# Mechanical Properties of baryons

How does gravity probe mechanical properties of baryons?

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Intensive Lecture
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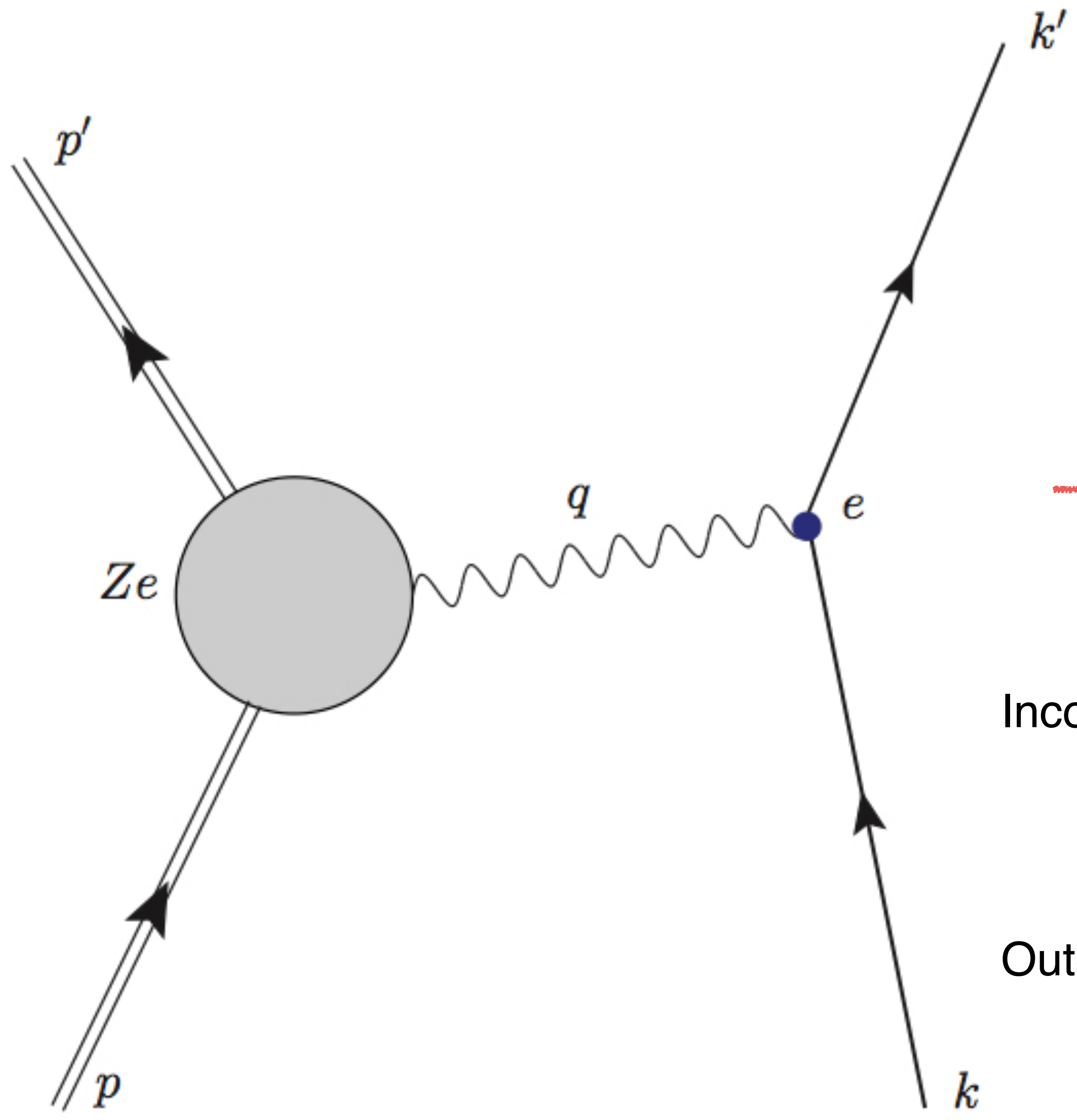
# Form factors

# What is a form factor?

Form factors tell you how the corresponding particle looks like in various aspects.

**Historic example: Rutherford scattering**




- Target is so heavy that the recoil effects are negligible.
- Elastic scattering

If $Z\alpha \ll 1\,(\alpha \approx 1/137)$

→ Born approximation can be used.

Incoming wave for the electron $\psi_i = \frac{1}{\sqrt{V}} e^{i\mathbf{k}\cdot\mathbf{r}/\hbar}$

Outgoing wave for the electron $\psi_f = \frac{1}{\sqrt{V}} e^{i\mathbf{k}'\cdot\mathbf{r}/\hbar}$

# What is a form factor?

**Quantum mechanical** definition of the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{V^2 E'^2}{(2\pi)^2(\hbar c)^4} \left|\langle\psi_f\left|\mathcal{H}_{\text{int}}\right|\psi_i\rangle\right|^2$$

$$\langle\psi_f|\mathcal{H}_{\text{int}}|\psi_i\rangle = \frac{e}{V}\int \phi(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}/\hbar}\,d^3x$$

$$= -\frac{e\hbar^2}{V|\mathbf{q}|^2}\int \nabla^2\phi(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}/\hbar}\,d^3x$$

$$\nabla^2\phi(\mathbf{r}) = -\frac{\rho_{\text{ch}}(\mathbf{r})}{\varepsilon_0}$$

$$\rho_{\text{ch}} = Ze\rho(\mathbf{r})$$

$$\langle\psi_f|\mathcal{H}_{\text{int}}|\psi_i\rangle = \frac{Z4\pi\alpha\hbar^3 c}{|\mathbf{q}|^2\cdot V}\int \rho(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}/\hbar}\,d^3x$$

$$F(\mathbf{q}) = \int \rho(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}/\hbar}\,d^3x$$

# What is a form factor?

**Rutherford scattering**

$$\rho(\mathbf{r}) = \delta(\mathrm{r})$$

: The particle taken as a point-like one

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Rutherford}} = \frac{4Z^2\alpha^2(\hbar c)^2 E'^2}{\mathbf{q}^4 c^4}$$

It decrease very quickly as q becomes larger.

$$E = E', \quad E \approx |\mathbf{k}|c \qquad |\mathbf{q}| = 2|\mathbf{k}|\sin\frac{\theta}{2}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Rutherford}} = \frac{Z^2\alpha^2(\hbar c)^2}{4E^2\sin^4\theta/2}$$

(Relativistic expression)

# What is a form factor?

**Mott scattering**

Electron spins are considered

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} \left(1 - \frac{v^2}{c^2}\sin^2\frac{\theta}{2}\right)$$

(Recoil effects are still neglected.)

It can be easily derived by using the interaction Lagrangian:

$$\mathcal{L}_{\text{int}} = -e\bar{\psi}(x)\gamma^{\mu}\psi(x)A^{\text{ext}}_{\mu}$$

HW. If you have time, please try to derive the Mott formula using the following S-matrix:

$$S_{fi} = -ie\int d^4x\bar{\psi}_f(x)\gamma^{\mu}\psi(x)_i A^{\text{ext}}_{\mu}(x)$$

Please, keep in mind that the Mott formula applies only
for structureless particles such as electrons etc.

# What is a form factor?

## Particles with internal structure

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} |F(\mathbf{q}^2)|^2$$

$$\begin{aligned}
F(\mathbf{q}^2) &= \int \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}/\hbar}\, d^3x \\
&= \int \rho(r) \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{i|\mathbf{q}|\mathbf{r}|\cos\theta}{\hbar}\right)^n d^3x \\
&= \int_0^\infty \int_{-1}^{1} \rho(r) \left[1 - \frac{1}{2}\left(\frac{|qr|}{\hbar}\right)^2 \cos^2\theta + \cdots\right] d\varphi d\theta r^2 dr \\
&= 4\pi \int_0^\infty \rho(r) r^2 dr - \frac{1}{6}\frac{|\mathbf{q}^2|}{\hbar^2} 4\pi \int_0^\infty \rho(r) r^4 dr + \cdots \\
&= 1 - \frac{1}{6}\frac{\mathbf{q}^2}{\hbar^2}\langle r^2\rangle + \cdots
\end{aligned}$$

$$\langle r^2\rangle = -6\hbar^2 \left.\frac{dF(\mathbf{q}^2)}{d\mathbf{q}^2}\right|_{\mathbf{q}^2=0}$$

# What is a form factor?

| Charge distribution $f(r)$ | | Form Factor $F(\boldsymbol{q}^2)$ | |
|---|---|---|---|
| point | $\delta(r)/4\pi$ | $1$ | constant |
| exponential | $(a^3/8\pi)\cdot\exp(-ar)$ | $\left(1+\boldsymbol{q}^2/a^2\hbar^2\right)^{-2}$ | dipole |
| Gaussian | $\left(a^2/2\pi\right)^{3/2}\cdot\exp\left(-a^2r^2/2\right)$ | $\exp\left(-\boldsymbol{q}^2/2a^2\hbar^2\right)$ | Gaussian |
| homogeneous sphere | $\begin{cases} 3/4\pi R^3 & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$ | $3\,\alpha^{-3}(\sin\alpha-\alpha\cos\alpha)$ with $\alpha=\lvert\boldsymbol{q}\rvert R/\hbar$ | oscillating |

An oscillating form factor corresponds to a homogeneous sphere with sharp edge!

Particles & Nuclei by Povh et al.

# What is a form factor?

| $\rho(r)$ | $\lvert F(\boldsymbol{q}^2)\rvert$ | Example |
|---|---|---|
| pointlike | constant | Electron |
| exponential | dipole | Proton |
| gauss | gauss | $^6Li$ |
| homogeneous sphere | oscillating | – |
| sphere with a diffuse surface | oscillating | $^{40}Ca$ |

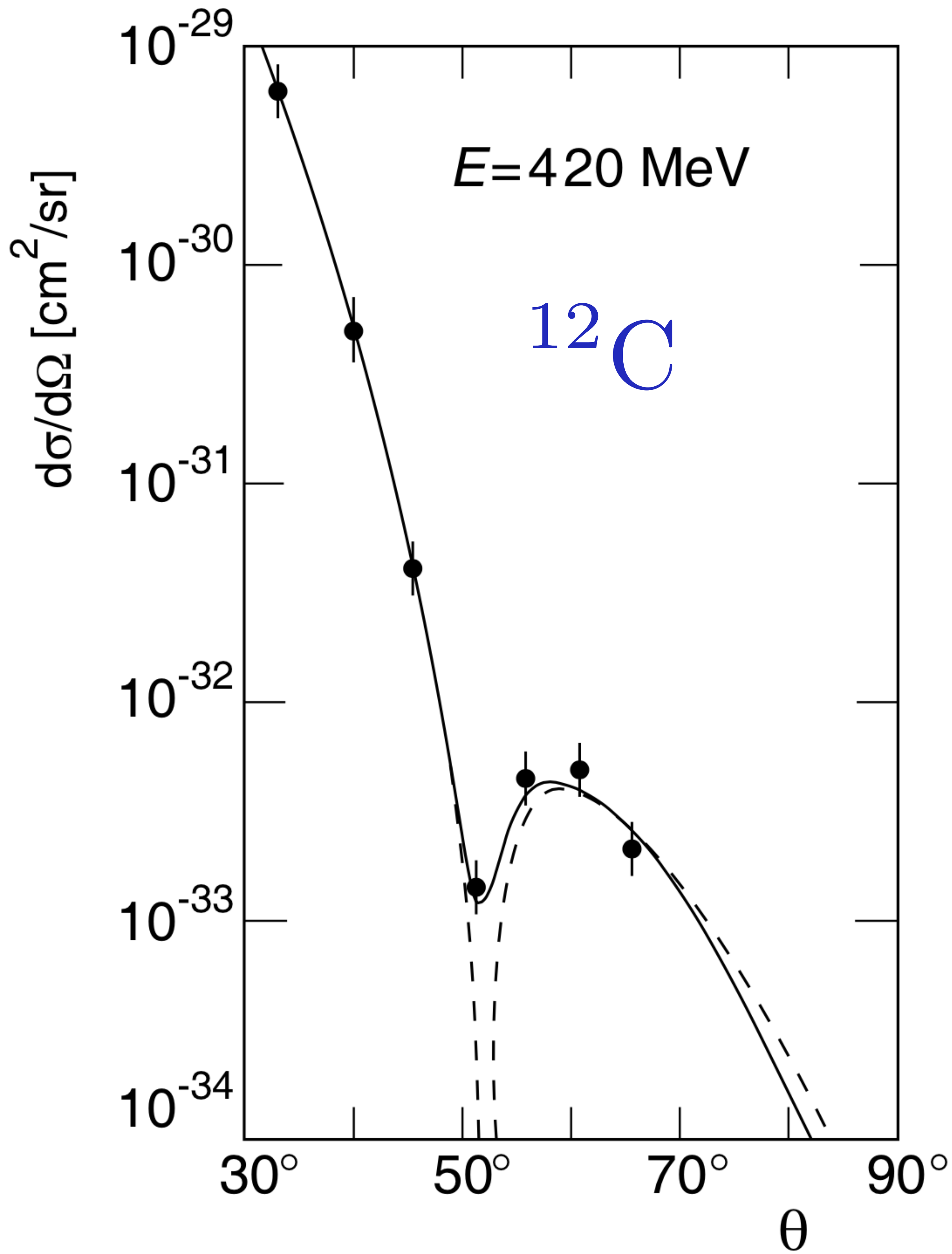
r ⟶ $\lvert q\rvert$ ⟶

$$F(\mathbf{q}) = \int \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}/\hbar}\, d^3x$$

$$\langle r^2 \rangle = -6\hbar^2 \frac{dF(\mathbf{q}^2)}{d\mathbf{q}^2}\bigg|_{\mathbf{q}^2=0}$$

Particles & Nuclei by Povh et al.

# EM Form factors and charge distributions

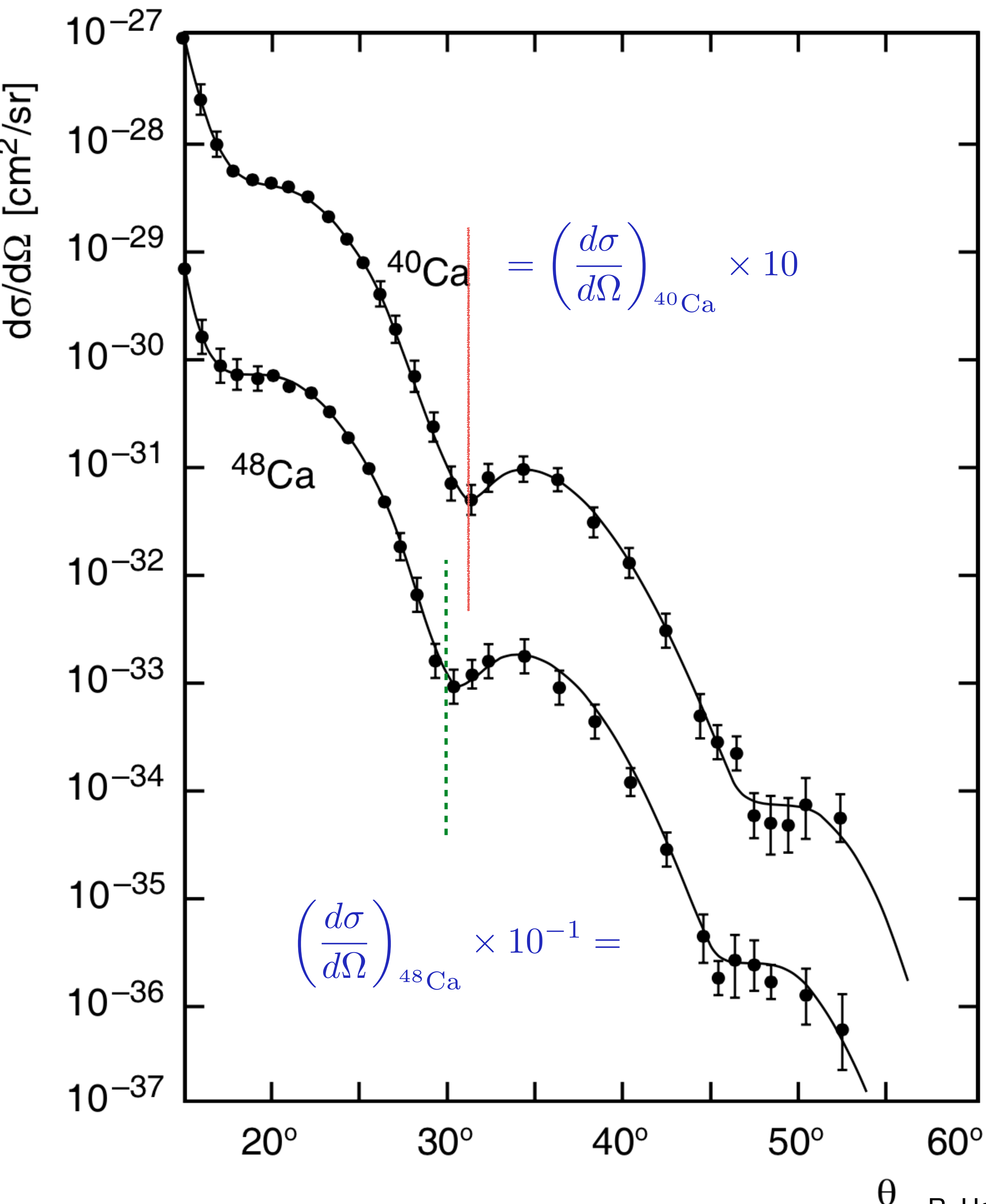



Measurement of the form factor of C12 by electron scattering.

Exact phase shift analysis

Scattering of a plane wave off an homogeneous sphere with a diffuse surface: Born approximation

R. Hofstadter, Ann. Rev. Nucl. Sci 7, 231 (1957)



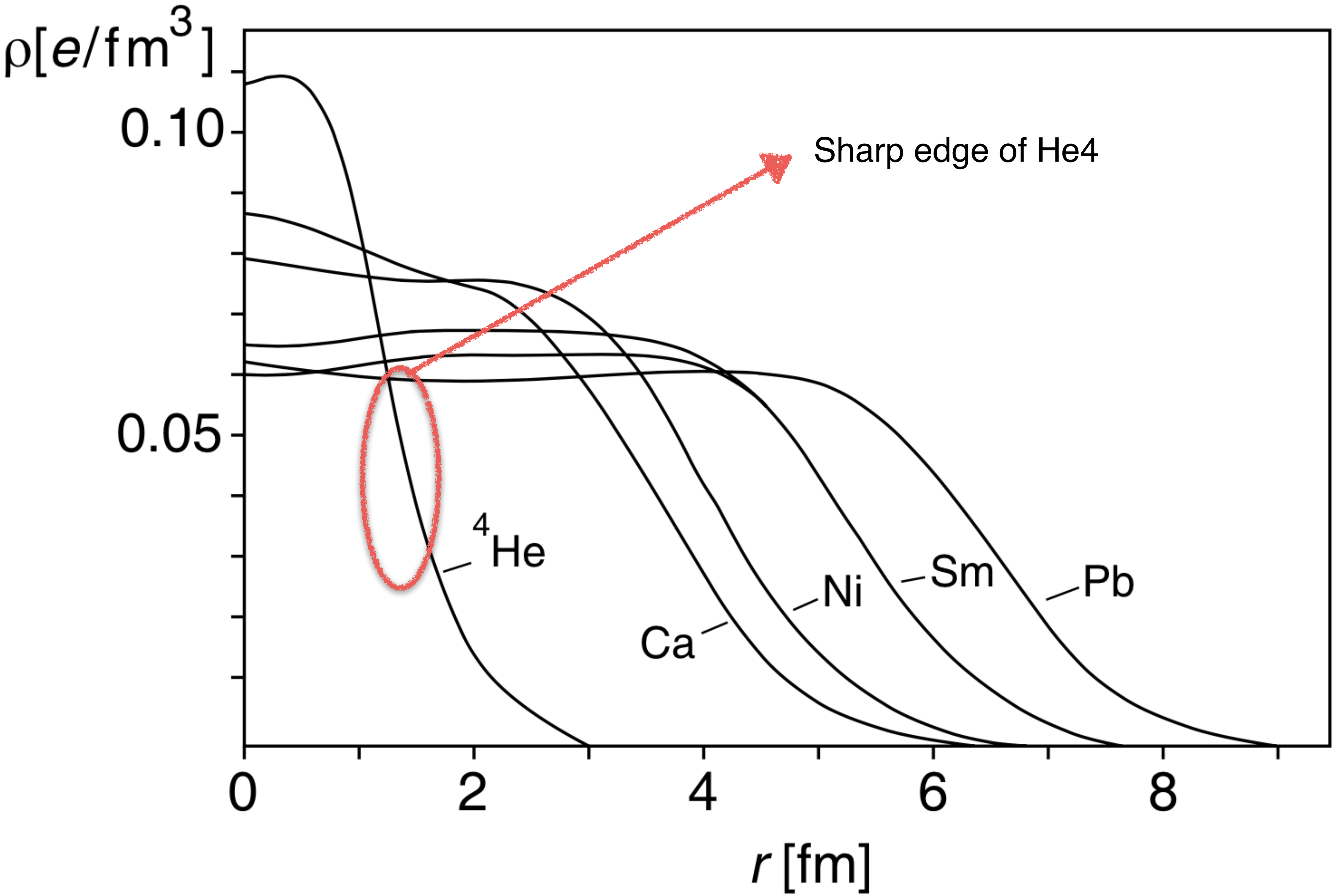

Those of which the form factors fall off faster, the corresponding sizes are larger!

