

# Isospin-symmetry breaking correction to nuclear $\beta$ -decay matrix elements for weak interaction studies

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# Outline

- 1 Motivation
- 2 Shell-model formalism
- 3 Shell-model inputs
- 4 Superallowed  $0^+ \rightarrow 0^+$  Fermi  $\beta$  decay
- 5 Gamow-Teller  $\beta$  decay

- Nadezda Smirnova (LP2I, Bordeaux),
- Michael Bender (LP2I, Lyon),
- Karim Bennaceur (LP2I, Lyon),
- Frédérick Nowacki (IPHC, Strasbourg),
- Alex Brown (MSU, Michigan),
- Yi Hua Lam (IMP, Lanzhou).

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# Nucleus as a laboratory to study weak interaction

- Sperallowed  $0^+ \rightarrow 0^+$  Fermi  $\beta$  decay of  $T = 1$  nuclei (CVC, CKM unitarity)
- ~~Fermi  $\beta$  decay of mirror nuclei (alternative to the first process)~~
- Gamow-Teller  $\beta$  decay of mirror nuclei (search for  $\mathcal{G}$ -parity violating terms induced by nuclear structure)
- ~~First forbidden  $\beta$  decay (search for structure beyond the  $V - A$  interaction).~~

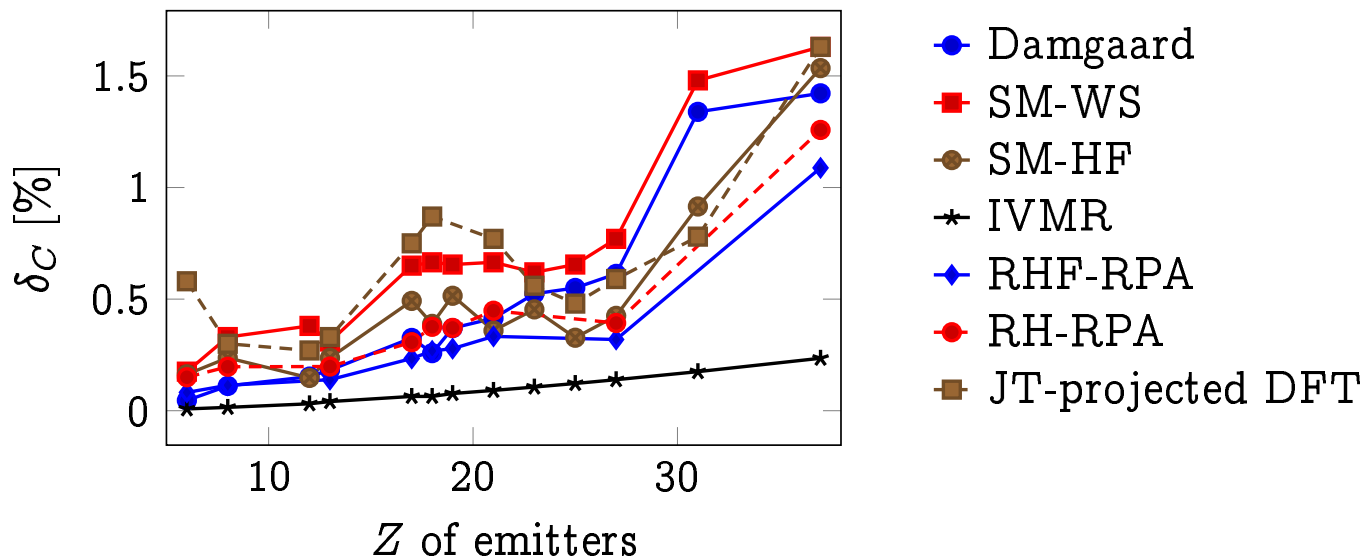
All this requires accurate description of the nuclear states, including isospin-symmetry breaking

# Nucleus as a laboratory to study weak interaction

- Superallowed  $0^+ \rightarrow 0^+$  Fermi  $\beta$  decay

$$Ft = ft(1 + \delta'_R)(1 - \delta_C + \delta_{NS}) = \frac{K}{2G_F^2 V_{ud}^2 (1 + \Delta_R^V)}$$

$\delta_C$  has become a limiting factor in the extraction of  $V_{ud}$ :



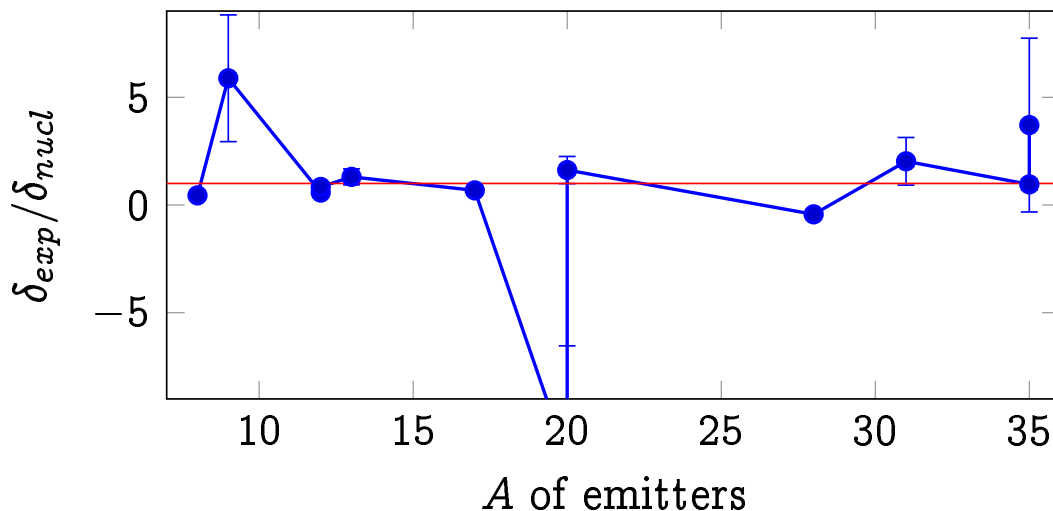
Towner, Hardy, PRC82, 065501 (2010)

# Nucleus as a laboratory to study weak interaction

- Gamow-Teller  $\beta$  decay of mirror nuclei

$$\delta = \frac{(ft)_+}{(ft)_-} - 1 = \delta_{nucl} + \delta_{ssc}, \quad \delta_{nucl} = \frac{(M^2)_-}{(M^2)_+} - 1$$

$\delta_{ssc} = 0$  if  $\mathcal{G}$  parity is conserved by weak interaction.



