Primordial tensor power spectrum and its scale dependence

OTSU YUTA

IN COLLABORATION WITH

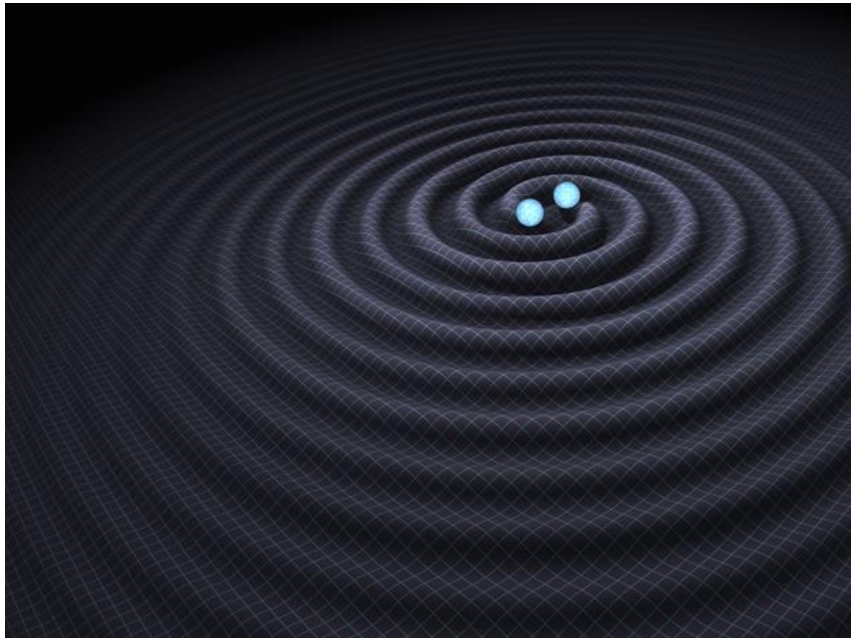
TATSUKI KODAMA, TAKUMI SHINOHARA, TOMO TAKAHASHI

SAGA UNIVERSITY

What are Gravitational Waves?

Phenomenon of space-time distortion propagates as waves.

ex)
- Binary black hole merger
- Binary neutron star merger
- etc

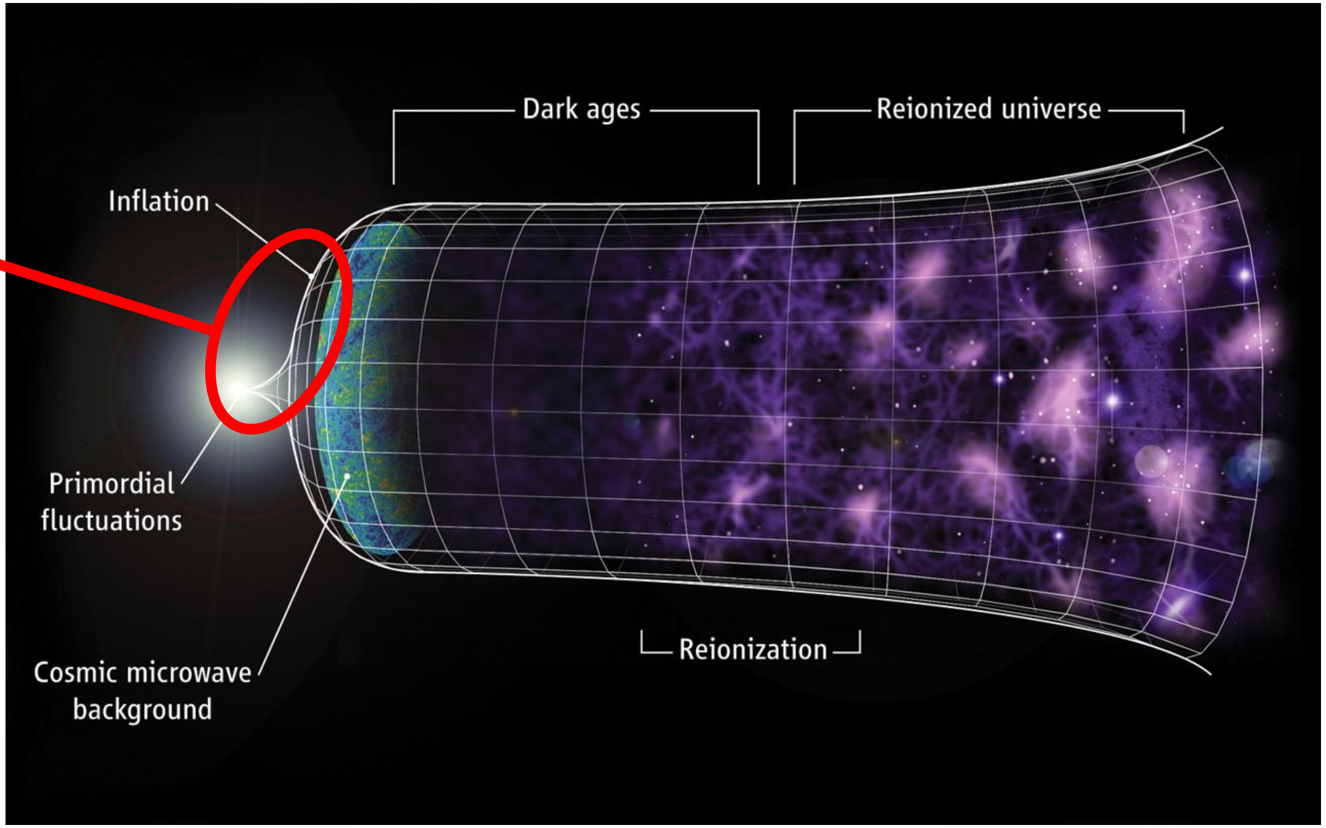


LIGO : https://www.ligo.caltech.edu/system/avm_image_sqls/binaries/154/page/GravWave.jpg?1637016727