↓

Ca48 is larger than Ca40

R. Hofstadter, Ann. Rev. Nucl. Sci 7, 231 (1957)

# Distributions of Nuclei




Particles & Nuclei by Povh et al.

# Nucleon structure

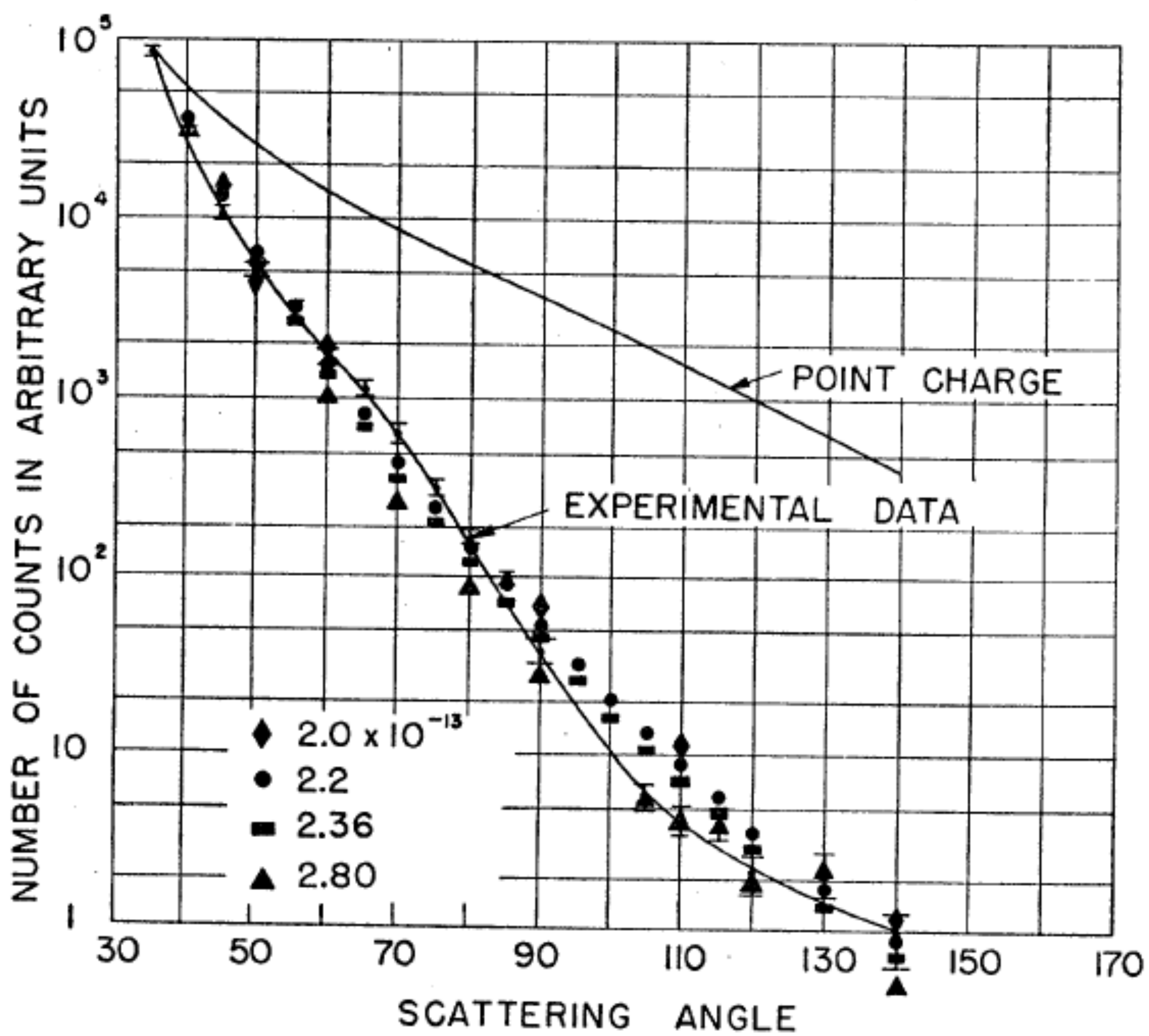



FIG. 11. The angular distribution of electrons scattered from a 2-mil gold foil at 125 Mev. The point charge calculation of Feshbach is indicated. Theoretical points based on the first Born approximation for exponential charge distributions are shown. Values of $\alpha=2.0$, 2.2, 2.36, $2.8\times10^{-13}$ cm are chosen to demonstrate the sensitivity of the angular distribution to change of radius. All curves are normalized arbitrarily at 35°.

Hofstadter et al., PR, 92, 978 (1953)

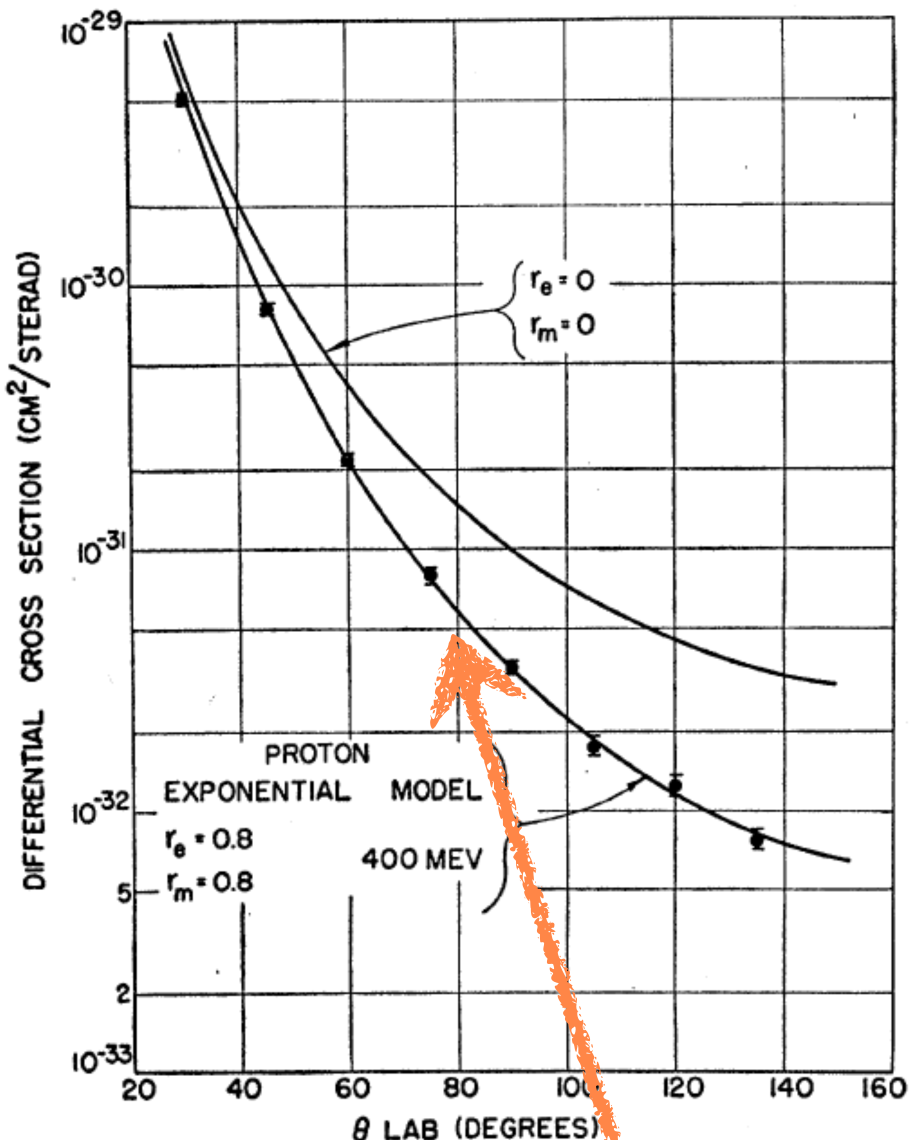



FIG. 26. Typical angular distribution for elastic scattering of 400-Mev electrons against protons. The solid line is a theoretical curve for a proton of finite extent. The model providing the theoretical curve is an exponential with rms radii$=0.80\times10^{-13}$ cm.

Hofstadter, RMP, 28, 214 (1956)

Nucleon has a size!

# Nucleus structure

## The Nobel Prize in Physics 1961

Robert Hofstadter

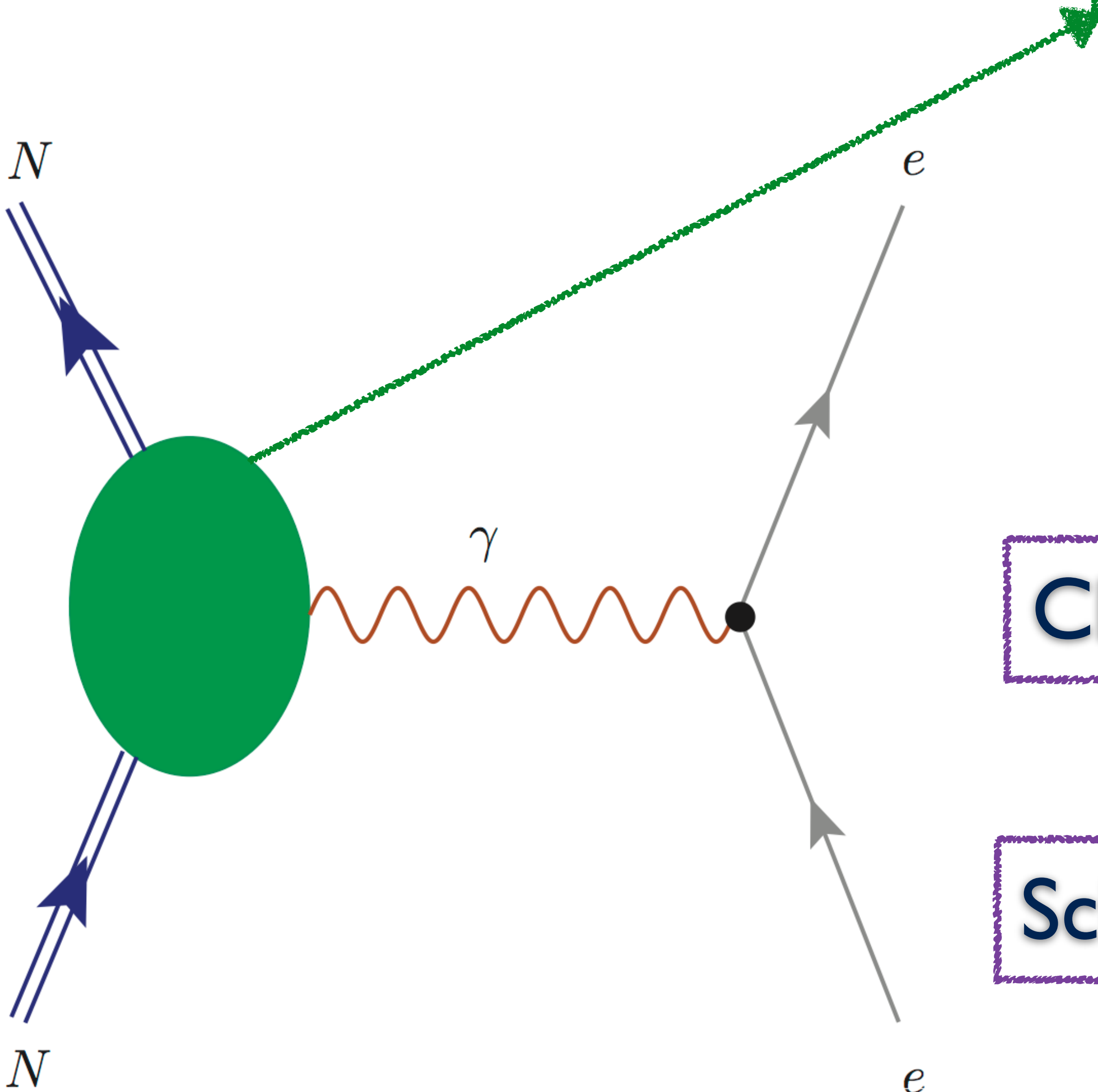
"For his pioneering studies of electron scattering in atomic nuclei and for his thereby achieved discoveries concerning the structure of the nucleons"

**Electron scattering: A clean-cut probe to the nucleon**

The electron is immune to the strong interaction
that contains a full of dirt.

# Interpretation of the Form factors

## Non-Relativistic picture of the EM form factors




Electromagnetic form factors

↑

3-D Fourier Transform

↑

Charge & Magnetisation Densities

↑

Schroedinger Eq. & Wave functions

However, this is not the end of our story.

# Nucleon

# Nucleon has internal structure!

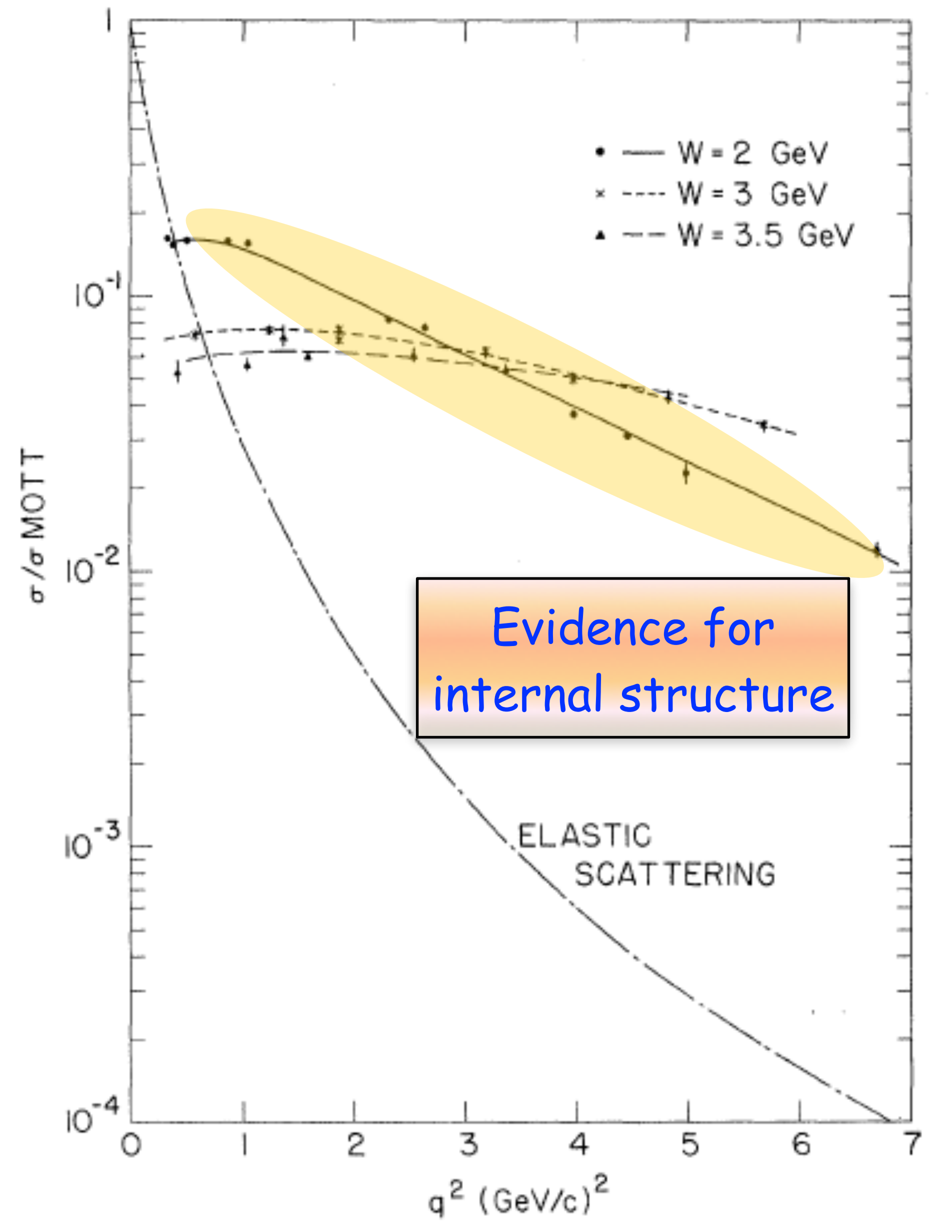
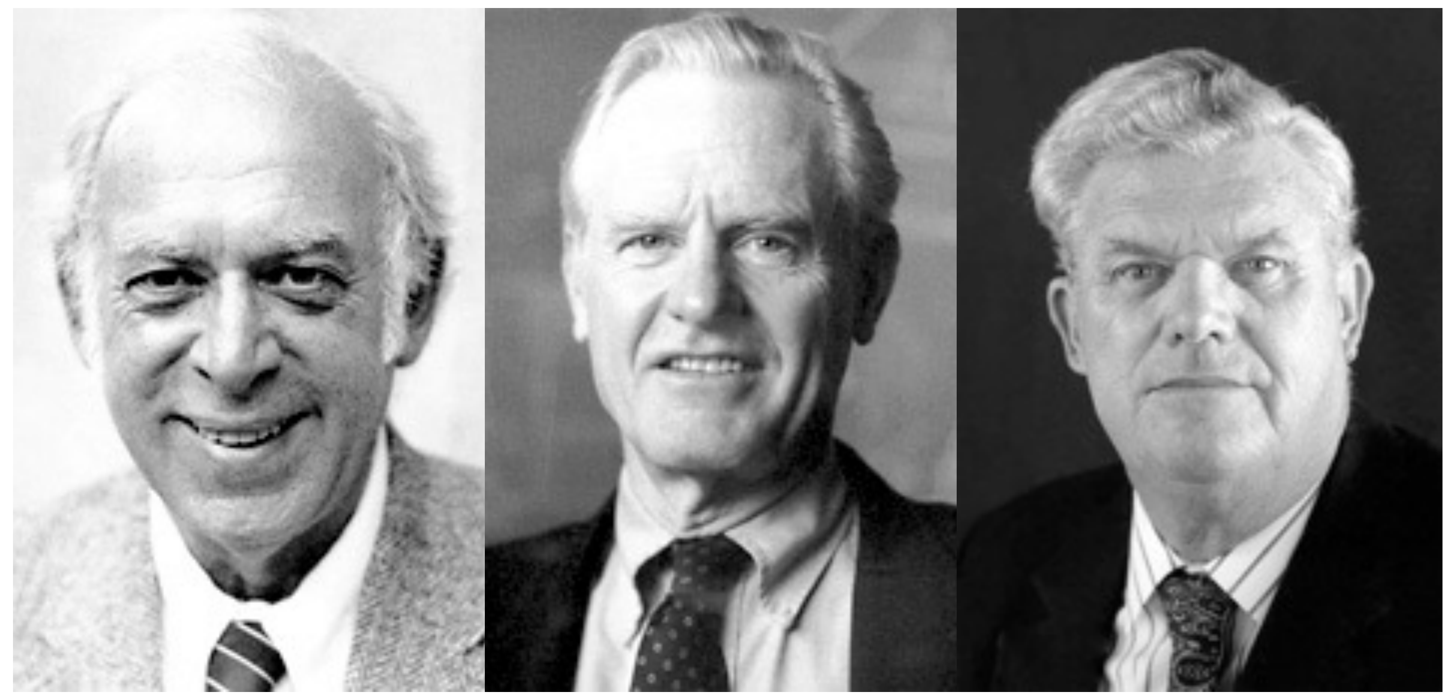



FIG. 1. $(d^2\sigma/d\Omega dE')/\sigma_{\rm Mott}$, in $\rm GeV^{-1}$, vs $q^2$ for $W$ $=2$, 3, and 3.5 GeV. The lines drawn through the data are meant to guide the eye. Also shown is the cross section for elastic $e$–$p$ scattering divided by $\sigma_{\rm Mott}$, $(d\sigma/d\Omega)/\sigma_{\rm Mott}$, calculated for $\theta = 10°$, using the dipole form factor. The relatively slow variation with $q^2$ of the inelastic cross section compared with the elastic cross section is clearly shown.

