Enhancement of the GW spectrum in early dark energy-like scenario

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in collaboration with
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(**TK,** T. Shinohara, T. Takahashi in preparation)

Saga-Yonsei Partnership XIX @ Yonsei University 20. Jan. 2023

Motivation and Contents

- How does Ω_{GW} behave if there was EDE in the early Universe?
- What are the parameter ranges that Ω_{GW} can be observed?

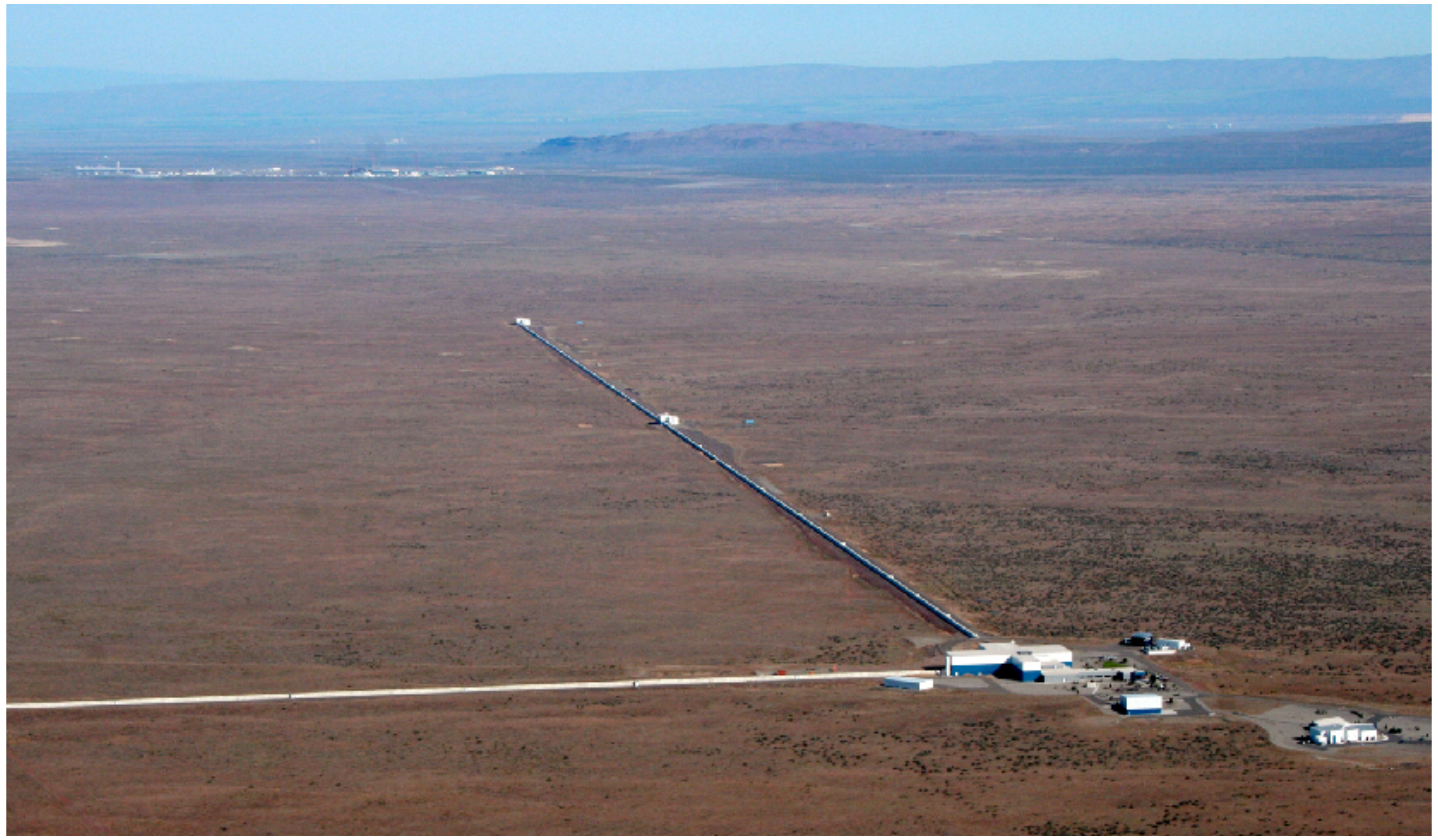
Table of contents

Introduction

✓ First detection

GW159014 (11. Feb. 2016)

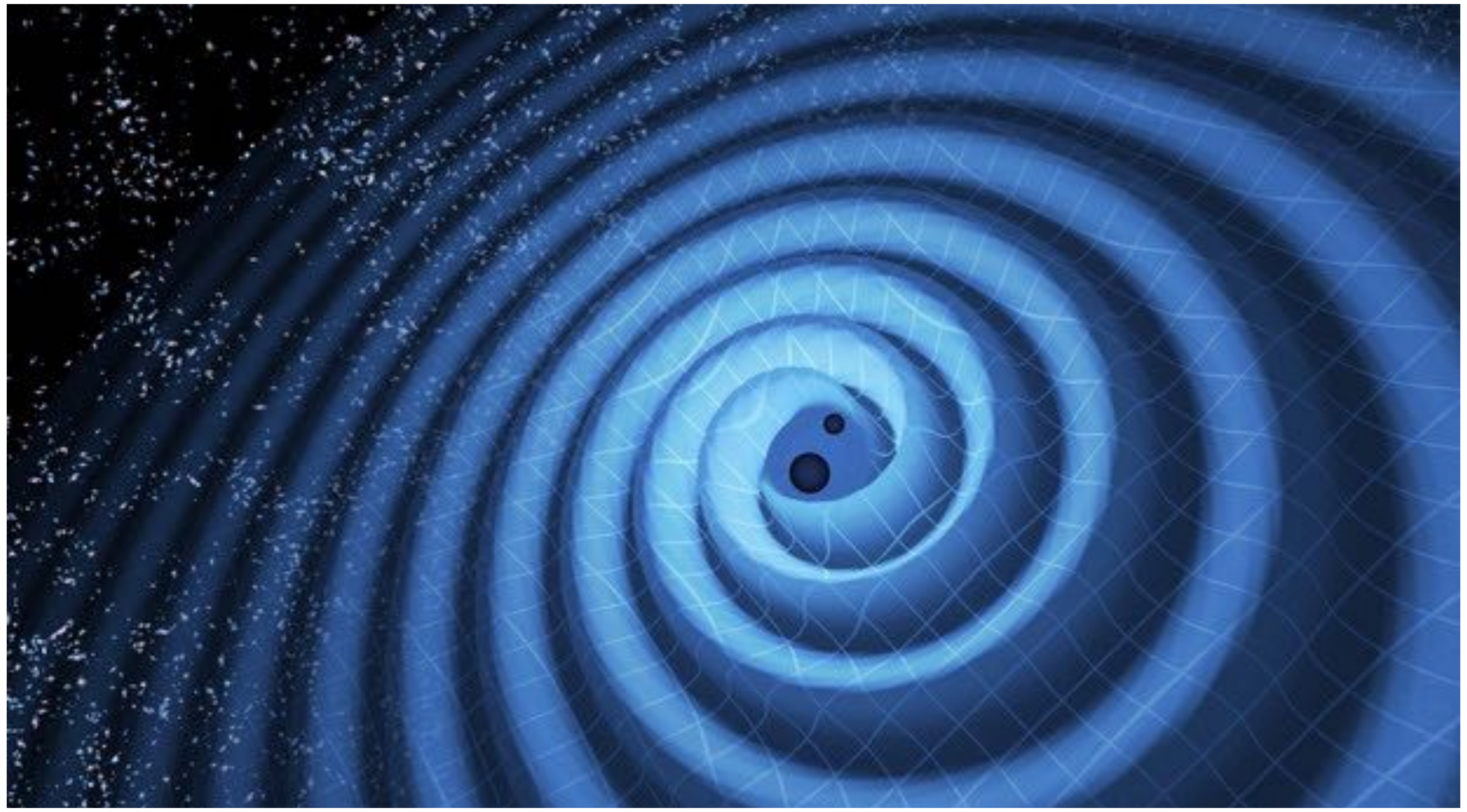
B. P. Abbott et al.
(LIGO Scientific Collaboration and Virgo Collaboration)
Phys. Rev. Lett. **116**, 061102



https://www.ligo.org/multimedia/gallery/lho.php

✓ What is gravitational waves?

⇒ waves propagating in space with varying the curvature



https://www.ligo.caltech.edu/image/ligo20160615f

Introduction

✓ What are gravitational waves?

e.g.) GWs with the background is Minkowski spacetime

- Line element $ds^2 = -dt^2 + (\delta_{ij} + \underbrace{h_{ij}}_{\text{Tensor perturbation}})dx^i dx^j$ (If $h_{ij} = 0$, it is Minkowski.)

- Einstein eq. $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$ $(c = 1)$

Linearize
(ignore $\mathcal{O}(h^2)$)

$$\boxed{\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right) h_{ij} = 0}$$

(TT gauge)

GWs propagate light speed in Minkowski spacetime

Introduction

✓ What are gravitational waves?

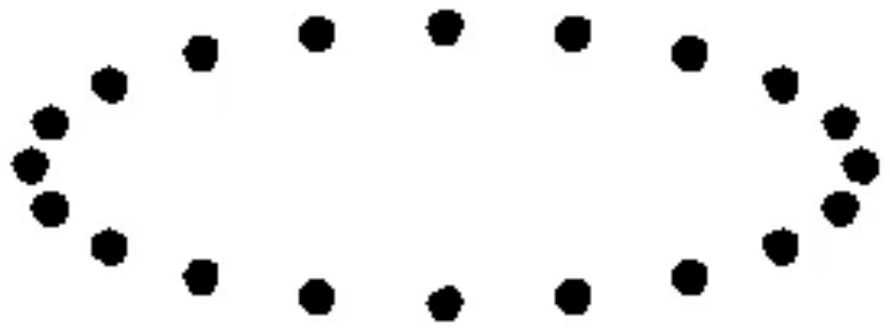
- Line element $$ds^2 = -dt^2 + (\delta_{ij} + \underbrace{h_{ij}}_{\text{Tensor perturbation}})dx^i dx^j$$

https://en.wikipedia.org/wiki/Gravitational_wave

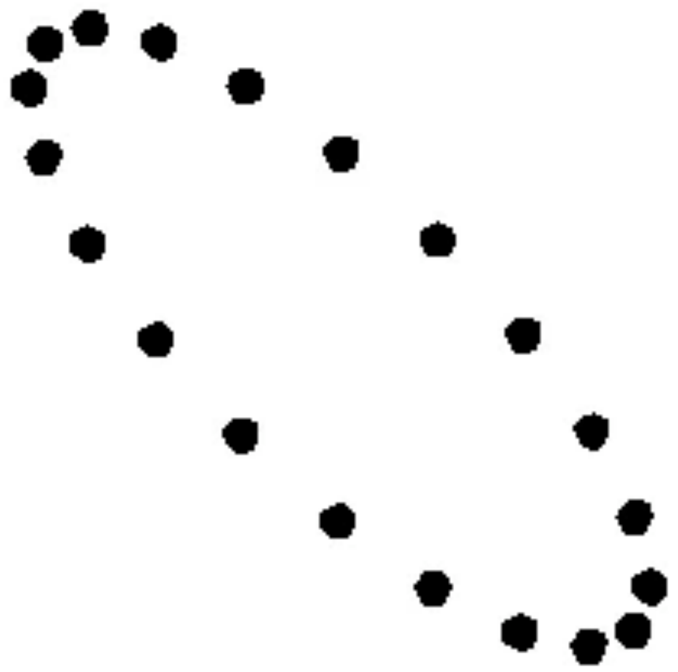
Consider GWs propagates along z axis

$$h_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} \cos(kz - \omega t)$$

Plus mode



Cross mode



e.g.) Proper distance is varied periodically

$$d\ell_x = \left(1 + \frac{1}{2}h_+(t)\right) dx\,, \quad d\ell_y = \left(1 - \frac{1}{2}h_+(t)\right) dy$$

Primordial GW power spectrum

✓ GWs in the FLRW background

FLRW line element $$ds^2 = -dt^2 + \underbrace{a^2(t)}_{\text{Scale factor}}\left[\delta_{ij} + h_{ij}\right]dx^i dx^j$$

Linearized Einstein eq.

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = 16\pi G \Pi_{ij}$$

EoM for tensor pert. $$\ddot{h}^{\lambda}_{\mathbf{k}} + 3H\dot{h}^{\lambda}_{\mathbf{k}} + \frac{k^2}{a^2}h^{\lambda}_{\mathbf{k}} = 0 \qquad (\Pi_{ij} = 0)$$

Primordial GW power spectrum

✓ Stochastic GW background and Observable

- GW spectrum … dimensionless quantity to characterize the strength of GW

$$\boxed{\Omega_{\rm GW} = \frac{1}{12}\left(\frac{k}{aH}\right)^2 \mathscr{P}_{\rm T,prim}(k) T_T^2(k)} \quad \left(\Omega_{\rm GW} \equiv \frac{1}{\rho_{\rm crit}} \frac{d\rho_{\rm GW}}{d\log k}\right)$$

$\mathscr{P}_{\rm T,prim}(k)$ … Primordial GW power spectrum,
$T_T(k)$ … Transfer function

Primordial GW power spectrum

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$\mathscr{P}_{\rm T,prim}(k)$ … Primordial GW power spectrum,

$T_T(k)$ … Transfer function

$$\mathscr{P}_{T,prim}(k) = A_T\left(\frac{k}{k_*}\right)^{n_T}$$

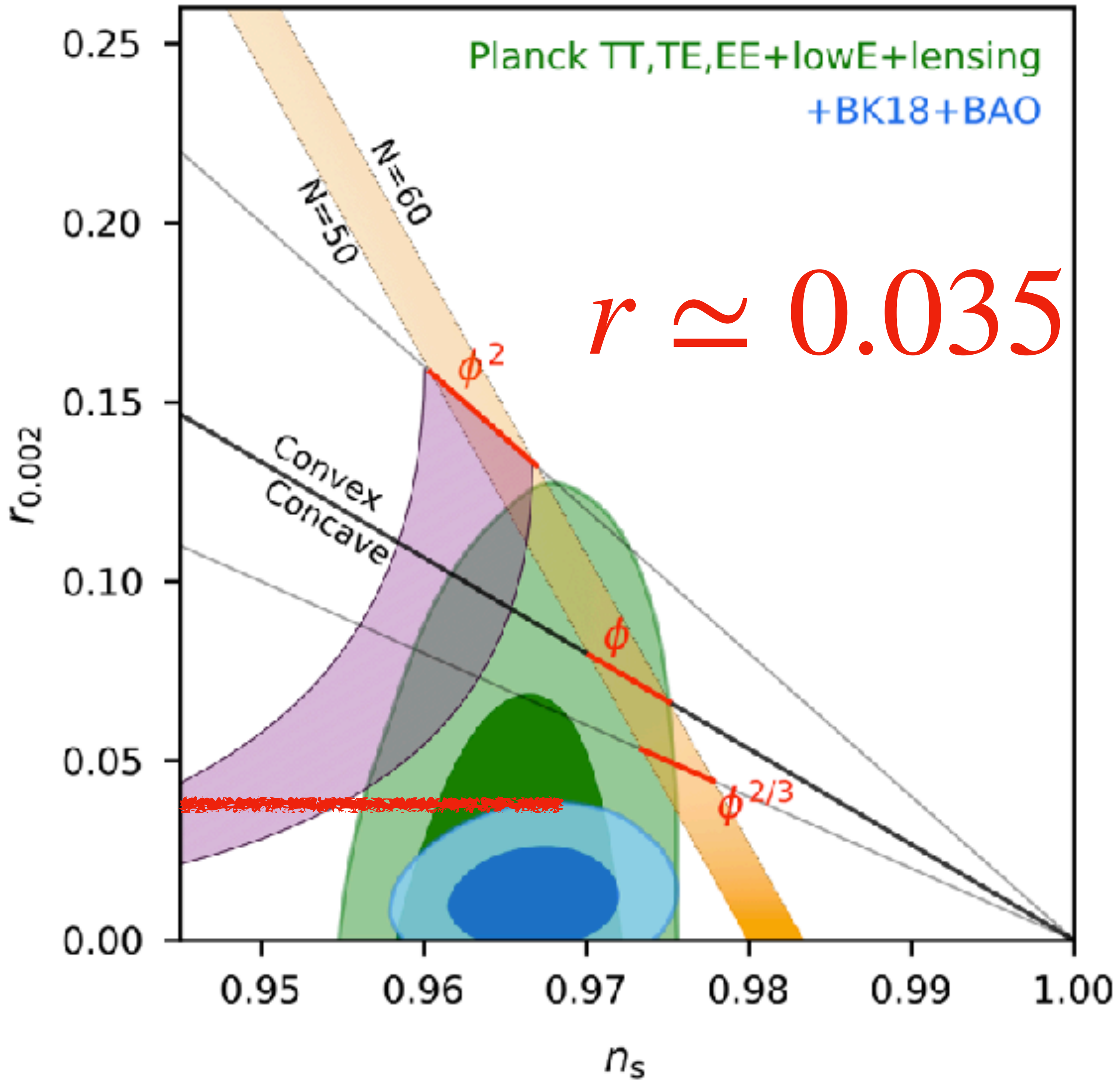
k_*: pivot scale

$$r = -8n_T$$

Consistency relation
(Single field inf.)