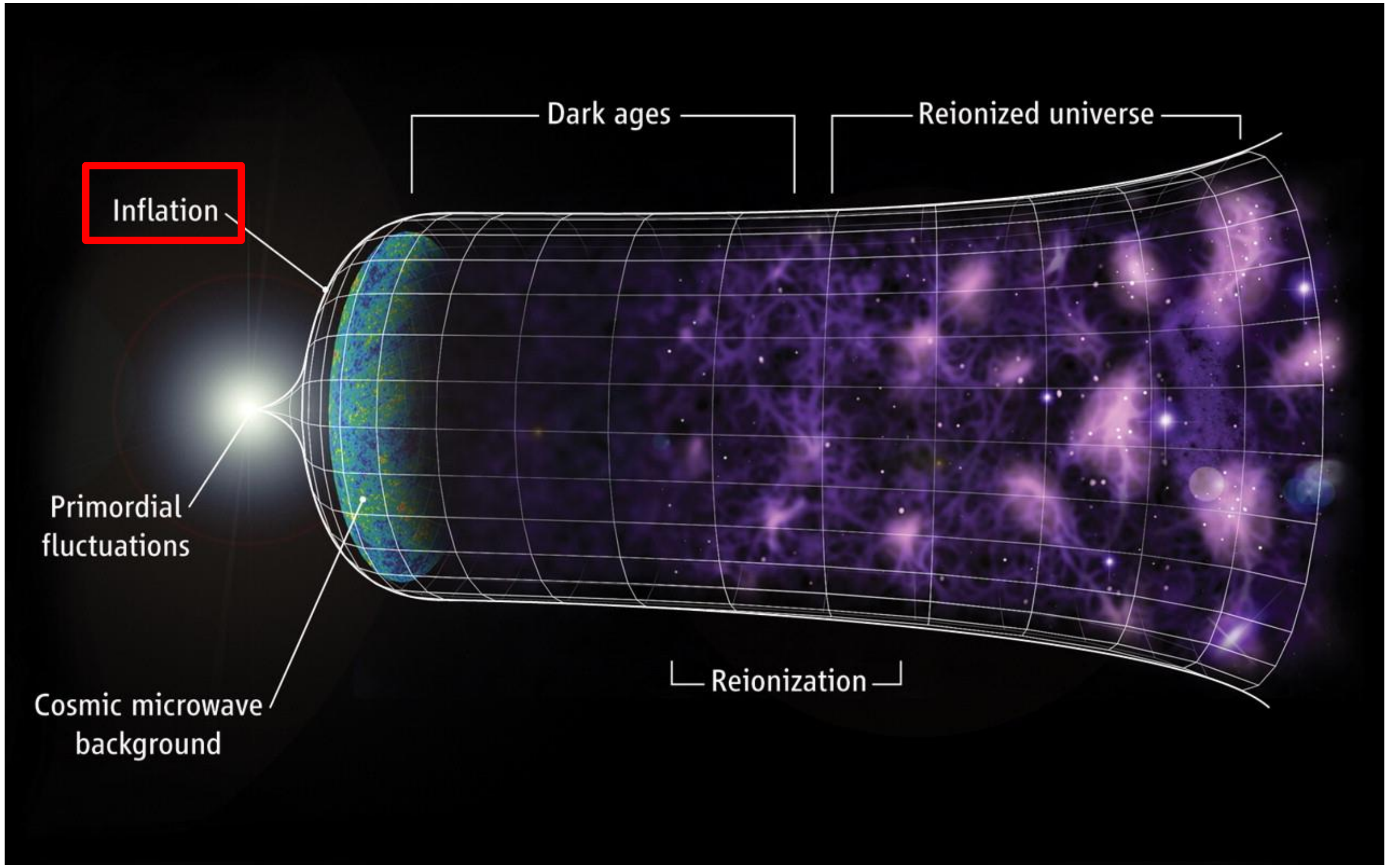
What are Primordial Gravitational Waves?

GWs from Inflation

- The size of the primordial Gravitational wave is decided by the energy scale.



What is Inflation?



CLAUDE-ANDRÉ FAUCHER-GIGUÈRE, ADAM LIDZ, AND LARS HERNQUIST, SCIENCE.319, NO.5859

Why do we consider inflation?

Problems of the standard Big bang cosmology

1. Horizon problem
2. Flatness problem
3. (The origin of density perturbations in the Universe)

→ solution

Inflation

1.Horizon problem

time dependence of the scale factor

$$a(t) \propto t^{\alpha} \qquad 0 < \alpha < 1$$ decelated expansion

$$RD : \alpha = \frac{1}{2} \qquad MD : \alpha = \frac{2}{3}$$

$$L_H(t) = \int_0^t \frac{dt'}{a(t')} \propto t^{1-\alpha}$$ → ✖ Causality

Why do different regions of the universe have the same density?