Smirnova, Volpe, NPA714, 441 (2003)

- Derive exact shell-model formalism for  $\delta_C$  (only *LO* terms were included in the previous calculations)
- Full model-space diagonalization (MCIA, University of Bordeaux, France)
- Existing well-established effective Hamiltonians (CKP, USD, GXPF1A, etc.)
- Selection of realistic basis functions (Woods-Saxon, Hartree-Fock)



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# Shell-model formalism

- Matrix element for a transition between  $|i\rangle$  and  $|f\rangle$ ,

$$M = \sum_{k_a k_b} \langle k_a \tau_a || \hat{O} || k_b \tau_b \rangle \text{OBTD}(k_a \tau_a k_b \tau_b i f \lambda)$$

with

$$\text{OBTD}(k_a \tau_a k_b \tau_b i f \lambda) = \frac{\langle f || [a_{k_b \tau_b}^\dagger \otimes \tilde{a}_{k_b \tau_b}]^{(\lambda)} || i \rangle}{\sqrt{2\lambda + 1}},$$

where  $\lambda$  is tensor rank of  $\hat{O}$ , and

$$\hat{O} = \begin{cases} \tau_{\pm}, & \text{for F} \\ \vec{\sigma}\tau_{\pm}, & \text{for GT} \end{cases}$$

the operator  $\tau_{\pm}$  only connects 2 isobaric analogue states, whereas  $\vec{\sigma}\tau_{\pm}$  involves a larger class of states

# Shell-model formalism

The single-particle matrix element

$$\langle k_a \tau_a || \hat{O} || k_b \tau_b \rangle = \theta(l_a l_b j_a j_b) \Omega_{k_a k_b}^{\tau_a \tau_b} \xi_{\tau_a \tau_b}$$

with isospin component  $\xi_{\tau_a \tau_b} = \langle \tau_a | \tau_{\pm} | \tau_b \rangle$ , angular component

$$\theta(l_a l_b j_a j_b) = \begin{cases} \hat{j}_a \delta_{l_a l_b} \delta_{j_a j_b}, & \text{for F} \\ (-1)^{l_a + j_a + \frac{3}{2}} \sqrt{6} \hat{j}_a \hat{j}_b \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ j_b & j_a & l_a \end{Bmatrix} \delta_{l_a l_b}, & \text{for GT} \end{cases}$$

and radial component

$$\Omega_{k_a k_b}^{\tau_a \tau_b} = \int_0^{\infty} R_{k_a}^{\tau_a}(r) R_{k_b}^{\tau_b}(r) r^2 dr$$

this integral is slightly deviated from unity because of Coulomb repulsion between protons

# Shell-model formalism

- To derive  $\delta_C$ , we define

$$D(k_a \tau_a k_b \tau_b i f \lambda) = \text{OBTD}_0(k_a \tau_a k_b \tau_b i f \lambda) - \text{OBTD}(k_a \tau_a k_b \tau_b i f \lambda)$$
$$\Lambda_{k_a k_b}^{\tau_a \tau_b} = 1 - \Omega_{k_a k_b}^{\tau_a \tau_b}.$$

then, rearrange  $M$  as

$$M = M_0 \left[ 1 - \frac{1}{M_0} \sum_{k_a k_b} \theta(l_a l_b j_a j_b) \Lambda_{k_a k_b}^{\tau_a \tau_b} \xi_{\tau_a \tau_b} \text{OBTD}_0(k_a \tau_a k_b \tau_b i f \lambda) \right. \\ \left. - \frac{1}{M_0} \sum_{k_a k_b} \theta(l_a l_b j_a j_b) \xi_{\tau_a \tau_b} D(k_a \tau_a k_b \tau_b i f \lambda) \right. \\ \left. + \frac{1}{M_0} \sum_{k_a k_b} \theta(l_a l_b j_a j_b) \Lambda_{k_a k_b}^{\tau_a \tau_b} \xi_{\tau_a \tau_b} D(k_a \tau_a k_b \tau_b i f \lambda) \right].$$

where  $M_0$  is the isospin-symmetry matrix element.

# Shell-model formalism

- According to  $M^2 = M_0^2(1 - \delta_C)$ , we obtain  $\delta_C = \sum_{i=1}^6 \delta_{C_i}$  where

$$\delta_{C1} = \frac{2}{M_0} \sum_{k_a k_b} \theta(l_a l_b j_a j_b) \xi_{\tau_a \tau_b} D(k_a \tau_a k_b \tau_b i f \lambda),$$

$$\delta_{C2} = \frac{2}{M_0} \sum_{k_a k_b} \theta(l_a l_b j_a j_b) \Lambda_{k_a k_b}^{\tau_a \tau_b} \xi_{\tau_a \tau_b} \text{OBTD}_0(k_a \tau_a k_b \tau_b i f \lambda),$$

$$\delta_{C3} = -\frac{2}{M_0} \sum_{k_a k_b} \theta(l_a l_b j_a j_b) \Lambda_{k_a k_b}^{\tau_a \tau_b} \xi_{\tau_a \tau_b} D(k_a \tau_a k_b \tau_b i f \lambda),$$

$$\delta_{C4} = -\frac{1}{4} (\delta_{C1} + \delta_{C2})^2,$$

$$\delta_{C5} = -\delta_{C3} \sqrt{|\delta_{C4}|},$$

$$\delta_{C6} = -\frac{1}{4} (\delta_{C3})^2.$$

$$\delta_{C1}, \delta_{C2} \equiv LO, \delta_{C3}, \delta_{C4} \equiv NLO, \delta_{C5} \equiv N^2LO, \delta_{C6} \equiv N^3LO.$$

# Shell-model formalism

- Therefore, the isospin-symmetry breaking occurs in 2 ways:

$$D(k_a \tau_a k_b \tau_b i f \lambda) \neq 0 \text{ and } \Lambda_{k_a k_b}^{\tau_a \tau_b} \neq 0.$$

- $LO$  approximation

Both  $D(k_a \tau_a k_b \tau_b i f \lambda)$  and  $\Lambda_{k_a k_b}^{\tau_a \tau_b}$  are expected to be small, then