(Yuta will talk!)

Planck collaboration
Y. Akrami et al.
[arXiv: 1807.06211]



Primordial GW power spectrum

✓ Stochastic GW background and Observable

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$$\boxed{\Omega_{\rm GW} = \frac{1}{12}\left(\frac{k}{aH}\right)^2 \mathscr{P}_{\rm T,prim}(k) T_T^2(k)} \qquad \left(\Omega_{\rm GW} \equiv \frac{1}{\rho_{\rm crit}} \frac{d\rho_{\rm GW}}{d\log k}\right)$$

$\mathscr{P}_{\rm T,prim}(k)$ … Primordial GW power spectrum,

$T_T(k)$ … Transfer function

The function that can relate
the present power spectrum
and
the primordial power spectrum

$$\mathscr{P}_{T,prim}(k) = A_T\left(\frac{k}{k_*}\right)^{n_T}$$

k_*: pivot scale

$$r = -8n_T$$

Consistency relation
(Single field inf.)

Early-dark energy (EDE)

✓ What is EDE?

⇒ Axion-like scalar field that can solve Hubble tension

(See Sora or Fumiya's presentation!)

(Note: we will consider scalar field different from the scalar field that can solve it)

$$\boxed{V(\phi) = V_0 \left(1 - \cos\frac{\phi}{f}\right)^n}$$

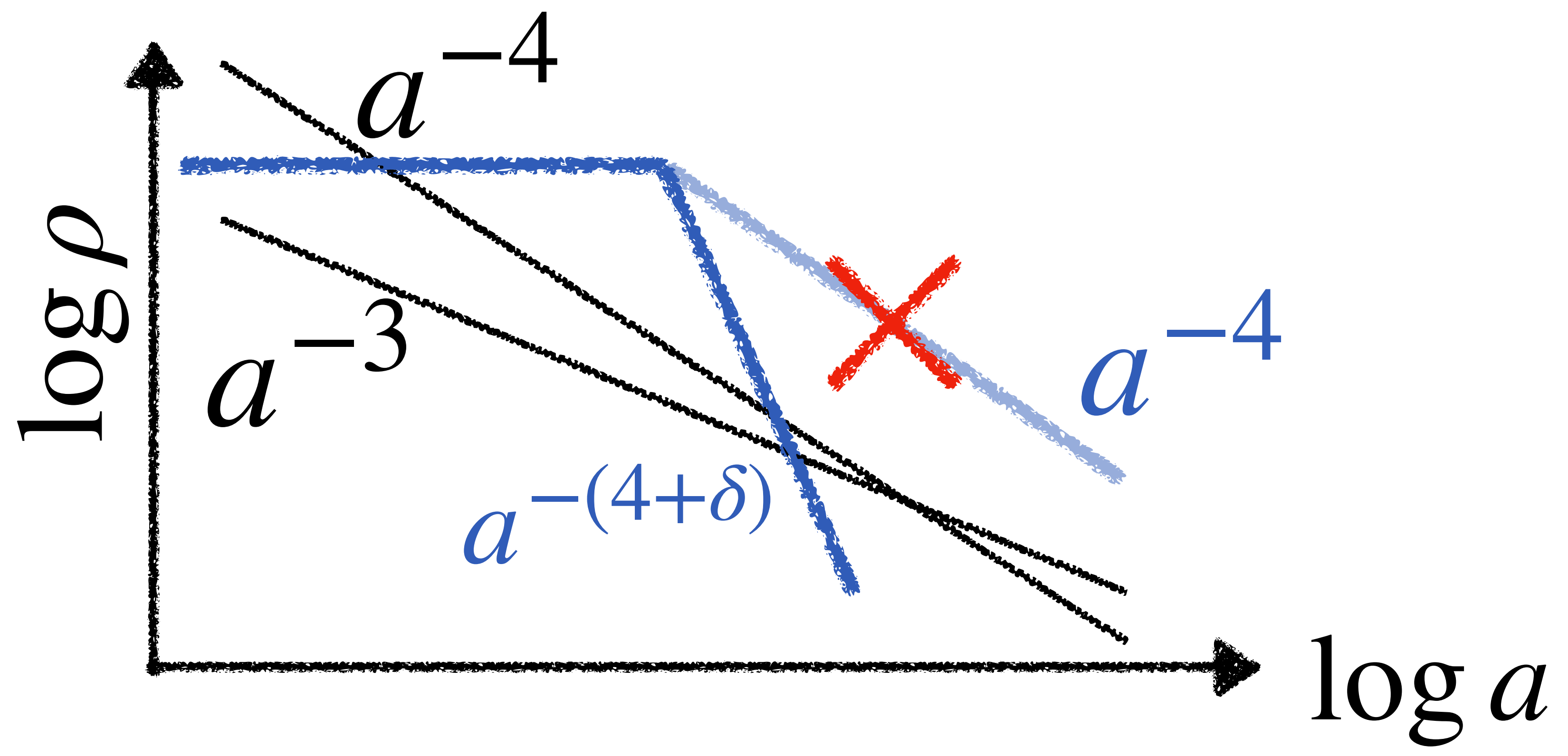
V. Poulin et al, [arXiv: 1806.10608]

Motivation & Set up

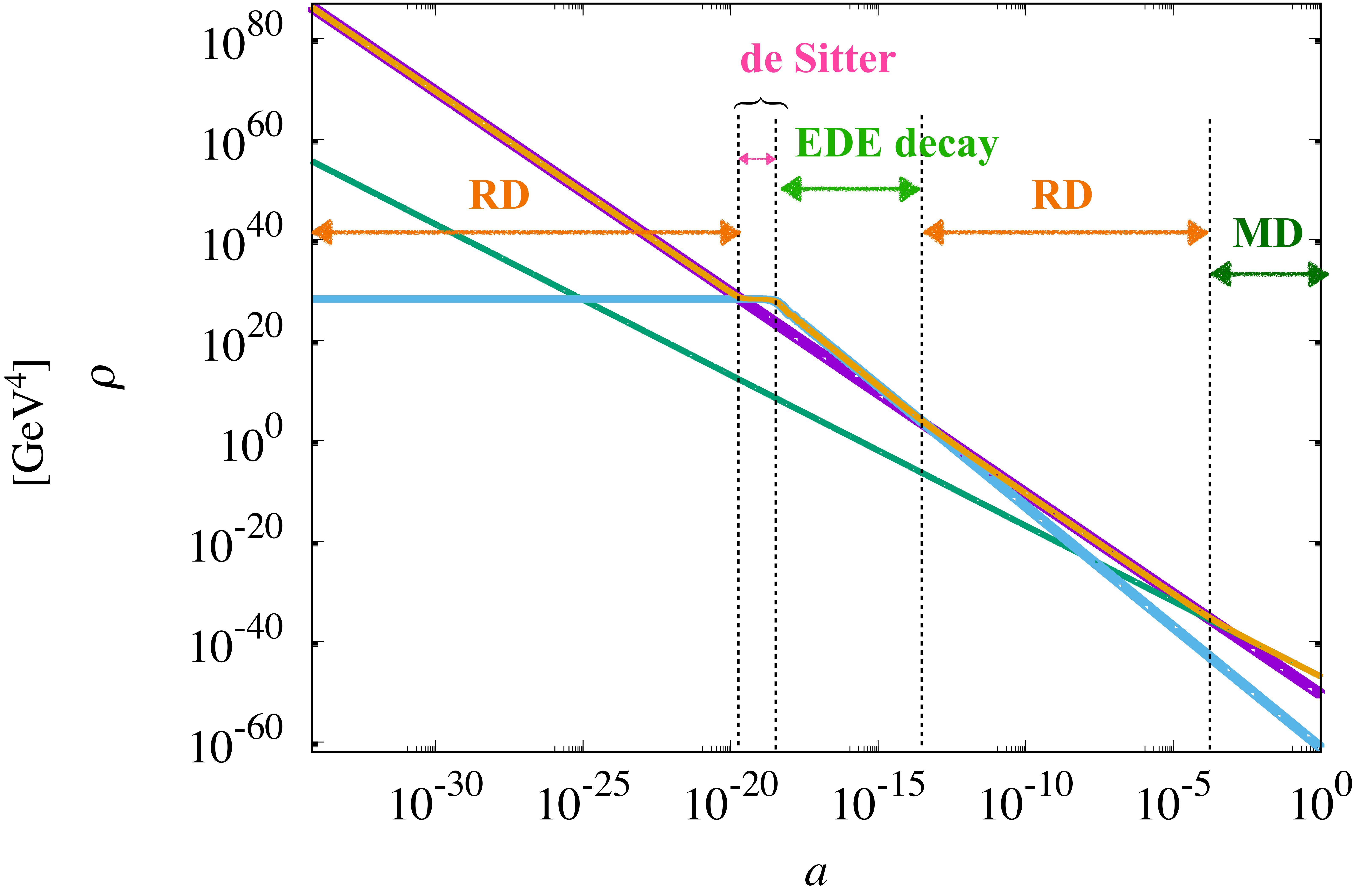
- How does Ω_{GW} behave if there was EDE in the early Universe?
- What are the parameter ranges that Ω_{GW} can be observed?

- EDE slow-roll on the potential
- EDE should decay faster than radiation to be consistent with current observation
- There are studies that the dependence of ρ is similar to ours, but mechanism is different

(Y. Gouttenoire et al. [arXiv: 2108.10328
R. T. Co et al. [arXiv: 2108.09299]
Y. Gouttenoire et al. [arXiv: 2111.01150])



Evolution of EDE



De Sitter phase appears between the radiation phase in our setup.

Early-dark energy (EDE)

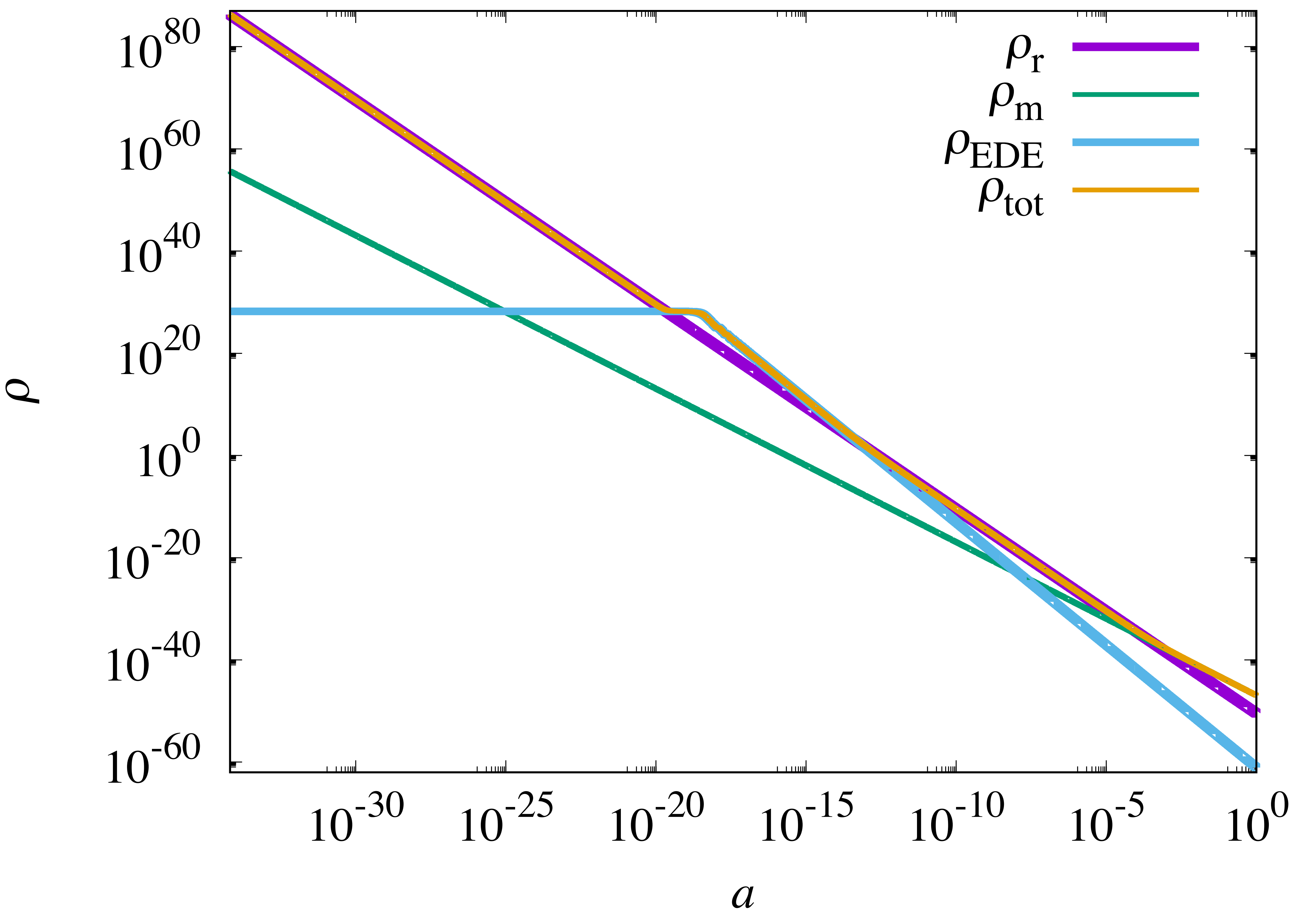
✓ How does $\Omega_{\rm GW}$ be affected by EDE?

