From Nuclei to Nuclear Equation of State

Yeunhwan Lim[1]

[1]Ewha Womans University

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Saga-Yonsei XIX Joint workshop

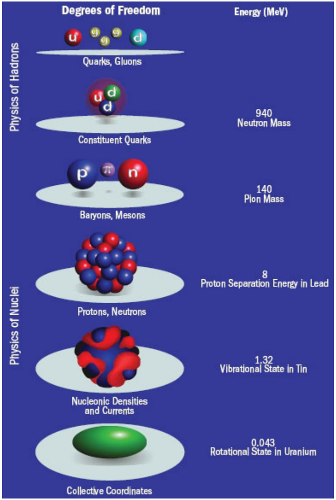
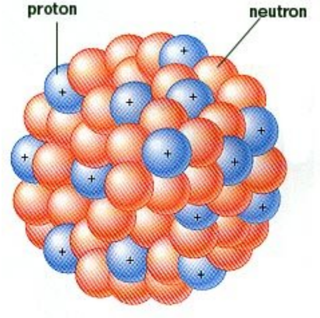


Figure 2 from W. Nazarewicz, Journal of Physics G 2016

- Nuclear star as a giant nucleus!

R



$R \sim 10\,\mathrm{km} = 10^4\,\mathrm{m}$

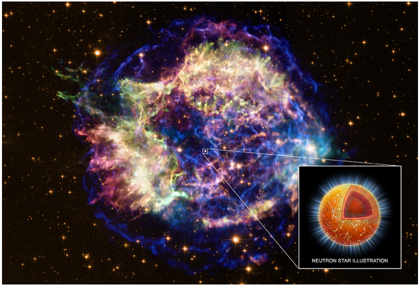


Figure 3 from NASA.

$R \simeq 1.12A^{1/3}\mathrm{fm}$ → Heavy nuclei ∼ 200 nucleons, Neutron stars ∼ 10^{57} nucleons.

Mass of a nucleus

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Total binding energy

- Nuclear Mass

$$m_{\mathrm{nuc}}(Z, N) = Zm_p + Nm_n - B(Z, N)$$

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- Atomic Mass

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AME

There are more than 2300 nuclei which are mass measured!

Binding energy per baryon

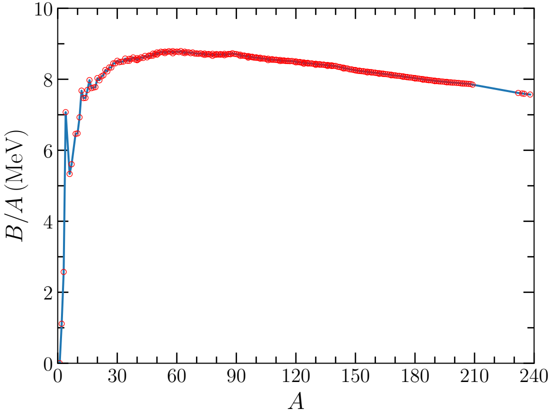

Figure: Binding energy per baryon as a function of mass number A.

Semi-empirical mass formula

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$$B(N, Z)$$

Semi-empirical mass formula

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$$B(N,Z) = aA$$

Semi-empirical mass formula

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$$B(N,Z) = aA - bA^{\frac{2}{3}}$$

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$$B(N,Z) = aA - bA^{\frac{2}{3}} - s\frac{(N-Z)^2}{A} - d\frac{Z^2}{A^{\frac{1}{3}}} - \frac{\delta}{\sqrt{A}} \qquad (1)$$

$$\begin{aligned} a &= 15.835\,\mathrm{MeV} \\ b &= 18.33\,\mathrm{MeV} \\ s &= 23.20\,\mathrm{MeV} \\ d &= 0.714\,\mathrm{MeV} \end{aligned}$$

$$\delta = \begin{cases} +11.2\,\mathrm{MeV} & \text{for odd-odd nuclei (i.e., odd } N\text{, odd } Z\text{)} \\ 0\,\mathrm{MeV} & \text{for even-odd nuclei (i.e., evn } N\text{, odd } Z\text{)} \\ -11.2\,\mathrm{MeV} & \text{for even-even nuclei (i.e., even } N\text{, even } Z\text{)} \end{cases} \qquad (2)$$

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It is assumed that the density($n_0 = 0.16\,\mathrm{fm}^{-3}$) is uniform for the center to the surface of nucleus. $R \simeq r_0 A^{1/3}$.