2. Flatness problem

Curvature radius of comoving coordinate

$$\mathcal{R} = K^{-1/2}$$

Hubble radius of co-moving coordinate

$$a(t) \propto t^{\alpha}$$
$$\alpha < 1$$

$$D_H = 1/aH \propto t^{1-\alpha}$$

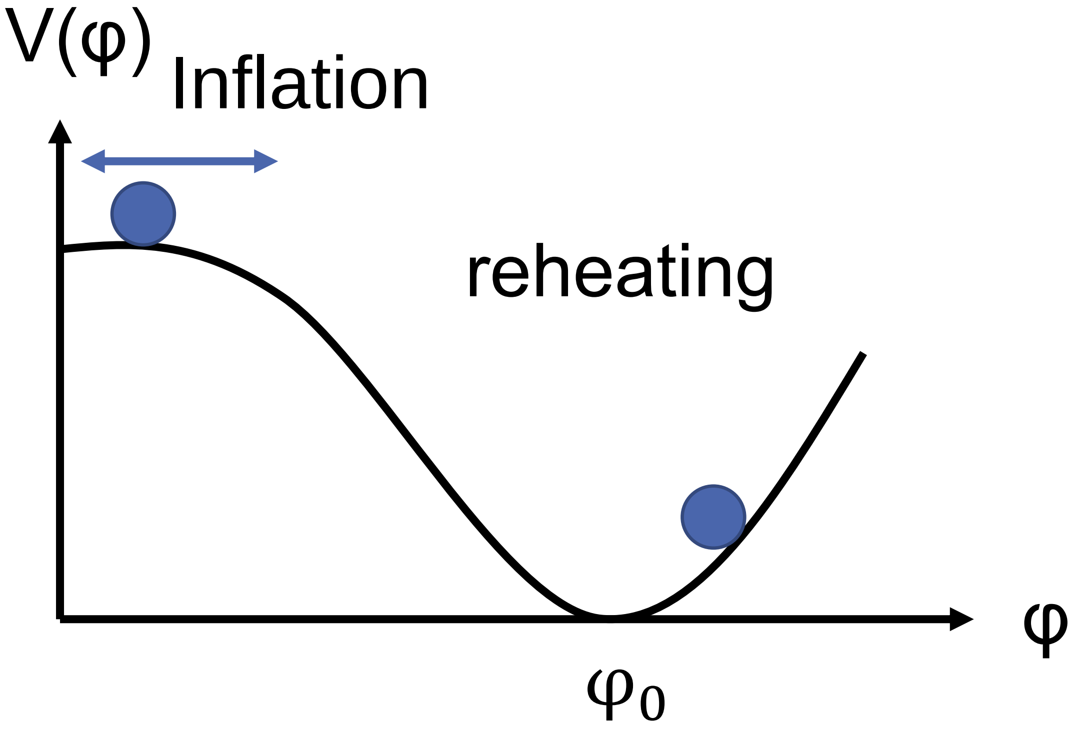
the ratio between curvature radius and Hubble radius

$$\mathcal{R}/D_H = aH/K^{1/2} \propto t^{-(1-\alpha)}$$

we need the fine-tuning of the curvature

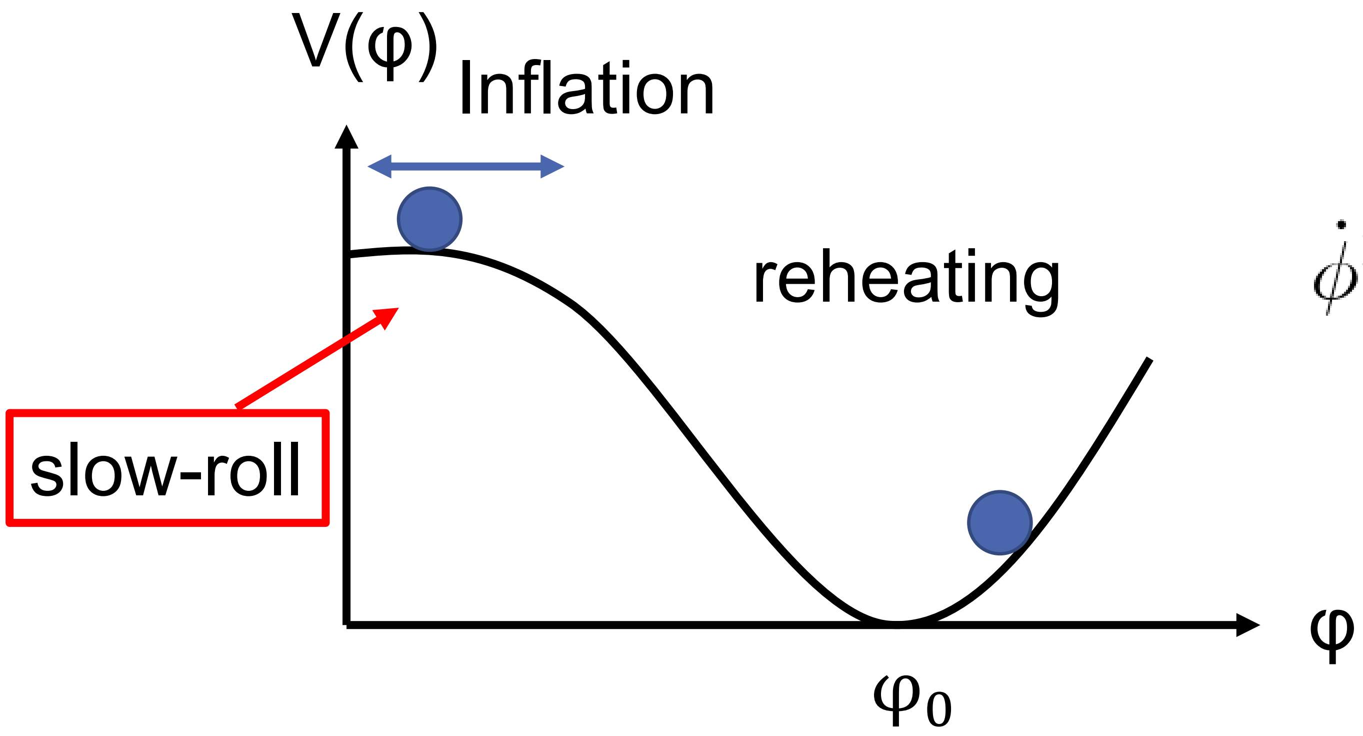
Inflation

Mechanism of Inflation



Inflation

Mechanism of Inflation



slow-roll conditions

$$\dot{\phi}^2 \ll 2V(\phi), \quad |\ddot{\phi}| \ll 3H|\dot{\phi}|$$

Purpose

- We investigate whether the Taylor expansion of the power spectrum gives a good description of the scale dependence.
- We study tensor power spectra for some inflation models.