⇒ Thermal history changes from ΛCDM

⇒ $T_T(k)$ changes

$$\Omega_{\rm GW} = \frac{1}{12}\left(\frac{k}{aH}\right)^2 \mathscr{P}_{\rm T,prim}(k) T_T^2(k)$$

⇒ Therefore, EDE affects $\Omega_{\rm GW}$



Early-dark energy (EDE)

✓ ρ vs $a(t)$

$$V(\phi) = V_0 \left(1 - \cos\frac{\phi}{f}\right)^n$$

$\downarrow$

$$w_n = \frac{n-1}{n+1}$$

V. Poulin et al,
[arXiv: 1806.10608]

$\downarrow$

$$\rho \propto a^{-3(1+w_n)}$$

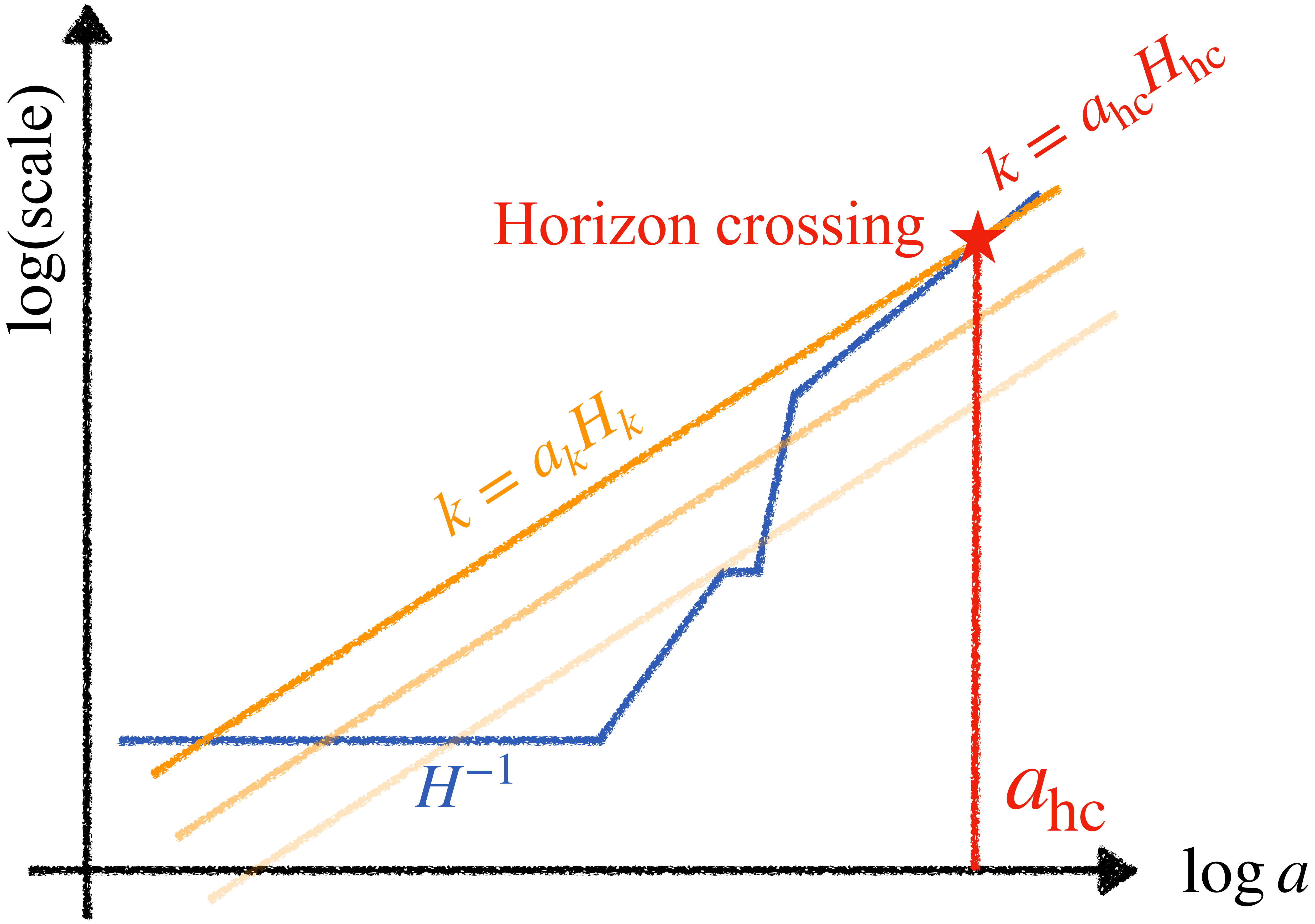
Table: Dependence of $a(t)$ for ρ

n		ρ	
1	$\rightarrow$	a^{-3}	Matter
2	$\rightarrow$	a^{-4}	Radiation
3	$\rightarrow$	$a^{-9/2}$	
$\vdots$		$\vdots$	
∞	$\rightarrow$	a^{-6}	Kination

Early-dark energy (EDE)

✓ Estimate k dependence of Ω_{GW}

$$\rho \propto a^{-3(1+w_n)} \rightarrow H \propto a^{-3(1+w_n)/2} \rightarrow a_{\mathrm{hc}} \propto k^{-2/(1+3w)}$$



$$\Omega_{\mathrm{GW},0} = \frac{1}{12}\left(\frac{k}{H_0}\right)^2 \mathscr{P}_{\mathrm{T,prim}}(k) T_T^2(k)$$

$$\propto k^2 a_{\mathrm{hc}}^2$$

$$\boxed{\Omega_{\mathrm{GW},0} \propto k^{2(-1+3w)/(1+3w)}}$$

Early-dark energy (EDE)

✓ Estimate k dependence of $\Omega_{\rm GW}$

$$\rho \propto a^{-3(1+w_n)} \rightarrow H \propto a^{-3(1+w_n)/2} \rightarrow a_{\rm hc} \propto k^{-2/(1+3w)}$$

$$\Omega_{\rm GW,0} = \frac{1}{12}\left(\frac{k}{H_0}\right)^2 \mathscr{P}_{\rm T,prim}(k) T_T^2(k)$$

$$\propto k^2 a_{\rm hc}^2$$

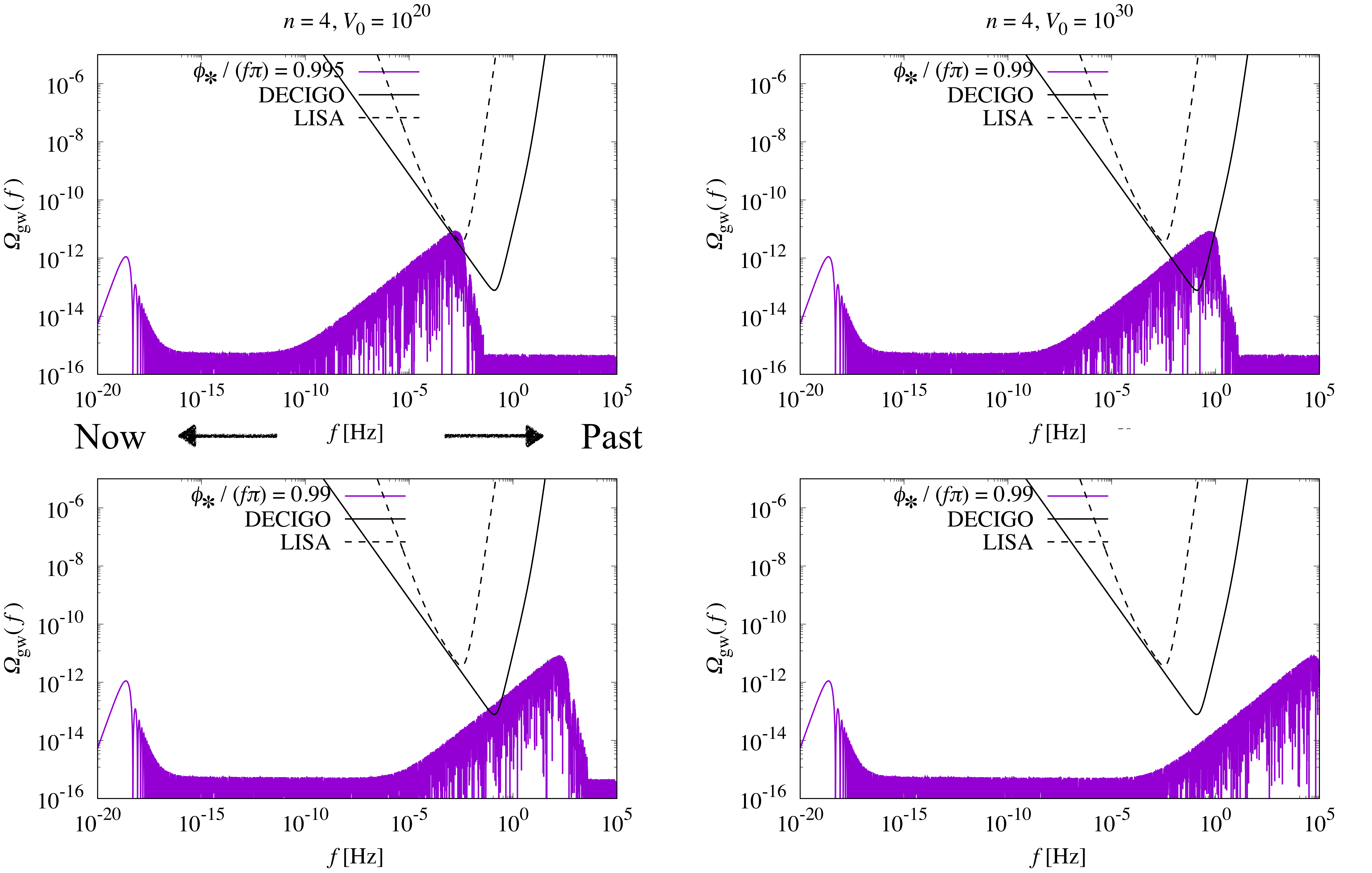
$$\boxed{\Omega_{\rm GW,0} \propto k^{2(-1+3w)/(1+3w)}}$$

n	ρ	$\Omega_{\rm GW}$
1	a^{-3}	k^{-2}
2	a^{-4}	k^{0}
3	$a^{-9/2}$	$k^{2/5}$
$\vdots$	$\vdots$	$\vdots$
∞	a^{-6}	k^{1}

When $n > 2$, $\Omega_{\rm GW}$ is enhanced

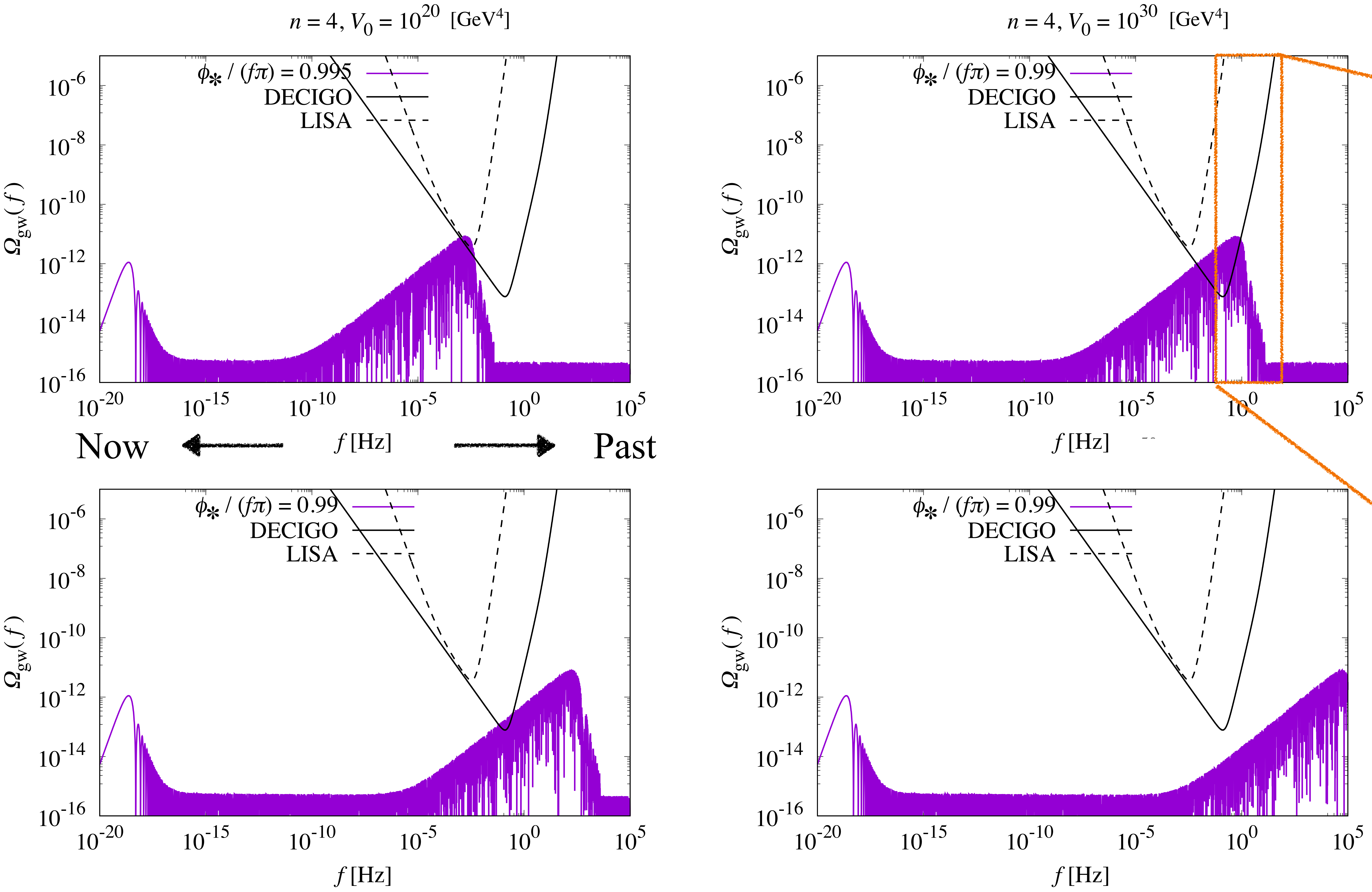
$$V(\phi) = V_0 \left(1 - \cos\frac{\phi}{f}\right)^n$$