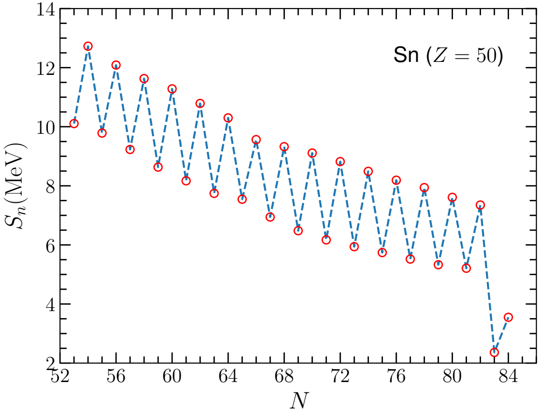

Figure: one neutron separation energy : $S_n = B(N, Z) - B(N-1, Z)$, pairing

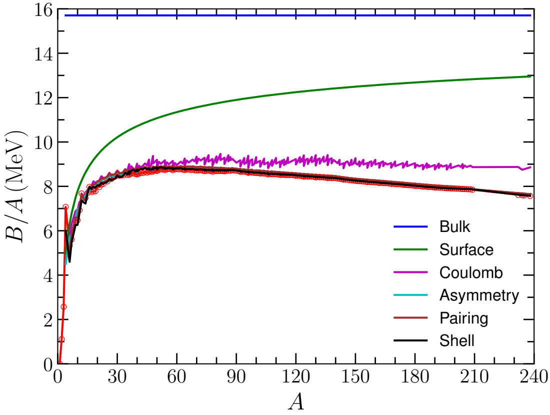

Figure: Binding energy per baryon from each contribution

Question : Is actually the proton density or neutron density constant through out a nucleus?

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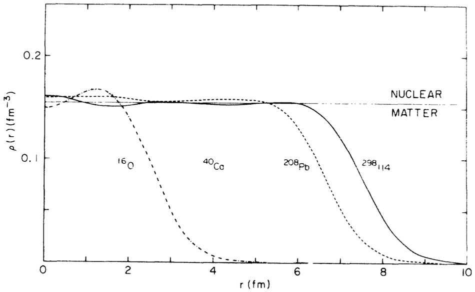

FIG. 1. Mass distributions $\rho(r)$ calculated with interaction I for the nuclei ^{16}O, ^{40}Ca, ^{208}Pb, and ^{298}114.

Figure: Nuclear density profile, Vauterin, Brink, 10.1103/PhysRevC.5.626

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$$F(A,Z) = \left[-B + S_v\delta^2\right]A + 4\pi r^2(\sigma_0 - \delta^2\sigma_\delta) + \frac{3}{5}\frac{Z^2e^2}{r}$$

Finite Nuclei : compressible model

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where $\delta = 1 - 2x$ and $\sigma = \sigma_0 - (1-2x)^2\sigma_\delta$ is surface tension .