Method

The equation of motion for inflaton ϕ

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

Slow-roll conditions

Slow-roll parameters

$$\epsilon_V \equiv \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2$$

$$\eta_V \equiv M_{Pl}^2 \frac{V''}{V}$$

$$\xi_V^2 \equiv M_{Pl}^4 \frac{V'V''}{V^2}$$

Inflation lasts as long as

$$\epsilon_V\,,|\eta_V| \ll 1$$

Inflation end

$$max\{\epsilon_V(\phi_{end}), |\eta_V|(\phi_{end})\} = 1$$

Hubble parameter

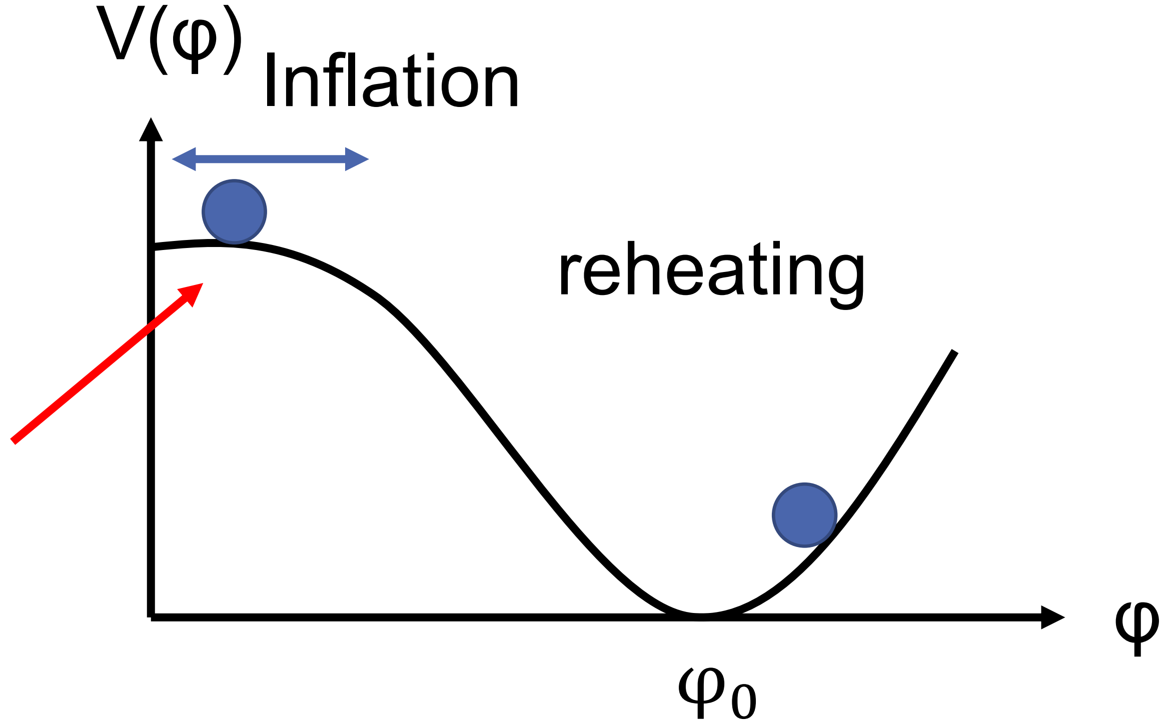
$$H^2(t) \simeq \frac{V}{3M_{Pl}^2}$$

where reduced Planck mass

$$M_{\rm Pl} = 1/\sqrt{8\pi G}$$

e-folding number

$$\mathrm{N} \equiv \ln(a_{end}/a)$$



The primordial power spectra of tensor perturbations

Within the slow-roll approximation

$$\mathcal{P}_T \simeq [1-(C+1)\epsilon_H]^2 \frac{2}{\pi^2 M_{Pl}^2} H^2|_{k=aH}$$

