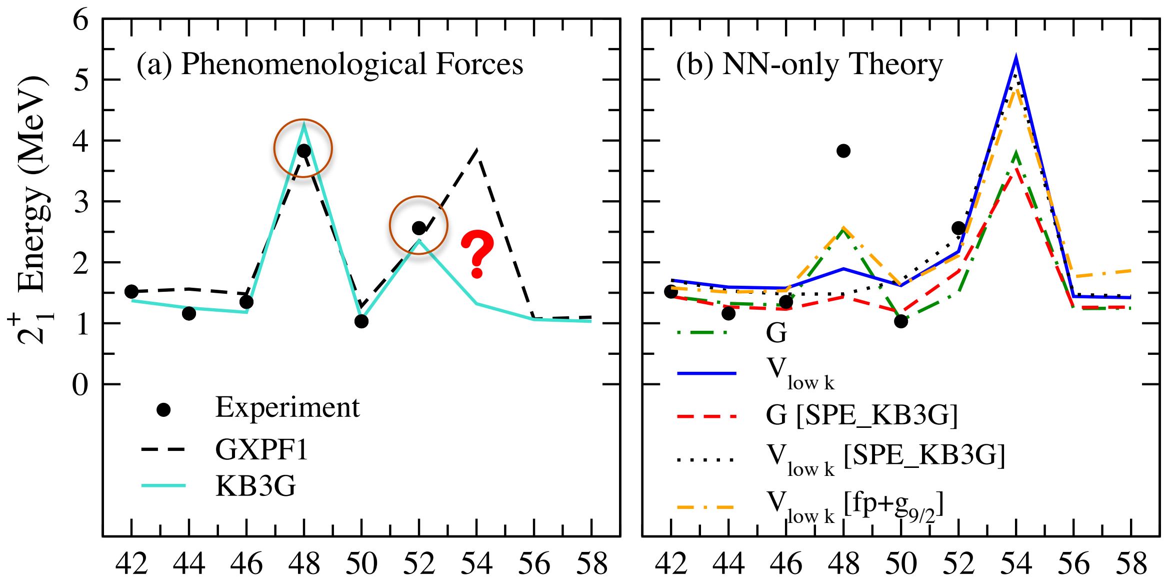
Upcoming: general operators M1, E2, GT, double-beta decay Stroberg et al.

Calcium Isotopes: Magic Numbers





GXPF1: Honma, Otsuka, Brown, Mizusaki (2004)
KB3G: Poves, Sanchez-Solano, Caurier, Nowacki (2001)



Phenomenological Forces

Large gap at ^{48}Ca

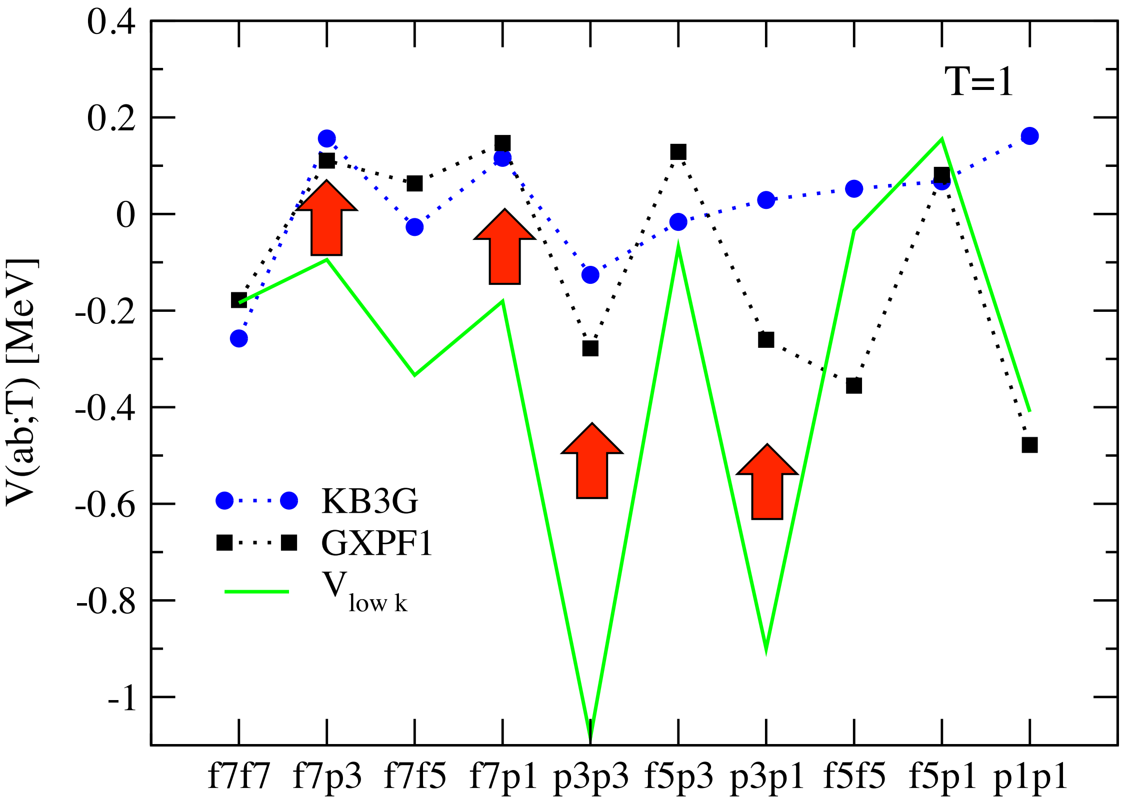
Discrepancy at $N=34$

Microscopic NN Theory

Small gap at ^{48}Ca

N=28: first standard magic number not reproduced in microscopic NN theories

Phenomenological vs. Microscopic



Compare monopoles from:

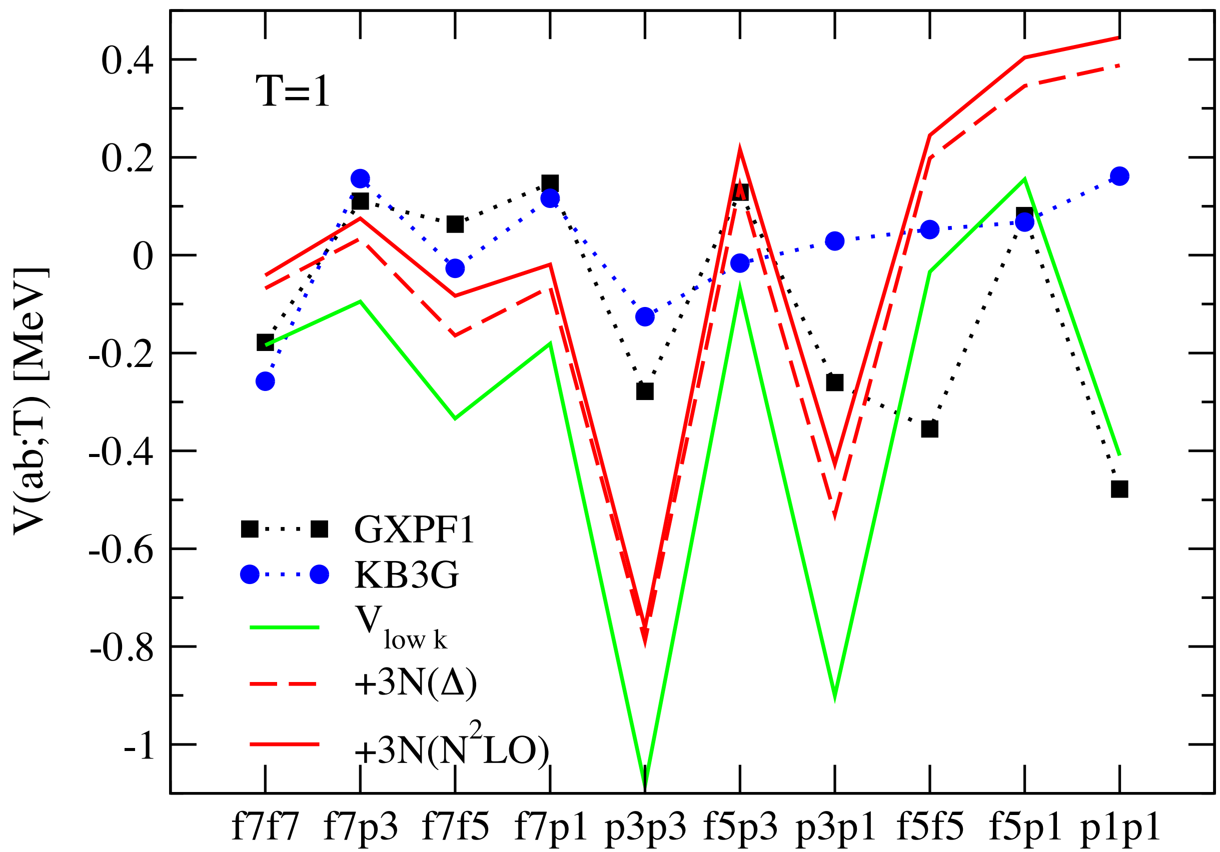
Microscopic **low-momentum** interactions

Phenomenological **KB3G, GXPF1** interactions

Shifts in **low-lying orbitals**:

- T=1 repulsive shift

Two-body 3N: Monopoles in *pf*-shell



Dominant effect from **one-Δ** – as expected from cutoff variation

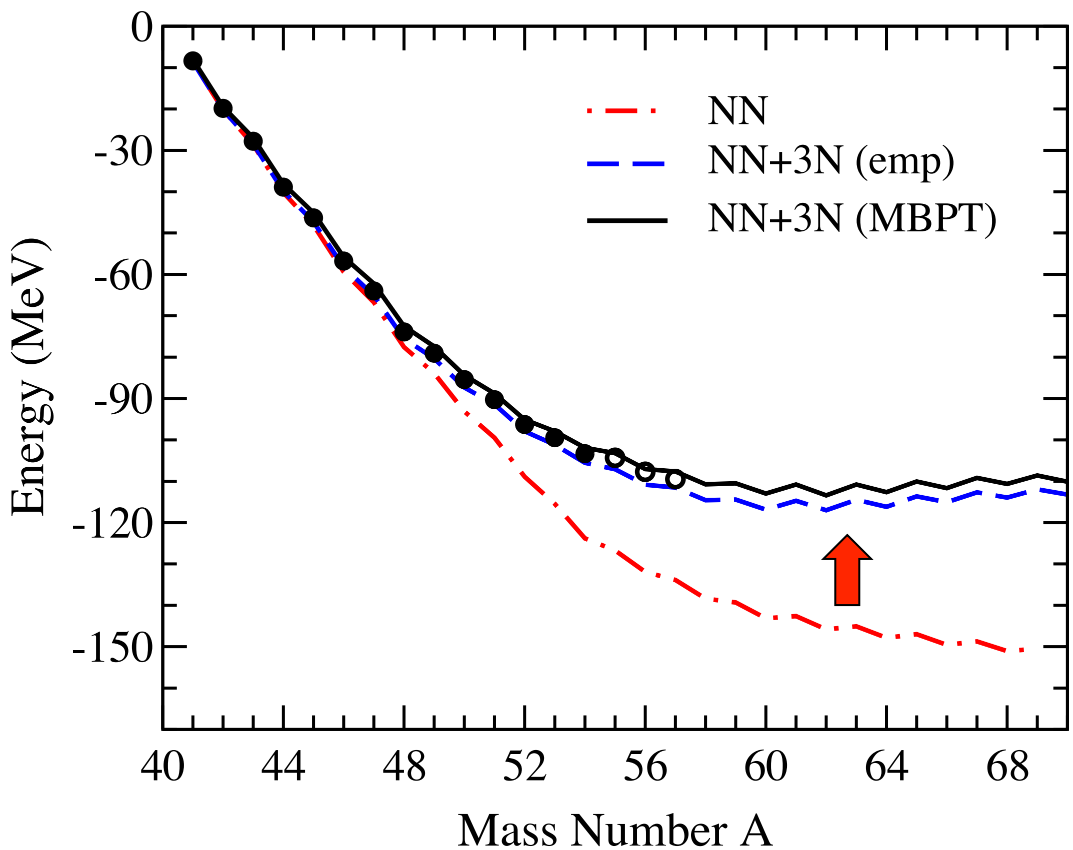
3N forces produce clear repulsive shift in monopoles

Similar to *sd*-shell

First calculations to show missing monopole strength due to neglected 3N

Calcium Ground State Energies and Dripline

Signatures of shell evolution from ground-state energies?