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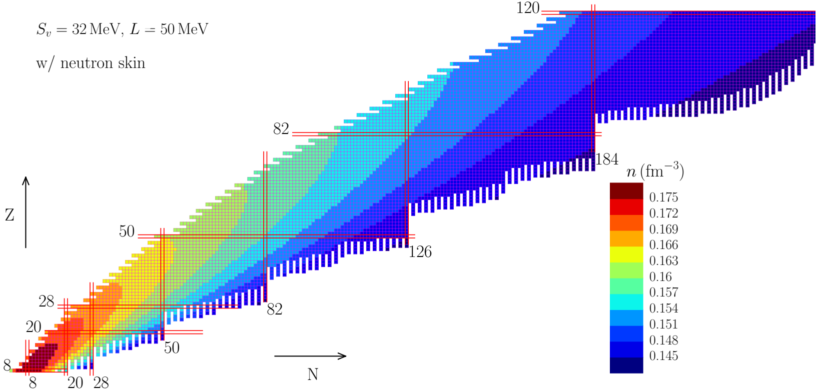
gives

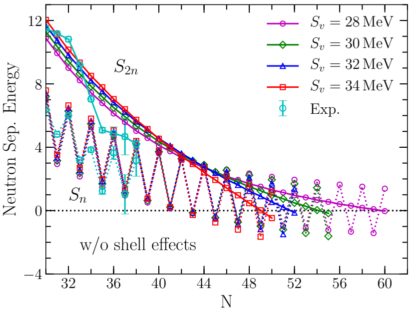
$$2\cdot 4\pi r^2\left[\sigma_0 - \sigma(1-2x)^2\right] - \frac{3}{5}\frac{Z^2e^2}{r} = 0, \quad (5)$$

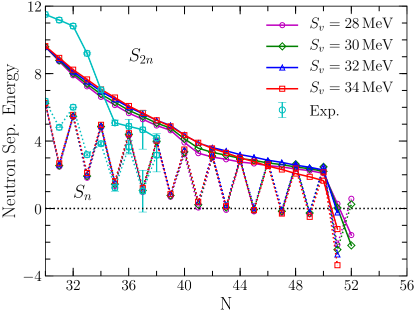
$$\frac{4\pi r^3}{3}n = A, \quad \left(x = \frac{Z}{A}\right). \quad (6)$$

- Nuclear mass table from LDM ($S_v = 32\,\mathrm{MeV}$, $L = 50\,\mathrm{MeV}$)

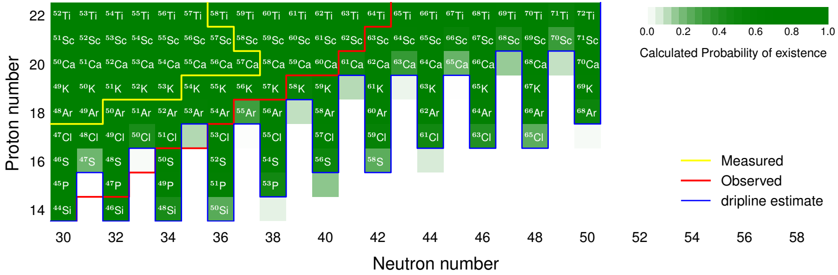


- Neutron drip line study using LDM, Calcium (Z=20); without shell effect (left), with shell effect(right)