$$\delta_C \approx \delta_{C1} + \delta_{C2}$$

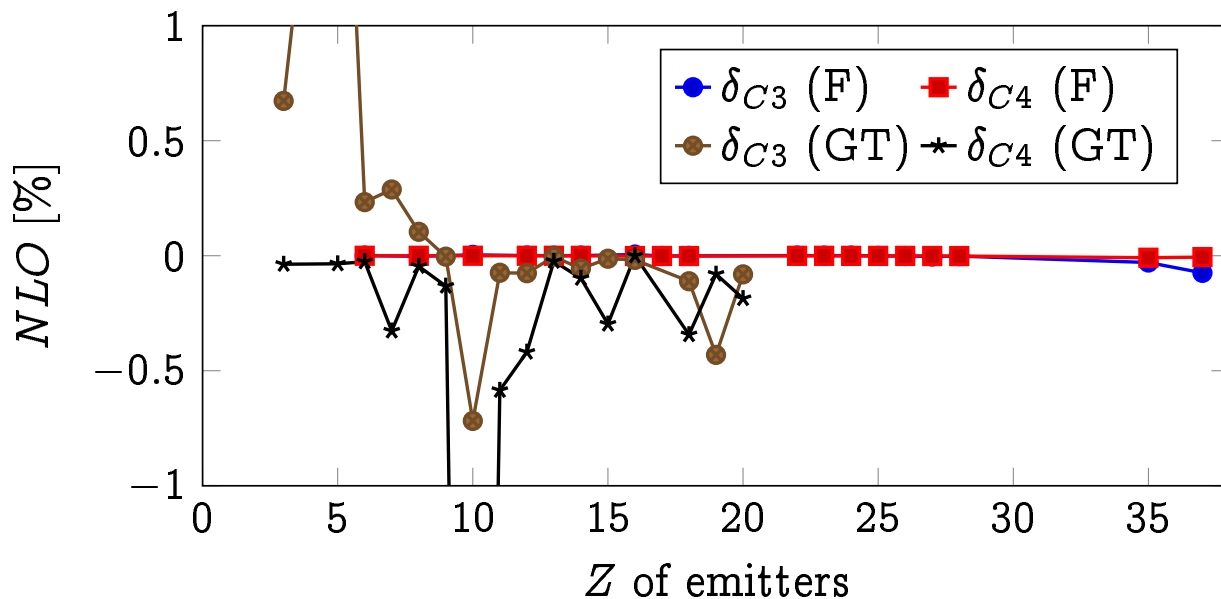
where  $\delta_{C1}$  accounts for  $D(k_a \tau_a k_b \tau_b i f \lambda) \neq 0$ , whereas  $\delta_{C2}$  accounts for  $\Lambda_{k_a k_b}^{\tau_a \tau_b} \neq 0$ .

Although this separation ansatz is convenient to handle, the higher order contribution has never been checked.

Towner, Hardy, PRC77, 025501 (2008)

# Shell-model formalism

- Higher order contribution



*NLO* are negligible for F. Contrary it can be as large as 7 % for GT.

Xayavong, Smirnova, submitted to PRC.

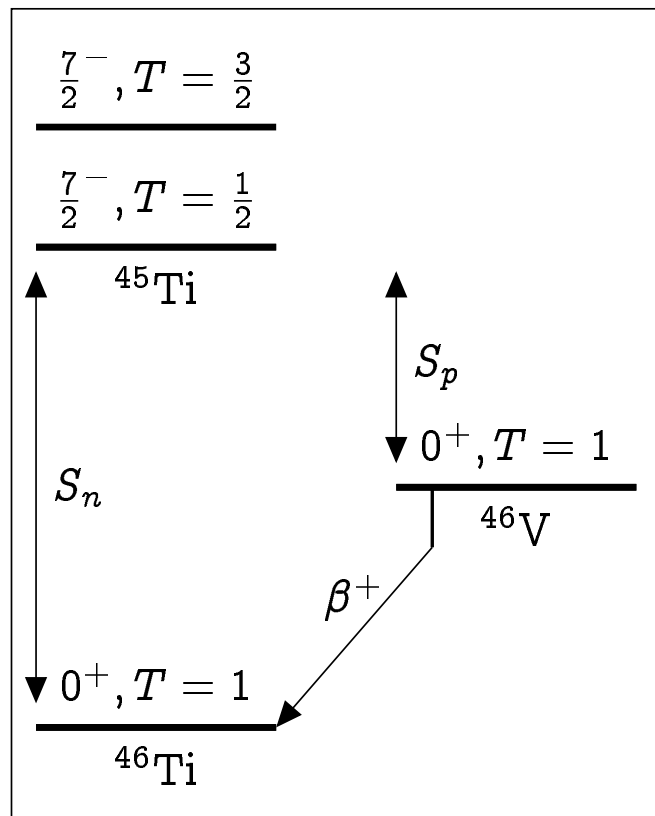
# Shell-model formalism

- We expand OBTDs in terms of intermediate states  $|\pi\rangle$  :

$$\text{OBTD}(k_a \tau_a k_b \tau_b i f \lambda) = \sum_{\pi} \Theta(j_a j_b J_i J_f J_{\pi} \lambda) \times \langle i || a_{k_b \tau_b}^{\dagger} || \pi \rangle \langle f || a_{k_a \tau_a}^{\dagger} || \pi \rangle,$$

where

$$\Theta(j_a j_b J_i J_f J_{\pi} \lambda) = (-1)^{J_f + J_{\pi} + j_a + \lambda} \left\{ \begin{array}{ccc} J_i & J_f & \lambda \\ j_b & j_a & J_{\pi} \end{array} \right\}$$



so that radial wave functions can be evaluated with excitation energies of  $|\pi\rangle$



# Shell-model formalism

- Subsequently  $\delta_{C2}, \delta_{C3}$  are evaluated as

$$\delta_{C2} = \frac{2}{M_0} \sum_{k_a k_b \pi} \theta(l_a l_b j_a j_b) \Lambda_{k_a k_b}^{\tau_a \tau_b \pi} \xi_{\tau_a \tau_b} \Theta(j_a j_b J_i J_f J_\pi \lambda) \\ \times A_0(f; \pi k_a \tau_a) A_0(i; \pi k_b \tau_b),$$

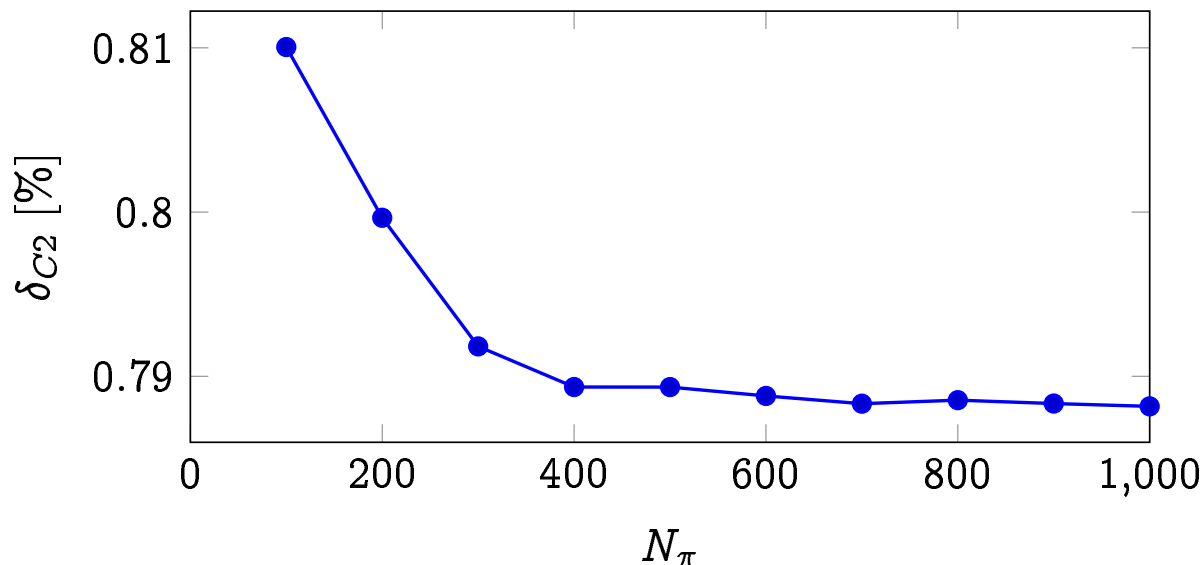
$$\delta_{C3} = -\delta_{C2} + \frac{2}{M_0} \sum_{k_a k_b \pi} \theta(l_a l_b j_a j_b) \Lambda_{k_a k_b}^{\tau_a \tau_b \pi} \xi_{\tau_a \tau_b} \\ \times \Theta(j_a j_b J_i J_f J_\pi \lambda) A(f; \pi k_a \tau_a) A(i; \pi k_b \tau_b).$$