## 1990, Nobel Laureates



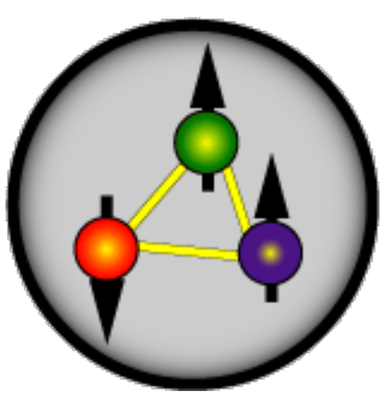
J. Friedman H. Kendall R. Taylor

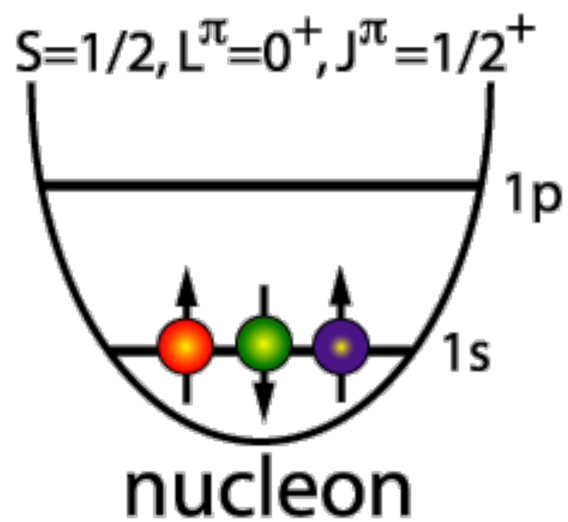
"For their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics"

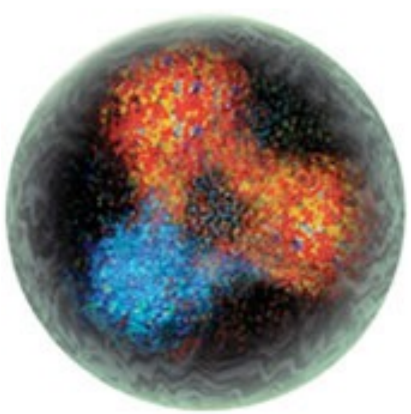
# What we know about the Nucleon

- Charge
  - Proton: $Q_p$ = +1
  - Neutron: $Q_n$ = 0

- Miss: $M_p$ = 938.272046 +- 0.000021 MeV/$c^2$
  $M_n$ = 939.565379 +- 0.000021 MeV/$c^2$
  - Proton + neutron make up 99.9% of the mass of the visible universe



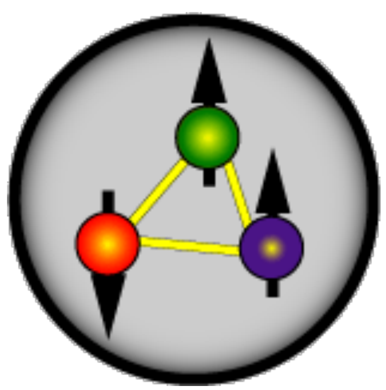


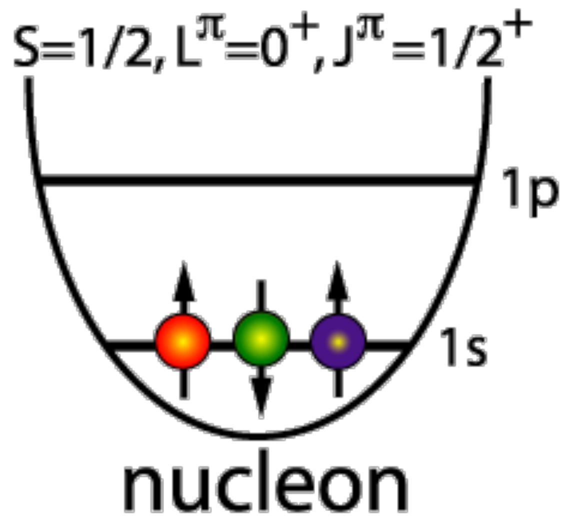



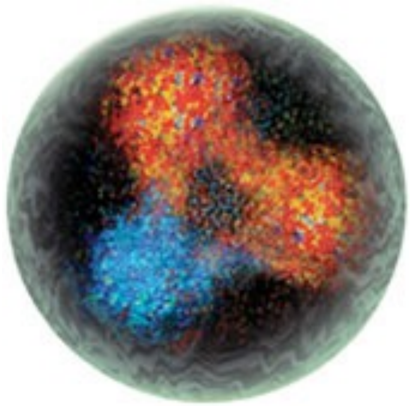
The origin of the nucleon mass is still unknown.

# What we know about the Nucleon

- Spin: $s = \frac{1}{2}\hbar$
  - Magnetic moment $\mu_p = 2.79\mu_N$, $\mu_p = -1.91\mu_N$
  - Anomalous magnetic moment $\kappa_p = 1.79\mu_N$, $\kappa_n = -1.91\mu_N$



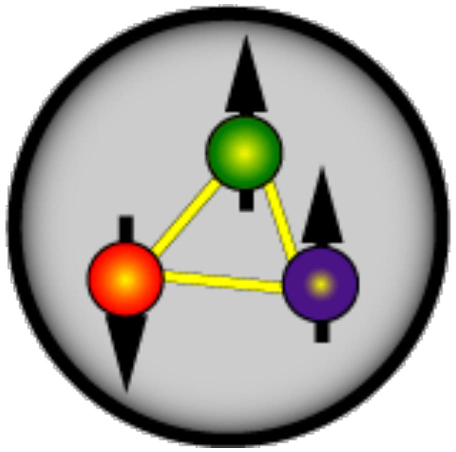


The origin of the nucleon spin is still unknown.

# How the Nucleon looks like

## The Non-Relativistic Quark Model



$$|N\rangle \sim |qqq\rangle$$

$$|N\rangle \sim |q_\uparrow q_\uparrow q_\downarrow\rangle$$

Amazingly successful!

SU(3) flavor Symmetry
+
Spin symmetry

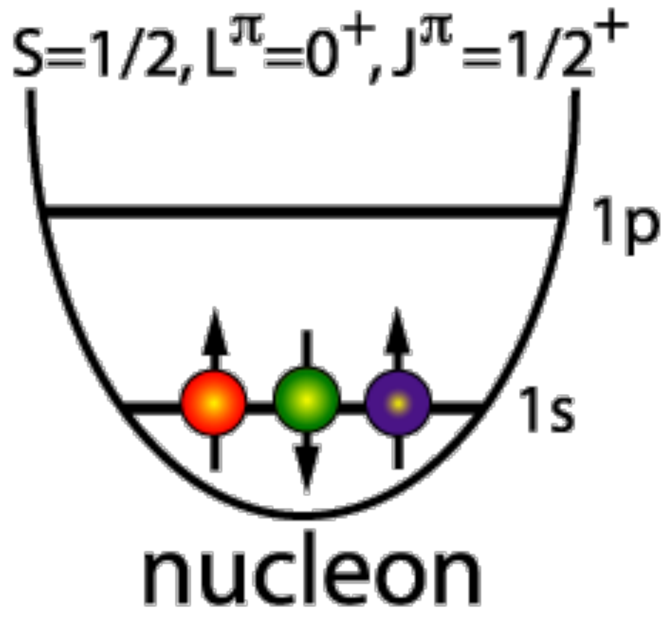
Constituent quark mass

$$M_q \simeq 350\,\mathrm{MeV}$$

$$M_N \approx 3M_q$$

- No explanation was given why the quark mass is so large.
- No interaction and dynamics were considered.

# How the Nucleon looks like

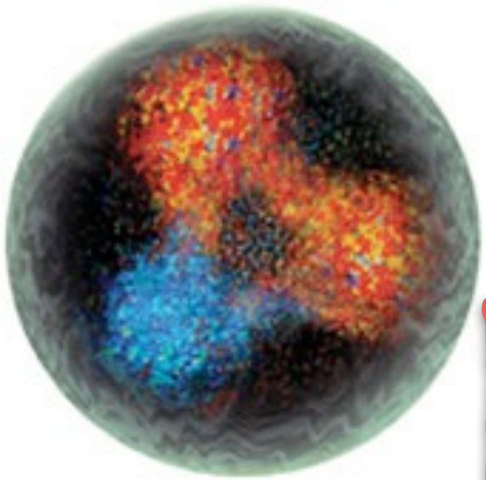



The Nucleon as three quarks
in an instantaneous potential

- Symmetry + Phenomenological dynamics
- Nucleon excited states can be described (confinement potential)
- Many properties were nicely explained.

- Potential originated from heavy-quark systems, not for the light-quark system.
- Failure of explaining strong decays (correct feature for resonances)
- Not fully relativistic (No seq quark, one needs field theory).

# How the Nucleon looks like

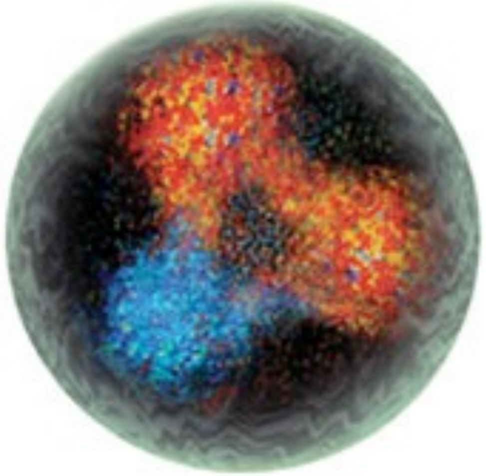


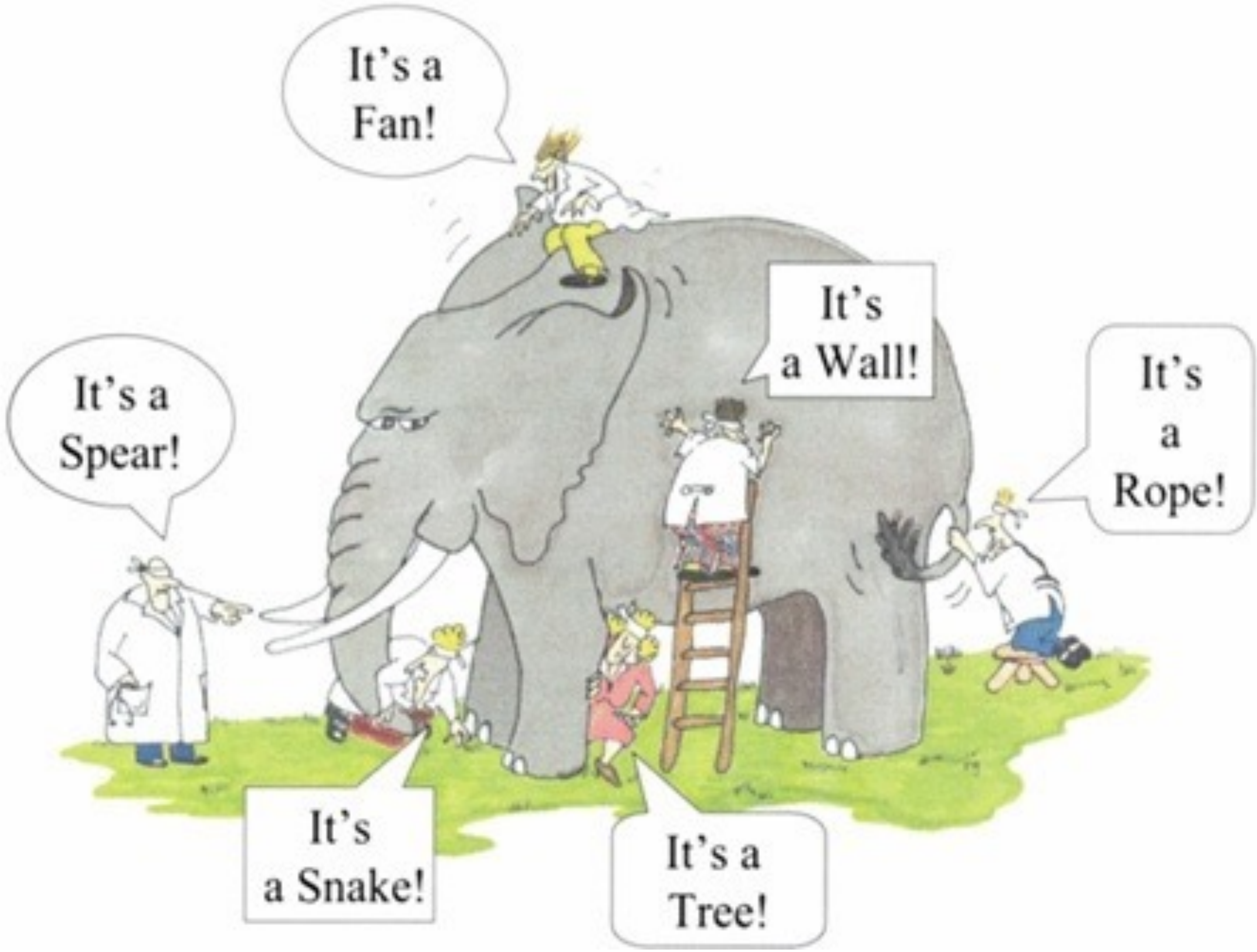
The Nucleon, the most messy object in the Universe

- Valence quarks + Sea quarks + gluons + …
- Too complicated to solve?
- Brute force way: Lattice QCD
- Hadrons as relevant degrees of freedom (Effective Field Theory)
- Holographic QCD (5D QCD)
- Instantons
- Monopoles
- Large Nc QCD
- Skyrme models, NJL models, Chiral quark models…

**Each approach has pros and cons.**

# How the Nucleon looks like





# New definition
# of
# the form factors

# Three Missions of the EIC

1. How does the mass of the nucleon arise?
2. How does the spin of the nucleon arise?
3. What are the emergent properties of dense systems of gluons?

Taken from the EIC CDR (2021)

- **Key words**

1. The fundamental structure of the nucleon
2. Gluons inside the nucleon
3. Gluons under extreme conditions

→ Behind the scene

Gravitational form factors of the nucleon &
Energy-momentum tensor densities

# Critical view on Nucleon form factors

## Traditional interpretation of the nucleon form factors

$$F_1(Q^2) = \int d^3x e^{i\mathbf{Q}\cdot\mathbf{x}} \rho(\mathbf{r}) \rightarrow \quad \rho(\mathbf{r}) = \sum \psi^\dagger(\mathbf{r})\psi(\mathbf{r})$$

However, the initial and final momenta are different in a relativistic case.
Thus, the initial and final wave functions are different.

→ **Probability interpretation is wrong in a relativistic case!**

$$r \sim 0.8\,\mathrm{fm}$$

$$\delta r \sim \frac{\hbar}{M_N c} \approx 0.2\,\mathrm{fm} \qquad r \sim \delta r$$

↓

We need a correct interpretation of the form factors.


Belitsky & Radyushkin, Phys.Rept. **418**, 1 (2005)

G.A. Miller, PRL **99,** 112001 (2007)

C. Lorce, PRL **125,** 232002 (2020)

R. L. Jaffe, PRD **103,** 016017 (2021)

# Critical view on Nucleon form factors

R: Size of the system
M: Mass of the system

## Non-Relativistic description

$$M_{\rm atom} R_{\rm atom} = M_{\rm atom}/(m_e \alpha) \sim 10^5$$

$$\|Q\| \ll M_{\rm atom} \qquad 1/\|Q\| \leq R$$

$$\rho(\mathbf{r}) = \sum \Psi^\dagger(\mathbf{r})\Psi(\mathbf{r})$$

Particle number fixed.

Form factors can be measured and well interpreted (almost no recoil effect).

## Relativistic description

$$M_N R_N \sim 4 \qquad \|Q\| \geq M_N$$

Particle creation & annihilation

Initial and final momenta are different!

⇨ Nucleon cannot be treated non-relativistically!

Belitsky & Radyushkin, Phys.Rept. **418**, 1 (2005)

# Critical view on Nucleon form factors

- A question arises: Then what is a physical meaning of the 3D charge distribution of the nucleon?

Quasi-probabilistic meaning of the 3D charge distribution in the BF

$$\langle \hat{O} \rangle_\psi = \int \frac{d^3P}{(2\pi)^3} d^3R \rho_\psi(\boldsymbol{R}, \boldsymbol{P}) \langle \hat{O} \rangle_{\boldsymbol{P},\boldsymbol{R}}$$

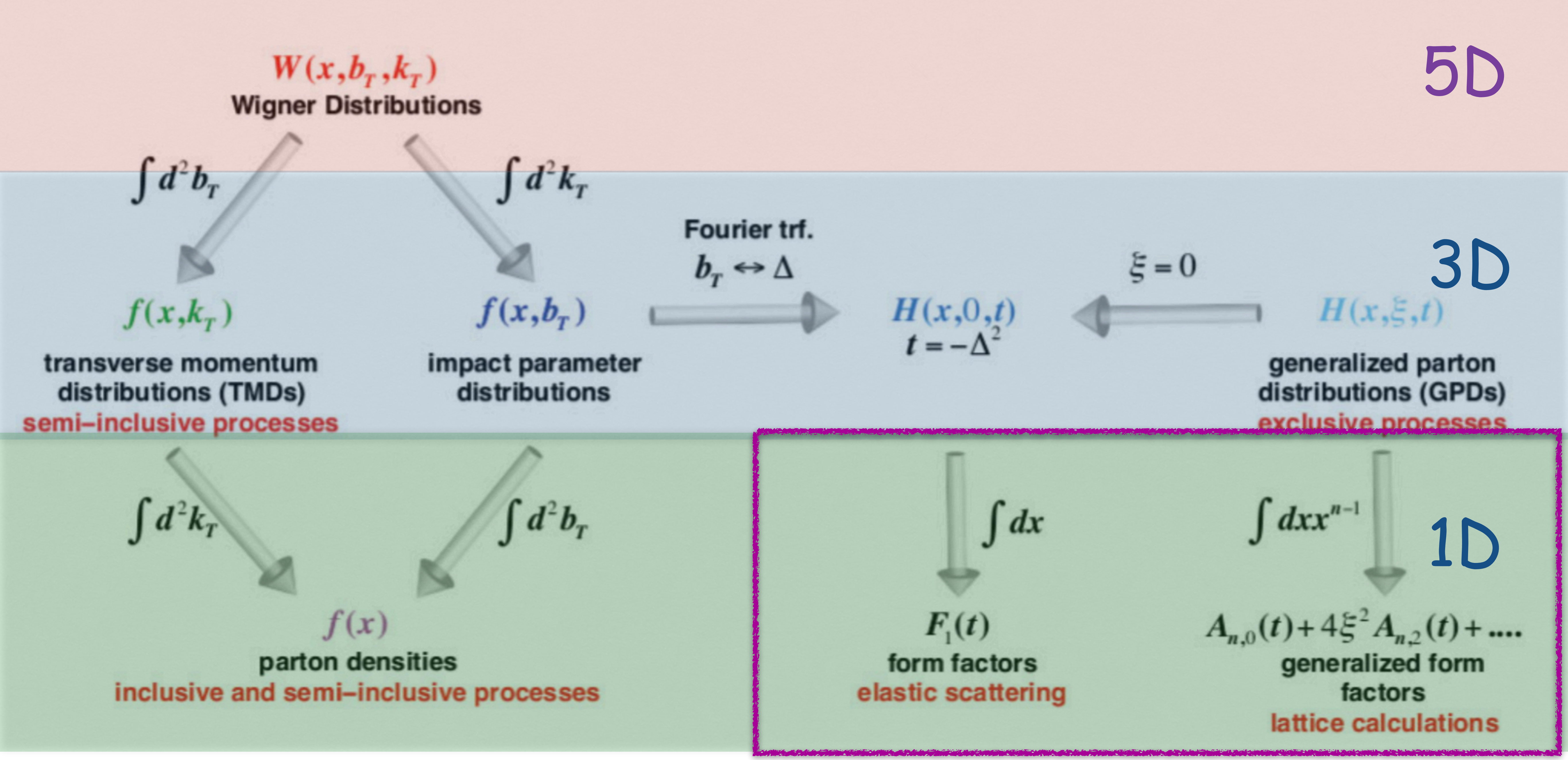
$$\rho_\psi(\boldsymbol{R},\boldsymbol{P}) \equiv \int d^3 z e^{-i\boldsymbol{P}\cdot\boldsymbol{z}} \psi^*\left(\boldsymbol{R} - \frac{\boldsymbol{z}}{2}\right) \psi\left(\boldsymbol{R} + \frac{\boldsymbol{z}}{2}\right)$$ : Wigner distribution.