Probability of bound nuclei for $Z \leq 22$ constructed by LDM

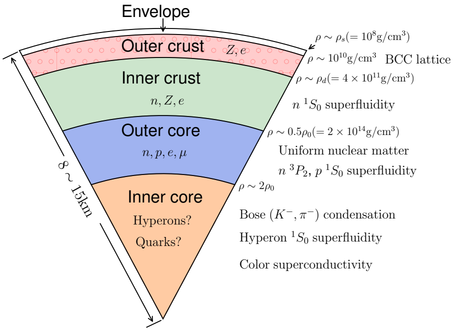


Neutron Stars

- Formed after core collapsing supernovae.
- Suggested by Walter Baade and Fritz Zwicky (1934) - Only a year after the discovery of the neutron by James Chadwick
- Jocelyn Bell Burnell and Antony Hewish observed pulsar in 1965.
- Neutron star is cold after 30s $\sim$ 60s of its birth
  - inner core, outer core, inner crust, outer crust, envelope
  - R : $\sim$ 10km, M : $1.2 \sim 2.x\ M_\odot$ ($2.14^{+0.1}_{-0.09}\ M_\odot$ PSR J0704+6620; $2.01 \pm 0.04\ M_\odot$ PSR J0348+0432 ; $1.97 \pm 0.04\ M_\odot$ PSR J614-2230)
  - 2×10^{11} earth g $\rightarrow$ General relativity
  - B field : $10^8 \sim 10^{12}$G.
  - Central density : $3 \sim 10\rho_0 \rightarrow$ Nuclear physics!!
- TOV equations for macroscopic structure

$$\frac{dp}{dr} = -\frac{G(M(r) + 4\pi r^3 p/c^2)(\epsilon + p)}{r(r - 2GM(r)/c^2)c^2}, \qquad (7)$$
$$\frac{dM}{dr} = 4\pi \frac{\epsilon}{c^2} r^2,$$

- Inner structure of neutron stars



- Neutron Stars:
  - Dense nuclear matter physics

Neutron star core

- In the core of neutron star, uniform nuclear matter exists. Because of charge neutrality, electrons or muons exist depending on the condition. Now we want to find the ground state-which we are usually interested. Let's apply the Lagrange multiplier method and see the conclusion.

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  4. Take derivatives w.r.t. n_n, n_p, n_e, n_μ, λ_1, and λ_2
  $$\frac{\partial G}{\partial n_n} = 0, \frac{\partial G}{\partial n_p} = 0, \frac{\partial G}{\partial n_e} = 0, \frac{\partial G}{\partial n_\mu} = 0, \frac{\partial G}{\partial \lambda_1} = 0, \frac{\partial G}{\partial \lambda_2} = 0. \tag{11}$$

LDM for neutron star crust

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- Write energy contribution and apply Lagrange Multiplier Method with constraints !

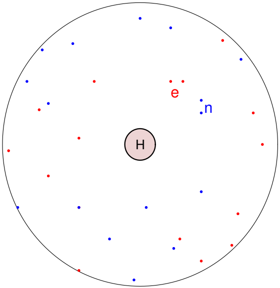
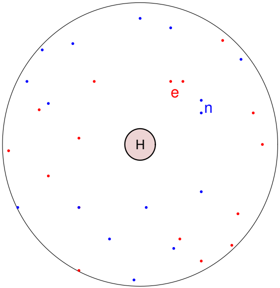


Figure: Nuclei in the crust of neutron stars



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- F_i : Energy density of a heavy nucleus

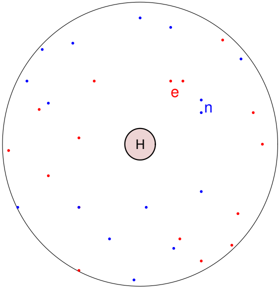

Figure: Nuclei in the crust of neutron stars

- F_i : Energy density of a heavy nucleus
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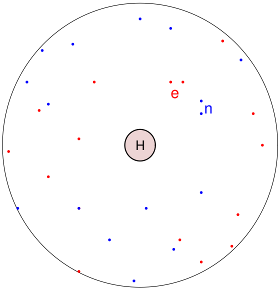

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- F_i : Energy density of a heavy nucleus
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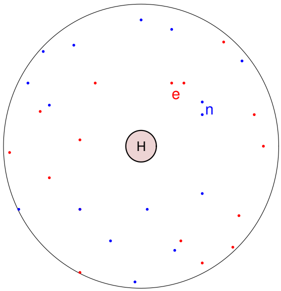

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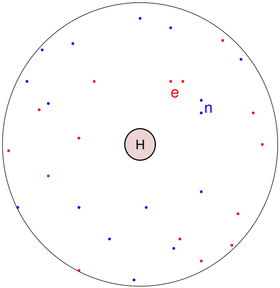

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- F_e : Electron energy density