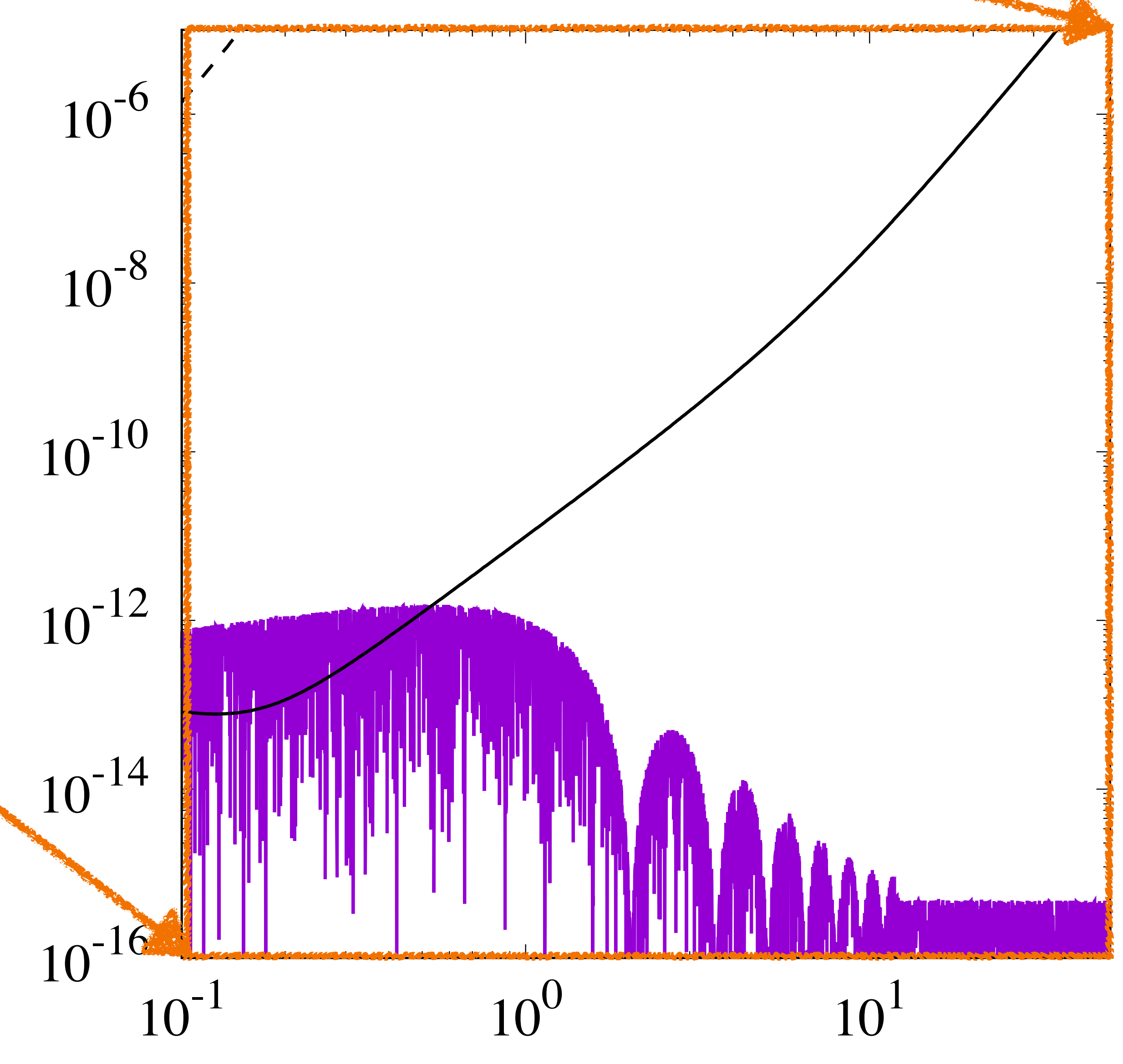
GW spectrum in EDE scenario



The peak Ω_{GW} moves high frequency as the energy scale increases.

GW spectrum in EDE scenario

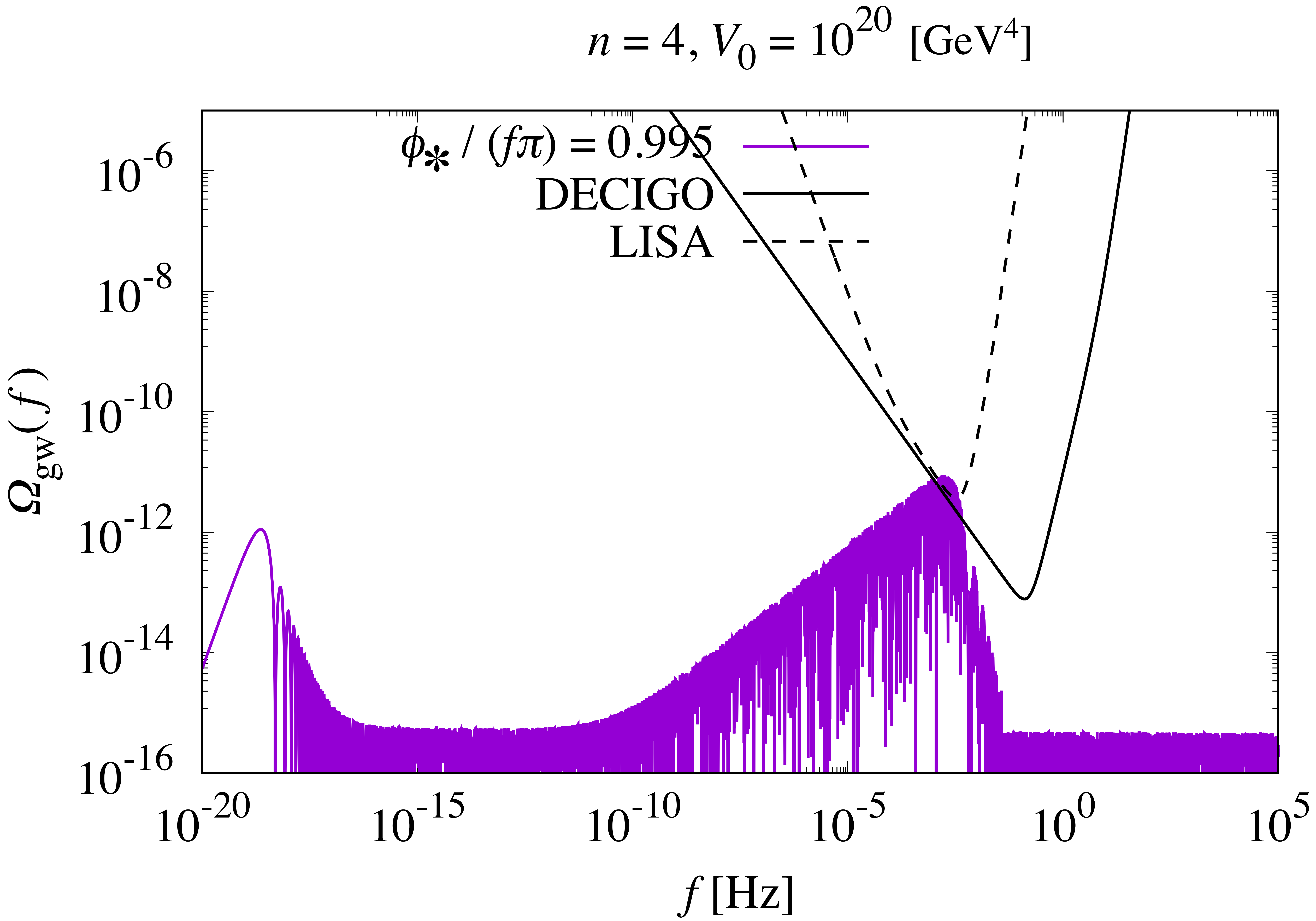


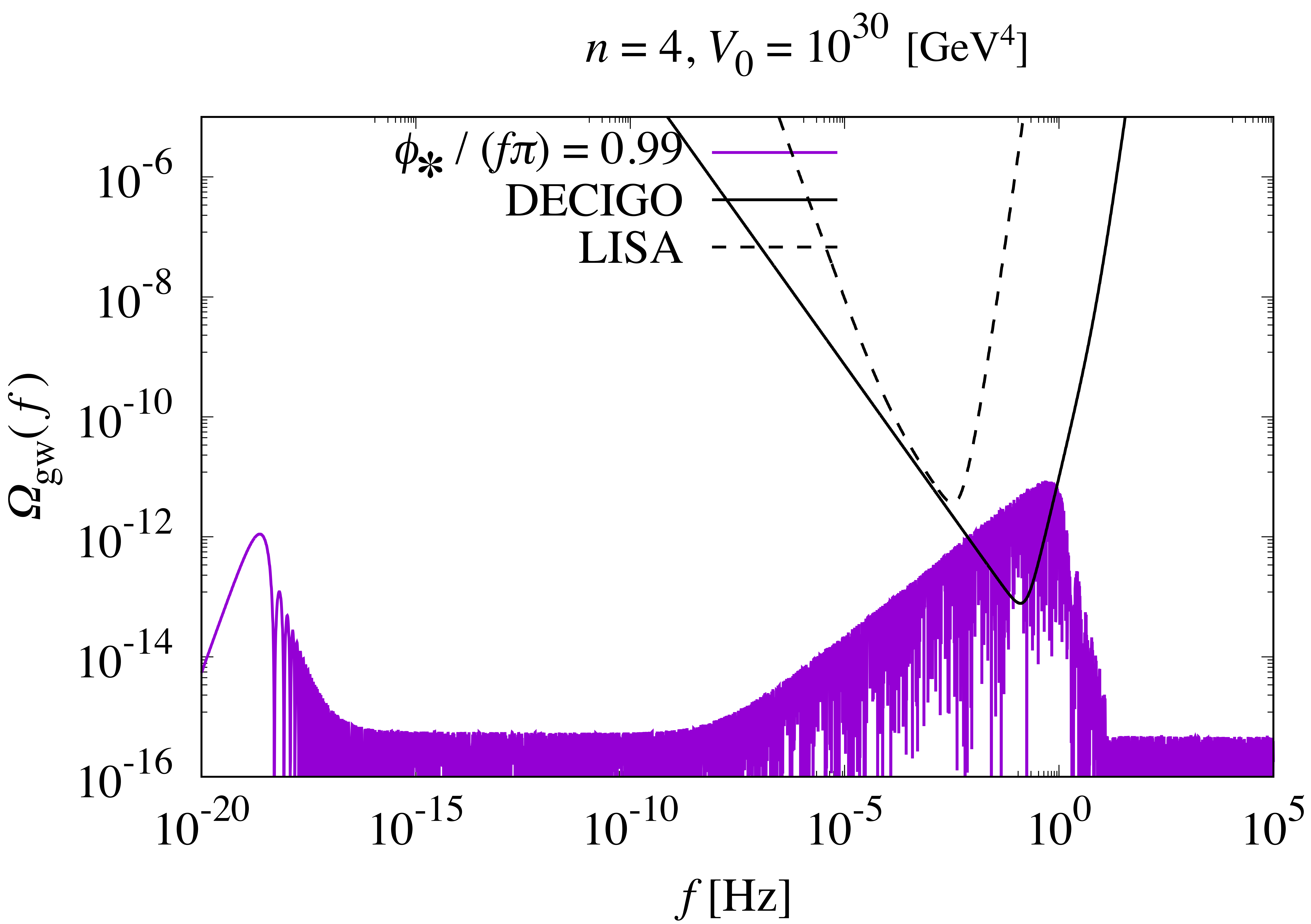


(Shi Pi et al. [arXiv: 1904.06304])

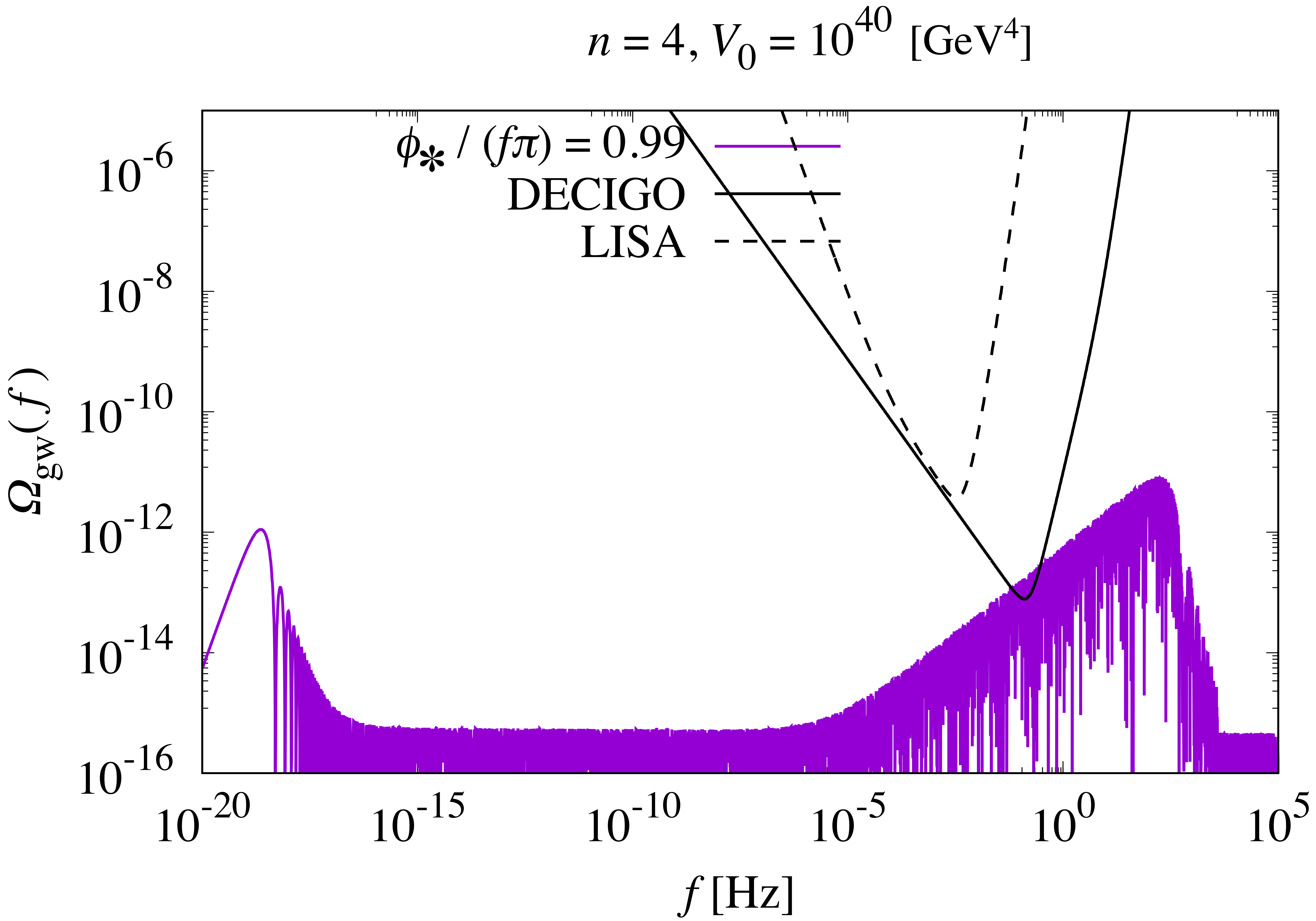
Oscillation in the de Sitter period

The peak Ω_{GW} moves high frequency as the energy scale increases.