$$= \int \frac{d^3 q}{(2\pi)^3} e^{-i\boldsymbol{q}\cdot\boldsymbol{R}} \tilde{\psi}^*\left(\boldsymbol{P} + \frac{\boldsymbol{q}}{2}\right) \tilde{\psi}\left(\boldsymbol{P} - \frac{\boldsymbol{q}}{2}\right)$$

$$\langle \hat{O} \rangle_{\boldsymbol{R},\boldsymbol{P}} \equiv \int \frac{d^3\Delta}{(2\pi)^3} e^{i\boldsymbol{\Delta}\cdot\boldsymbol{R}} \left\langle \boldsymbol{P} + \frac{\boldsymbol{\Delta}}{2} \middle| \hat{O} \middle| \boldsymbol{P} - \frac{\boldsymbol{\Delta}}{2} \right\rangle$$

Wigner, PR 40 (1932); C. Lorce, PRL 125 (2020)

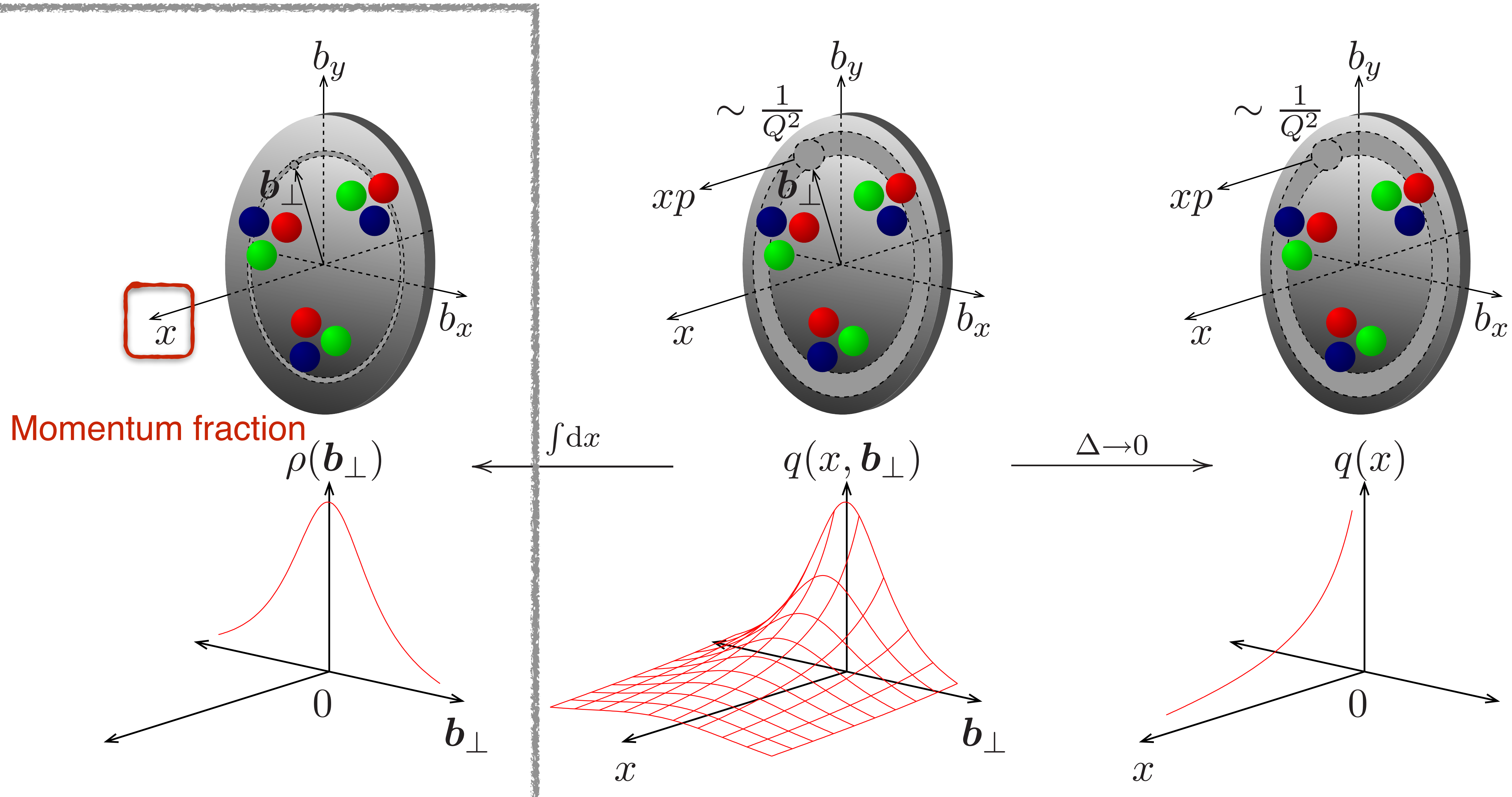
# Stitching together a 5D Image of the Nucleon




I will concentrate on 2D transverse distributions.

Figure taken from Eur. Phys. J. A (2016) 52: 268

# Modern Understanding on Nucleon form factors




Transverse densities of Form factors

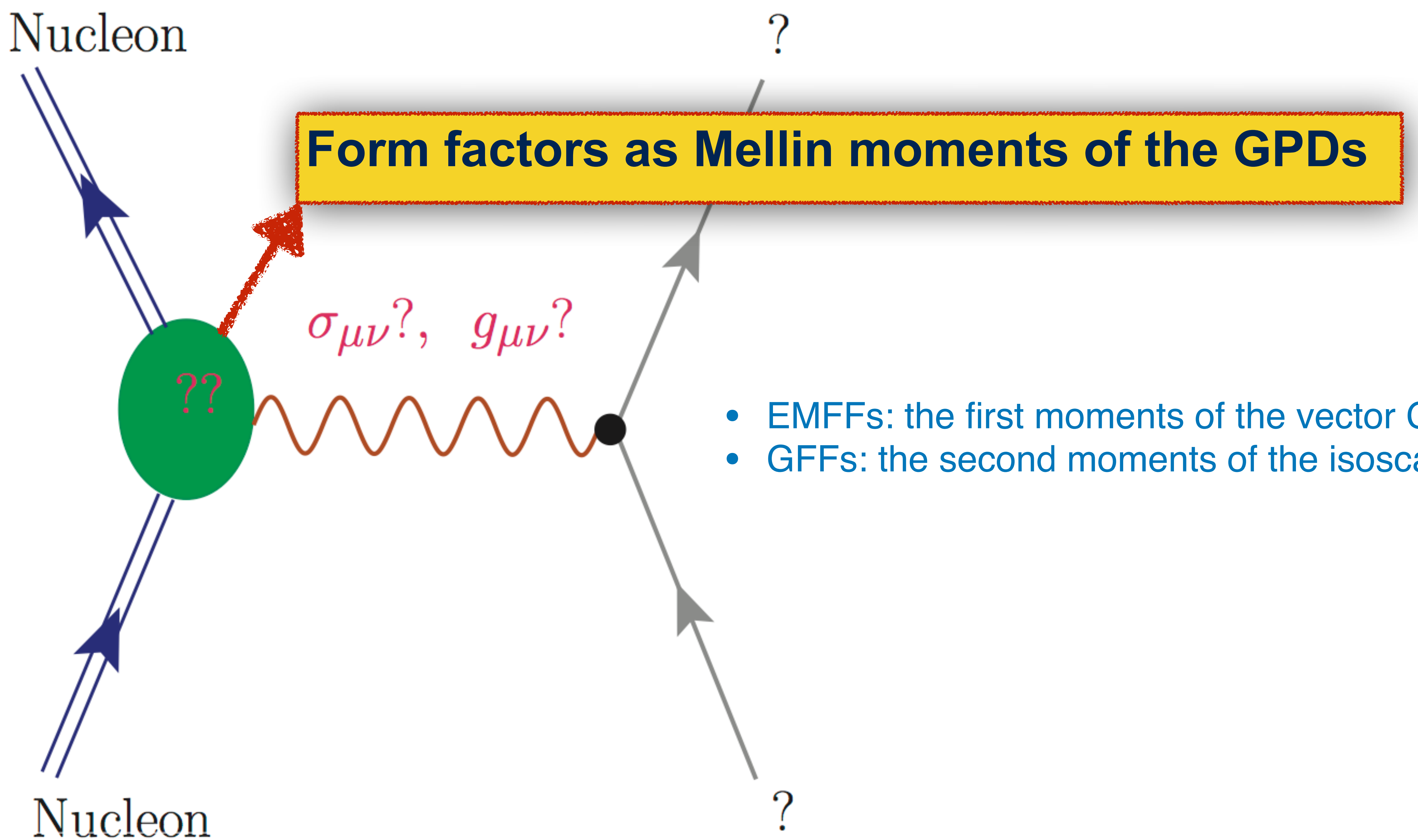
GPDs
Nucleon Tomography

Structure functions
Parton distributions

D. Brömmel, Dissertation (Regensburg U.)

# Modern Understanding on Nucleon form factors

Probes are unknown for **Tensor form factors**
and the **Gravitational form factors!**




- EMFFs: the first moments of the vector GPD
- GFFs: the second moments of the isoscalar GPD

# Gravitational form factors
# of
# the Pion

# Energy-Momentum Tensor

Hilbert-Einstein Action (Hilbert, 1915)

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\;R + \int d^4x\sqrt{-g}L_M$$

Changing the metric in the long-wave approximation

$$g^{\mu\nu} = \eta^{\mu\nu} + \delta g^{\mu\nu}(\boldsymbol{r}) \qquad \lambda_{\text{grav}} \gg M_N^{-1}$$

we find the EMT that characterizes the response of the nucleon to the static change of the space-time metric:

Energy-Momentum Tensor (EMT): $T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta g_{\mu\nu}}$

# Gravitational form factors of the pion

- Energy-momentum tensor (Gravitational) form factor: $A_{20}$, $A_{22}$

M. V. Polyakov and C. Weiss, Phys. Rev. D 60, 114017 (1999)

Isoscalar vector GPD

$$2\delta^{ab}H_\pi^{I=0}(x,\xi,t) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda(P\cdot n)} \langle \pi^a(p')|\bar{\psi}(-\lambda n/2)\not{n}[-\lambda n/2, \lambda n/2]\psi(\lambda n/2)|\pi^b(p)\rangle$$

Its second moments

$$\int dx\, x H_\pi^{I=0}(x,\,\xi,t) \;=\; A_{2,0}(t) + 4\xi^2 A_{2,2}(t)$$

Energy-momentum tensor form factor

$$\langle \pi^a(p')|T_{\mu\nu}(0)|\pi^b(p)\rangle = \frac{\delta^{ab}}{2}\left[(t g_{\mu\nu} - q_\mu q_\nu)\Theta_1(t) + 4P_\mu P_\nu \Theta_2(t)\right],$$

H.D. Son & HChK, Phys. Rev. D90 (2014) 111901(R)

# Gravitational form factors of the pion

- Energy-momentum tensor operator

$$T_{\mu\nu}(x) = \frac{1}{2}\bar{\psi}(x)\gamma_{\{\mu}i\overleftrightarrow{\partial}_{\nu\}}\psi(x).$$

- The pion mass

$$\langle \pi^a(p)|T_{00}(0)|\pi^b(p)\rangle\Big|_{t=0} = 2m_\pi^2\Theta_2(0)\delta^{ab}$$

- Spatial component → Pressure of the pion

$$\langle \pi^a(p)|T_{ii}(0)|\pi^b(p)\rangle\Big|_{t=0} = \delta^{ab}\frac{3}{2}t\,\Theta_1(t)\Big|_{t=0}$$

The pressure of any particle must be equal to zero:
Stability condition (von Laue condition)

H.D. Son & HChK, Phys. Rev. D90 (2014) 111901(R)

# Pressure of the pion

- Pressure of the pion

(In the chiral limit, the pressure vanishes trivially.)

$$\mathcal{P} = \langle \pi^a(p) | T_{ii}(0) | \pi^a(p) \rangle$$
$$= \frac{12N_c m M}{f_\pi^2} \int d\tilde{l} \, \frac{-l^2}{[l^2 + \overline{M}^2]^2} + \frac{12 N_c M^2}{f_\pi^2} \int d\tilde{l} \int_0^1 dx \, \frac{-p^2 l^2}{[l^2 + x(1-x)p^2 + \overline{M}^2]^3}$$

$$p^2 = -m_\pi^2$$

Quark condensate

$$i\langle \psi^\dagger \psi \rangle = 8N_c \int d\tilde{l} \frac{\overline{M}}{[l^2 + \overline{M}^2]}$$

Pion decay constant

$$f_\pi^2 = 4N_c \int_0^1 dx \int d\tilde{l} \, \frac{M\overline{M}}{[l^2 + \overline{M}^2 + x(1-x)p^2]^2}$$

The pressure of any particle must be equal to zero:
Stability condition (von Laue condition)

$$\boxed{\mathcal{P} = \frac{3M}{f_\pi^2 \overline{M}} \left( m \langle \bar{\psi}\psi \rangle + m_\pi^2 f_\pi^2 \right) = 0}$$

(By the Gell-Mann-Oakes-Renner relation)

H.D. Son & HChK, Phys. Rev. D90 (2014) 111901(R)

# Low-Energy Constants

- LECs in the curved space: $L_{11}, L_{12}, L_{13}$

$$\Theta_1(t) = 1 + \frac{2}{f_\pi^2}\left[t(4L_{11} + L_{12}) - 8m_\pi^2(L_{11} - L_{13})\right]$$

$$\Theta_2(t) = 1 - \frac{2t}{f_\pi^2} L_{12}.$$

$$L_{11} = \frac{N_c}{192\pi^2} = 1.6 \times 10^{-3}, L_{12} = -2L_{11} = -3.2 \times 10^{-3},$$

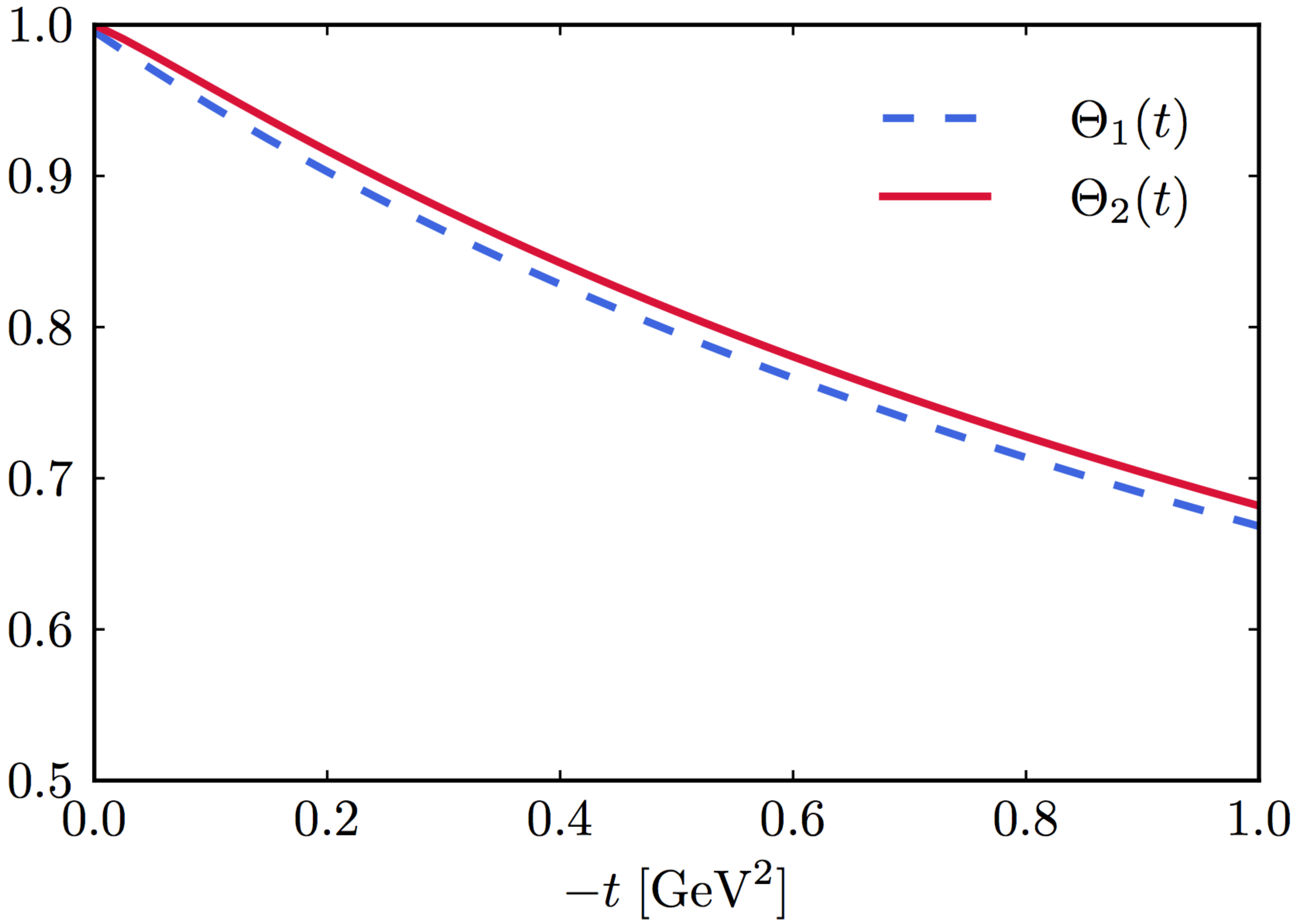
$$L_{13} = -\frac{N_c}{96\pi^2}\frac{M}{B_0}\Gamma\left(0, \frac{M^2}{\Lambda^2}\right) = 0.84 \times 10^{-3}$$

In XPT, $(L_{11} = 1.4, \quad L_{12} = -2.7, \quad L_{13} = 0.9 \text{ in unit of } 10^{-3})$

J. F. Donoghue and H. Leutwyler, Z. Phys. C 52, 343 (1991)

H.D. Son & HChK, Phys. Rev. D90 (2014) 111901(R)

# EMT form factor of the pion




H.D. Son & HChK, Phys. Rev. D90 (2014) 111901(R)

# Mechanical properties
# of
# Baryons

# EMT densities

- Static EMT distributions in the BF (M.V. Polyakov, PLB 555 (2003))

$$T^{\mu\nu}(\boldsymbol{r}) = \int \frac{d^3\Delta}{(2\pi)^3 2E} e^{-i\boldsymbol{r}\cdot\boldsymbol{\Delta}} \langle p'|T^{\mu\nu}(0)|p\rangle$$

These EMT distributions of the nucleon are also hampered by ambiguous relativistic corrections. The localized wavefunction of the nucleon is ill-defined.

- However, the 3D distributions in the BF can be projected on the 2D transverse plane by the Abel transform, which yields the 2D light-front distributions. (Panteleeva & Polyakov, ArXiv: 2102.10902)

→ Equivalence of the 2D & 3D distributions

↓

3D BF distributions can be understood by Abel-transforming from 3D to 2D unambiguously, which gives the 2D distributions in the IMF, and vice versa.