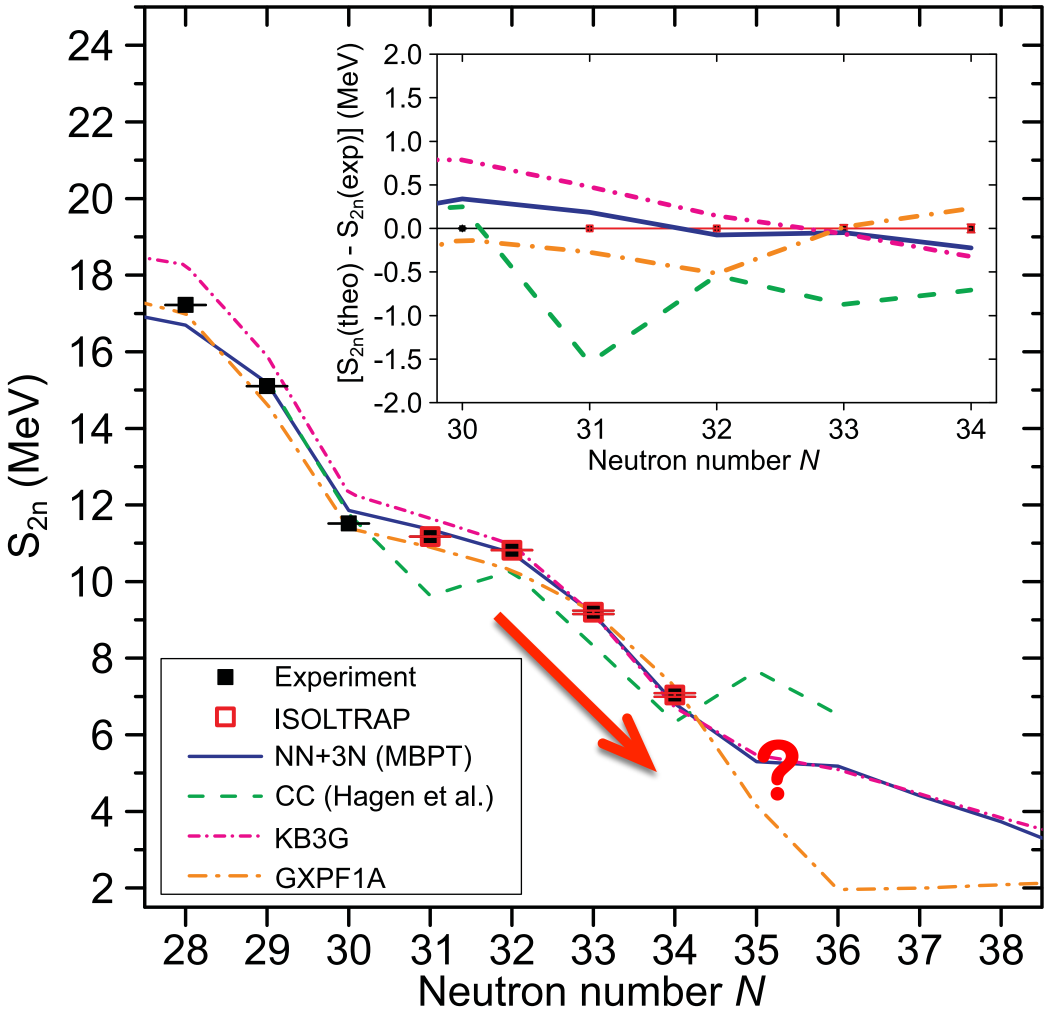
Holt, Otsuka, Schwenk, Suzuki, JPG (2012)

No clear dripline; flat behavior past ^{54}Ca – **Halos beyond ^{60}Ca?**

$S_{2n} = -\left[BE(N,Z) - BE(N-2,Z)\right]$ **sharp decrease indicates shell closure**

Experimental Connection: Mass of ^{54}Ca

New precision mass measurement of ^{53,54}Ca at **ISOLTRAP**: multi-reflection ToF



Wienholtz et al., Nature (2013)