$$\epsilon_H \equiv 2M_{Pl}^2 (H'/H)^2$$

Consider the leading order in the slow-roll parameters

$$\mathcal{P}_T \simeq \frac{2}{\pi^2 M_{Pl}^2} H^2|_{k=aH}$$

Taylor expansion calculation

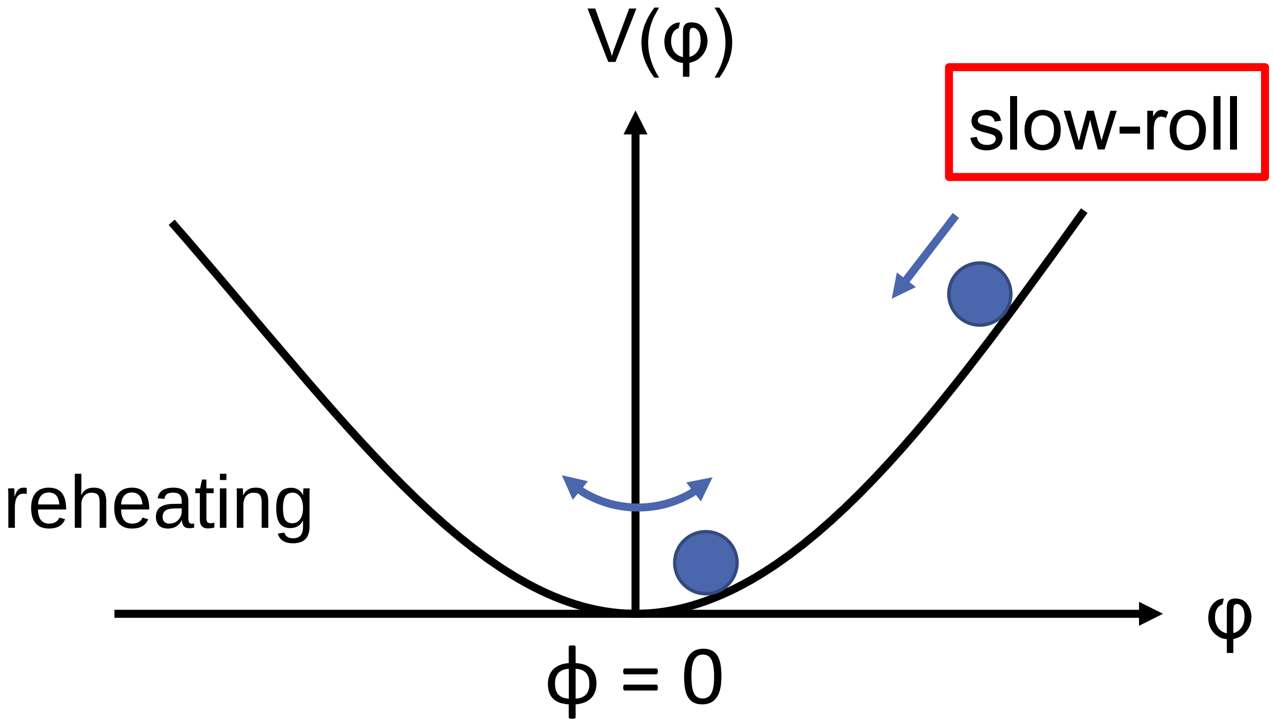
By the Taylor expansion in terms of the logarithm of the wave number

Kuroyanagi and Takahasi 2011

$$\mathcal{P}_T(k) = \mathcal{P}_{T\star}\exp\Big[n_{T\star}\ln\frac{k}{k_\star} + \frac{1}{2!}\alpha_{T\star}\ln^2\frac{k}{k_\star} + \frac{1}{3!}\beta_{T\star}\ln^3\frac{k}{k_\star} + \frac{1}{4!}\gamma_{T\star}\ln^4\frac{k}{k_\star} + \frac{1}{5!}\delta_{T\star}\ln^5\frac{k}{k_\star} + \frac{1}{6!}\theta_{T\star}\ln^6\frac{k}{k_\star} + \cdots\Big]$$

$$k_\star = 0.002\mathrm{Mpc}^{-1}$$

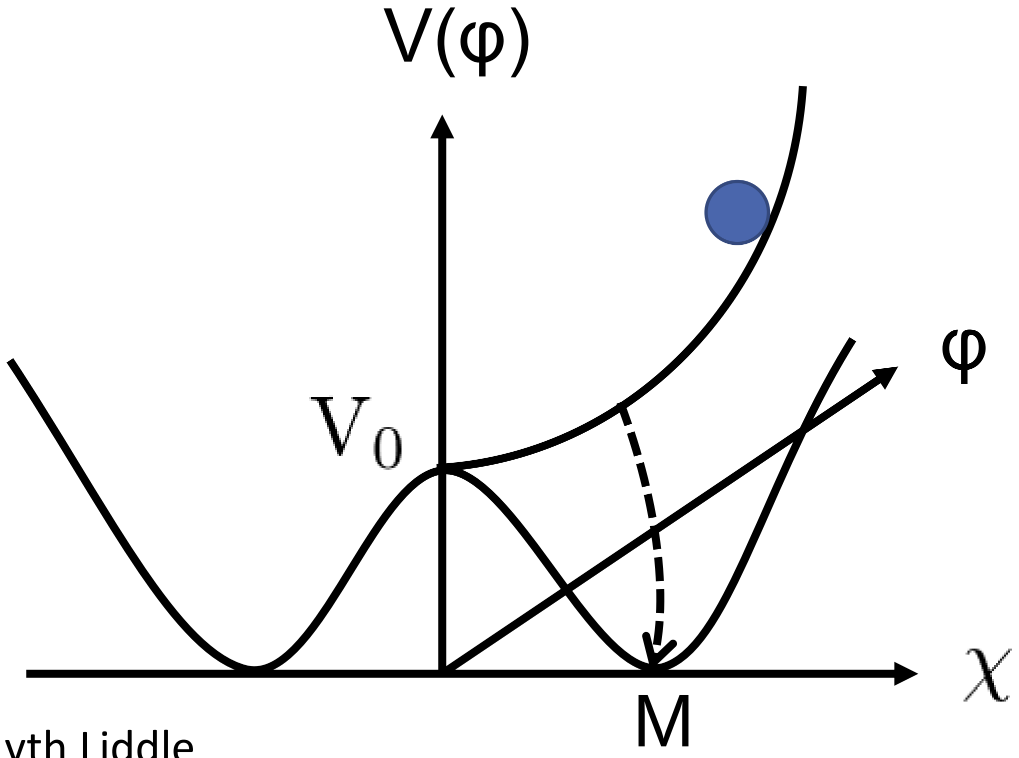
Inflation models – chaotic inflation



$$V = \frac{1}{2} m^2 \phi^2$$

After inflation, inflaton decays into radiation and the Universe is reheated.