→ Real tomography of the nucleon

# Gravitational form factors

- EMT current in QCD & GFFs

Kobzarev et al. 1962; Pagels, 1966

$$T_q^{\mu\nu} = \frac{1}{4}\overline{\psi}_q\left(-i\overleftarrow{\mathcal{D}}^\mu\gamma^\nu - i\overleftarrow{\mathcal{D}}^\nu\gamma^\mu + i\overrightarrow{\mathcal{D}}^\mu\gamma^\nu + i\overrightarrow{\mathcal{D}}^\nu\gamma^\mu\right)\psi_q - g^{\mu\nu}\overline{\psi}_q\left(-\frac{i}{2}\overleftarrow{\not{\mathcal{D}}} + \frac{i}{2}\overrightarrow{\not{\mathcal{D}}} - m_q\right)\psi_q,$$

$$T_g^{\mu\nu} = F^{a,\mu\eta}\,F^{a,}{}_\eta{}^\nu + \frac{1}{4}\,g^{\mu\nu}F^{a,\kappa\eta}\,F^{a,}{}_{\kappa\eta}.$$

D(Druck)-term Weiss & Polyakov, 1999

$$\langle p'|T^{\mu\nu}(0)|p\rangle = \bar{u}(p')\left[A^a(t)\frac{P^\mu P^\nu}{M_N} + J^a(t)\frac{iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M_N} + D^a(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M_N} + M_N\bar{c}^a(t)g^{\mu\nu}\right]u(p)$$

$\delta g^{00}$ → Mass: $\sum_a A^a(0) = 1$

$\delta g^{0i}$ → Spin: $\sum_a J^a(0) = \frac{1}{2}$

$\delta g^{ij}$ → Deformation of space = mechanical properties of the nucleon

Non-conservation of EMT pieces

Pressure & Shear-force distributions (pressure anisotropy)

# EMT distributions

The matrix element of the EMT current in terms of the Wigner distribution

$$\langle \hat{T}^{\mu\nu}(\boldsymbol{r})\rangle = \int \frac{d^3\boldsymbol{P}}{(2\pi)^3}\int d^3\boldsymbol{R}\, W(\boldsymbol{R},\boldsymbol{P})\langle \hat{T}^{\mu\nu}(\boldsymbol{r})\rangle_{\boldsymbol{R},\boldsymbol{P}}$$

Wigner distribution

$$W(\boldsymbol{R},\boldsymbol{P}) = \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{R}}\tilde{\psi}^*\left(\boldsymbol{P}+\frac{\boldsymbol{\Delta}}{2}\right)\tilde{\psi}\left(\boldsymbol{P}-\frac{\boldsymbol{\Delta}}{2}\right)$$
$$= \int d^3\boldsymbol{z}\, e^{-i\boldsymbol{z}\cdot\boldsymbol{P}}\psi^*\left(\boldsymbol{R}-\frac{\boldsymbol{z}}{2}\right)\psi\left(\boldsymbol{R}+\frac{\boldsymbol{z}}{2}\right)$$

$$\boldsymbol{R} = (\boldsymbol{r}'+\boldsymbol{r})/2$$

$$\boldsymbol{P} = (\boldsymbol{p}'+\boldsymbol{p})/2$$

$$\boldsymbol{\Delta} = \boldsymbol{p}'-\boldsymbol{p}$$

$$\int \frac{d^3\boldsymbol{P}}{(2\pi)^3} W_N(\boldsymbol{R},\boldsymbol{P}) = |\psi_N(\boldsymbol{R})|^2, \quad \int d^3\boldsymbol{R}\, W_N(\boldsymbol{R},\boldsymbol{P}) = |\tilde{\psi}_N(\boldsymbol{P})|^2$$

Wave functions

$$\psi(\boldsymbol{r}) = \langle \boldsymbol{r}|\psi\rangle = \int \frac{d^3\boldsymbol{p}}{(2\pi^3)} e^{i\boldsymbol{p}\cdot\boldsymbol{r}}\tilde{\psi}(\boldsymbol{p}), \quad \tilde{\psi}(\boldsymbol{p}) = \frac{1}{\sqrt{2p^0}}\langle p|\psi\rangle$$

Localized at r at t.

$$\langle p'|p\rangle = 2p^0(2\pi)^3\delta^{(3)}(\boldsymbol{p}'-\boldsymbol{p}) \qquad \langle \boldsymbol{r}'|\boldsymbol{r}\rangle = \delta^{(3)}(\boldsymbol{r}'-\boldsymbol{r}) \qquad |\boldsymbol{r}\rangle = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3\sqrt{2p^0}} e^{-i\boldsymbol{p}\cdot\boldsymbol{r}}|p\rangle.$$

# Energy distribution

$$\langle \hat{T}^{\mu\nu}(\boldsymbol{r})\rangle_{\boldsymbol{R},\boldsymbol{P}} = \langle \hat{T}^{\mu\nu}(0)\rangle_{-\boldsymbol{x},\boldsymbol{P}} = \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3} e^{-i\boldsymbol{x}\cdot\boldsymbol{\Delta}} \frac{1}{\sqrt{2p^0}\sqrt{2p'^0}} \boxed{\langle p', J_3'|\hat{T}^{\mu\nu}(0)|p, J_3\rangle}$$

$$\boldsymbol{x} = \boldsymbol{r} - \boldsymbol{R}$$

- 3D energy-momentum tensor distributions in the Breit frame

$$T^{\mu\nu}_{\mathrm{BF},B}(\boldsymbol{r}, J_3', J_3) = \int \frac{d^3\Delta}{(2\pi)^3 2P_0} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \langle B, p', J_3'|\hat{T}^{\mu\nu}(0)|B, p, J_3\rangle$$

$$T^{00}_{\mathrm{BF},B}(\boldsymbol{r}, J_3', J_3) = \varepsilon^B(r)\delta_{J_3'J_3} = m_B \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \left[A^B(t) - \frac{t}{4m_B^2}\left(A^B(t) - 2J^B(t) + D^B(t)\right)\right]\delta_{J_3'J_3}$$

Normalization for the mass form factor

$$\int d^3r T^{00}_{\mathrm{BF},B}(\boldsymbol{r}, J_3', J_3) = m_B A^B(0) = m_B$$

- Mass radius $\langle r_\varepsilon^2\rangle_B = \dfrac{\int d^3r\, r^2 \varepsilon^B(r)}{\int d^3r\, \varepsilon^B(r)} = \dfrac{6}{A^B(0)} \dfrac{dA^B(t)}{dt}\Big|_{t=0}$

# Angular momentum distribution

The 0k-component of the EMT current

$$J^i_B(\boldsymbol{r}, J_3', J_3) = \epsilon^{ijk} r^j T^{0k}_{\mathrm{BF},B}(\boldsymbol{r}, J_3', J_3)$$
$$= 2S^j_{J_3' J_3} \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \left[ \left( J^B(t) + \frac{2}{3} t \frac{dJ^B(t)}{dt} \right) \delta^{ij} + \left( \Delta^i \Delta^j - \frac{1}{3} \boldsymbol{\Delta}^2 \delta^{ij} \right) \frac{dJ^B(t)}{dt} \right]$$

Angular-momentum distribution

$$\rho^B_J(r) := \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \left[ \left( J^B(t) + \frac{2}{3} t \frac{dJ^B(t)}{dt} \right) \right]$$

Spin normalization

$$\int d^3r J^i_B(\boldsymbol{r}, J_3', J_3) = 2\hat{S}^i_{J_3' J_3} \int d^3r\, \rho^B_J(r) = 2\hat{S}^i_{J_3' J_3} J^B(0) = \hat{S}^i_{J_3' J_3}$$

# Pressure & Shear-force distributions

$$T^{a}_{ij}(\boldsymbol{r},\sigma',\sigma) = p^{a}(r)\delta^{ij}\delta_{\sigma'\sigma} + s^{a}(r)\left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)\delta_{\sigma'\sigma}$$

- 3D Shear-force density in the BF

$$s^{a}(r) = -\frac{1}{4M_B} r \frac{d}{dr}\frac{1}{r}\frac{d}{dr}\tilde{D}^{a}(r)$$

- 3D Pressure density in the BF   M.V. Polyakov, PLB555 (2003)

$$p^{a}(r) = \frac{1}{6M_B}\frac{1}{r^2}\frac{1}{dr}r^2\frac{d}{dr}\tilde{D}^{a}(r) - M_B\int\frac{d^3\Delta}{(2\pi)^3}e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}}\bar{c}^{a}(t)$$

$$\tilde{D}^{a}(r) = \int\frac{d^3\Delta}{(2\pi)^3}e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}}D^{a}(t)$$

- This term is related to forces between quark and gluon subsystems (Polyakov & Son, 2018).
- It contributes to gluon and quark parts of energy density (mass decomposition). (Lorce, 2018)
- It vanishes for Goldstone bosons (P. Schweitzer & M.V. Polyakov, 2019).

# Stability conditions

- Conservation of the statice EMT current → Global & local stability conditions

$$\partial^i T_{ij} = \frac{r_j}{r}\left[\frac{2}{3}\frac{\partial s(r)}{\partial r} + \frac{2s(r)}{r} + \frac{\partial p(r)}{\partial r}\right] = 0$$

- Von Laue condition: Global stability condition

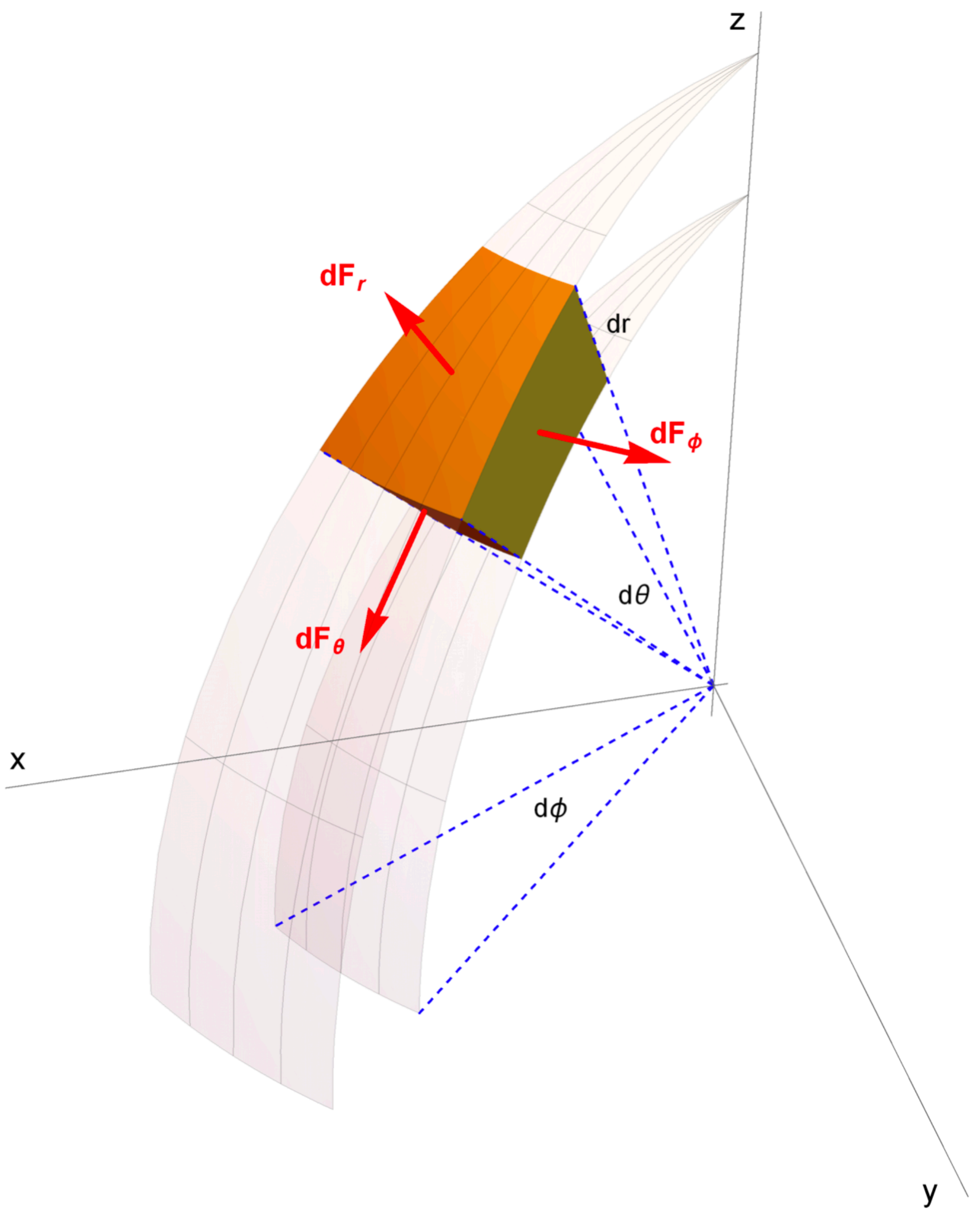
$$\int_0^\infty dr\ r^2 p(r) = 0$$

$$dF^i_{(r,\theta,\phi)} = T^{ij} dS_{(r,\theta,\phi)} \boldsymbol{e}^j_{(r,\theta,\phi)}$$

$$p_r(r) := \frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r),$$

$$p_\theta(r) := \frac{dF_\theta}{dS_\theta} = -\frac{1}{3}s(r) + p(r),$$

$$p_\phi(r) := \frac{dF_\phi}{dS_\phi} = -\frac{1}{3}s(r) + p(r)$$



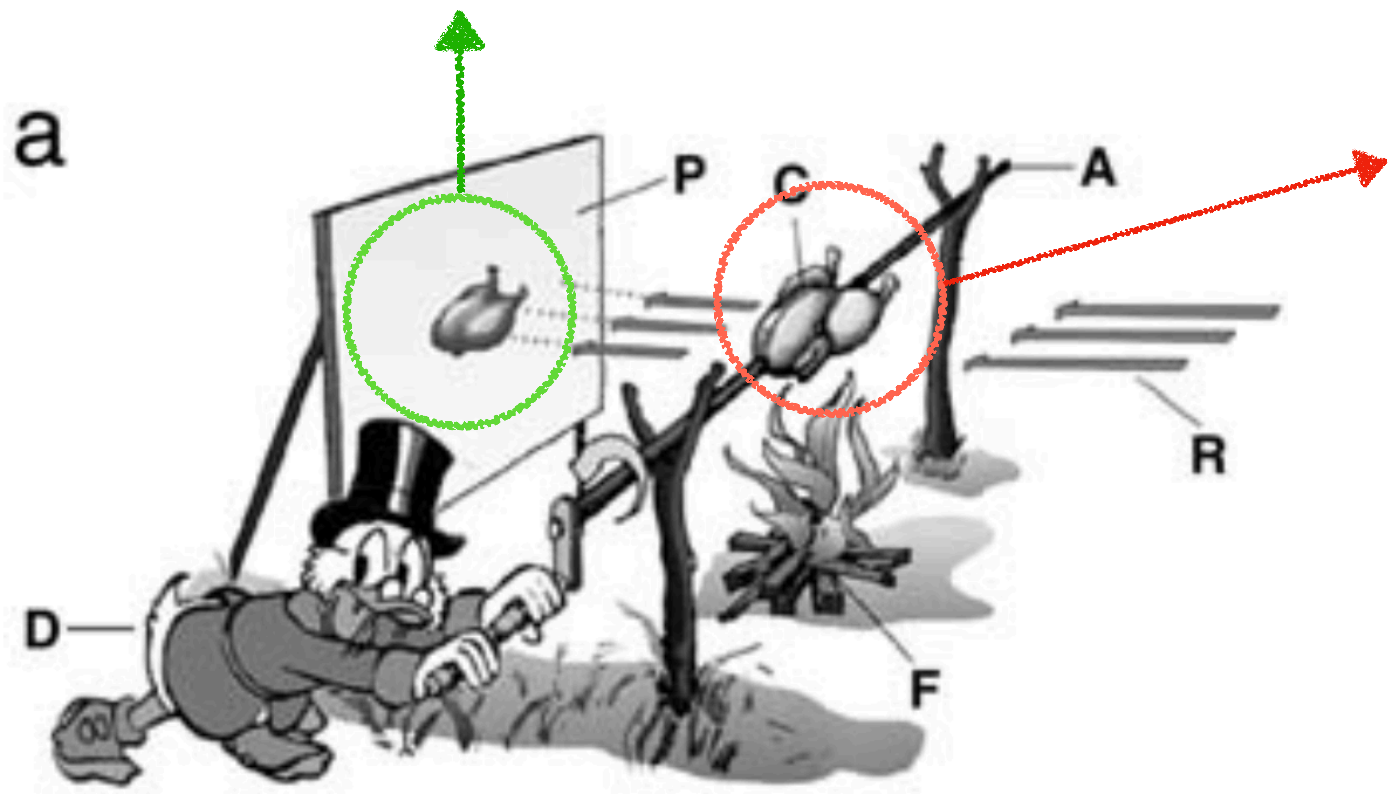

Local stability conditions $p_r(r) > 0$

A. Perevalova, M. V. Polyakov and P. Schweitzer, PRD 94 (2016)

# Abel transform & Nucleon tomography

2D transverse distributions in the IMF

3D distributions in the BF




- Nucleon tomography is indeed analogous to medical tomography!
- When baryons with higher spins are considered,
  we need to use the Radon transform.

"Electron tomography, edited by J. Frank"

# Abel transforms

- Abel transform from 3D in the BF to 2D in the IMF (Also invertible)

$$\mathcal{E}(x_\perp) = 2\int_{x_\perp}^{\infty} \left(\varepsilon(r) + \frac{3}{2}p(r) + \frac{1}{4m}\partial^2\left[\tilde{A}(r) - 2\tilde{J}(r)\right]\right)\frac{r\,dr}{\sqrt{r^2 - x_\perp^2}}$$

$$\rho_J^{(2D)}(x_\perp) = 3\int_{x_\perp}^{\infty} \frac{\rho_J(r)}{r}\frac{x_\perp^2\,dr}{\sqrt{r^2 - x_\perp^2}}$$

$$\mathcal{S}(x_\perp) = \int_{x_\perp}^{\infty} \frac{s(r)}{r}\frac{x_\perp^2\,dr}{\sqrt{r^2 - x_\perp^2}}$$

$$\frac{1}{2}\mathcal{S}(x_\perp) + \mathcal{P}(x_\perp) = \frac{1}{2}\int_{x_\perp}^{\infty} \left(\frac{2}{3}s(r) + p(r)\right)\frac{r\,dr}{\sqrt{r^2 - x_\perp^2}}$$

- Abel transform is used for tomography of spherically symmetric systems (spin 0 & 1/2 hadrons).
- For non-spherical objects (spin > 1/2), the Radon transform comes into play.

Panteleeva & Polyakov, ArXiv: 2102.10902; Kim et al., in preparation

# Equivalence of the 3D BF & 2D LF distributions

- Von Laue Conditions

$$\int_0^\infty dr\ r^2 p(r) = 0 \quad \Longleftrightarrow \quad \int d^2x_\perp \mathcal{P}(x_\perp) = 0$$

- Local stability Conditions

$$\frac{2}{3}s(r) + p(r) > 0 \quad \Longleftrightarrow \quad \frac{1}{2}\mathcal{S}(x_\perp) + \mathcal{P}(x_\perp) > 0$$

Geometric factor

$$\Omega_d = \frac{\sqrt{\pi}}{2}\frac{\Gamma\left(\frac{d+1}{2}\right)}{\Gamma\left(\frac{d+2}{2}\right)}$$

- Mechanical radius

$$\langle x_\perp^2 \rangle_{\text{mech}} = \frac{\int d^2x_\perp x_\perp^2 \left(\frac{1}{2}\mathcal{S}(x_\perp) + \mathcal{P}(x_\perp)\right)}{\int d^2x_\perp \left(\frac{1}{2}\mathcal{S}(x_\perp) + \mathcal{P}(x_\perp)\right)} = \frac{4D(0)}{\int_{-\infty}^0 dt D(t)} = \frac{2}{3}\langle r^2 \rangle_{\text{mech}}$$

- D(Druck)-terms

$$D(0) = -\frac{4M_N}{15}\int d^3r r^2 s(r) = m \int d^3r r^2 p(r) \quad \Longleftrightarrow \quad D(0) = -m\int d^2x_\perp x_\perp^2 \mathcal{S}(x_\perp) = 4m\int d^2x_\perp x_\perp^2 \mathcal{P}(x_\perp)$$

Panteleeva & Polyakov, ArXiv: 2102.10902

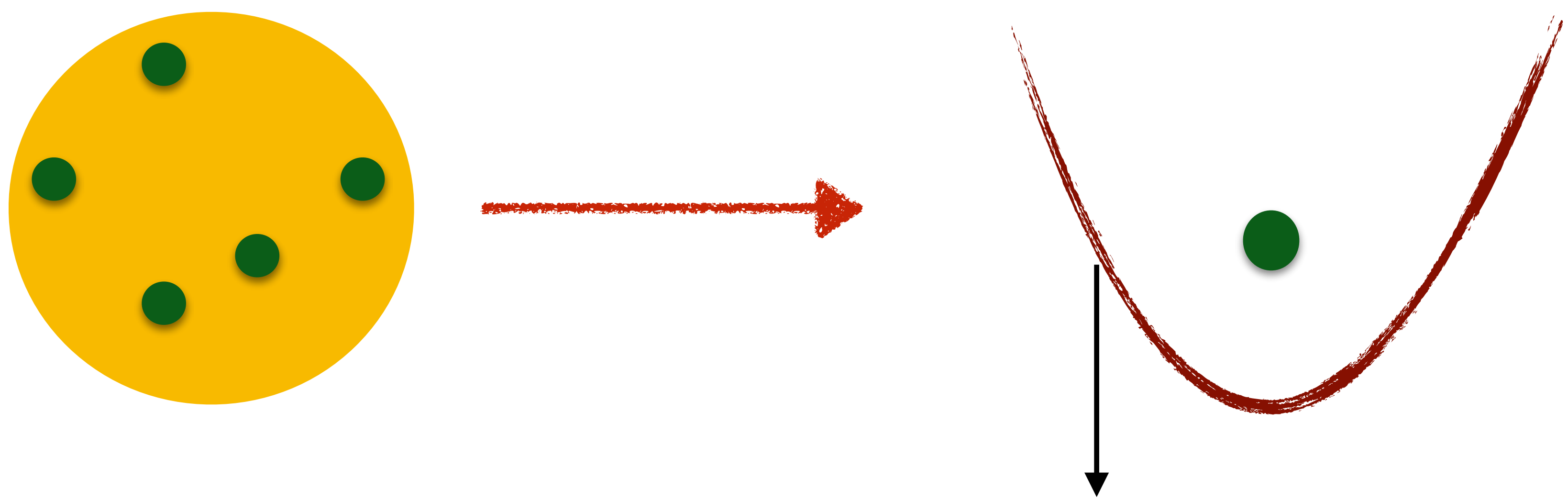
# Pion Mean-field approach

# Mean fields

Given action $S[\phi]$,

$$\left.\frac{\delta S}{\delta \phi}\right|_{\phi=\phi_0} = 0$$ : Solution of this saddle-point equation $\phi_0$

**This classical solution is regarded as a mean field.**



Mean-field potential that is produced by all other particles.