where

$$A(i; \pi k_b \tau_b) = \frac{\langle i || a_{k_b \tau_b}^\dagger || \pi \rangle}{\hat{J}_i}, \quad A(f; \pi k_a \tau_a) = \frac{\langle f || a_{k_a \tau_a}^\dagger || \pi \rangle}{\hat{J}_f}$$

# Shell-model formalism

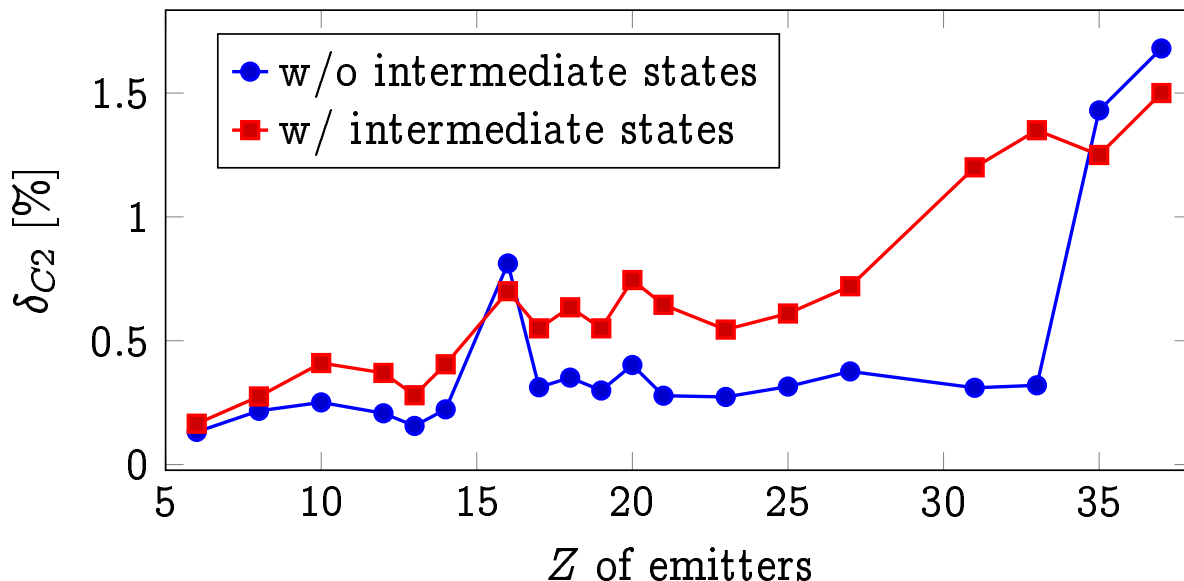
- Cut-off for the  $\sum_{\pi}$



Usually, a thousand lowest states (for a given  $J^{\pi}$ ) must be included.  $\delta_{C3}$  converges much faster.

# Shell-model formalism

- Influence of intermediate states



Usually  $\delta_{C2}$  becomes larger and agrees better with the Standard Model, when intermediate states are included.

**Towner, Hardy, PRC77, 025501 (2008)**

# Shell-model formalism

- Isospin structure of  $\delta_{C2}$  for  $T_i = T_f$  (F transitions)

$$\delta_{C2} = \frac{\sqrt{(T_i + 2t_{az}T_{iz} + 1)(T_i - 2t_{az}T_{iz})}}{T_i M_0} \sum_{k_a k_b} \theta(l_a l_b j_a j_b) \xi_{\tau_a \tau_b}$$

$$\times \left[ \sum_{\pi'} \Theta(j_a j_b J_i J_f J_{\pi'}) \Lambda_{k_a k_b}^{\tau_a \tau_b \pi'} \sqrt{S_0(f \pi' k_a) S_0(i \pi' k_b)} \right.$$

$$\left. - \frac{T_i}{(T_i + 1)} \sum_{\pi''} \Theta(j_a j_b J_i J_f J_{\pi''}) \Lambda_{k_a k_b}^{\tau_a \tau_b \pi''} \sqrt{S_0(f \pi'' k_a) S_0(i \pi'' k_b)} \right],$$

where  $\pi'$  and  $\pi''$  denote intermediate states with  $T_{\pi} = T_i - \frac{1}{2}$  and  $T_{\pi} = T_i + \frac{1}{2}$ , respectively.

Smaller  $\delta_{C2}$  is expected for Fermi transitions.

- Isospin structure of  $\delta_{C2}$  for  $T_i \neq T_f$  (GT transitions)