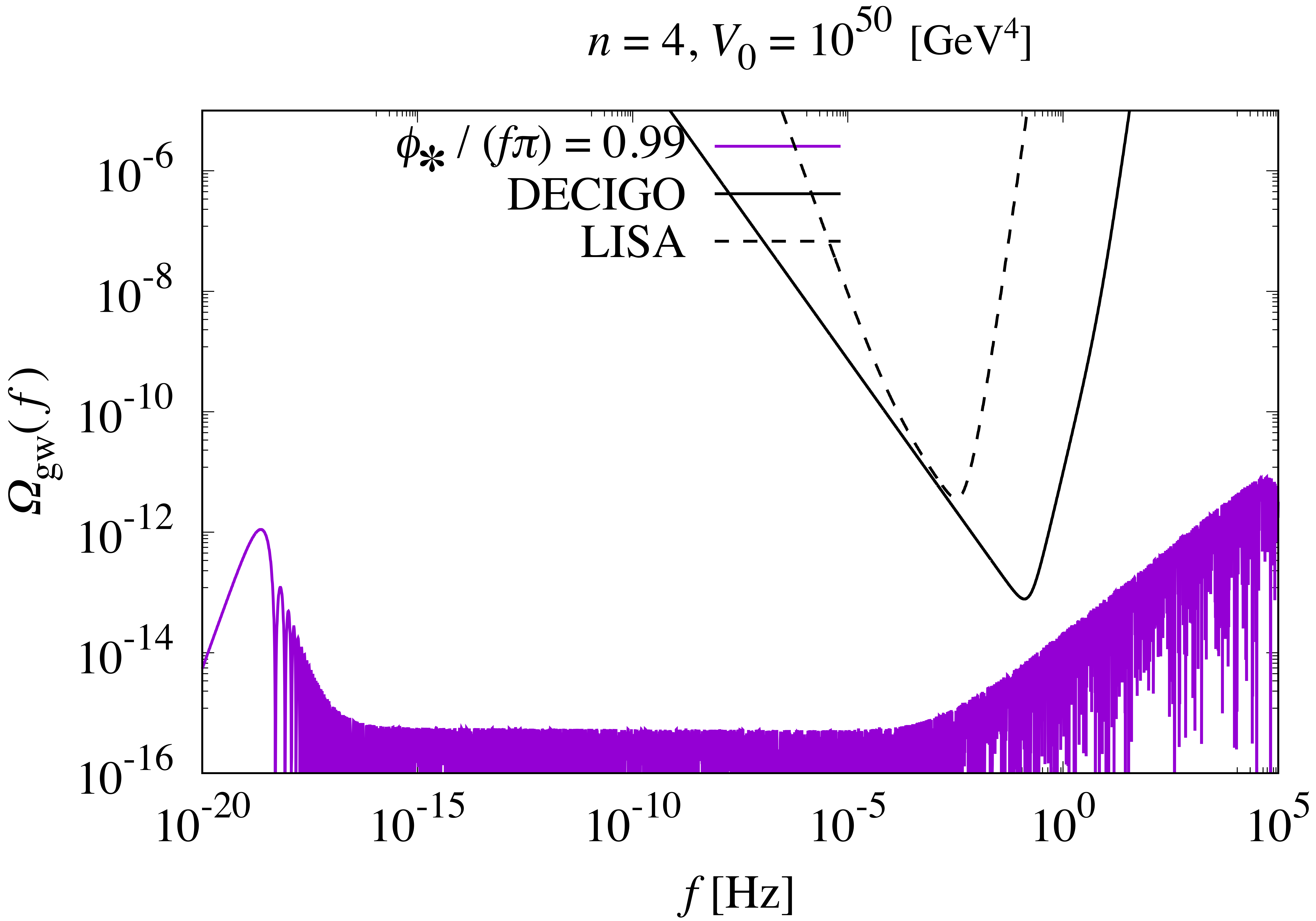

$n = 4$, $V_0 = 10^{20}$ [GeV^4]
$\phi_* / (f\pi) = 0.995$
DECIGO
LISA
$\Omega_{\rm gw}(f)$
f [Hz]
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-20}
10^{-15}
10^{-10}
10^{-5}
10^{0}
10^{5}



$n = 4$, $V_0 = 10^{30}$ [GeV^4]
$\phi_* / (f\pi) = 0.99$
DECIGO
LISA
$\Omega_{\rm gw}(f)$
f [Hz]
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-20}
10^{-15}
10^{-10}
10^{-5}
10^{0}
10^{5}

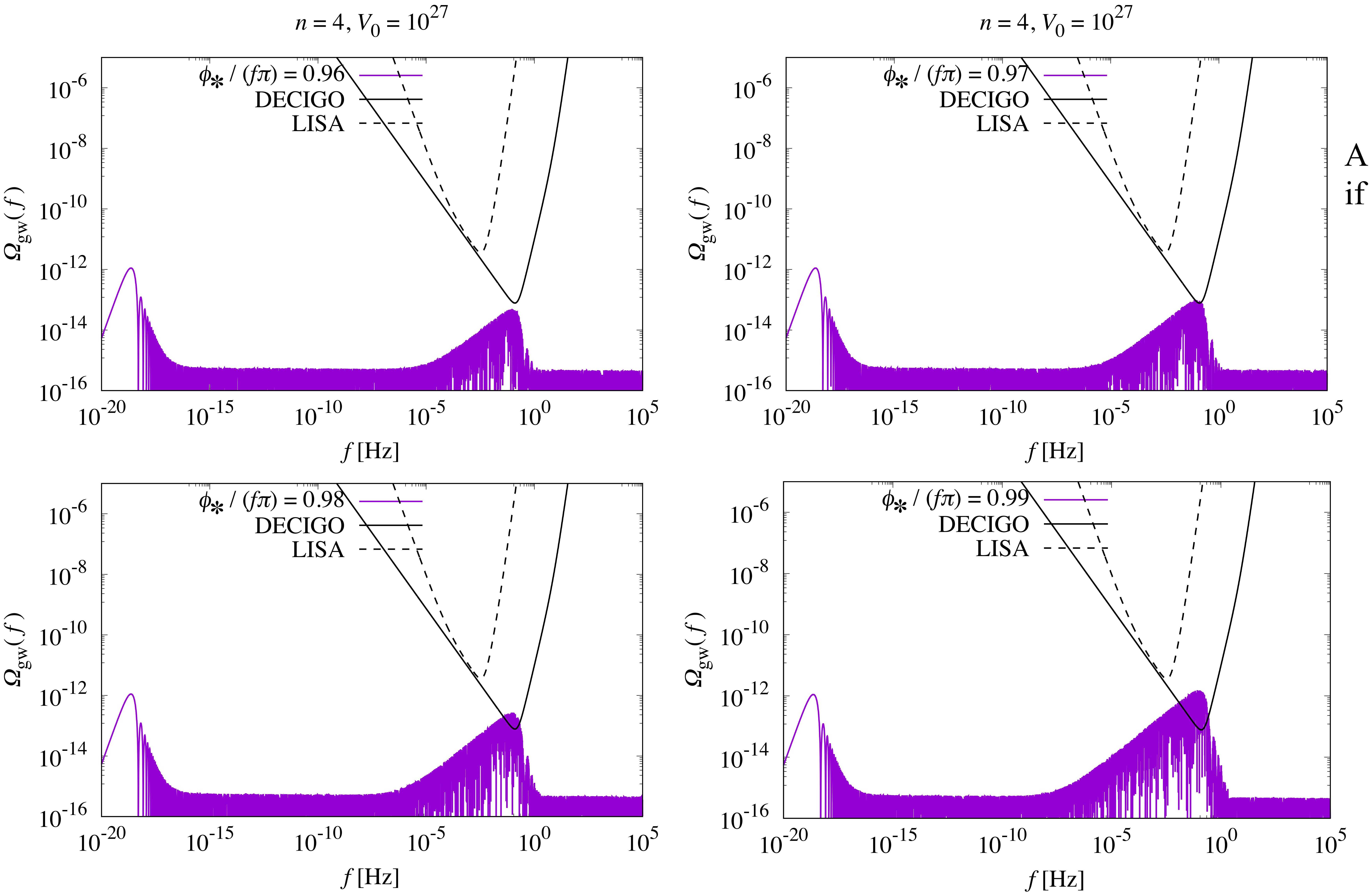


$n = 4, V_0 = 10^{40}$ [GeV^4]
$\phi_* / (f\pi) = 0.99$
DECIGO
LISA
$\Omega_{gw}(f)$
f [Hz]
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-20}
10^{-15}
10^{-10}
10^{-5}
10^{0}
10^{5}

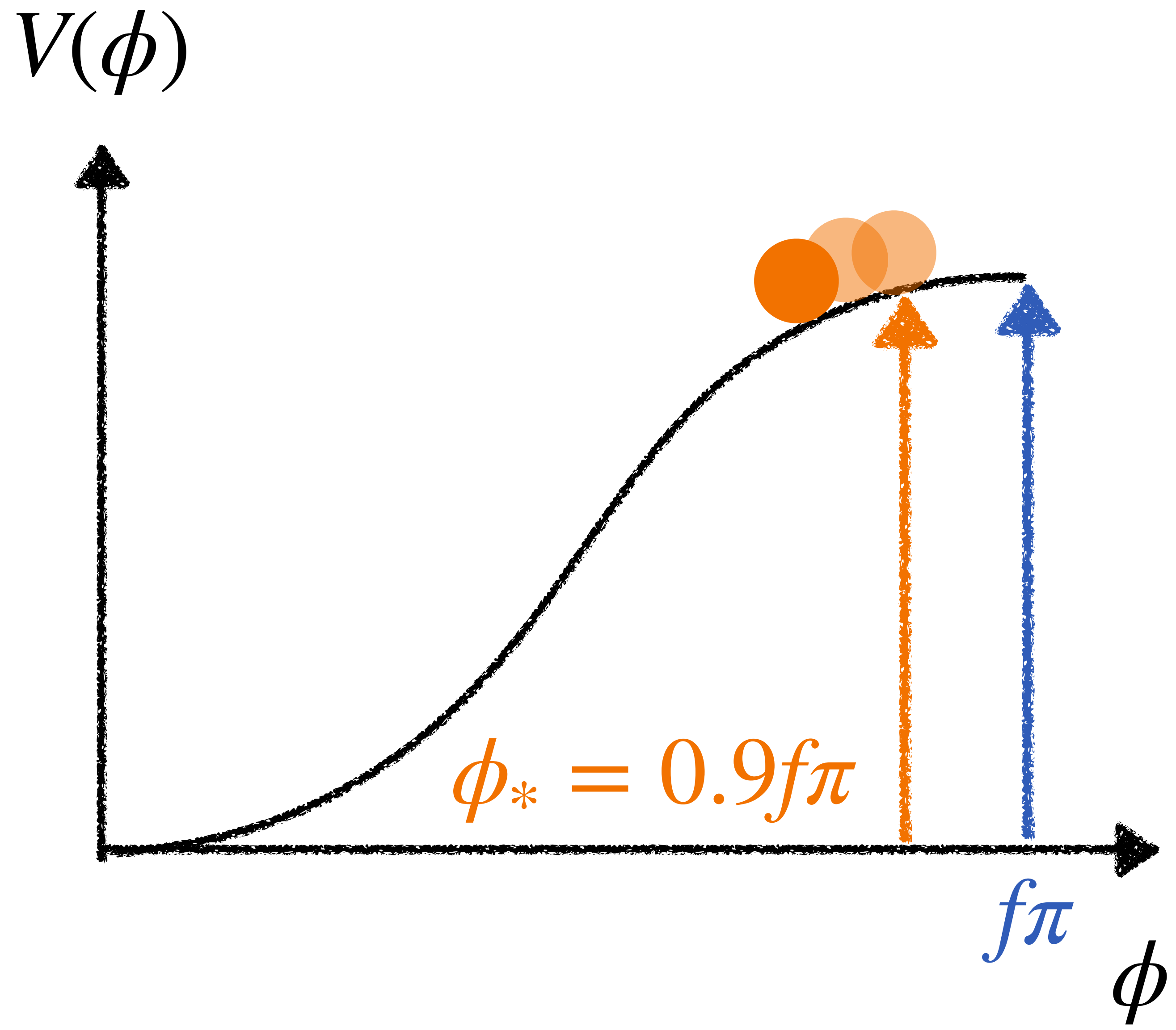


$n = 4$, $V_0 = 10^{50}$ [GeV^4]
$\phi_* / (f\pi) = 0.99$
DECIGO
LISA
$\Omega_{\rm gw}(f)$
f [Hz]
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-20}
10^{-15}
10^{-10}
10^{-5}
10^{0}
10^{5}