- Nuclear shell models
- Ginzburg-Landau theory for superconductivity
- Quark potential models for baryons

# Pion mean-field approach (Chiral Quark-Soliton model)

* Baryons as a state of Nc quarks bound by mesonic mean fields.

  Effective chiral action:

$$S_{\rm eff}[\pi^a] = -N_c {\rm Trlog}\left(i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m}\right)$$

* Key point: Hedgehog Ansatz

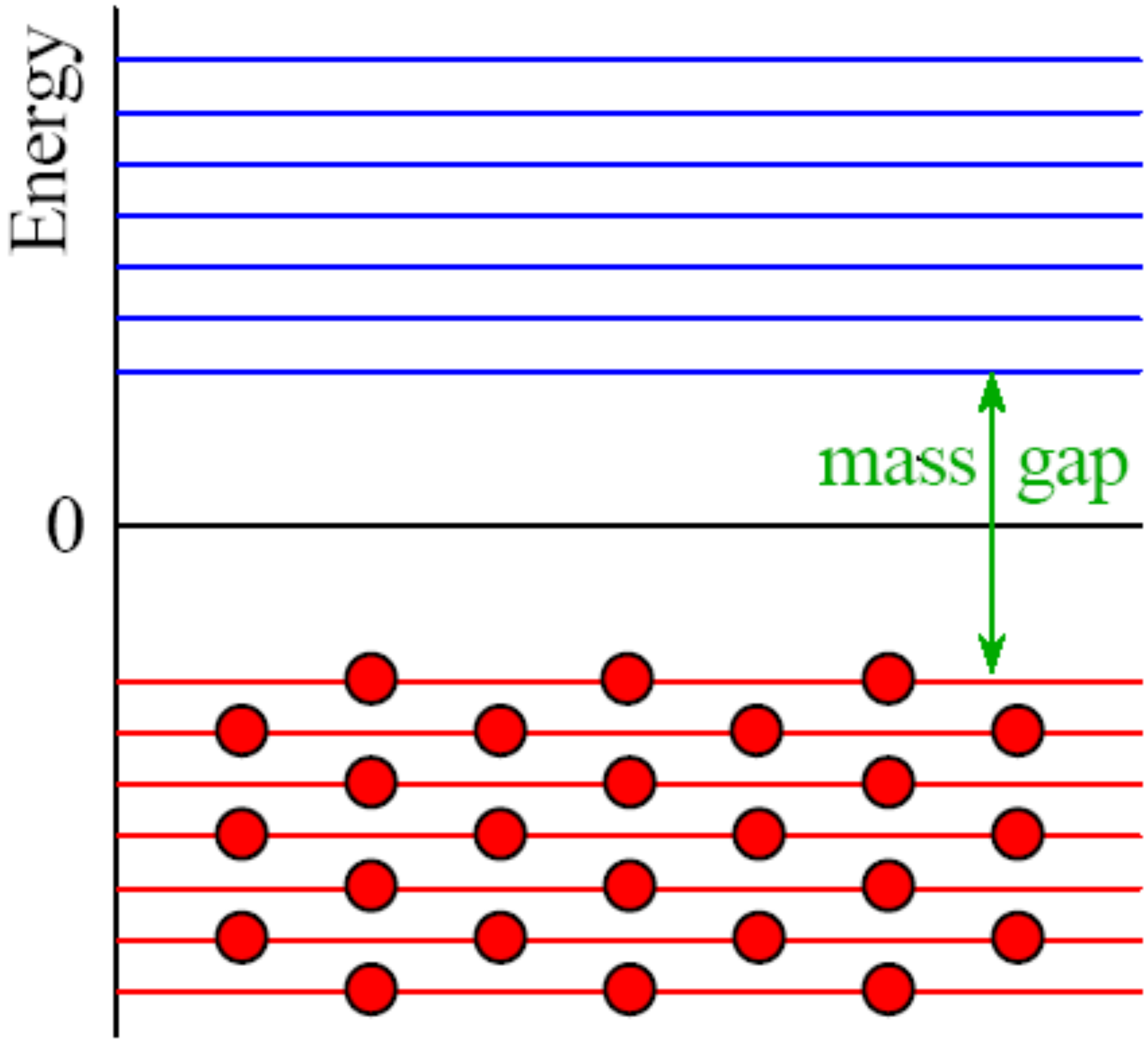
$$\pi^a(\boldsymbol{r}) = \begin{cases} n^a F(r),\ n^a = x^a/r, & a = 1,\,2,\,3 \\ 0, & a = 4,\,5,\,6,\,7,\,8. \end{cases}$$

It breaks spontaneously $\mathrm{SU}(3)_{\rm flavor} \otimes \mathrm{O}(3)_{\rm space} \to \mathrm{SU}(2)_{\rm isospin+space}$

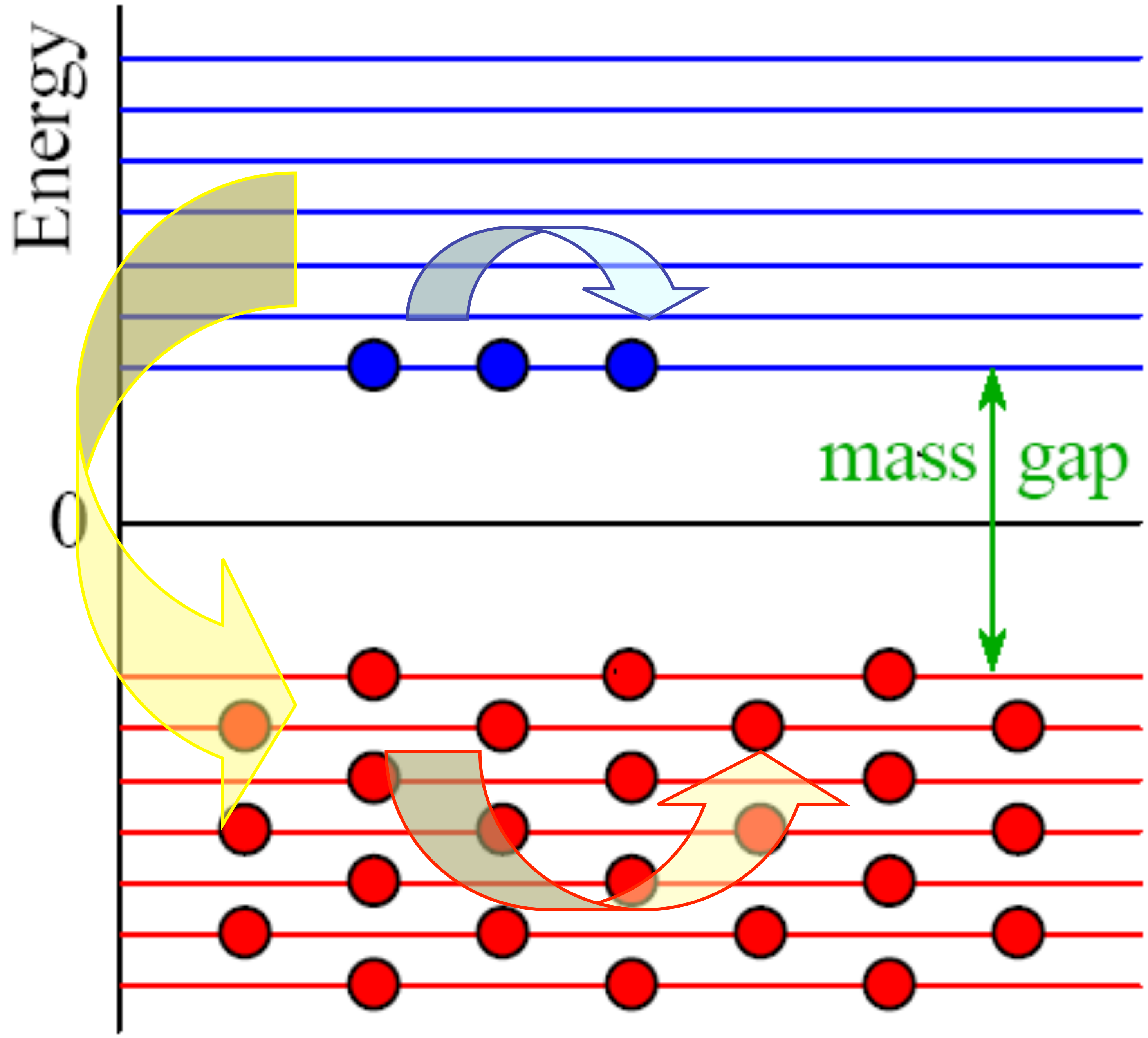
**Witten's trivial embedding**

$$U_o = \begin{pmatrix} e^{i\boldsymbol{n}\cdot\boldsymbol{\tau}P(r)} & 0 \\ 0 & 1 \end{pmatrix}$$

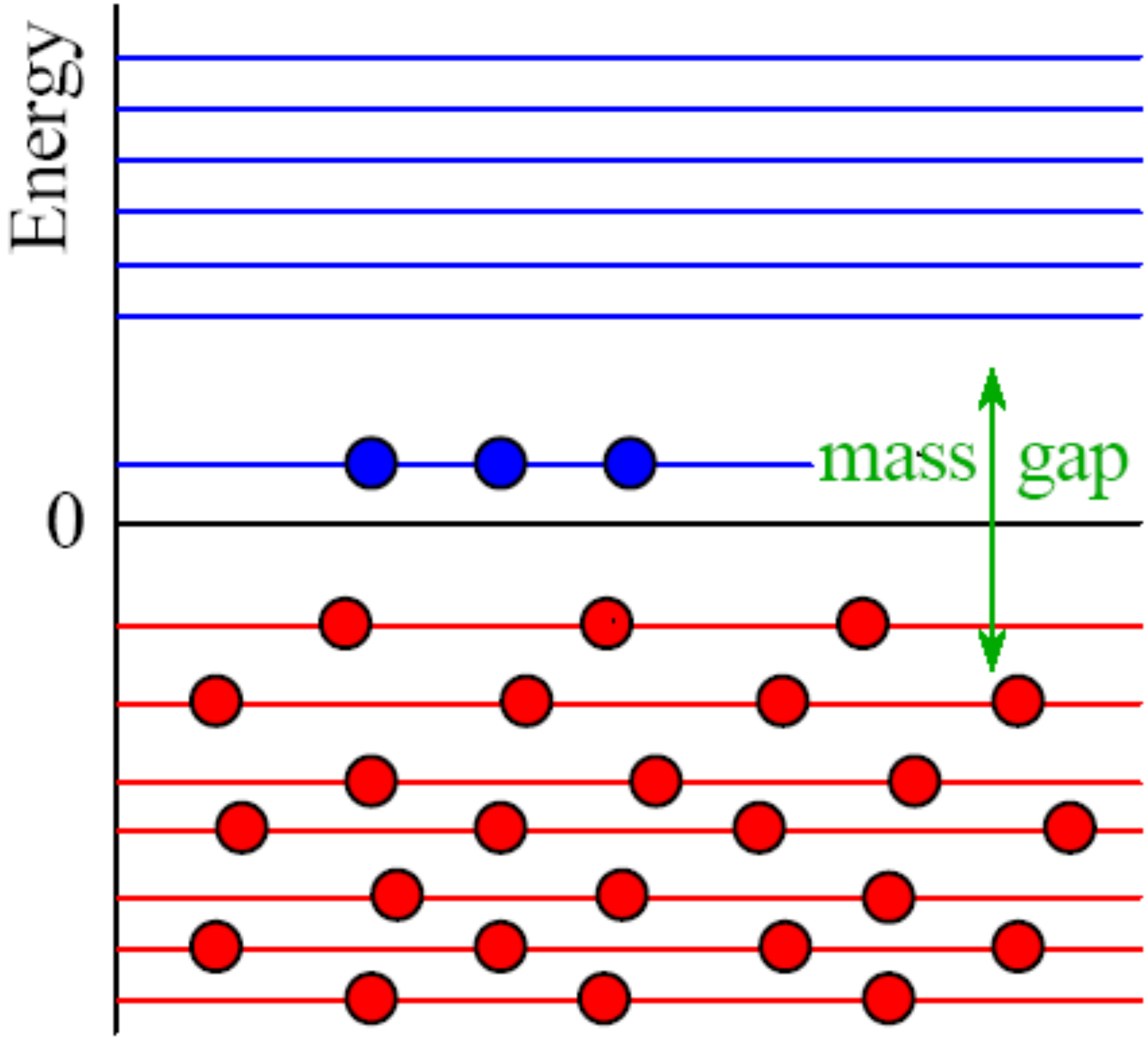
# Schematic view of baryons in the XQSM



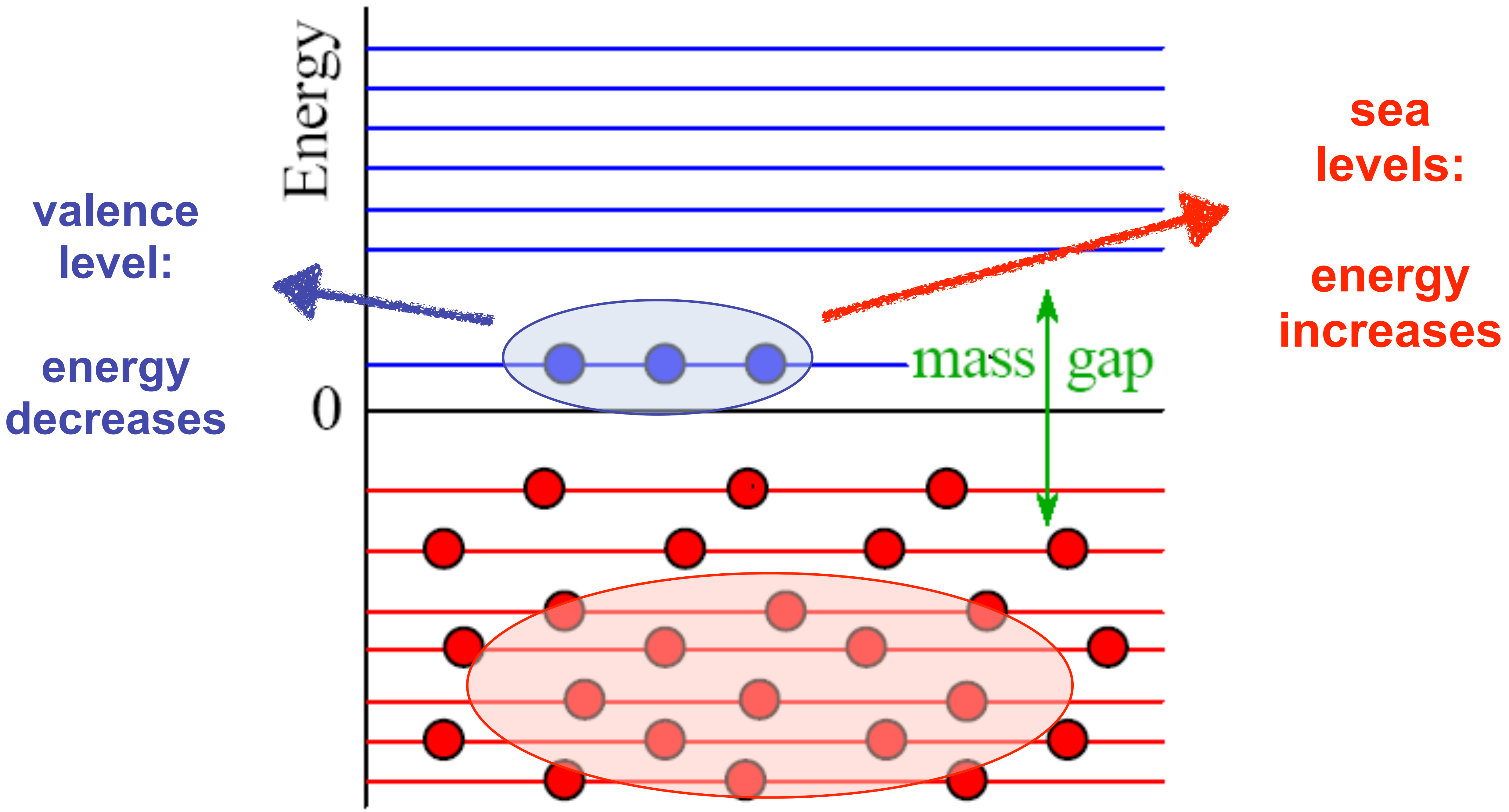
# Schematic view of baryons in the XQSM



# Schematic view of baryons in the XQSM



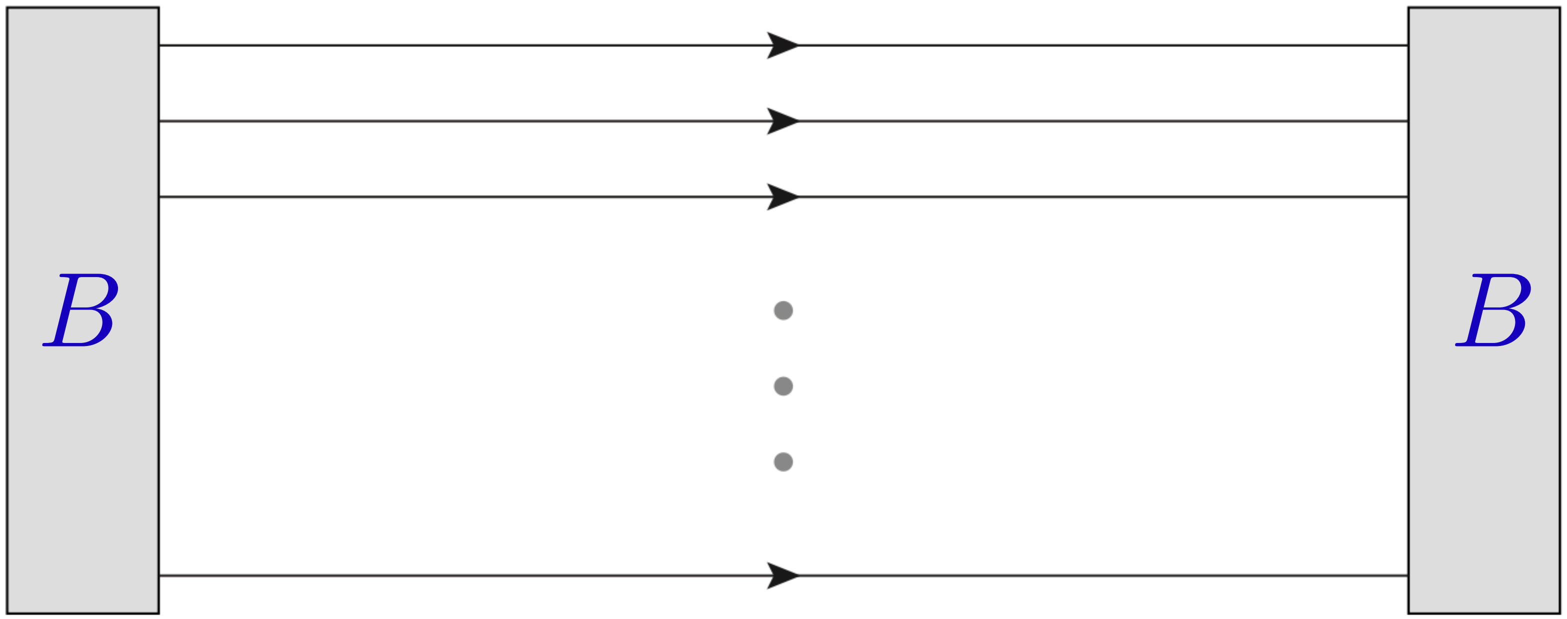
# Schematic view of baryons in the XQSM




**system is stabilized**

# Baryon correlation function

**Baryon as Nc valence quarks bound by pion mean fields**




$$\langle J_B J_B^\dagger \rangle_0 \sim e^{-N_c E_{\rm val} T}$$

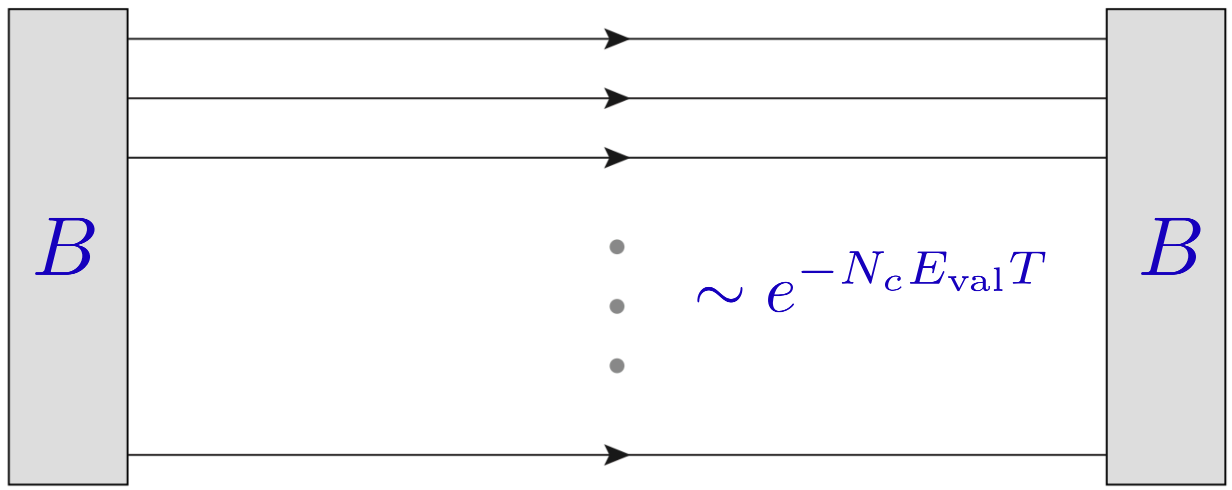
$$\Pi_N(\vec{x},t) \;=\; \Gamma_N^{\{f\}}\Gamma_N^{\{g\}*}\frac{1}{Z}\int dU \prod_{i=1}^{N_c}\left\langle 0,T/2\left|\frac{1}{D(U)}\right|0,-T/2\right\rangle_{f,g} \; e^{-S_{\rm eff}}$$

Presence of Nc quarks will polarize the vacuum or create mean fields.