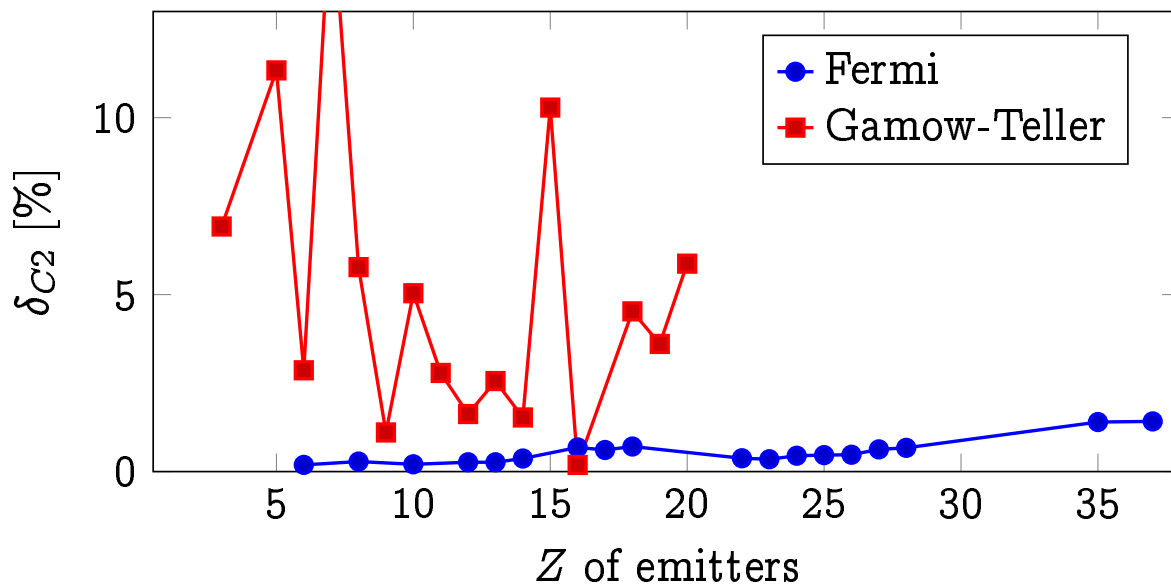
$$\delta_{C2} = - \frac{\sqrt{(T - 2t_{az}T_{iz})(T + 2t_{bz}T_{iz} + 1)}}{(T + 1)M_0} \times \sum_{k_a k_b \tilde{\pi}} \theta(l_a l_b j_a j_b) \Theta(j_a j_b J_i J_f J_{\tilde{\pi}}) \Lambda_{k_a k_b}^{\tau_a \tau_b \tilde{\pi}} \xi_{\tau_a \tau_b} \sqrt{S_0(f \tilde{\pi} k_a) S_0(i \tilde{\pi} k_b)}.$$

$\tilde{\pi}$  denotes intermediate states with  $T_{\pi} = T + \frac{1}{2}$  where  $T = \min(T_i, T_f)$ .

No cancellation occurs! Hence larger  $\delta_{C2}$  is expected for GT.

# Shell-model formalism

- Comparison of  $\delta_{C2}$  for Fermi and Gamow-Teller transitions



In general,  $\delta_{C2}$  for GT is much larger, and varies strongly from transition to transition.

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# Shell-model inputs

- Model spaces and effective Hamiltonians,

Generally, the nuclear many-body problem cannot be solved in the full Hilbert space due to computational limit.

Hamiltonian	model space	mass range
CKPcd/CKIcd/CKIIcd	$p$ shell	$A \leq 14$ ,
REWILcd/ZBMcd	$1p_{\frac{1}{2}} 1d_{\frac{5}{2}} 2s_{\frac{1}{2}}$	$15 \leq A \leq 22$ ,
USDcd/USDAcd/USDBcd	$sd$ shell	$23 \leq A \leq 34$ ,
ZBM2+Coulomb	$2s_{\frac{1}{2}} 1d_{\frac{3}{2}} 1f_{\frac{7}{2}} 2p_{\frac{3}{2}}$	$35 \leq A \leq 46$ ,
GXPF1AcD/FPD6cd/KB3Gcd	$pf$ shell	$47 \leq A \leq 66$ ,
JUN45+Coulomb	$2p_{\frac{3}{2}} 2p_{\frac{1}{2}} 1f_{\frac{5}{2}} 1g_{\frac{9}{2}}$	$A \geq 69$ .

We employ well-established empirical effective Hamiltonians



# Shell-model inputs

- Woods-Saxon potential,

$$V(r) = -V_0 f(r, a_0, r_0) - V_0 \lambda \frac{\hbar^2}{4\mu^2 c^2} \frac{1}{r} \frac{d}{dr} f(r, a_s, r_s) \langle l \cdot \sigma \rangle + V_{coul}(r)$$

with

$$V_{coul}(r) = (Z - 1)e^2 \begin{cases} \frac{1}{r}, & r > R_c \\ \frac{1}{R_c} \left( \frac{3}{2} - \frac{r^2}{2R_c^2} \right), & \text{otherwise,} \end{cases}$$

$R_c$  is extracted from charge radii  $R_{ch}$  via,

$$R_c^2 = \frac{5}{3} R_{ch}^2 + \text{corr. terms}$$

$V_0$  and  $r_0$  are readjusted to fix separation energies and ch. radii, respectively.

# Shell-model inputs

- Hartree-Fock potential using Skyrme interaction,

$$v_{Sk} = t_0(1 + x_0 P_\sigma)\delta + \frac{1}{2}t_1(1 + x_1 P_\sigma)(\mathbf{k}'^2\delta + \delta\mathbf{k}^2) + t_2(1 + x_2 P_\sigma)\mathbf{k}' \cdot \delta\mathbf{k} \\ + \frac{1}{6}t_3(1 + x_3 P_\sigma)\rho^\alpha(\mathbf{R})\delta + iW_0(\sigma_i + \sigma_j) \cdot \mathbf{k}' \times \delta\mathbf{k} + v_{coul}$$

We employed the transformation  $R_\alpha^L(r) = [m/m^*]^{\frac{1}{2}} R_\alpha(r)$  to get a local equivalent potential,

$$V^L(r, \epsilon_\alpha) = V^0(r, \epsilon_\alpha) + V^{so}(r) \langle \mathbf{l} \cdot \boldsymbol{\sigma} \rangle + V_{coul}(r)$$

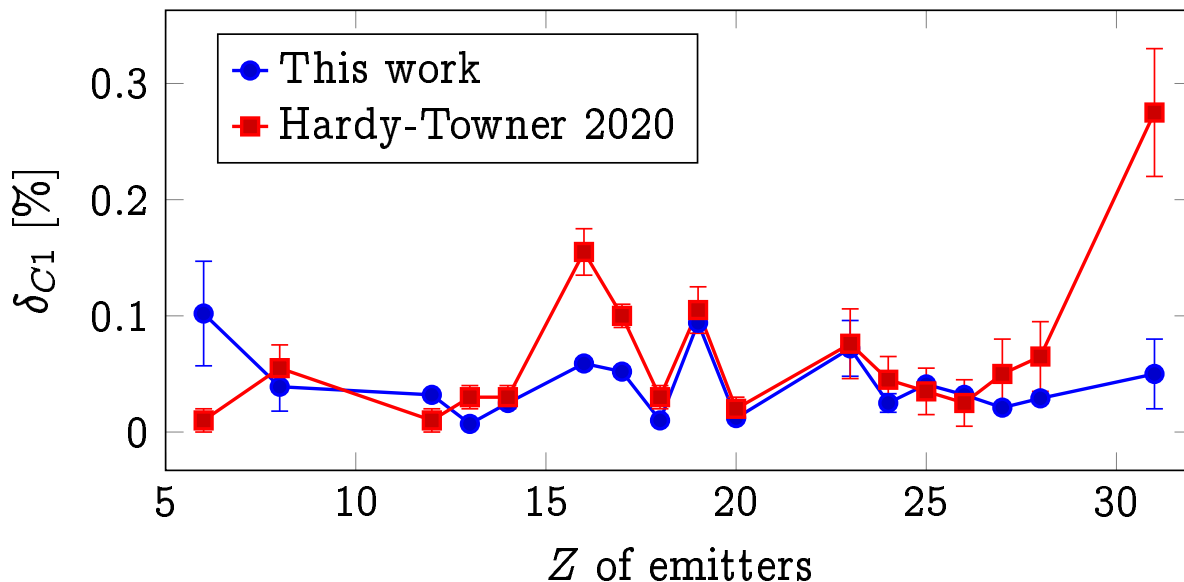
then, scaling  $V^0(r, \epsilon_\alpha)$  to reproduce separation energies.