GW spectrum in EDE scenario

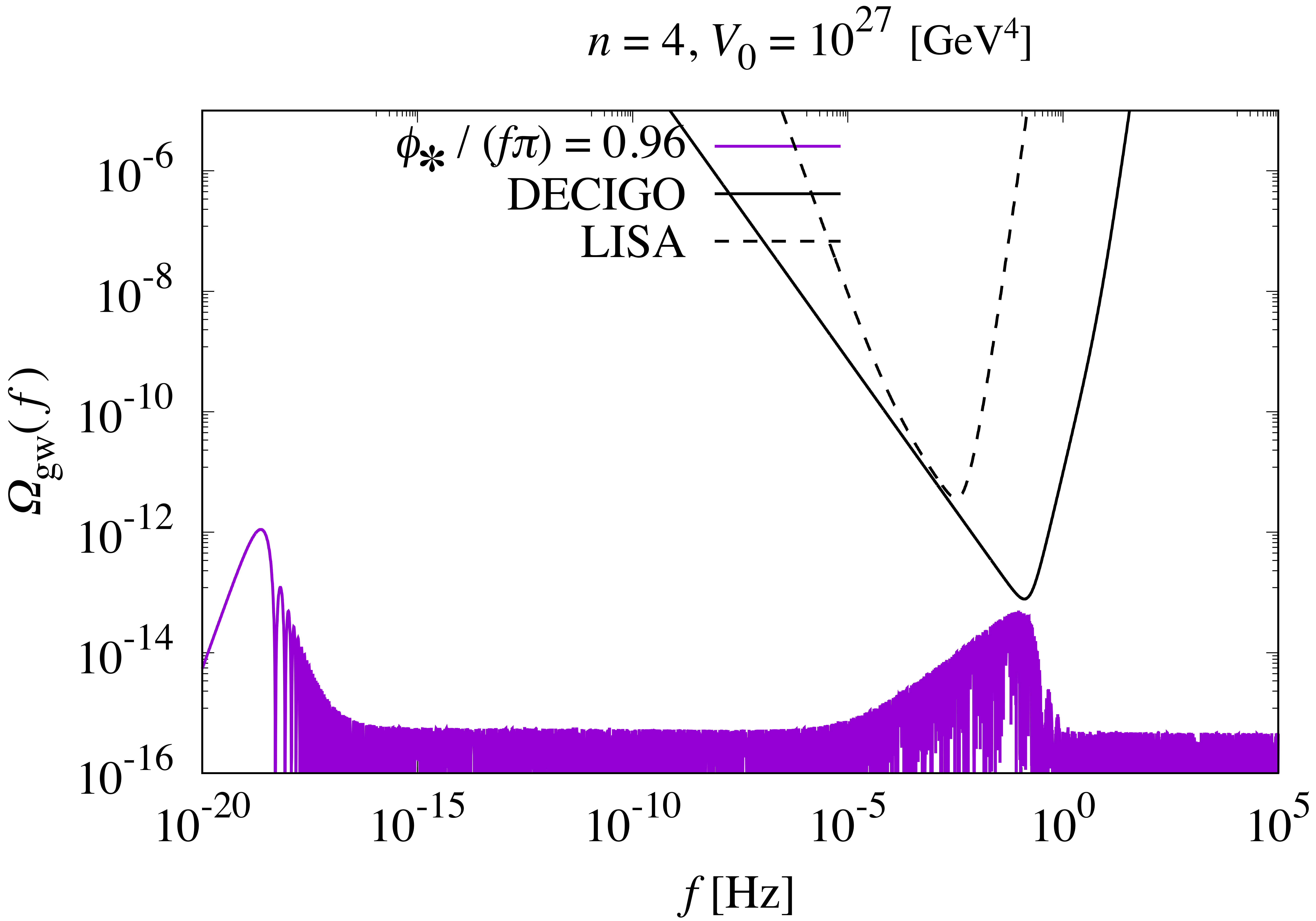


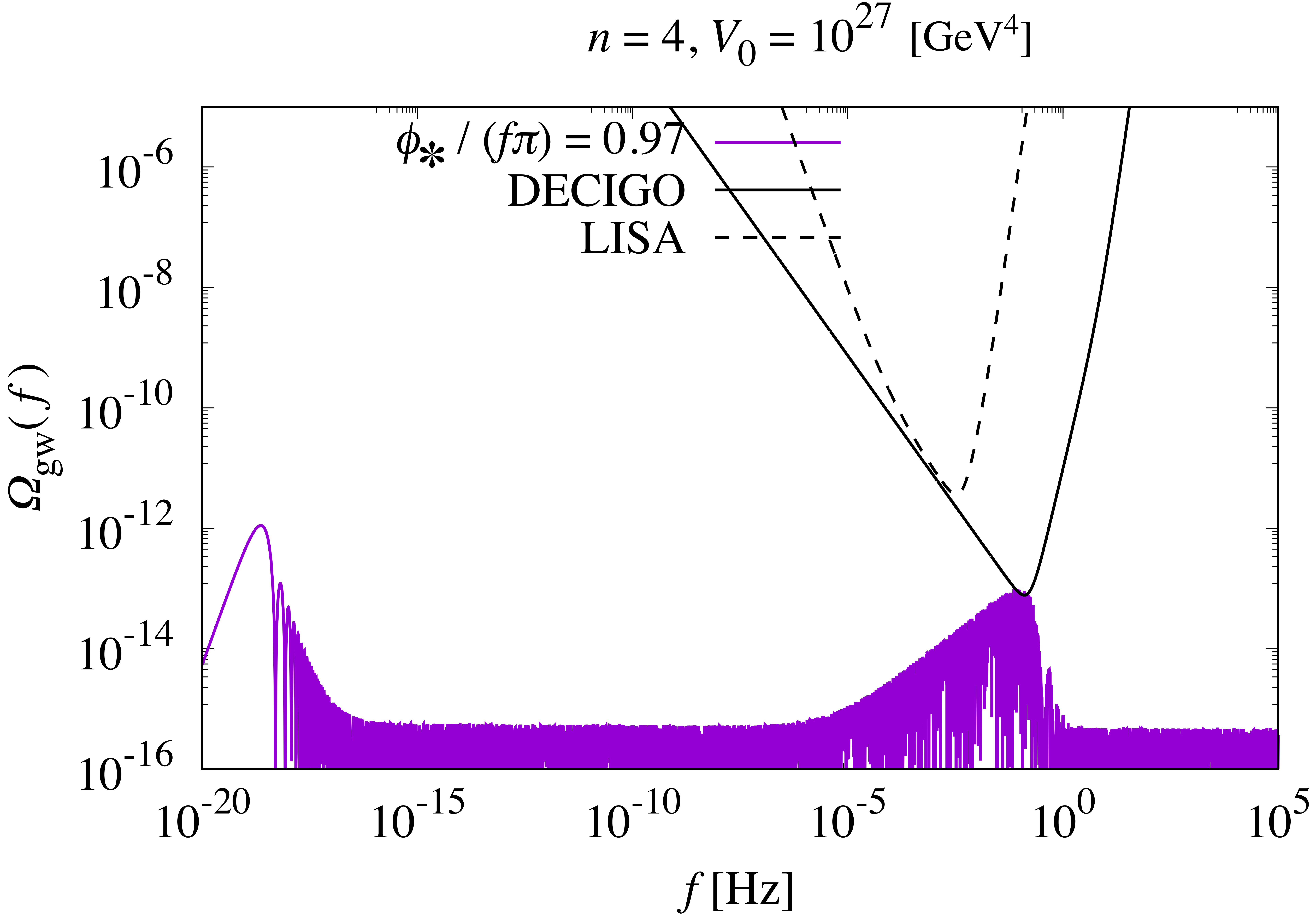
A period of slow-rolling is getting longer if you put it next to the top on the potential.

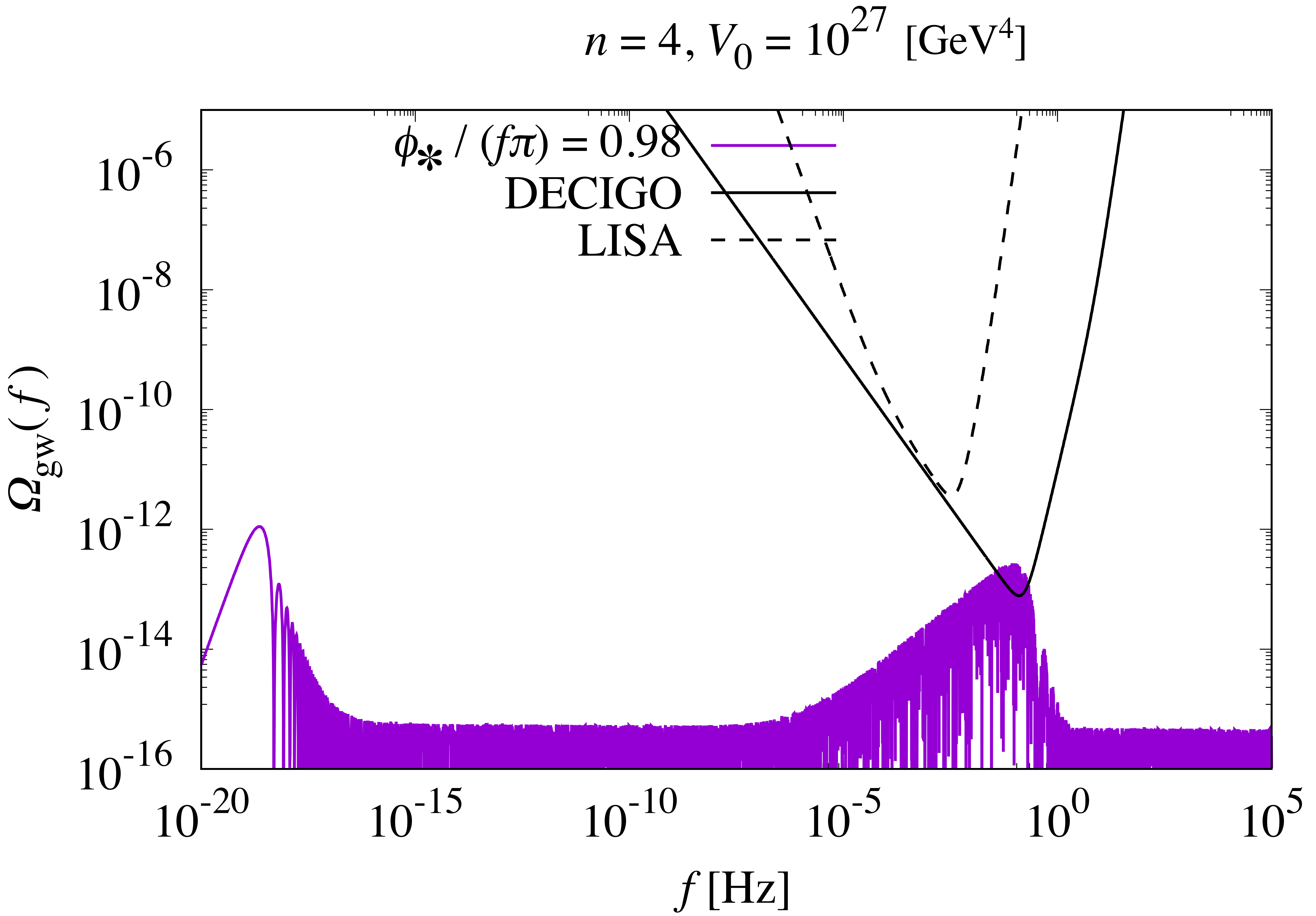


Since the EDE-dominated era is getting longer Ω_{GW} can further enhance.

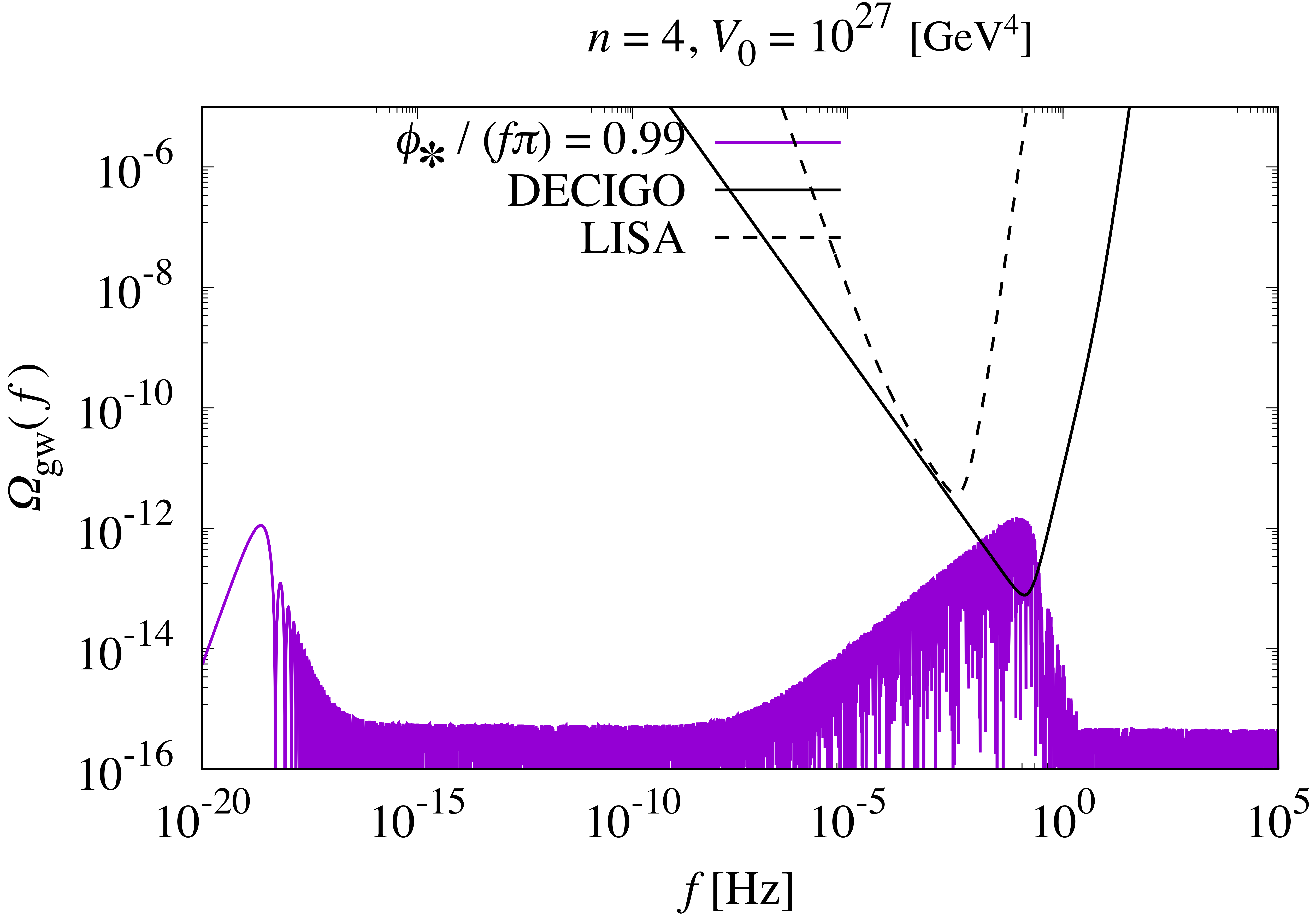
The peak height Ω_{GW} enhances by ϕ_* putting close to the top of potential.


$n = 4, V_0 = 10^{27}$ [GeV^4]
$\phi_* / (f\pi) = 0.96$
DECIGO
LISA
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
$\Omega_{gw}(f)$
10^{-20}
10^{-15}
10^{-10}
10^{-5}
10^{0}
10^{5}
f [Hz]


$n = 4$, $V_0 = 10^{27}$ [GeV^4]
$\phi_* / (f\pi) = 0.97$
DECIGO
LISA
$\Omega_{gw}(f)$
f [Hz]
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-20}
10^{-15}
10^{-10}
10^{-5}
10^{0}
10^{5}



n = 4, V0 = 10^27 [GeV^4]
φ* / (fπ) = 0.98
DECIGO
LISA
Ωgw(f)
f [Hz]
10^-6
10^-8
10^-10
10^-12
10^-14
10^-16
10^-20
10^-15
10^-10
10^-5
10^0
10^5