TITAN Measurement

Flat trend from $^{50-52}$Ca

Mass ^{52}Ca 1.74 MeV from AME

ISOLTRAP ***Measurement***

Sharp decrease past ^{52}Ca

Unambiguous closed-shell ^{52}Ca

Test predictions of various models

MBPT NN+3N

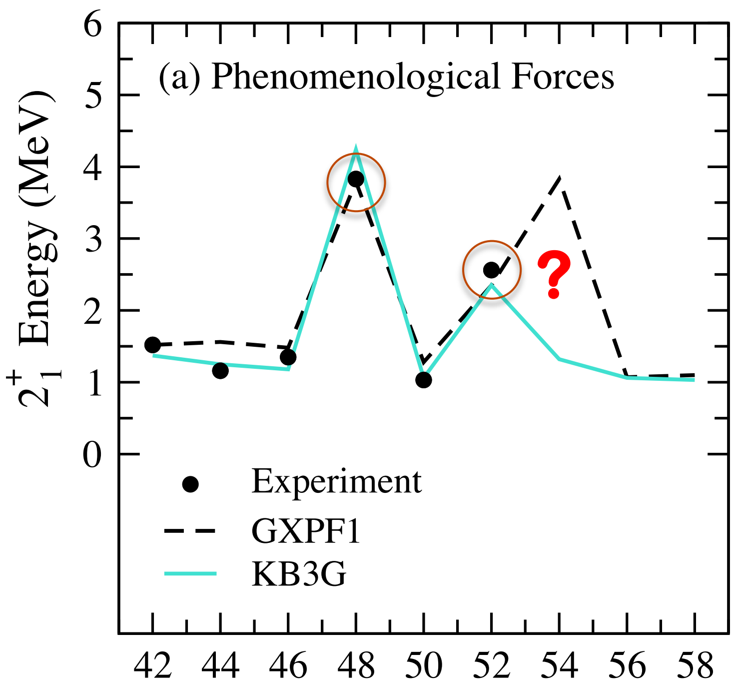
Excellent agreement with new data

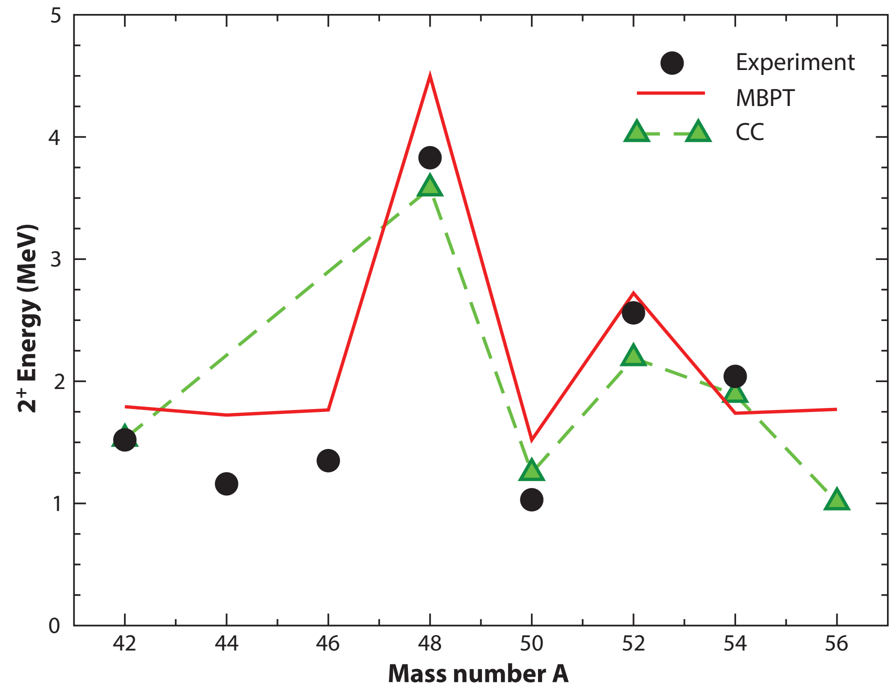
Reproduces closed-shell ^{48,52}Ca

Weak closed sell signature past ^{54}Ca

N=34 magic number in calcium?

Calcium Isotopes: Magic Numbers





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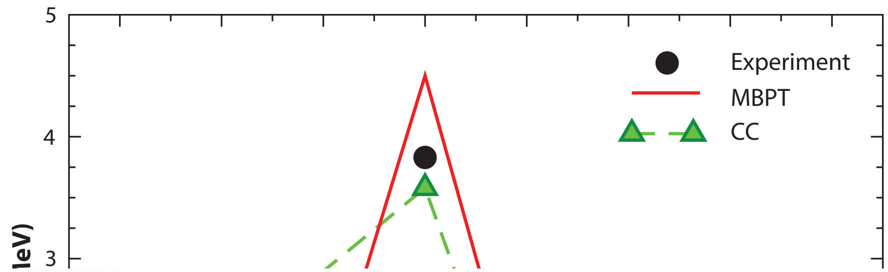
Phenomenological Models

Large gap at ^{48}Ca, discrepancy at *N=34*

Ab initio theories

Reproduce all new magic numbers, **consistent predictions**

Calcium Isotopes: Magic Numbers



PHYSICS

NEWS & VIEWS RESEARCH

Heavy calcium nuclei weigh in

The configurations of calcium nuclei make them good test cases for studies of nuclear properties. The measurement of the masses of two heavy calcium nuclei provides benchmarks for models of atomic nuclei. SEE LETTER P.346

ALEXANDRA GADE

quarks and gluons, which interact to form

LETTER

doi:10.1038/nature12226

Masses of exotic calcium isotopes pin down nuclear forces

F. Wienholtz[1], D. Beck[2], K. Blaum[3], Ch. Borgmann[3], M. Breitenfeldt[4], R. B. Cakirli[3,5], S. George[1], F. Herfurth[2], J. D. Holt[6,7], M. Kowalska[8], S. Kreim[3,8], D. Lunney[9], V. Manea[9], J. Menéndez[6,7], D. Neidherr[2], M. Rosenbusch[1], L. Schweikhard[1], A. Schwenk[7,6], J. Simonis[6,7], J. Stanja[10], R. N. Wolf[1] & K. Zuber[10]