Free energy density in the crust of neutron stars

$$F = u n_i f_i + \frac{\sigma(x_i) u d}{r_N} + 2\pi (n_i x_i e r_N)^2 u f_d(u)$$

$$F = un_i f_i + \frac{\sigma(x_i)ud}{r_N} + 2\pi(n_i x_i e r_N)^2 u f_d(u) + (1-u)n_{no} f_o$$

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with constraints

$$\begin{aligned} n - un_i - (1-u)n_{no} &= 0, \\ nY_p - un_i x_i &= 0, \\ nY_p - n_e &= 0. \end{aligned} \tag{13}$$

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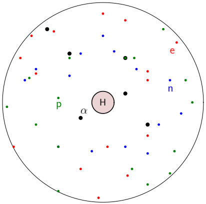
$$\frac{\partial G}{\partial u} = \frac{\partial G}{\partial n_o} = \frac{\partial G}{\partial n_{no}} = \frac{\partial G}{\partial n_i} = \frac{\partial G}{\partial x_i} = \frac{\partial G}{\partial n_e} = \frac{\partial G}{\partial \lambda_1} = \frac{\partial G}{\partial \lambda_2} = \frac{\partial G}{\partial \lambda_3} = 0. \tag{15}$$

For hot dense matter equation of state,

- Realistic nuclear force model
  Bulk energy $(-B + S_v\delta^2)$ should be replaced. Energy density functional (n, x, T)
- Wigner Seitz Cell method + Liquid drop approach



- Heavy nuclei at the center, n,p, α, e, exist outside + d, t, h.

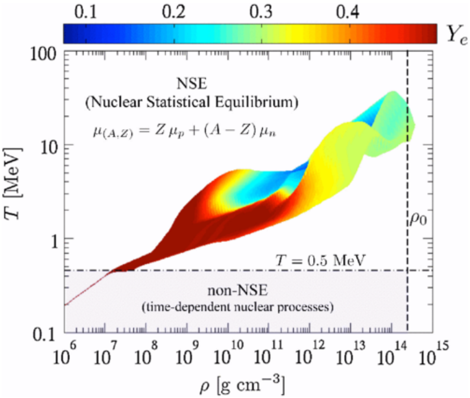
$$\langle A\rangle \simeq \sum_{A,Z} A n_{A,Z}, \quad \langle Z\rangle \simeq \sum_{A,Z} Z n_{A,Z}. \tag{16}$$

SN simulation

- Simple artist view of supernovae ▸ Link
- Computational simulation ▸ Link

Nuclear Equation of State

- Nuclear EOS is thermodynamic relations for given ρ, Y_e, T with wide range of varibles (Fig: Oertel et al., Rev. Mod. Phys. 89, 015007).
(ρ : $10^4 \sim 10^{14}$g/cm^3, Y_e : $0.01 \sim 0.56$, T : $0.5 \sim 200$MeV)



- Nuclear EOS is important to simulate core collapsing supernova explosion, proto-neturon stars, and compact binary mergers involving neutron stars.

Items in EOS tables

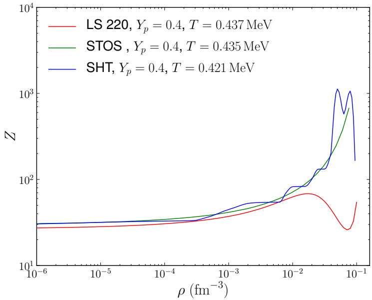
1 - Total pressure P
2 - Total free energy per baryon f
3 - Total entropy per baryon s
4 - Total internal energy per baryon e
5 - Neutron chemical potential wrt rest mass
6 - Proton chemical potential wrt rest ma
7 - Neutron mass fraction (external to nuclei)
8 - Proton mass fraction (external to nucl
9 - Alpha particle mass fraction
10 - Baryon pressure
11 - Baryon free energy per baryon
12 - Baryon entropy per baryon
13 - Baryon internal energy per baryon
14 - Nuclei filling factor u
15 - Baryon density inside heavy nucleus
16 - dP/dn
17 - dP/dT
18 - dP/dYe
19 - ds/dT
20 - ds/dYe
21 - Mass number of heavy nucleus
22 - Proton fraction of heavy nucleus
23 - Number of neutrons in neutron skin of heavy nucleus
24 - Baryon density of nucleons external to heavy nucleus and alpha particles
25 - Proton fraction of nucleons external to heavy nucleus and alpha particles
26 - Out of whackness:
$\mu_n - \mu_p - \mu_e + 1.293$ MeV