$n = 4$, $V_0 = 10^{27}$ [GeV^4]
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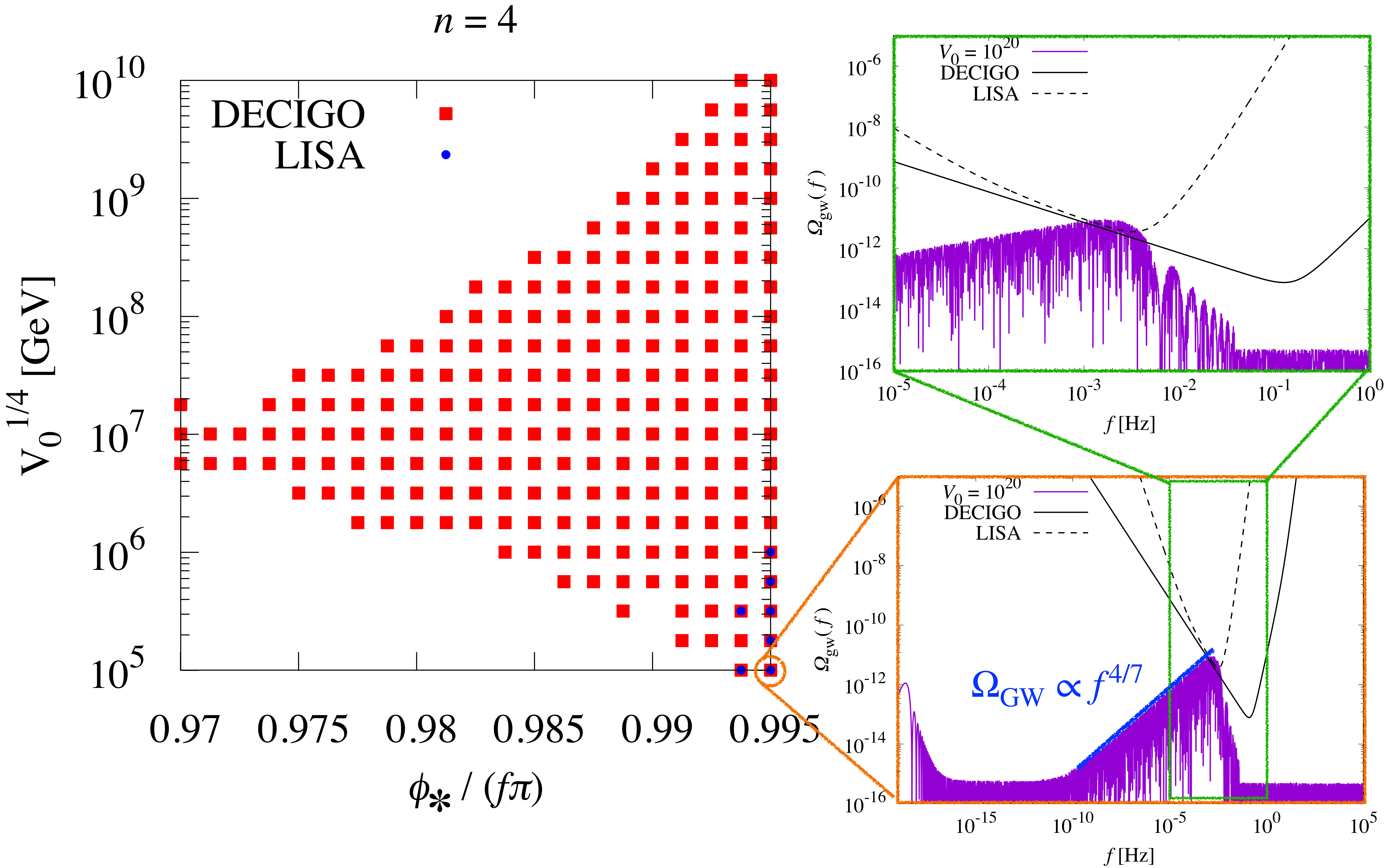
DECIGO
LISA
$\Omega_{\rm gw}(f)$
f [Hz]
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-20}
10^{-15}
10^{-10}
10^{-5}
10^{0}
10^{5}

Detectable parameter region



Summary

- We investigated $\Omega_{\rm GW}$ with EDE-like matter in the Universe.
- $\Omega_{\rm GW}$ can be enhanced by EDE-like matter decaying faster than radiation.
- The energy scale of potential controls the period at $\Omega_{\rm GW}$ enhances.
- The more ϕ_* is fine-tuned on top of potential, the more $\Omega_{\rm GW}$ is enhanced.

Auxiliary slides

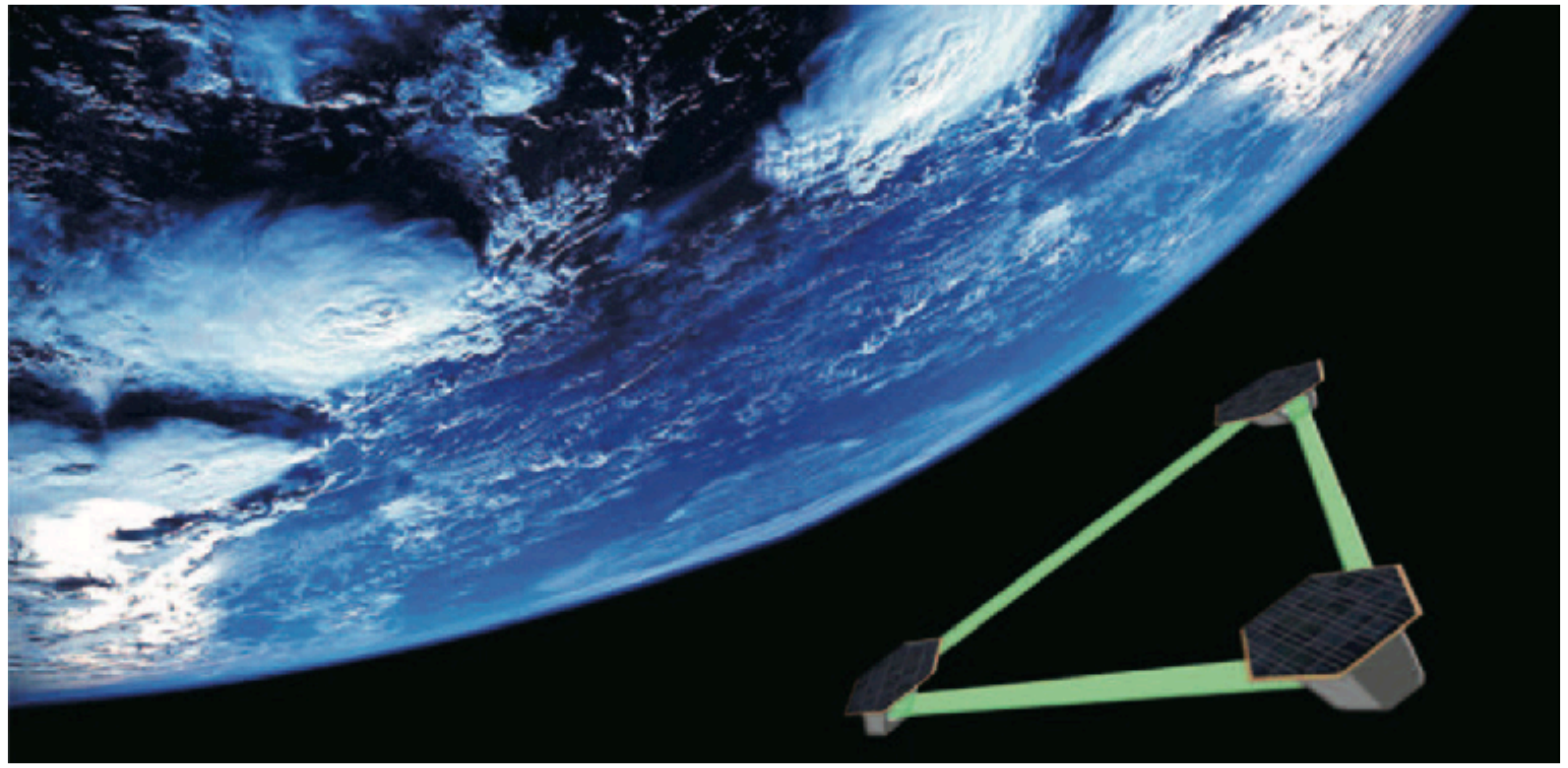
Introduction

https://www.ligo.caltech.edu/image/ligo20160615f

✓ Detector of GWs

・LIGO/Virgo/KAGRA

T. Nakamura et al.
Prog. Theor. Exp. Phys. 2016, 129301



・LISA/DECIGO/天琴(TianQin)

It would detect the primordial
GW background in the future.

LISA & DECIGO

LISA …The launch date will be postponed to 2037.

B-DECIGO…The predecessor of DECIGO will be launched in the 2030s.

DECIGO … The launch after B-DECIGO

LISA The Laser Interferometer Space Antenna Observing gravitational waves from space	
Cosmic Vision Themes	The Gravitational Universe
Primary goals	Observing low-frequency gravitational waves (from 0.1 mHz to 0.1 Hz) and studying their various sources from across the cosmos
Orbit	Three spacecraft in an Earth-trailing heliocentric orbit about 50 million km from Earth (inter-spacecraft separation of 2.5 million km)
Launch	2037
Lifetime	Four years, with possible six-year extension
Type	L-class mission

https://sci.esa.int/web/lisa/-/61367-mission-summary

4. Schedule of DECIGO and B-DECIGO

We plan to launch B-DECIGO as a precursor to DECIGO in the 2030s to demonstrate the technologies required for DECIGO, as well as to obtain fruitful scientific results to expand multi-messenger astronomy further. Then we hope to launch DECIGO at a later time, incorporating lessons learned from B-DECIGO.

S. Kawamura et al.
Prog. Theor. Exp. Phys. 2021, 05A105

The tensor primordial power spectrum

$$\ddot{h}^{\lambda}_{\mathbf{k}} + 3H\dot{h}^{\lambda}_{\mathbf{k}} + \frac{k^2}{a^2}h^{\lambda}_{\mathbf{k}} = 0$$

$$\left\langle h_{\mathbf{k}}(t)h_{\mathbf{k}'}(t)\right\rangle \equiv \frac{k^3}{2\pi^2}\mathscr{P}_{\mathrm{T,prim}}(k)\delta_D^{(3)}(\mathbf{k}+\mathbf{k}')$$

This term is model-dependent.

$$H^2 = \frac{1}{3}\rho = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

depends on inflationary models

e.g.)

Chaotic inflation:

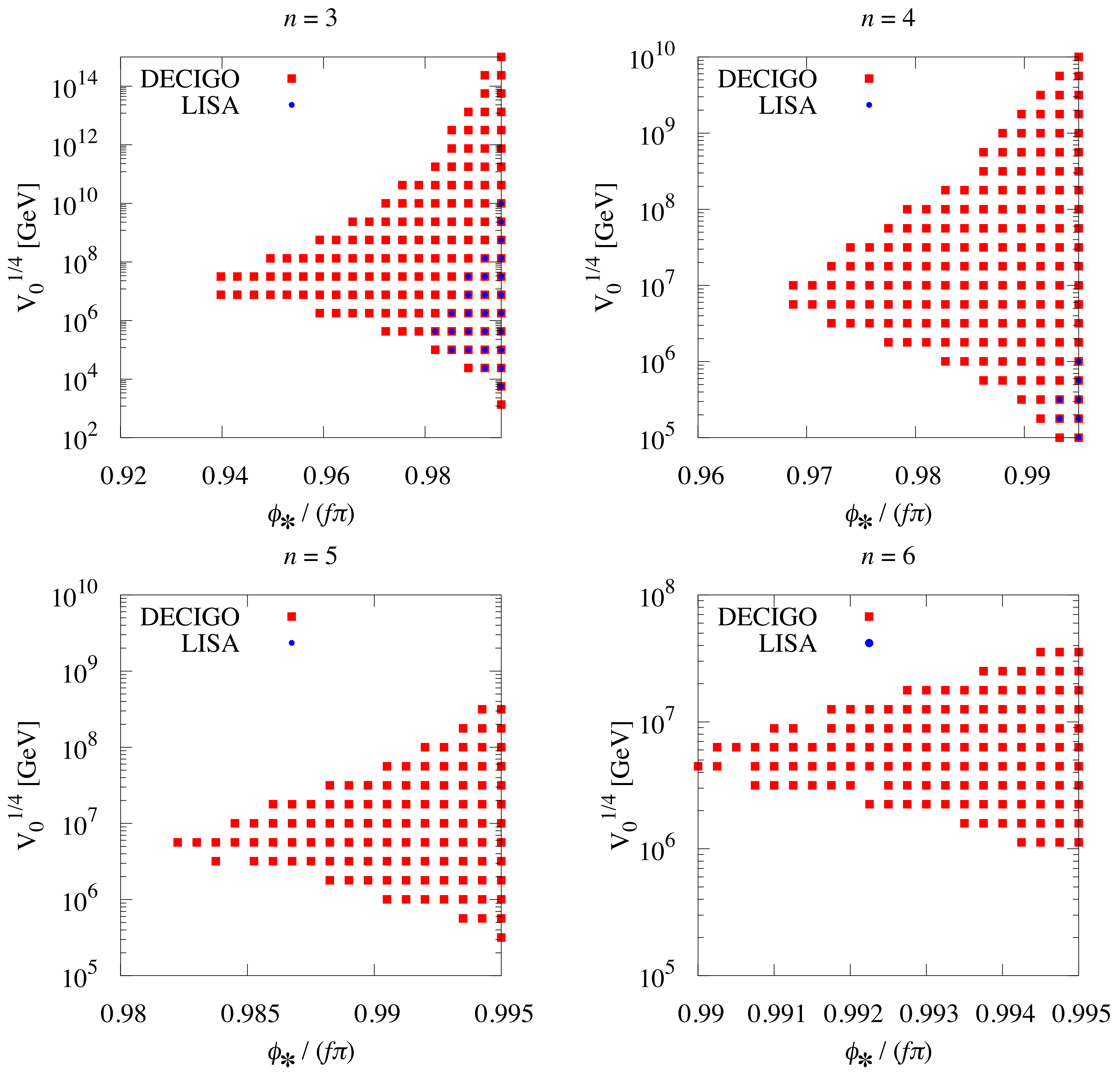
$$V(\phi) = \frac{1}{2}m^2\phi^2$$

Natural inflation:

$$V(\phi) = \Lambda^4\left(1 - \cos\frac{\phi}{f}\right)$$

There are a lot of inflationary models…

Detectable parameter region



$$V(\phi) = V_0 \left(1 - \cos\frac{\phi}{f}\right)^n$$

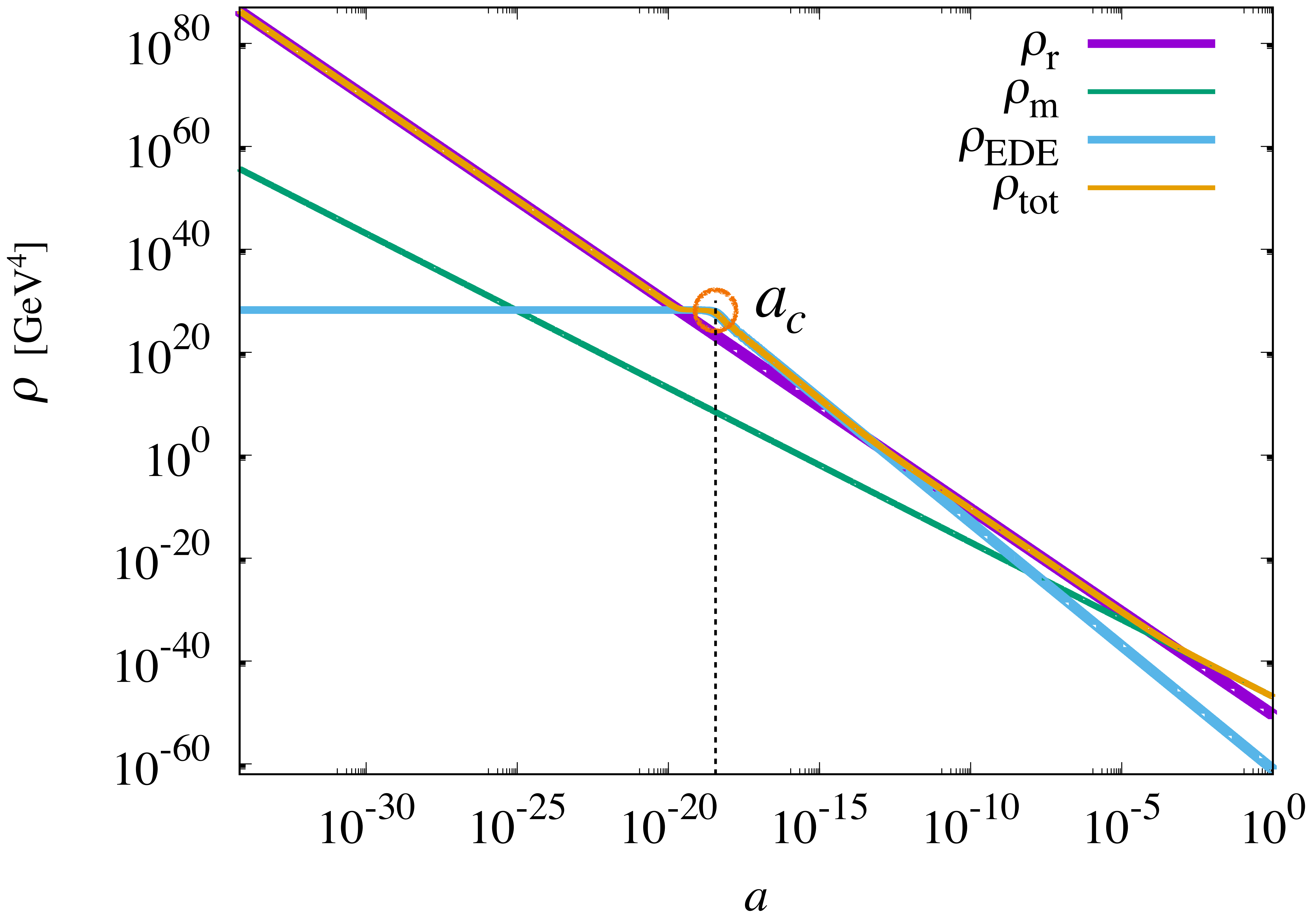
- ***n* is large**

→ m of $\Omega_{\mathrm{GW},0} \propto f^m$ is large.
Does the GW spectrum become large?