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# Superallowed $0^+ \rightarrow 0^+$ Fermi $\beta$ decay

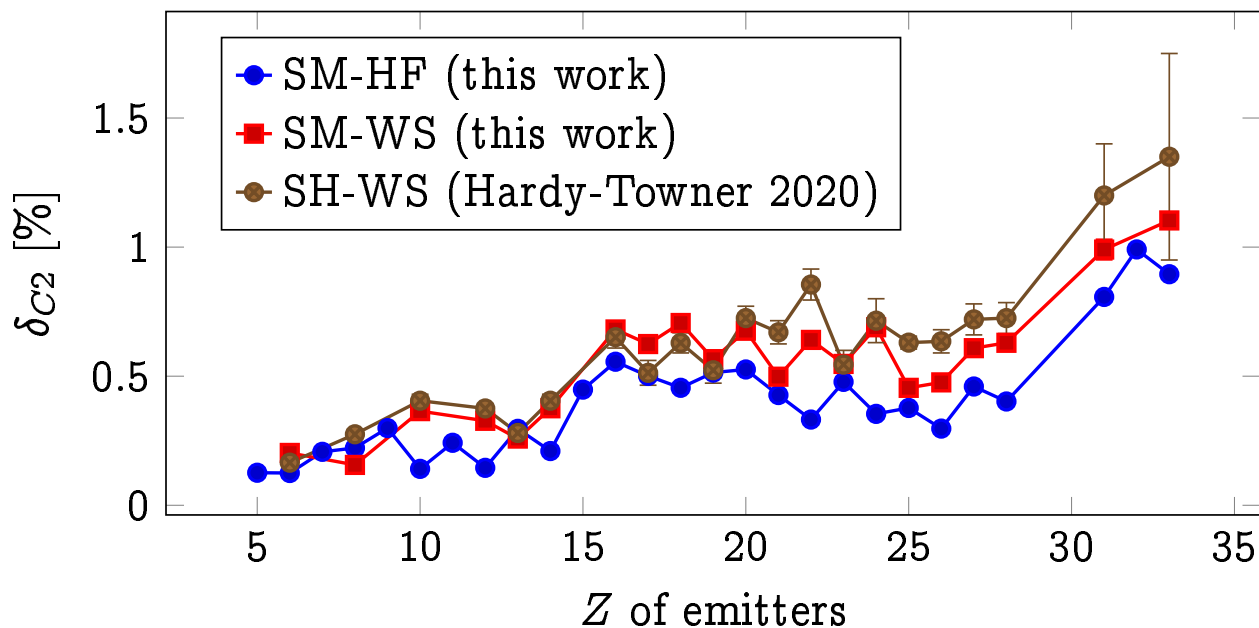
- Our preliminary result for  $\delta_{C1}$



HT readjusted interaction to reproduce IMME coefficients. Their result agrees very well with the Standard Model. **Our values are too large for  $^{10}\text{C}$  and too small for  $^{30}\text{S}$ ,  $^{34}\text{Cl}$ ,  $^{62}\text{Ga}$ . The interactions obtained from a global fit are not accurate enough.**

# Superallowed $0^+ \rightarrow 0^+$ Fermi $\beta$ decay

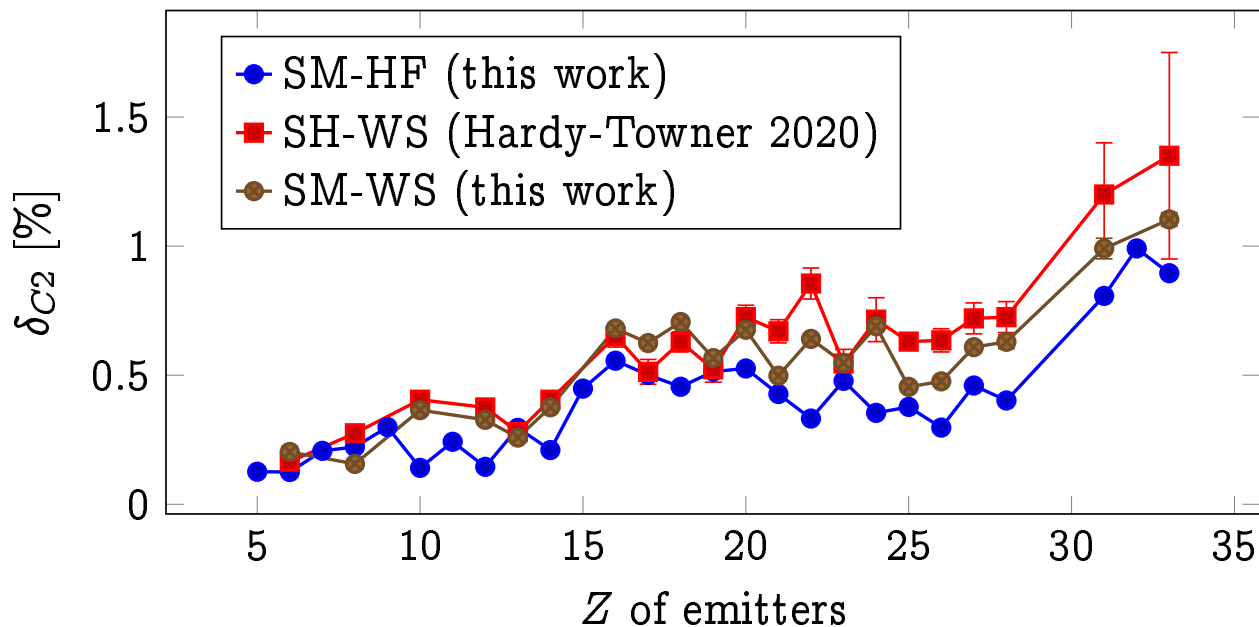
- The results for  $\delta_{C2}$



For SM-WS, our result agrees well with the latest result of Hardy-Towner, except for some cases in  $pf$  shell for which a full model space calculation is still challenging.