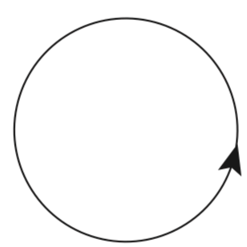
Nc valence quarks → Vacuum polarization or meson mean fields

# Baryon correlation function

**Baryon as Nc valence quarks bound by pion mean fields**




$$E_{\mathrm{cl}} = N_c E_{\mathrm{val}} + E_{\mathrm{sea}}$$



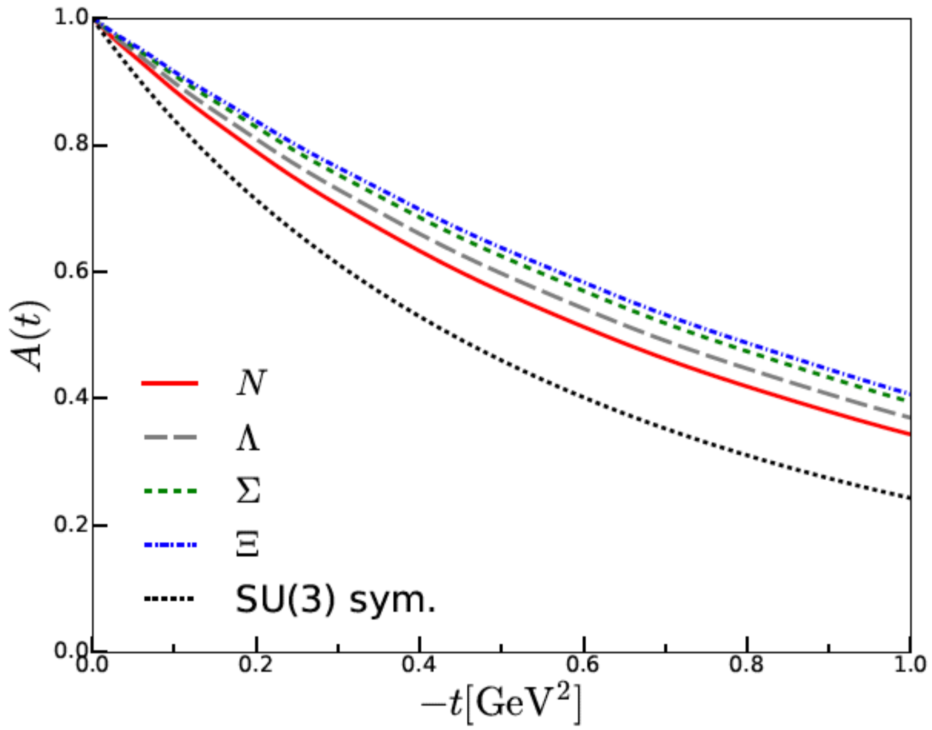
$$\sim e^{-E_{\mathrm{sea}} T}$$

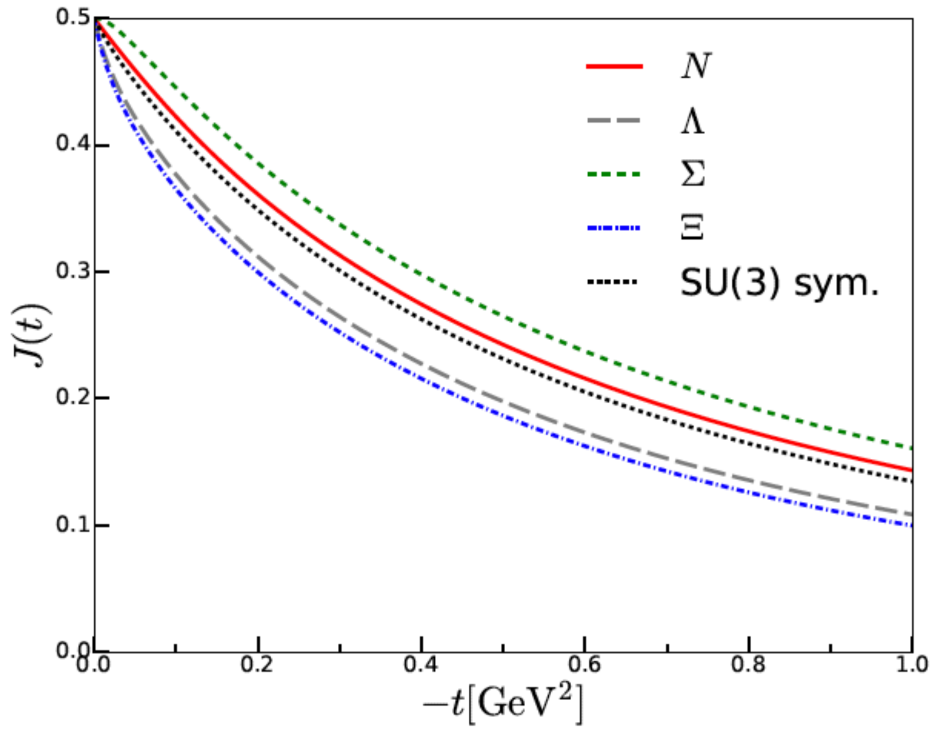
Classical Nucleon mass is described by the Nc valence quark energy and sea-quark energy.

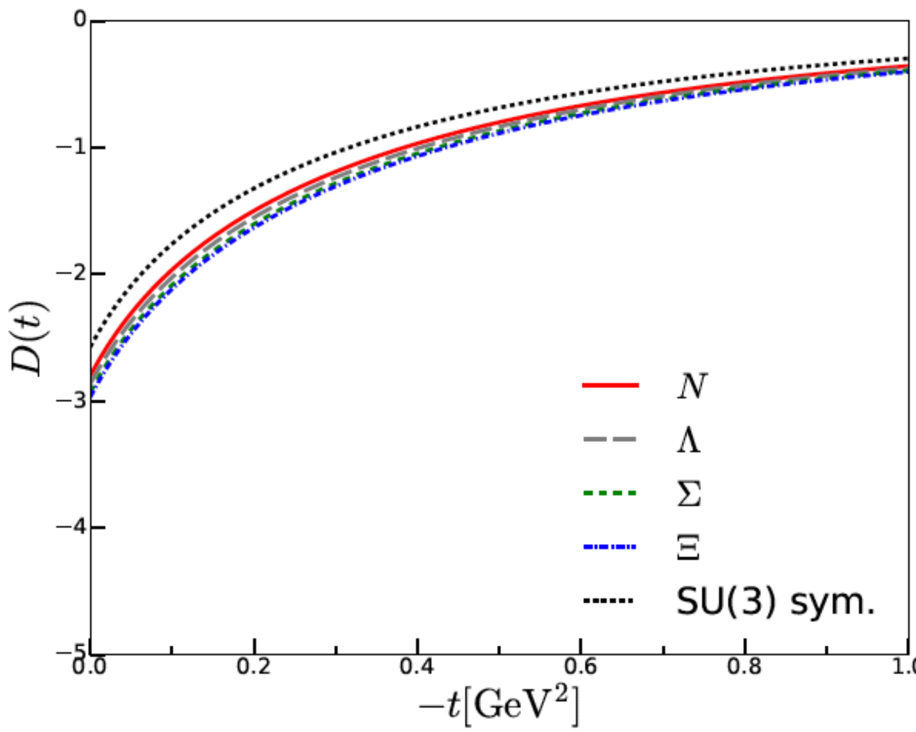
$$\frac{\delta E_{cl}}{\delta U} = 0 \longrightarrow M_{\mathrm{cl}} \longrightarrow P(r)$$

P(r): Soliton profile function
or Soliton field

# Gravitational form factors



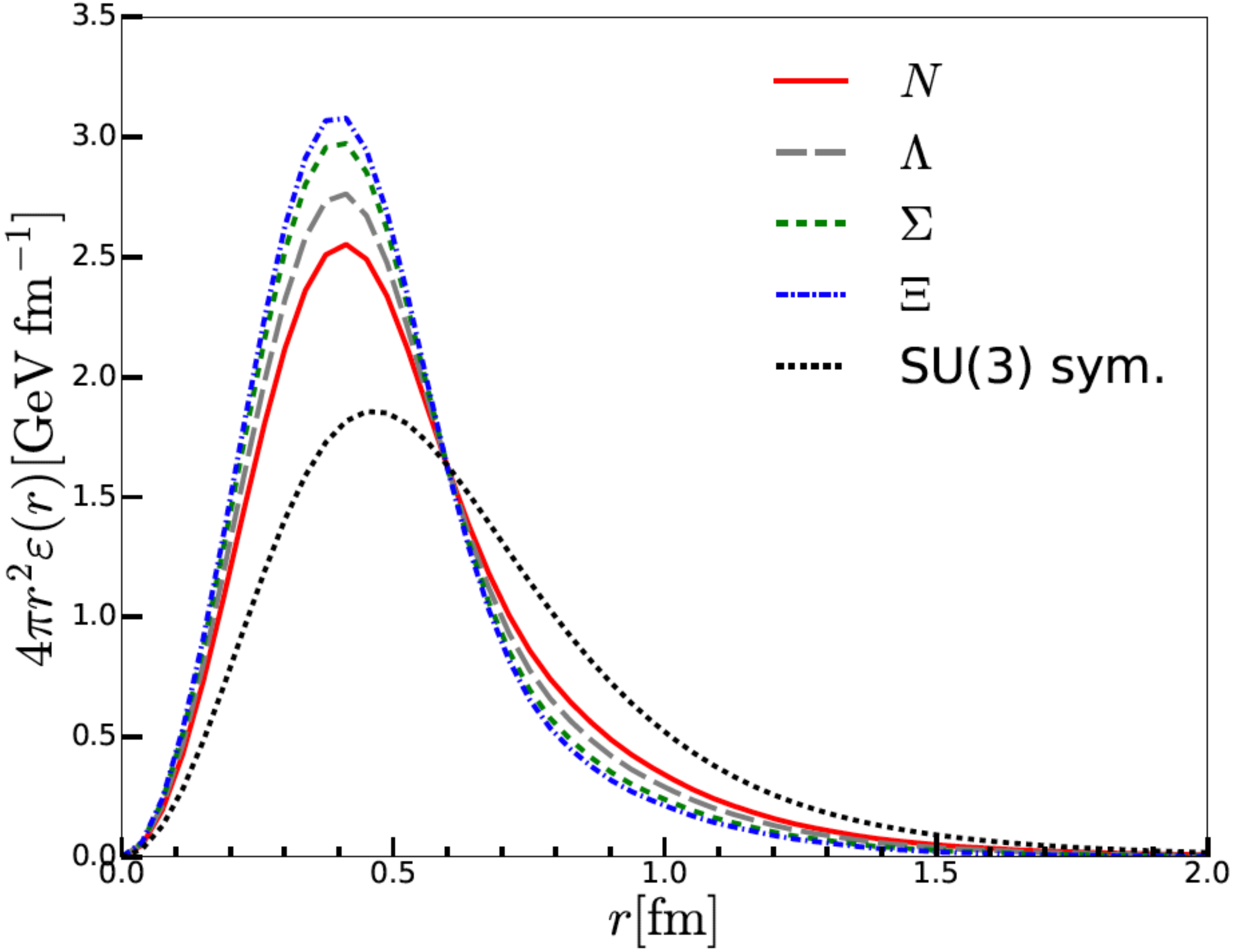

$$D(t) < 0$$

Essential for the stability

H.W. Won, J.-Y. Kim, HChK, PRD 106 (2022) 114009

# Mass distribution

$$\int d^3r\varepsilon(r) = M^{(\Omega^0, m_s^0)} + M^{(\Omega^0, m_s^1)} = M_B$$

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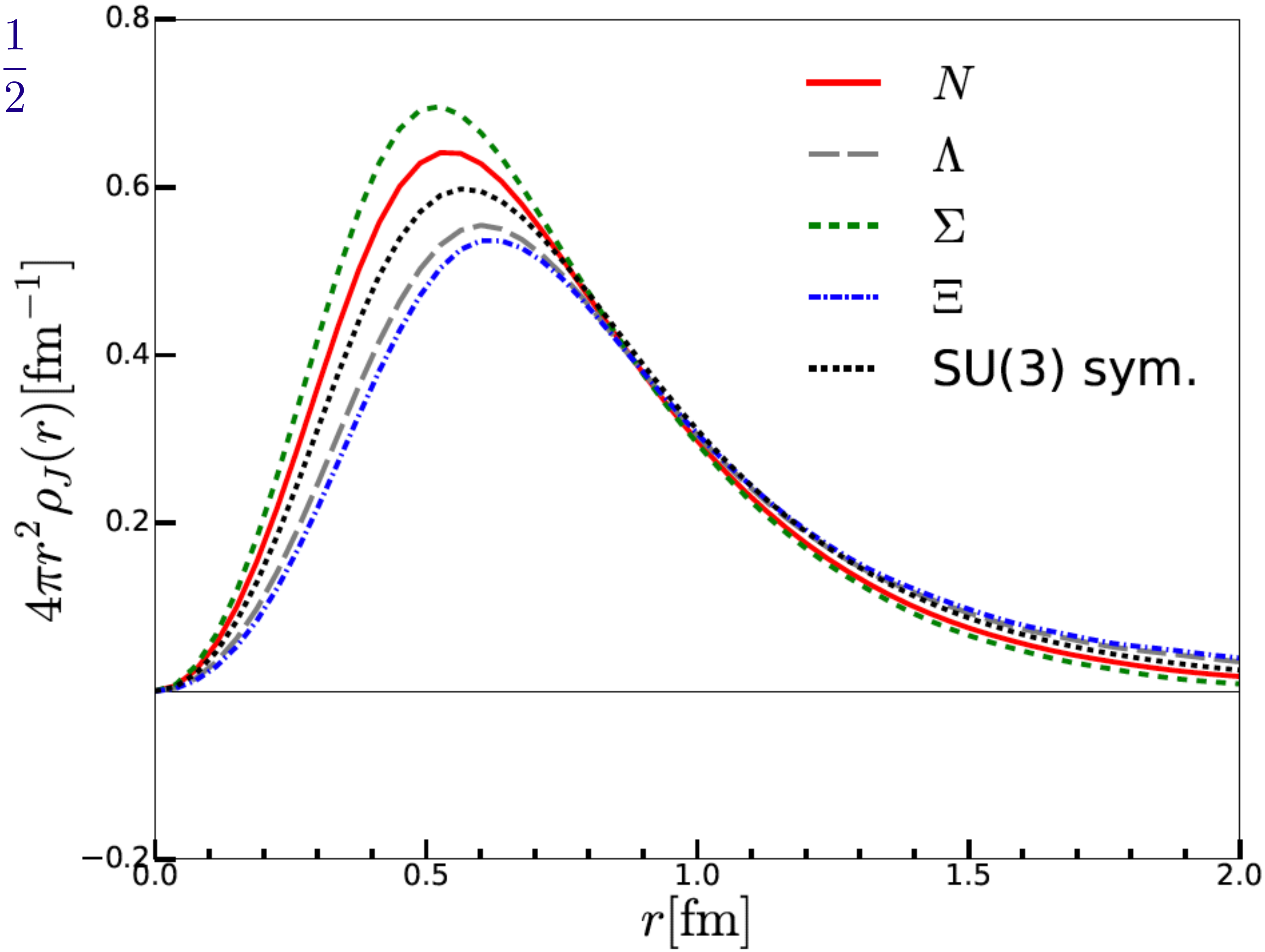
# Spin distribution