Inflation models – hybrid inflation



Lyth Liddle

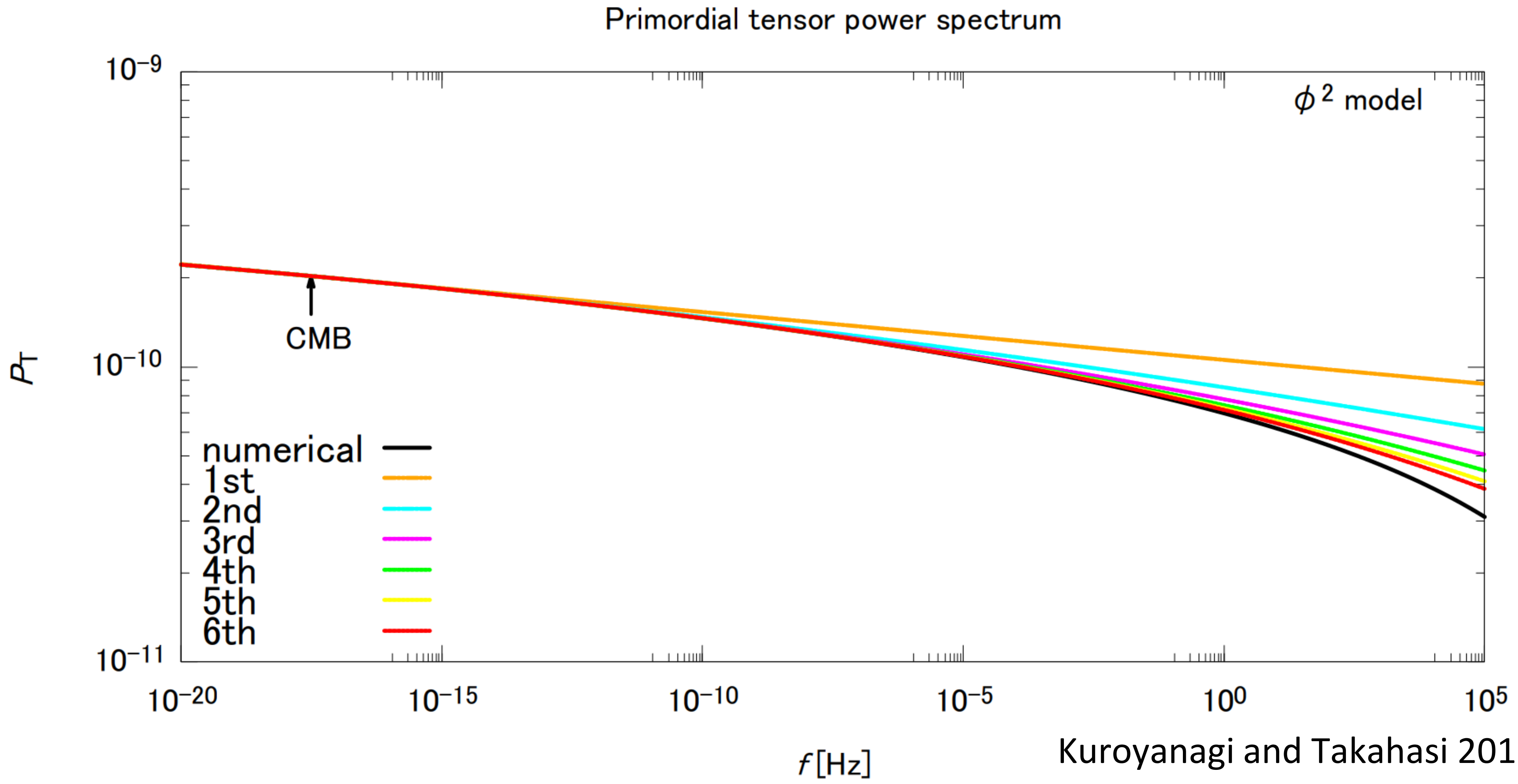
$$\mathrm{V}(\phi) = V_0 + \frac{1}{2}m^2\phi^2$$

$$\epsilon_V \equiv \frac{M_{\mathrm{Pl}}^2}{2}\left(\frac{V'}{V}\right)^2 = \frac{M_{\mathrm{Pl}}^2}{2}\left(\frac{m^2\phi}{V_0+\frac{1}{2}m^2\phi^2}\right)^2$$

$$\eta_V \equiv M_{\mathrm{Pl}}^2\frac{V''}{V} = M_{\mathrm{Pl}}^2\frac{m^2}{V_0+\frac{1}{2}m^2\phi^2}$$