LETTER

doi:10.1038/nature12522

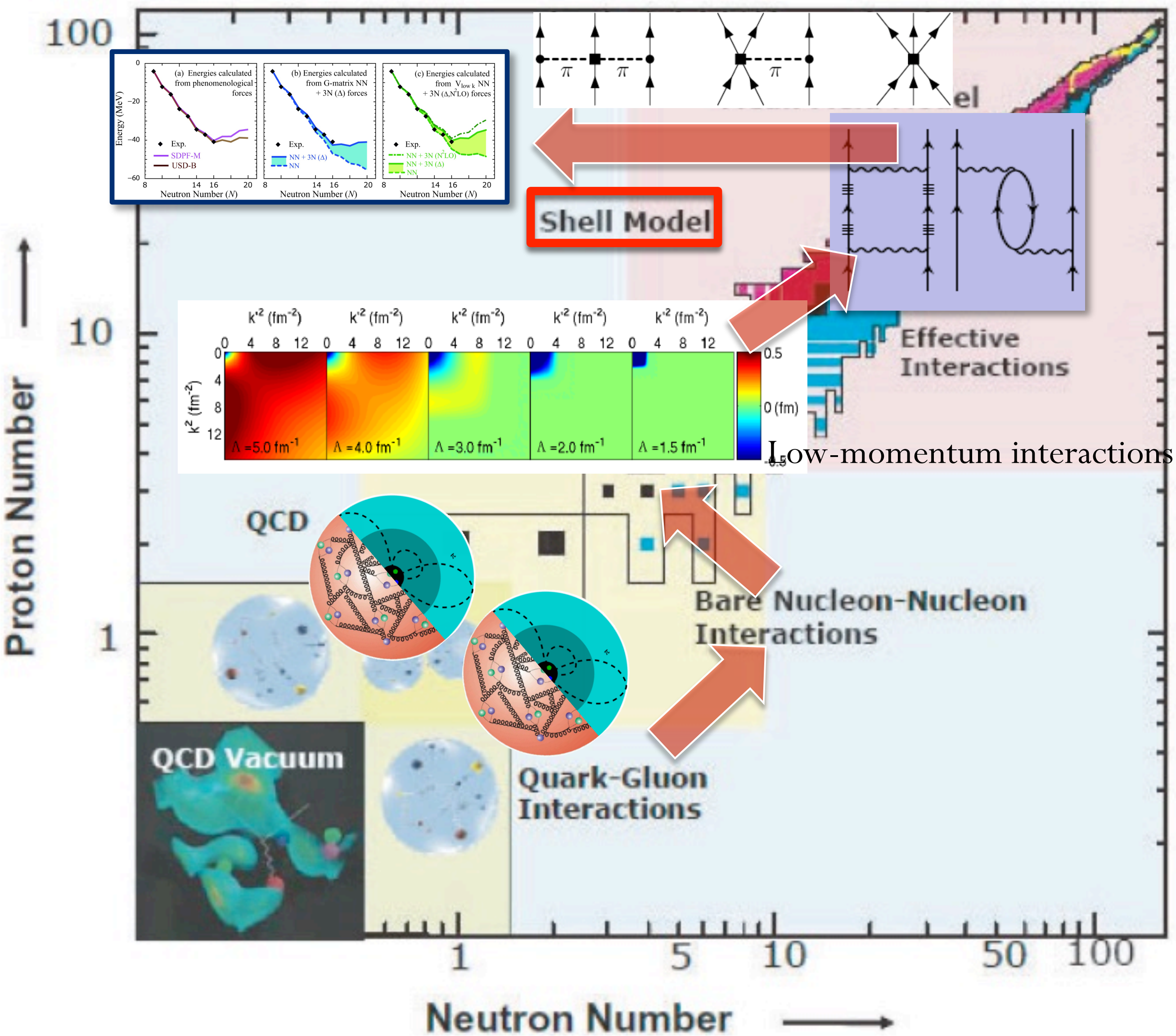
Evidence for a new nuclear 'magic number' from the level structure of ^{54}Ca

D. Steppenbeck[1], S. Takeuchi[2], N. Aoi[3], P. Doornenbal[2], M. Matsushita[1], H. Wang[2], H. Baba[2], N. Fukuda[2], S. Go[1], M. Honma[4], J. Lee[2], K. Matsui[5], S. Michimasa[1], T. Motobayashi[2], D. Nishimura[6], T. Otsuka[1,5], H. Sakurai[2,5], Y. Shiga[7], P.-A. Söderström[2], T. Sumikama[8], H. Suzuki[2], R. Taniuchi[5], Y. Utsuno[9], J. J. Valiente-Dobón[10] & K. Yoneda[2]

t predictions

The Challenge of Microscopic Nuclear Theory

To understand the properties of complex nuclei from elementary interactions



Three-Nucleon Forces

Clear path from symmetries of QCD to shell model

Ideas of:

Effective field theories

Renormalization group

Advances in many-body

Advances in computing

All essential for this progress

Still much to do!!

How will we approach this problem:

QCD → NN (3N) forces → Renormalize → Solve many-body problem → Predictions

New Directions and Outlook

Heavier semi-magic chains: MBPT as guide

Ab initio valence-shell Hamiltonians

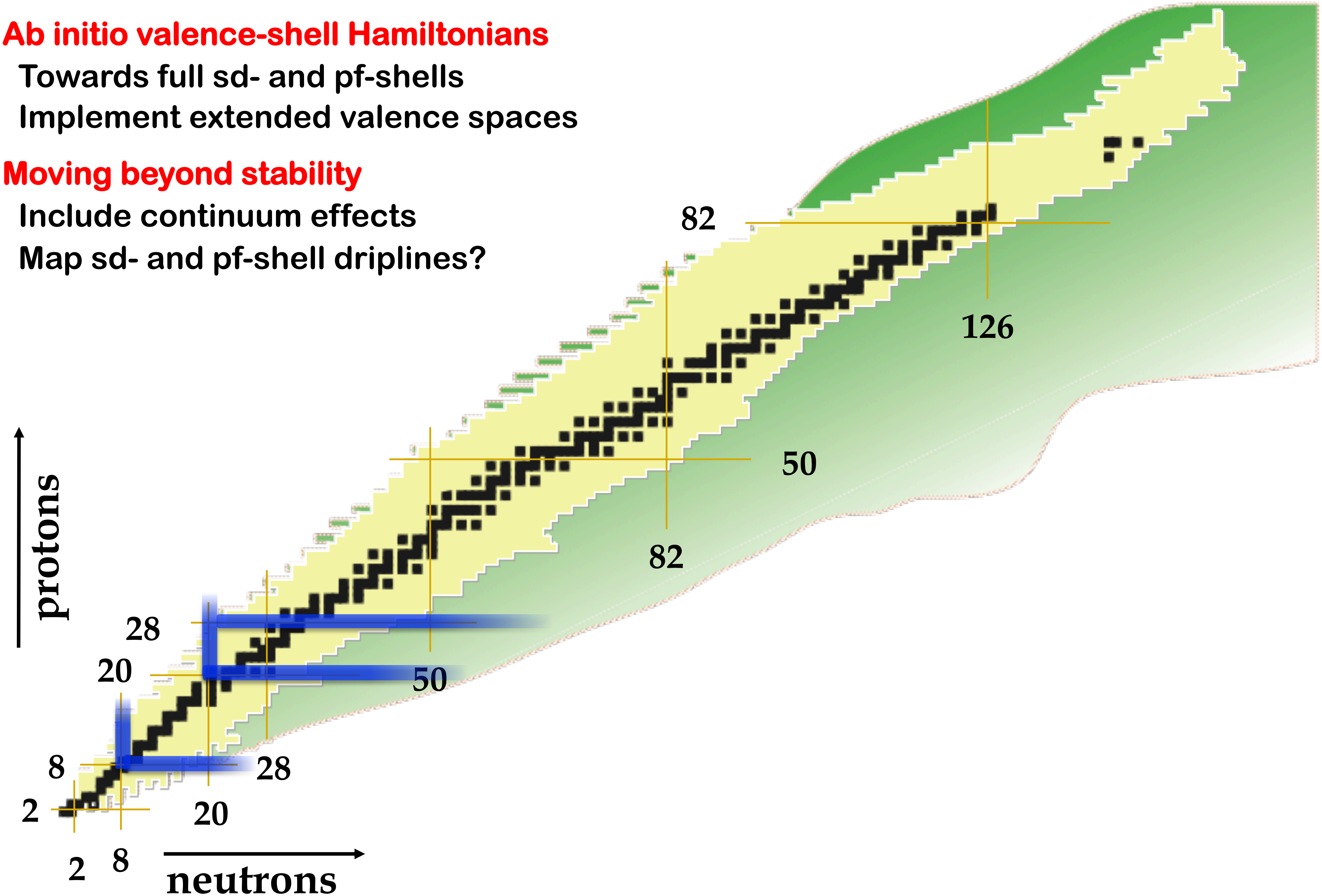
Towards full sd- and pf-shells

Implement extended valence spaces

Moving beyond stability

Include continuum effects

Map sd- and pf-shell driplines?



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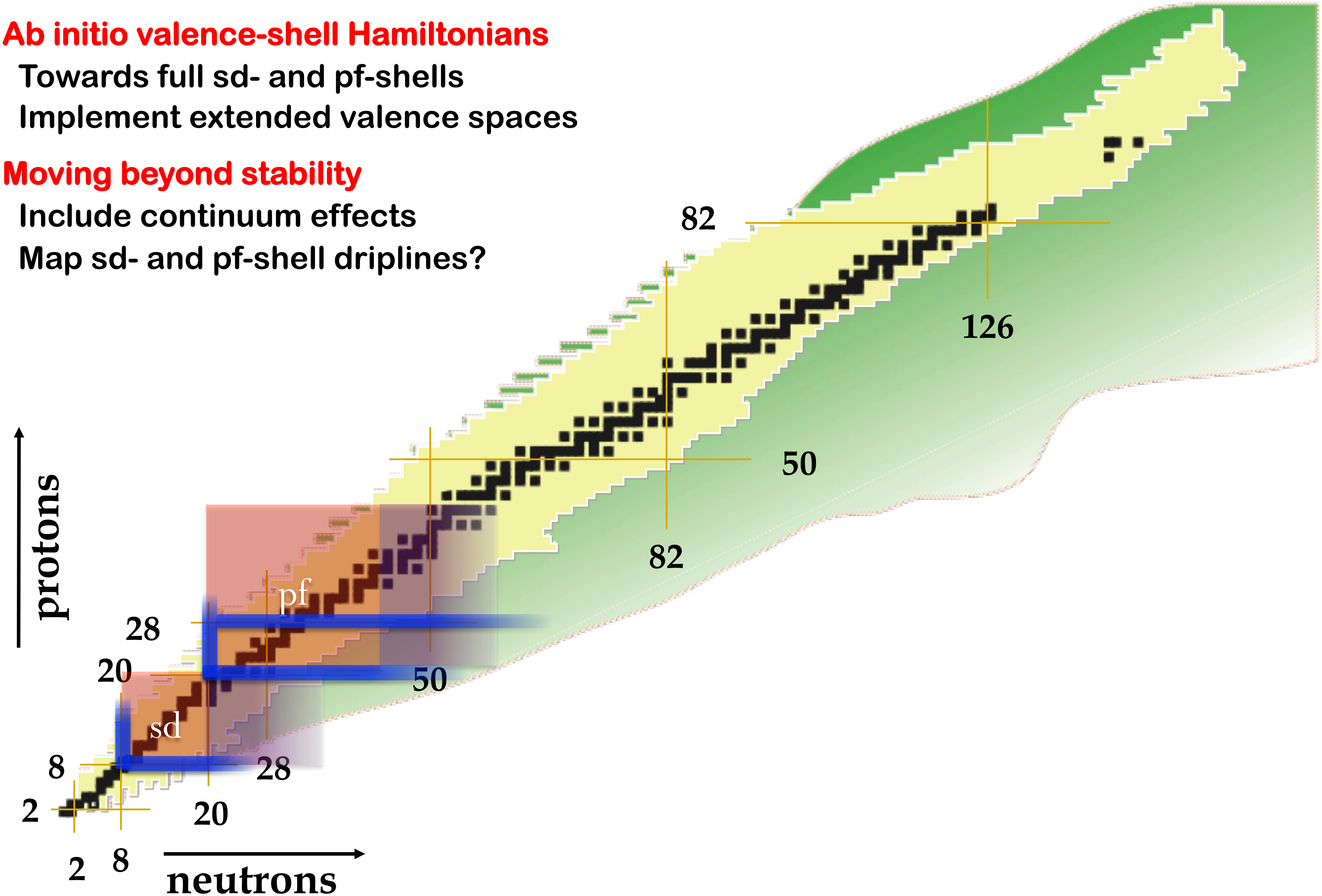