# Superallowed $0^+ \rightarrow 0^+$ Fermi $\beta$ decay

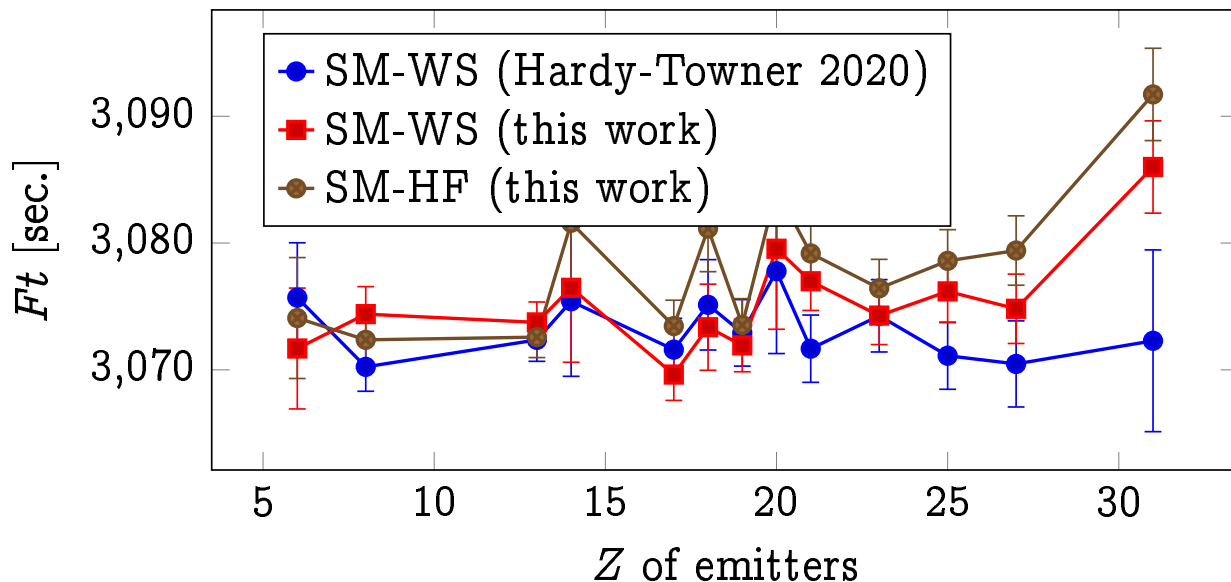
- The results for  $\delta_{C2}$



Unresolvable problem with HF: 1) sensitivity to beyond mean field contribution coming with readjustment, 2) spurious isospin mixing cannot be exactly suppressed.

# Superallowed $0^+ \rightarrow 0^+$ Fermi $\beta$ decay

- Constancy of corrected  $Ft$



To make a test on an equal footing, data on  $(\delta'_R, \delta_{NS})$  as well as theoretical uncertainties for  $\delta_C$  are taken from Hardy, Towner, PRC102, 045501 (2020).

# Superaligned $0^+ \rightarrow 0^+$ Fermi $\beta$ decay

- Constancy of corrected  $Ft$

Calculation	$\overline{Ft}$	$\chi^2/\nu$ ( $\nu = 14$ )
SM-WS (Hardy-Towner 2020)	3073.15(75)	0.493
SM-WS (this work)	3075.31(71)	1.869
SM-HF (this work)	3078.33(71)	4.040

The latest result of Hardy-Towner agrees excellently with the CVC hypothesis. The large  $\chi^2/\nu$  value for SM-HF reflects the remaining problems in the SM-HF protocol.



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# Gamow-Teller $\beta$ decay

- $p$  shell nuclei

Transition	$J_i^\pi, T_i$	$J_f^\pi, T_f$	$\delta_{C1}$	$\delta_{C2}$	$\delta_{C3}$	$\delta_{C4}$
${}^8\text{Li}(\beta^-){}^8\text{Be}$	$2^+, 1$	$2_1^+, 0$	-10.775	6.925	0.673	-0.037
${}^8\text{B}(\beta^+){}^8\text{Be}$	$2^+, 1$	$2_1^+, 0$	-7.597	11.336	2.751	-0.035
${}^9\text{Li}(\beta^-){}^9\text{Be}$	$\frac{3}{2}^-, \frac{3}{2}$	$\frac{3}{2}_1^-, \frac{3}{2}$	-4.785	1.559	0.227	-0.026
${}^9\text{C}(\beta^+){}^9\text{B}$	$\frac{3}{2}^-, \frac{3}{2}$	$\frac{3}{2}_1^-, \frac{3}{2}$	-6.082	2.862	0.233	-0.026
${}^{12}\text{B}(\beta^-){}^{12}\text{C}$	$1^+, 1$	$0_1^+, 0$	-5.602	8.597	0.279	-0.022
${}^{12}\text{N}(\beta^+){}^{12}\text{C}$	$1^+, 1$	$0_1^+, 0$	-4.926	16.356	0.288	-0.327
${}^{13}\text{B}(\beta^-){}^{13}\text{C}$	$\frac{3}{2}^-, \frac{3}{2}$	$\frac{1}{2}_1^-, \frac{1}{2}$	-3.589	2.464	0.096	-0.003
${}^{13}\text{O}(\beta^+){}^{13}\text{N}$	$\frac{3}{2}^-, \frac{3}{2}$	$\frac{1}{2}_1^-, \frac{1}{2}$	-1.489	5.777	0.104	-0.046

# Gamow-Teller $\beta$ decay

- $p - sd$  cross shell nuclei

Transition	$J_i^\pi, T_i$	$J_f^\pi, T_f$	$\delta_{C1}$	$\delta_{C2}$	$\delta_{C3}$	$\delta_{C4}$
$^{17}\text{N}(\beta^-)^{17}\text{O}$	$\frac{1}{2}^-, \frac{3}{2}$	$\frac{3}{2}^-, \frac{1}{2}$	13.729	1.481	-0.166	-0.578
$^{17}\text{Ne}(\beta^+)^{17}\text{F}$	$\frac{1}{2}^-, \frac{3}{2}$	$\frac{3}{2}^-, \frac{1}{2}$	49.537	5.039	-0.718	-7.446
$^{20}\text{F}(\beta^-)^{20}\text{Ne}$	$2^+, 1$	$2_1^+, 0$	6.156	1.106	-0.004	-0.132
$^{20}\text{Na}(\beta^+)^{20}\text{Ne}$	$2^+, 1$	$2_1^+, 0$	12.516	2.787	-0.074	-0.585
$^{20}\text{O}(\beta^-)^{20}\text{F}$	$0^+, 2$	$1_1^+, 1$	9.079	0.368	-0.013	-0.223
$^{20}\text{Mg}(\beta^+)^{20}\text{Na}$	$0^+, 2$	$1_1^+, 1$	11.32	1.626	-0.076	-0.419
$^{21}\text{F}(\beta^-)^{21}\text{Ne}$	$\frac{5}{2}^+, \frac{3}{2}$	$\frac{3}{2}^+, \frac{1}{2}$	5.967	1.072	-0.052	-0.124
$^{21}\text{Mg}(\beta^+)^{21}\text{Na}$	$\frac{5}{2}^+, \frac{3}{2}$	$\frac{3}{2}^+, \frac{1}{2}$	8.775	2.898	-0.267	-0.341
$^{24}\text{Ne}(\beta^-)^{24}\text{Na}$	$0^+, 2$	$1_1^+, 1$	-3.316	0.252	-0.016	-0.023
$^{24}\text{Si}(\beta^+)^{24}\text{Al}$	$0^+, 2$	$1_1^+, 1$	4.742	1.529	-0.056	-0.098