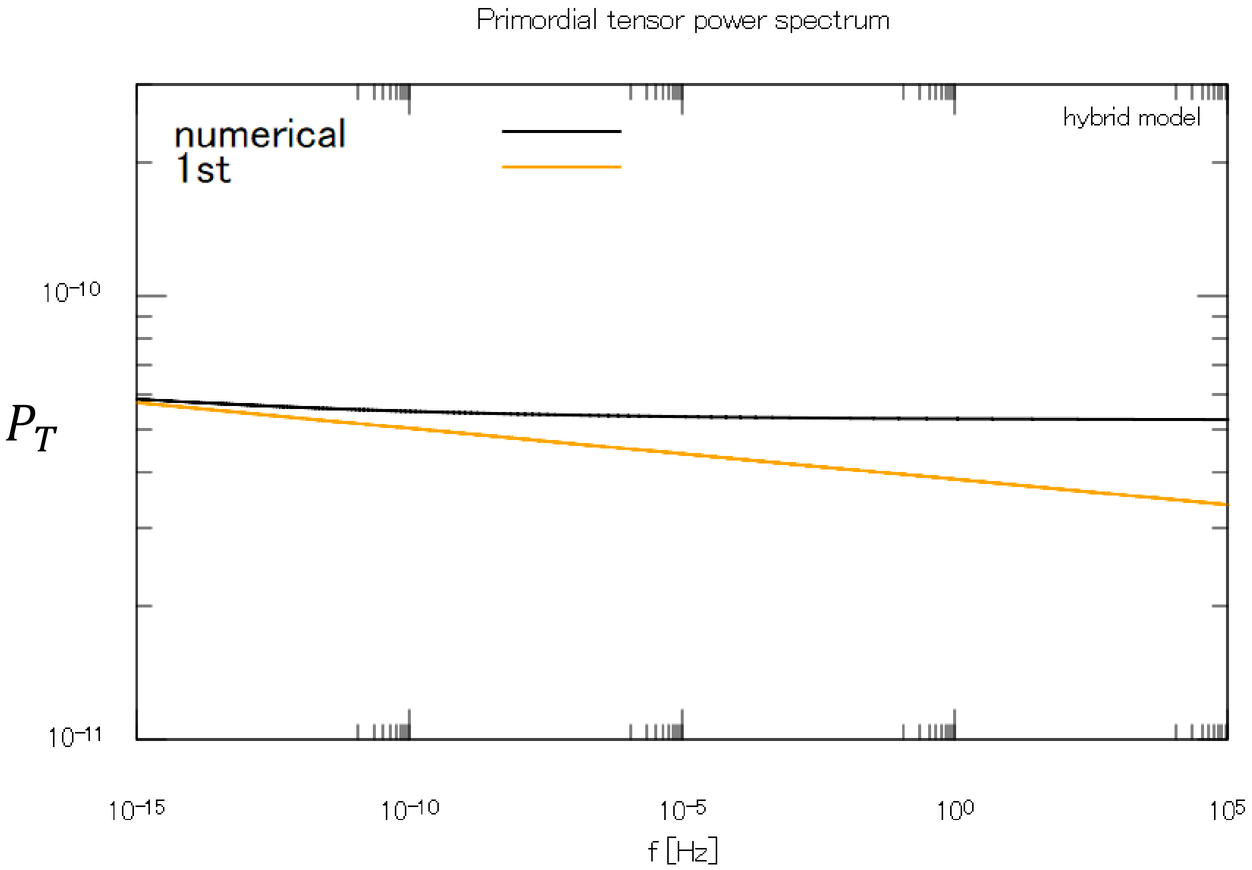
$$\xi_V^2 \equiv M_{\mathrm{Pl}}^4\frac{V'V'''}{V^2} = 0$$

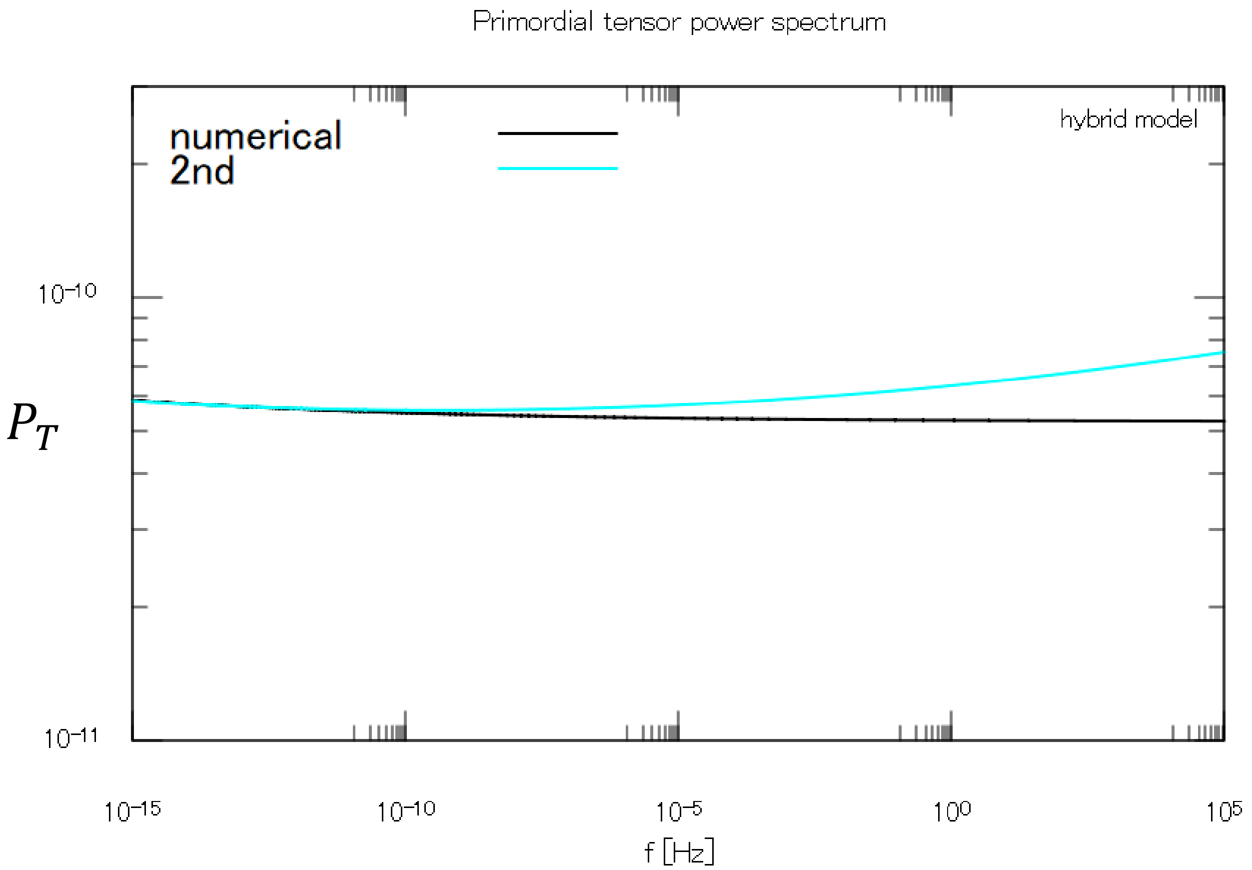
Chaotic model – results&discussion



Kuroyanagi and Takahasi 2011

Hybrid model – results&discussion





$$n_T(k) \equiv \frac{d\,ln\,P_T(k)}{d\,ln\,k} \simeq -2\epsilon_V$$

$$\alpha_T(k) \equiv \frac{d\,ln\,n_T(k)}{d\,ln\,k} \simeq -4\epsilon_V[2\epsilon_V - \eta_V] > 0$$

Conclusion

- We have investigated the validity of the description of the Taylor expansion for the tensor power spectrum by comparing it with numerical calculations using some inflation model.
- In some models, the truncation at low order in the expansion does not give a good description, but when we include higher order terms, the Taylor expansion formula works.