New Directions and Outlook

Heavier semi-magic chains: MBPT as guide

Ab initio valence-shell Hamiltonians

Towards full sd- and pf-shells

Implement extended valence spaces

Moving beyond stability

Include continuum effects

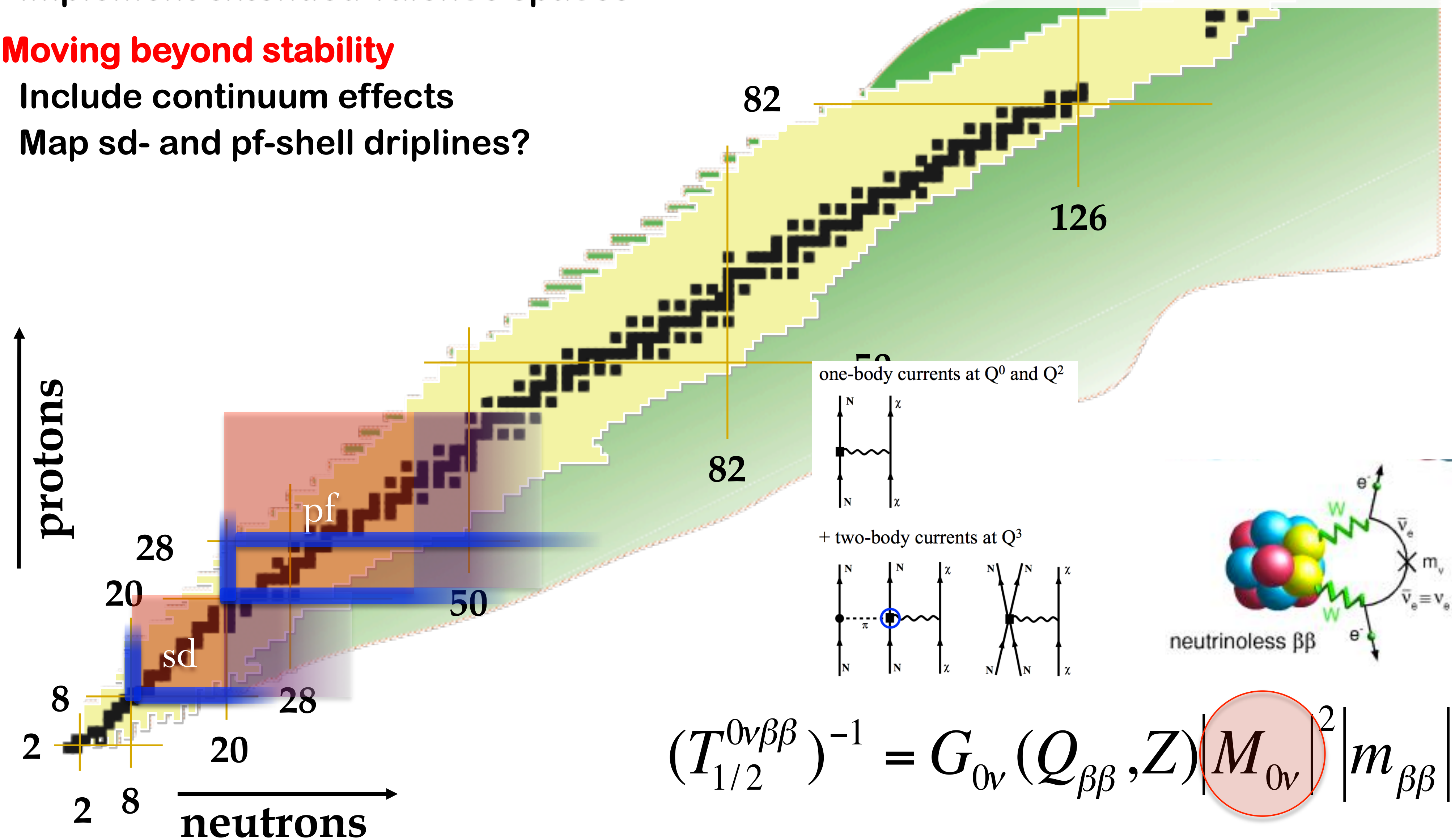
Map sd- and pf-shell driplines?

Fundamental symmetries

Effective electroweak operators

ab initio calculation of $0\nu\beta\beta$ decay

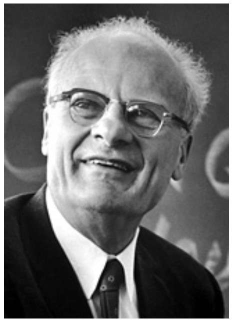
WIMP-nucleus scattering



$$(T_{1/2}^{0\nu\beta\beta})^{-1} = G_{0\nu}(Q_{\beta\beta}, Z)\left|M_{0\nu}\right|^2 \left|m_{\beta\beta}\right|^2$$

Final Thought

"Very soft (NN) potentials must be excluded because they do not give saturation; they give too much binding and too high density."
 - ***H. Bethe***



How might you respond?

Final Thought

"Very soft (NN) potentials must be excluded because they do not give saturation; they give too much binding and too high density."
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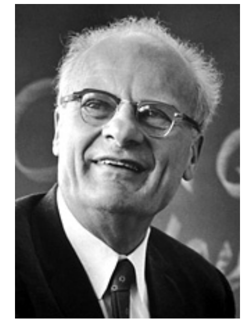

How might you respond?

Further Reading

Lepage, nucl-th/9706029 (1997)

Epelbaum, Hammer, Meißner, Rev. Mod. Phys. (2009)

Machleidt, Entem, Phys. Rep. (2011)

Bogner, Furnstahl, Schwenk, Prog. Part. Nucl. Phys. (2010)

Hebeler, Holt, Menendez, Schwenk, Ann. Rev. Nucl. Part. Sci. (2015)