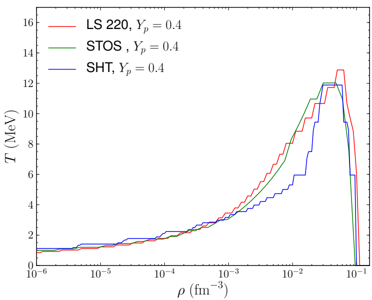
- Representative EOSs.

- LS EOS (Lattimer Swesty 1991)
  Use Skyrme type potential with Liquid droplet approach
  - Consider phase transition, several K
- STOS EOS (H. Shen, Toki, Oyamastu, Sumiyoshi 1998), new version (2011)
  Use RMF with TF approximation and parameterized density profile (PDP)
  - Old : awkward grid spacing
  - New : finer grid spacing, adds Hyperon($\Lambda, \Sigma^{+,-,0}$)
- SHT EOS (G. Shen, Horowitz, Teige 2010)
  Use RMF with Hartree approximation
- HSB (M. Hempel and J. Schaffner-Bielich). 2010, 2012
  - Use Relativistic mean field model (TM1, TMA, FSUgold)
  - Nuclear statistical equilibrium (alpha, deutron, triton, helion)

	LS 220	STOS	SHT
$\rho(\mathrm{fm}^{-3})$	$10^{-6} \sim 1$ (121)	$7.58 \times 10^{-11} \sim 6.022$ (110)	$10^{-8} \sim 1.496$ (328)
Y_p	$0.01 \sim 0.5$ (50)	$0 \sim 0.65$ (66)	$0 \sim 0.56$ (57)
T (MeV)	$0.3 \sim 30$ (50)	$0.1 \sim 398.1$ (90)	$0 \sim 75.0$ (109)

- STOS & SHT tables don't provide second derivative $\left(\frac{\partial(P,S)}{\partial(T,\rho,Y_p)}\right)$.





How can we construct EOS table ?
We need nuclear force model and numerical method.

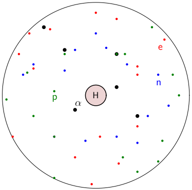
Nuclear force model	Numerical technique
Skyrme Force model (non-relativistic potential model)	Liquid Drop(let) approach (LDM)
Relativistic Mean Field model (RMF)	Thomas Fermi Approximatoin (TF)
Finite-Range Force model	Hartree-Fock Approximation (HF)
	Nuclear Statistical Equilibrium (NSE)

- LS EOS ⇒ Skyrme force + LDM (without neutron skin)
- STOS ⇒ RMF + Semi TF (parameterized density profile)
- SHT ⇒ RMF + HARTREE
- HSB ⇒ RMF + NSE

Nuclear force model should be picked up to represent both finite nuclei and neutron star observation + Neutron matter calculation.