Future Plan

- Validate with other models
  - Mixed inflation and curvaton model

- Another description method

Thank you for your attention!

Back up

Numerical calculation

We calculate power spectra by the below formula

Low of conservation of energy

$$\dot{\rho_r} + 4H\rho_r = \Gamma\rho_\phi$$
$$\dot{\rho_\phi} + 3H\rho_\phi = -\Gamma\rho_\phi$$

The energy density of the field

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

The equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

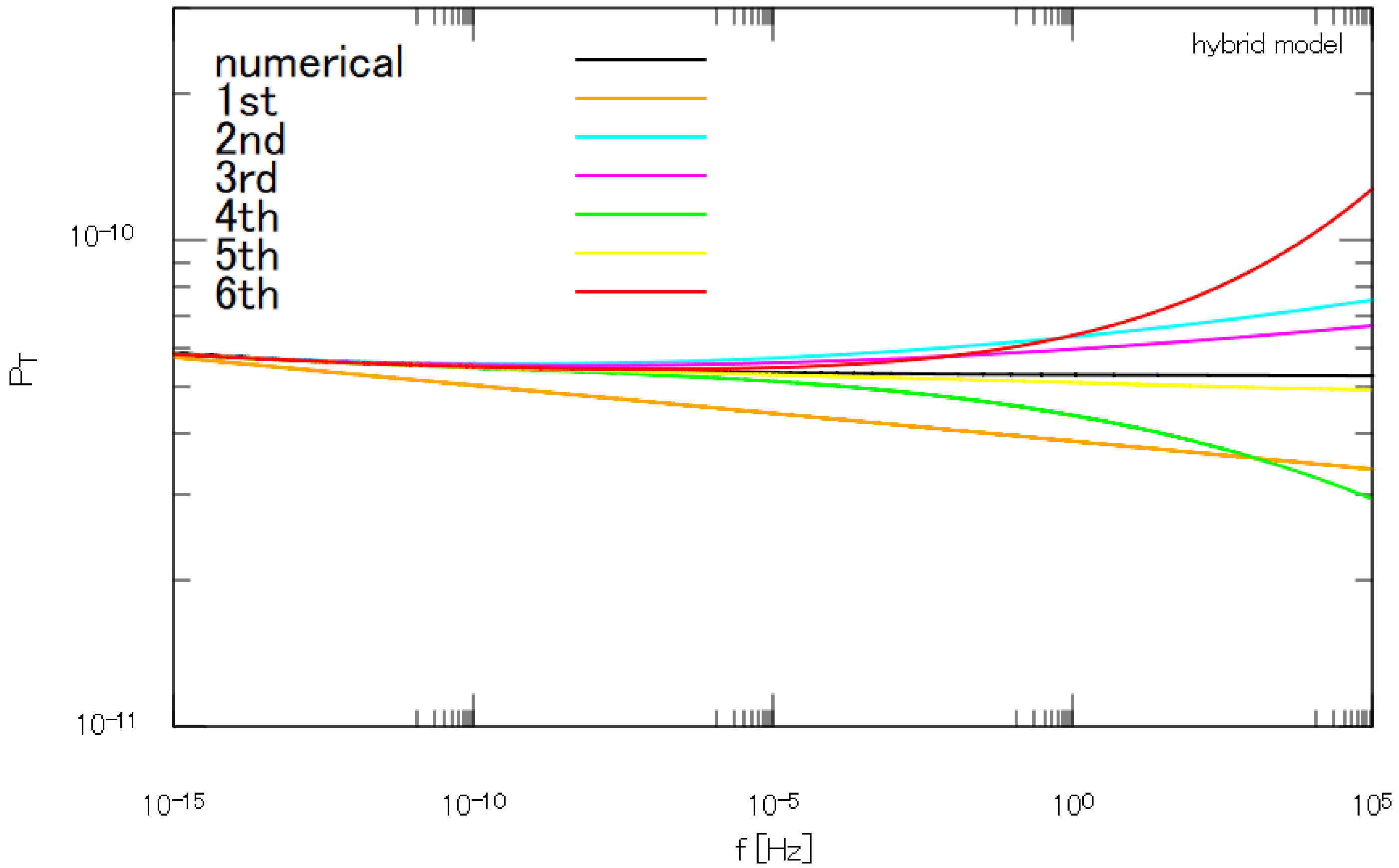
coefficient

$$n_T(k) \equiv \frac{d\,ln\,P_T(k)}{d\,ln\,k} \simeq -2\epsilon_V$$

$$\alpha_T(k) \equiv \frac{d\,ln\,n_T(k)}{d\;ln\,k} \simeq -4\epsilon_V[2\epsilon_V - \eta_V]$$

$$\beta_T(k) \equiv \frac{d\,ln\,\alpha_T(k)}{d\;ln\,k} \simeq -4\epsilon_V[16\epsilon_V^2 + 2\eta_V^2 - 14\epsilon_V\eta_V + \xi_V^2]$$

Hybrid model – 6th order



There is a deviation from numerical at higher orders.