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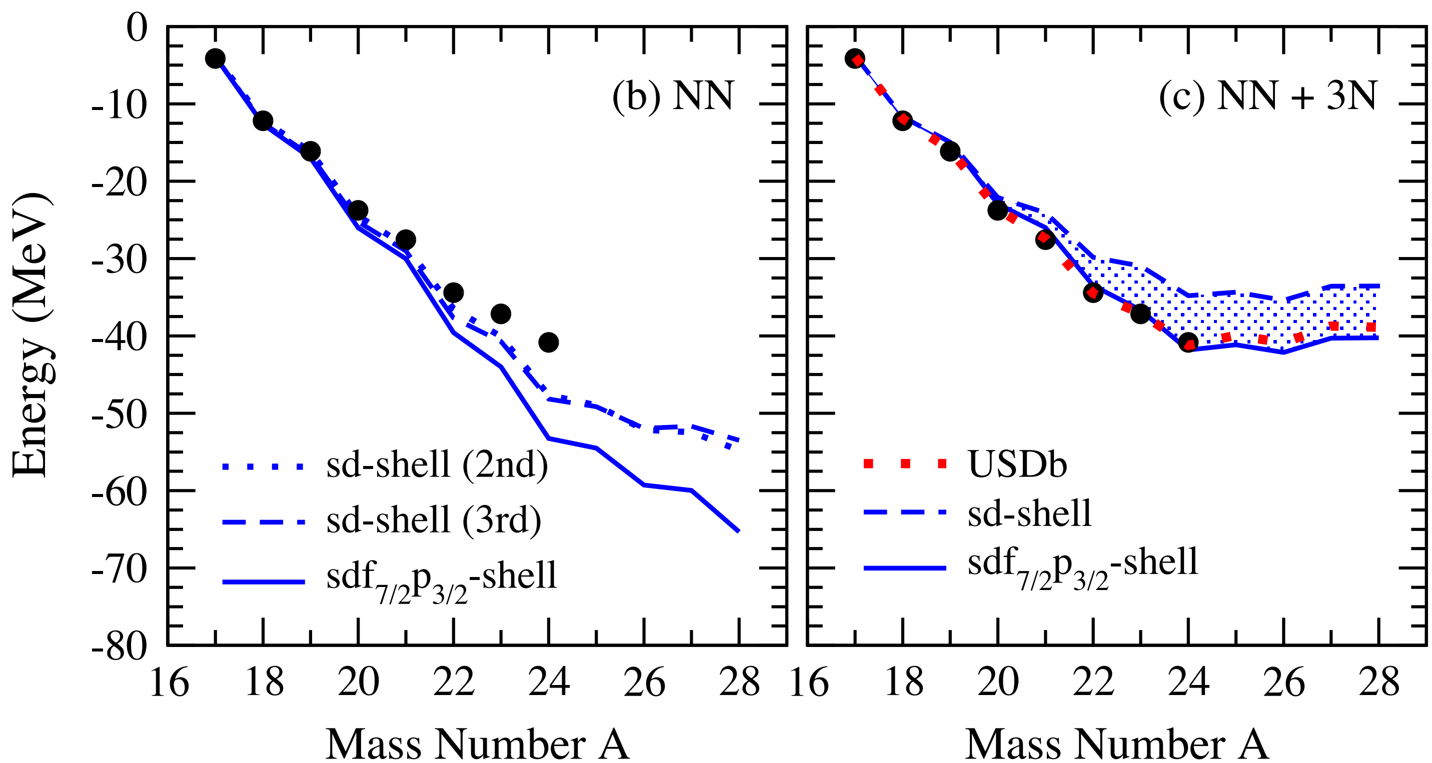
Scott Bogner, Angelo Calci, Thomas Duguet, Dick Furnstahl, Alex Gezerlis, Gaute Hagen, Kai Hebeler, Heiko Hergert, Herman Krebs, Javier Menendez, Petr Navratil, Achim Schwenk, Johannes Simonis, Ragnar Stroberg

Ground-State Energies of Oxygen Isotopes

Valence-space interaction and SPEs from NN+3N-fit



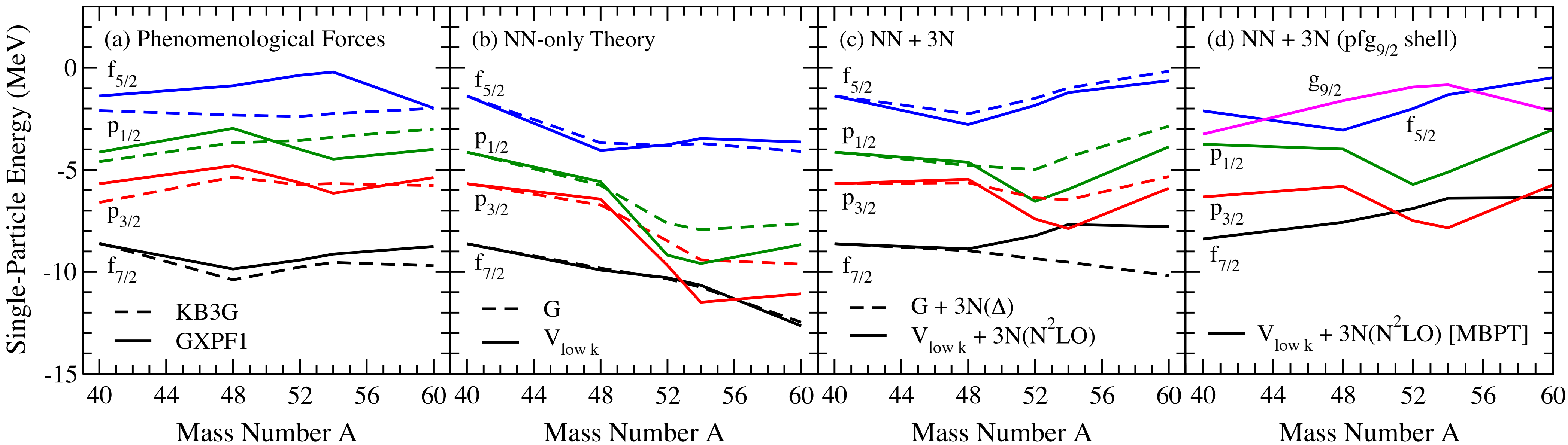
JDH, Menendez, Schwenk, EPJA (2013)

Repulsive character improves agreement with experiment

sd-shell results underbound; improved in **extended space** $sdf_{7/2}\ p_{3/2}$

Evolution of Shell Structure

SPE evolution with 3N forces in *pf* and $pfg_{9/2}$ spaces:



NN+3N *pf*-shell: JDH, Otsuka, Schwenk, Suzuki JPG (2012)

Trend across: improved binding energies

Increased gap at ^{48}Ca: enhanced closed-shell features

Include $g_{9/2}$ orbit, calculated SPEs

Different behavior of ESPEs (not observable, model dependent)

Small gap can give large 2^+ energy: due to many-body correlations

Duguet, Hagen, PRC (2012)