→The EDE-dominated era will be shortened.

→GW spectrum becomes less amplified.

Period of the EDE decaying



$a_c \rightarrow$ end of slow-roll

$$\epsilon = \frac{M_{Pl}^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 = 1$$

Using the slow-roll approximated EoM

$$3H\dot{\phi} + V'(\phi) = 0$$

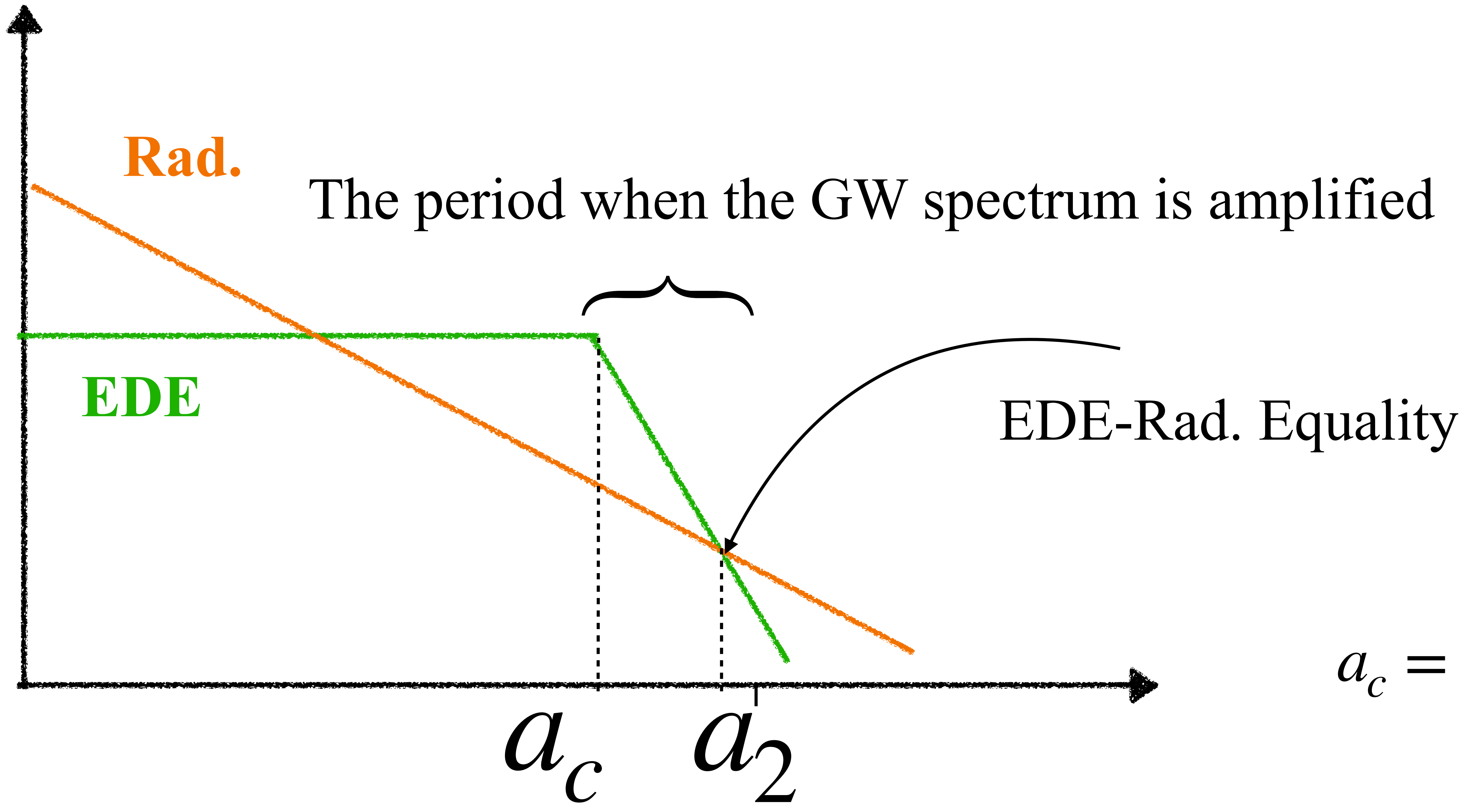
We describe a_c in terms of ϕ_*, a_*.

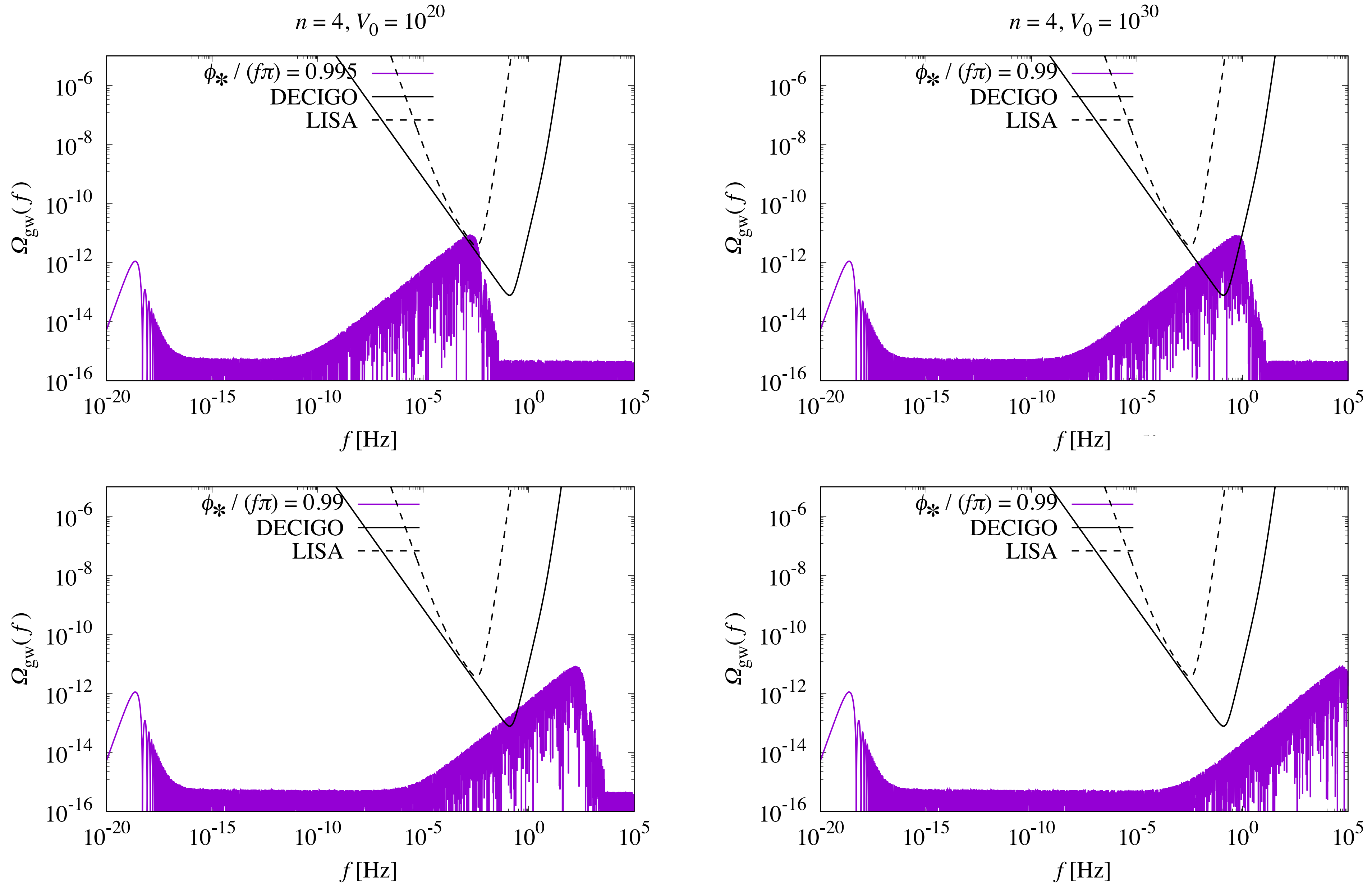
$_*$ denotes to the initial value

$$a_c = \left[\left(\frac{\rho_{r,0}}{V(\phi_*)} + a_*^4\right)\left(\frac{\cos[\phi_c/(2f)]}{\cos[\phi_*/(2f)]}\right)^{(8/n)(f/M_{Pl})^2} - 1\right]$$

Enhancement of GW spectrum independent on V_0

✓ The amplitude seems to be independent of the energy scale V_0

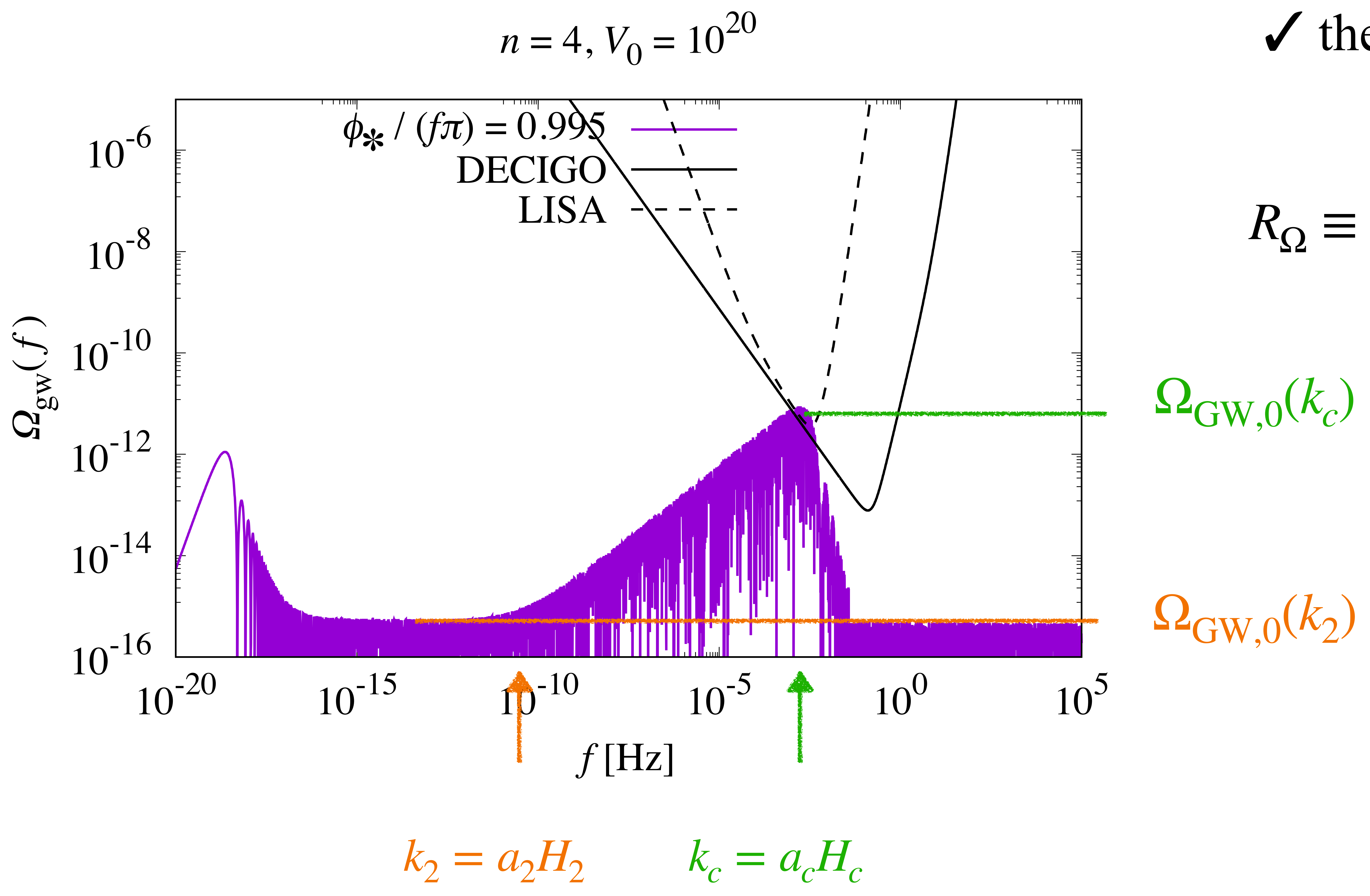




$$a_c = \left[\left(\frac{\rho_{r,0}}{V(\phi_*)} + a_*^4 \right) \left(\frac{\cos[\phi_c/(2f)]}{\cos[\phi_*/(2f)]} \right)^{(8/n)(f/M_{Pl})^2} - 1 \right]$$

$$a_2 = \left[a_c^{3(1+w)} \frac{\rho_c}{\rho_{r0}} \right]^{1/4}$$

Enhancement of GW spectrum independent on V_0



✓ the enhancement rate of GW spectrum

$$R_\Omega \equiv \frac{\Omega_{\rm GW,0}(k_c)}{\Omega_{\rm GW,0}(k_2)} = \frac{k_c^2 a_c^2 |_{k_c = a_c H_c}}{k_2^2 a_2^2 |_{k_2 = a_2 H_2}} = \frac{1}{2} a_c^4 \frac{\rho_c}{\rho_{r0}}$$

$$a_c \propto V_0^{-1/4}$$

$$\rho_{r0} \propto V_0$$

$$R_\Omega \propto (V_0^{-1/4})^{1/4} \cdot V_0 = (V_0)^0$$