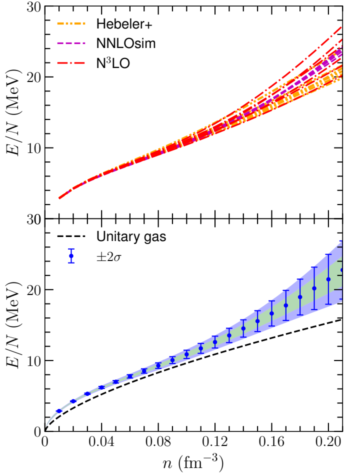
Calculation of EOS

- Schematic picture of inhomogeneous nuclear matter(neutron star crust)



- Liquid Drop Model(Fast and accurate)
- Most difficult part: inhomogeneous matter, low temperature
- Adopt state-of-the-art neutron matter results
  -ex) MBPT(Drischler *et al.*, PRL 2019), QMC(Tews *et al.*, PRC 2016)

- New energy density functional for the nuclear EOS. (Y. Lim, S. Huth, and A. Schwenk *in preparation*)



Free energy

Total free energy density consists of

$$F = F_N + F_o + F_\alpha + F_d + F_t + F_h + F_e + F_\gamma \tag{17}$$

where F_N, F_o, F_α, F_e, and F_γ are the free energy density of heavy nuclei, nucleons out the nuclei, alpha particles, electrons, and photons.

- $F_N = F_{bulk,i} + F_{coul} + F_{surf} + F_{trans}$
- $F_o = F_{bulk,o}$
- α, d, t, h particles : Non-interacting Boltzman gas
- e, γ : treat separately

For $F_{bulk,i}$, $F_{bulk,o}$, and F_{surf}, we use the same force model.
F_{surf} from the semi infinite nuclear matter calculation

The is the modification of LPRL (1985), LS (1991, No skin)

- Consistent calculation of surface tension
- Deuteron, triton, helion
- The most recent parameter set

Free energy minimization

For fixed independent variables (ρ, Y_p, T), we have the 11 dependent variables $(\rho_i, x_i, r_N, z_i, u, \rho_o, x_o, \rho_\alpha, \rho_d, \rho_t, \rho_h)$.

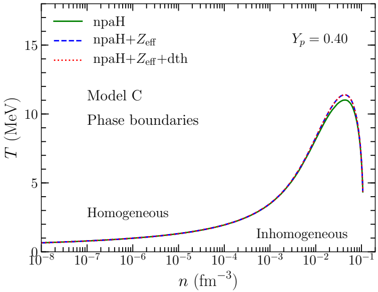
where i heavy nuclei, o nucleons outside, x proton fraction, u filling factor, and ν_n neutron skin density.

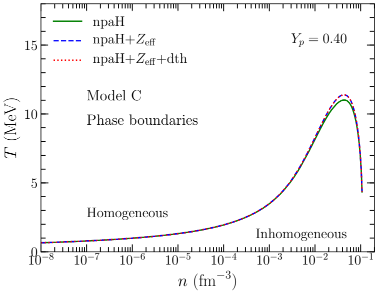
From baryon and charge conservation, we can eliminate x_o and ρ_o.

Free energy minimization, $\frac{\partial F}{\partial \rho_i} = \frac{\partial F}{\partial x_i} = \frac{\partial F}{\partial r_N} = \frac{\partial F}{\partial z_i} = \frac{\partial F}{\partial u} = \frac{\partial F}{\partial \rho_\alpha} = \frac{\partial F}{\partial \rho_d} = \frac{\partial F}{\partial \rho_t} = \frac{\partial F}{\partial \rho_h} = 0$.

- Finally, we have 6 equations to solve and 6 unknowns. $z = (\rho_i, \ln(\rho_{no}), \ln(\rho_{po}), x_i, \ln(u), z_i)$.
- The code (f90) is fast. $165(\rho) \times 101(T) \times 65(Y_p)$ zones < 20 mins in single machine. ($1.0 \times 10^{-8} \leq \rho \leq 1.6\,\mathrm{fm}^{-3}$, $0.1 \leq T \leq 100\,\mathrm{MeV}$, $0.01 \leq Y_p \leq 0.65$) - more grid points (maybe) needed
- Any Skyrme functionals (or any symmetric or neutron matter property EOSs) can be compared,

Phase Boundary





Particle fraction