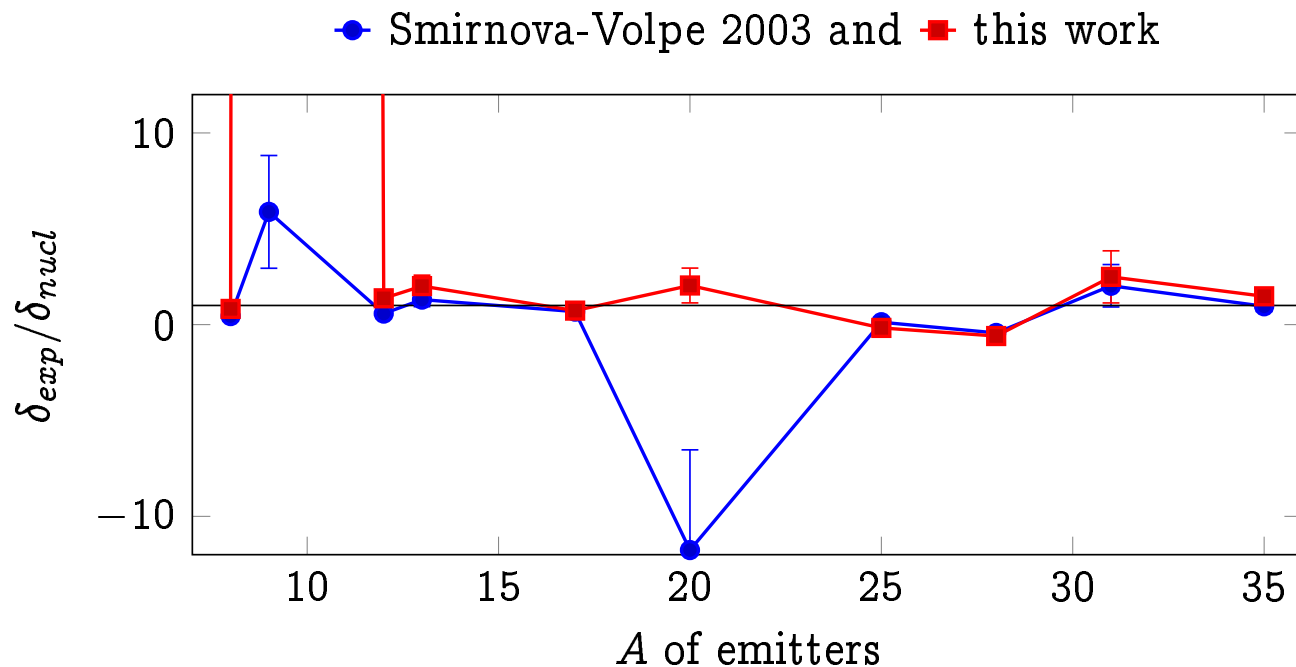
# Gamow-Teller $\beta$ decay

- $sd$  shell nuclei

Transition	$J_i^\pi, T_i$	$J_f^\pi, T_f$	$\delta_{C1}$	$\delta_{C2}$	$\delta_{C3}$	$\delta_{C4}$
$^{25}\text{Na}(\beta^-)^{25}\text{Mg}$	$\frac{5}{2}^+, \frac{3}{2}$	$\frac{5}{2}_1^+, \frac{1}{2}$	1.328	0.545	-0.014	-0.009
$^{25}\text{Si}(\beta^+)^{25}\text{Al}$	$\frac{5}{2}^+, \frac{3}{2}$	$\frac{5}{2}_1^+, \frac{1}{2}$	0.741	0.301	0.001	-0.003
$^{28}\text{Al}(\beta^-)^{28}\text{Si}$	$3^+, 1$	$2_1^+, 0$	0.547	2.558	-0.004	-0.024
$^{28}\text{P}(\beta^+)^{28}\text{Si}$	$3^+, 1$	$2_1^+, 0$	0.621	10.285	-0.013	-0.297
$^{31}\text{Al}(\beta^-)^{31}\text{Si}$	$\frac{5}{2}^+, \frac{5}{2}$	$\frac{3}{2}_1^+, \frac{3}{2}$	-1.77	1.261	0.023	-0.001
$^{31}\text{Ar}(\beta^+)^{31}\text{Cl}$	$\frac{5}{2}^+, \frac{5}{2}$	$\frac{3}{2}_1^+, \frac{3}{2}$	7.186	4.527	-0.111	-0.343
$^{35}\text{S}(\beta^-)^{35}\text{Cl}$	$\frac{3}{2}^+, \frac{3}{2}$	$\frac{3}{2}_1^+, \frac{1}{2}$	0.052	0.187	-0.018	0
$^{35}\text{K}(\beta^+)^{35}\text{Ar}$	$\frac{3}{2}^+, \frac{3}{2}$	$\frac{3}{2}_1^+, \frac{1}{2}$	2.045	3.607	-0.431	-0.08
$^{35}\text{P}(\beta^-)^{35}\text{S}$	$\frac{1}{2}^+, \frac{5}{2}$	$\frac{1}{2}_1^+, \frac{3}{2}$	-3.322	0.531	0.017	-0.019
$^{35}\text{Ca}(\beta^+)^{35}\text{K}$	$\frac{1}{2}^+, \frac{5}{2}$	$\frac{1}{2}_1^+, \frac{3}{2}$	2.755	5.875	-0.081	-0.186

# Gamow-Teller $\beta$ decay

- Asymmetry of mirror Gamow-Teller  $\beta$  decays



We got a better result for  $A = 20$ .

- We have derived a suitable exact shell-model formalism for  $\delta_C$
- Our shell-model diagonalizations were performed in full model spaces
- $\delta_{C1}$  is strongly interaction-dependent (generally, a global fitted interactions do not work)
- Calculation using Woods-Saxon basis (SM-WS) agrees well with the Standard Model
- Hartree-Fock suffers from a number of deficiencies (correlations, spontaneous symmetry breaking)
- Understanding of the isospin-symmetry breaking mechanism in Fermi and Gamow-Teller transitions