$$J = \int d^3r \rho_J(r) = \frac{1}{2}$$

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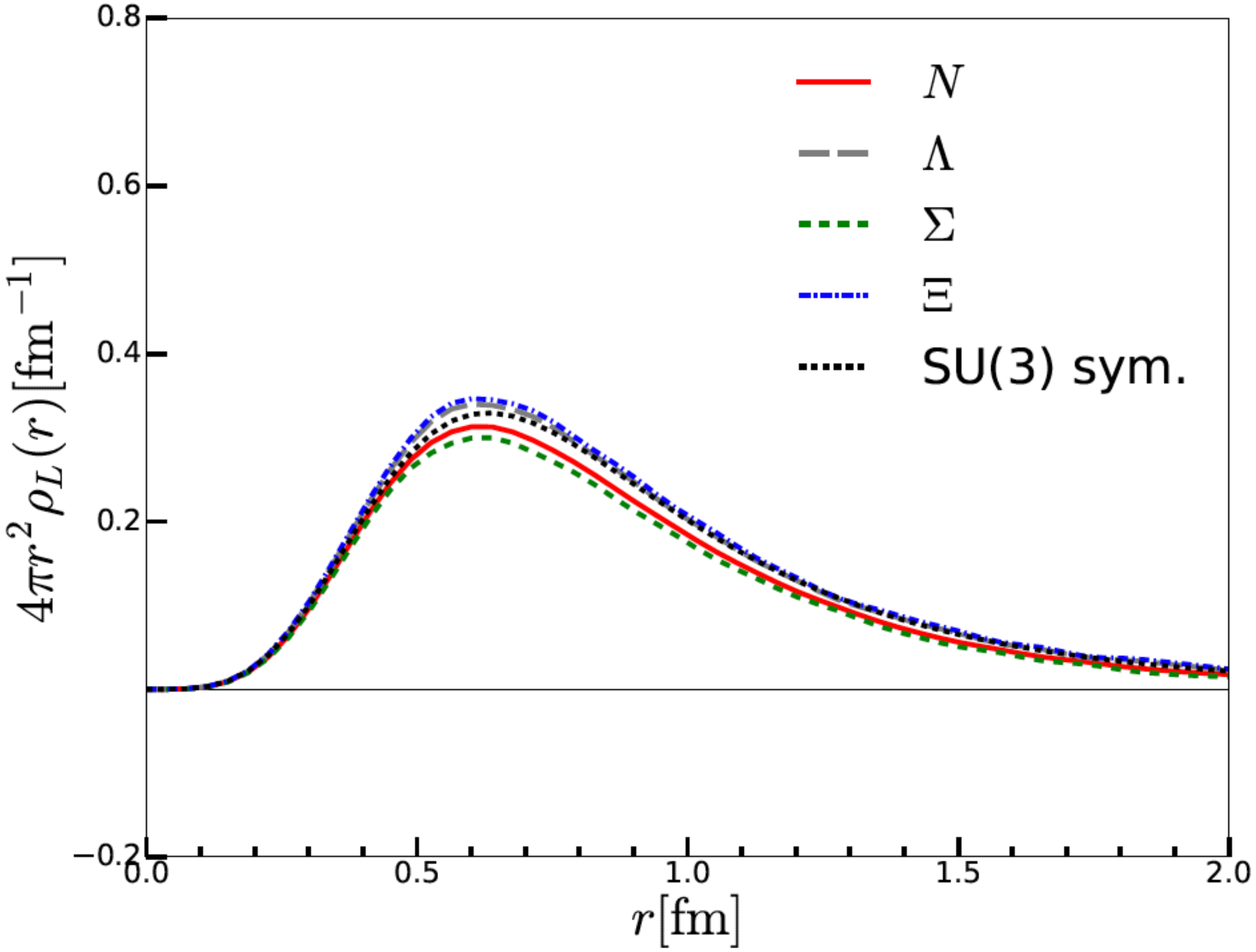
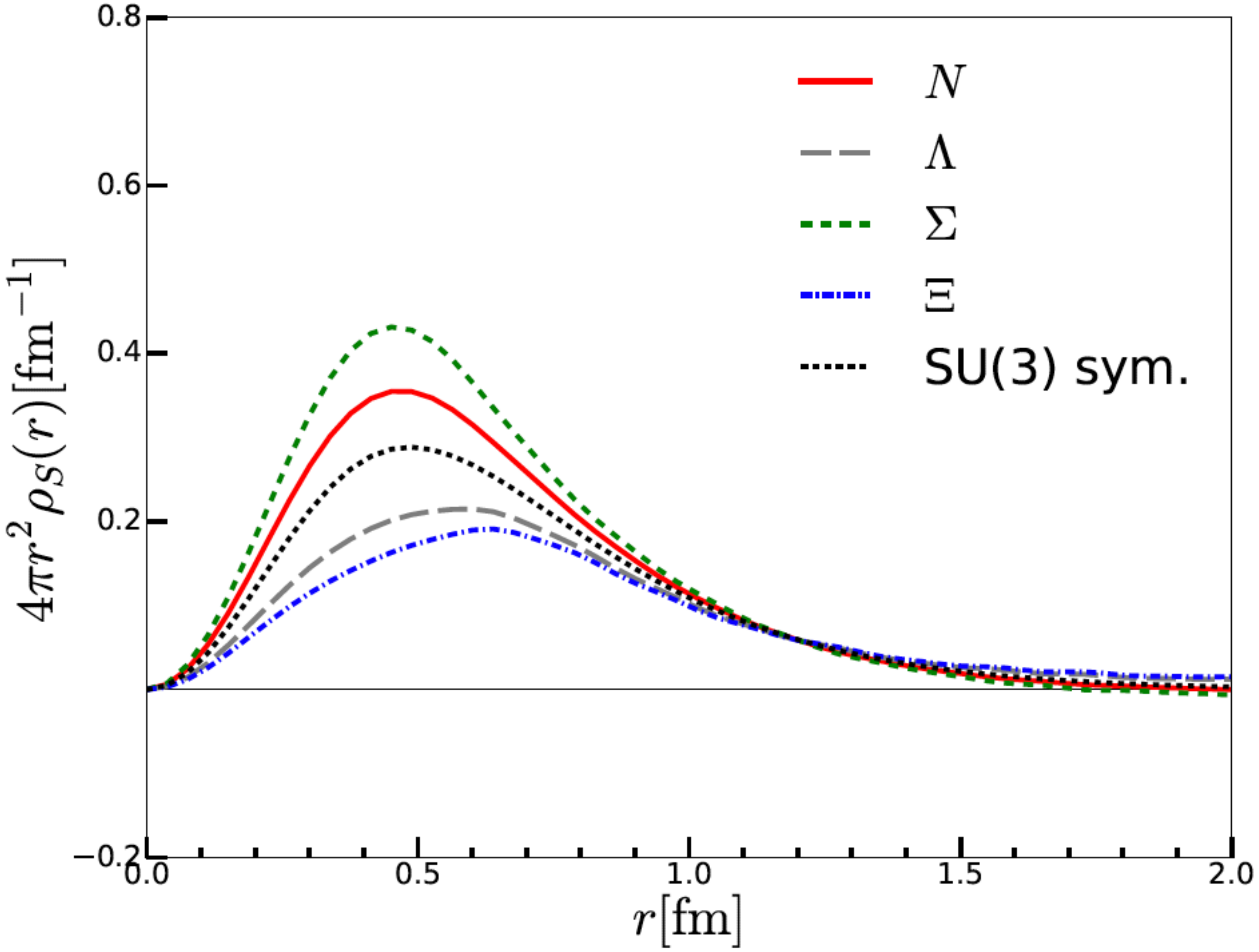
# Spin distribution

$$J = L^Q + S^Q$$

$$S^Q = \frac{1}{2} g_A^{(0)}$$

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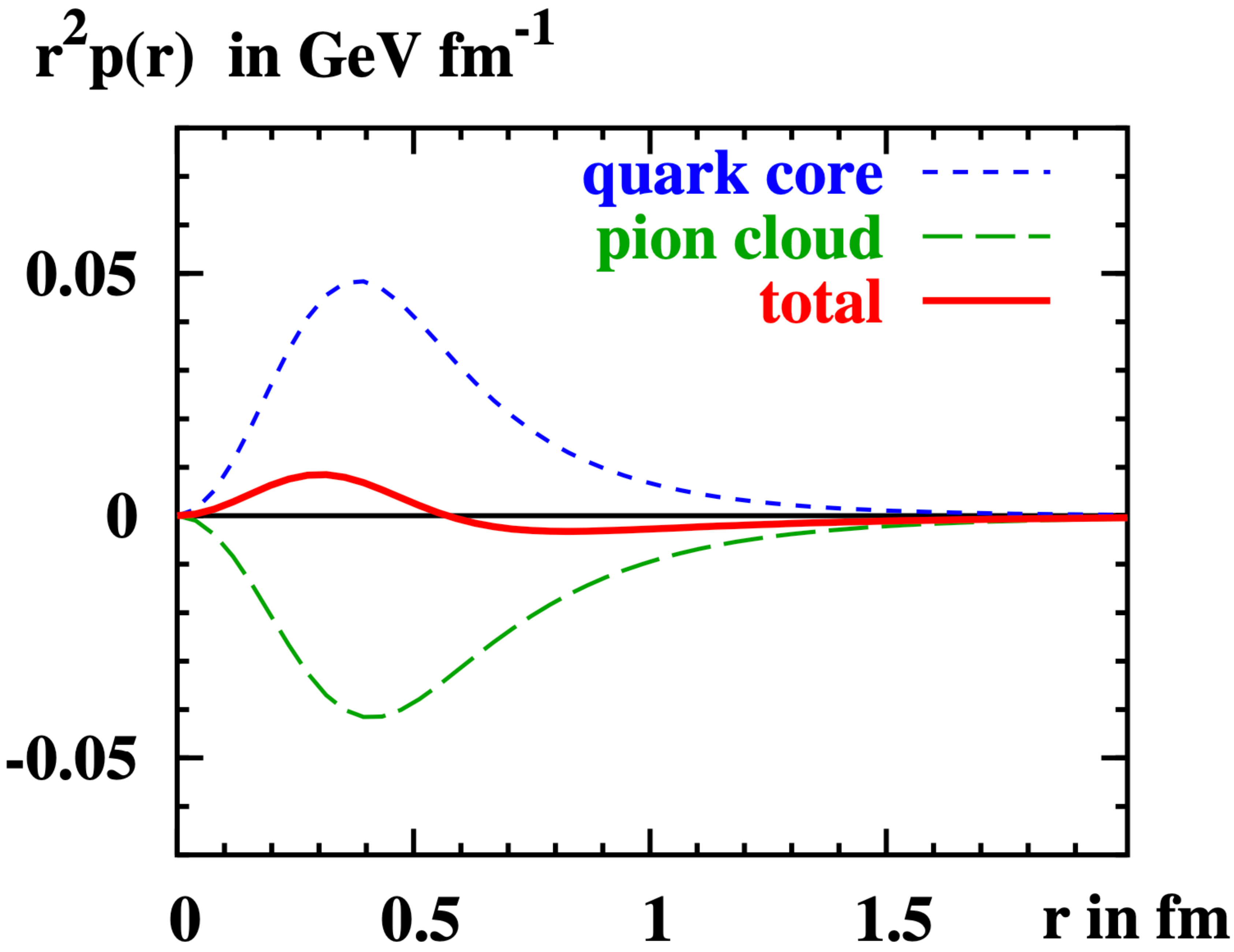
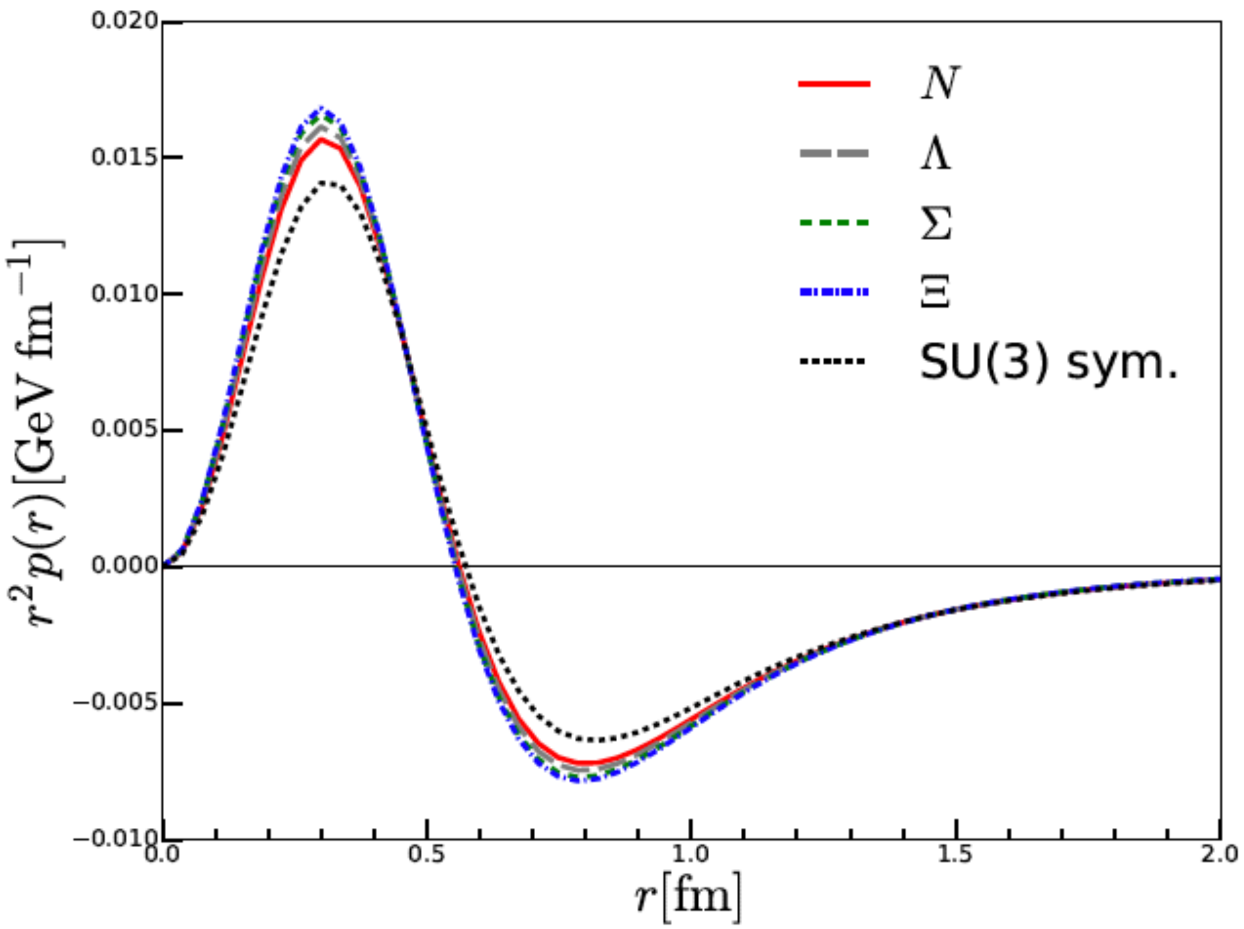
# The 3D BF pressure density



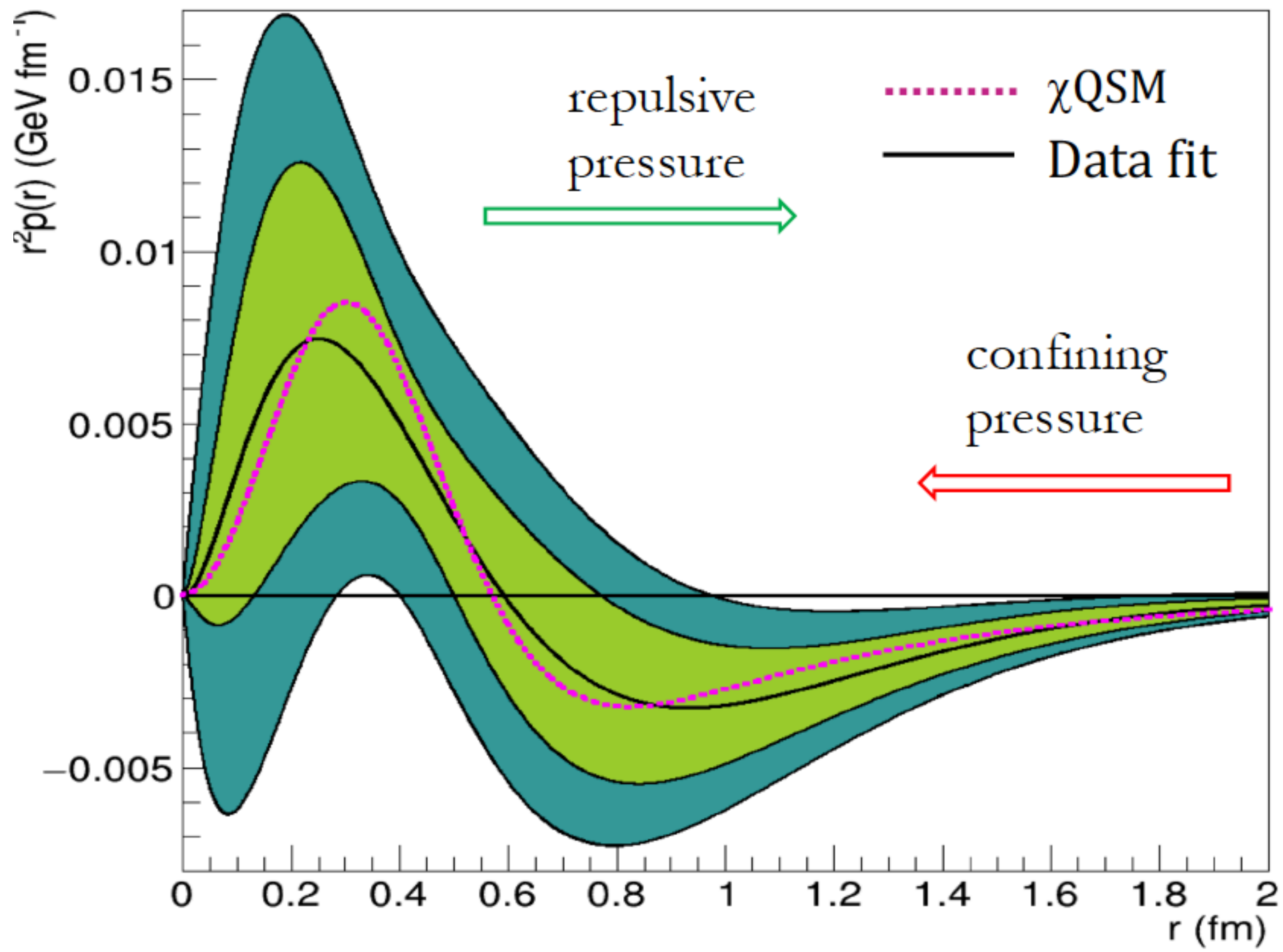

$$\int dr r^2 p(r) = 0$$

Goeke et al., PRD 75 (2007)    Kim et al., PRD 103 (2021)

# The 3D BF pressure density

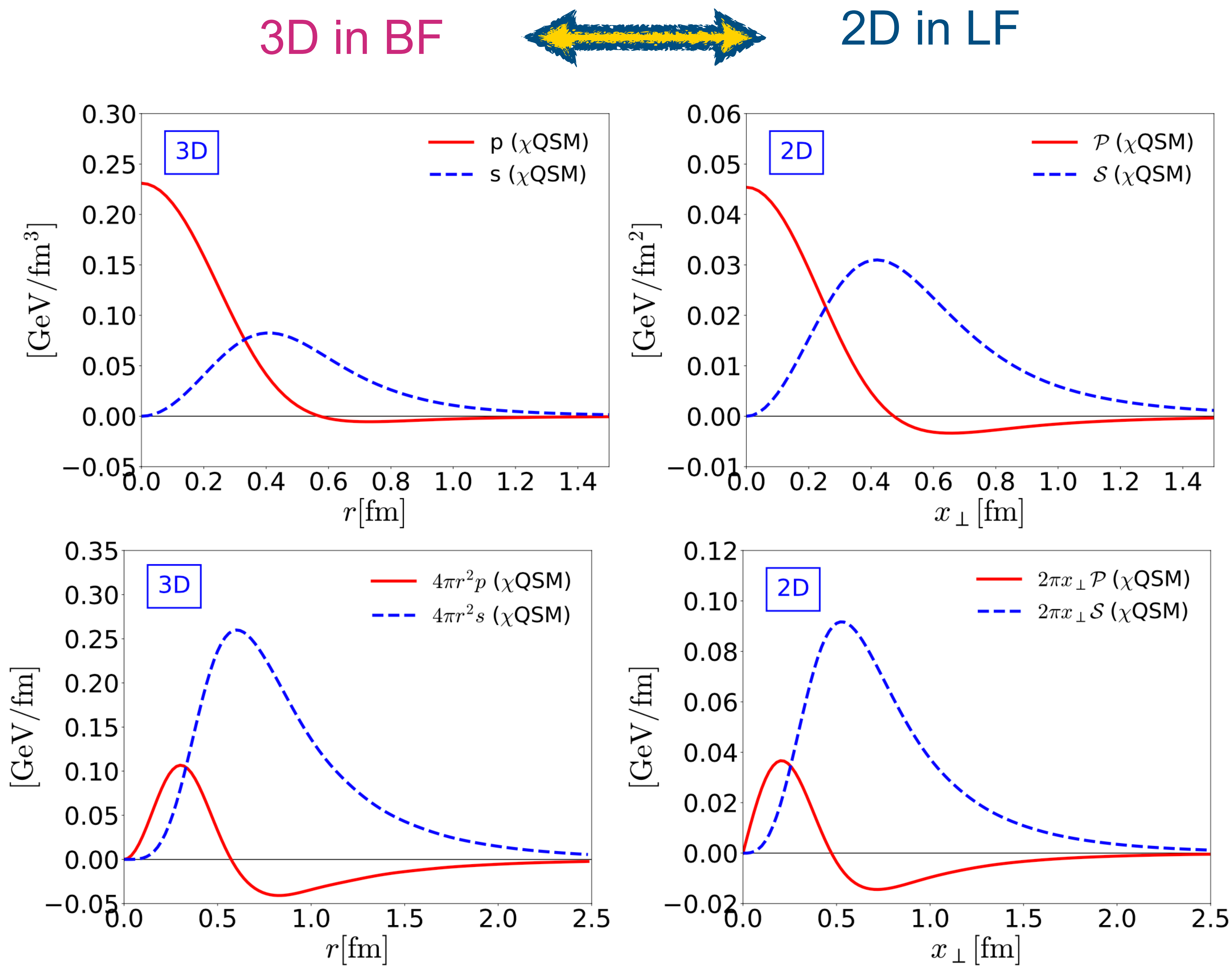



H.W. Won, J.-Y. Kim, HChK, PRD 106 (2022) 114009




*V.B., L. Elouadrhiri, F.X. Girod, Nature 557 (2018) 7705, 396*

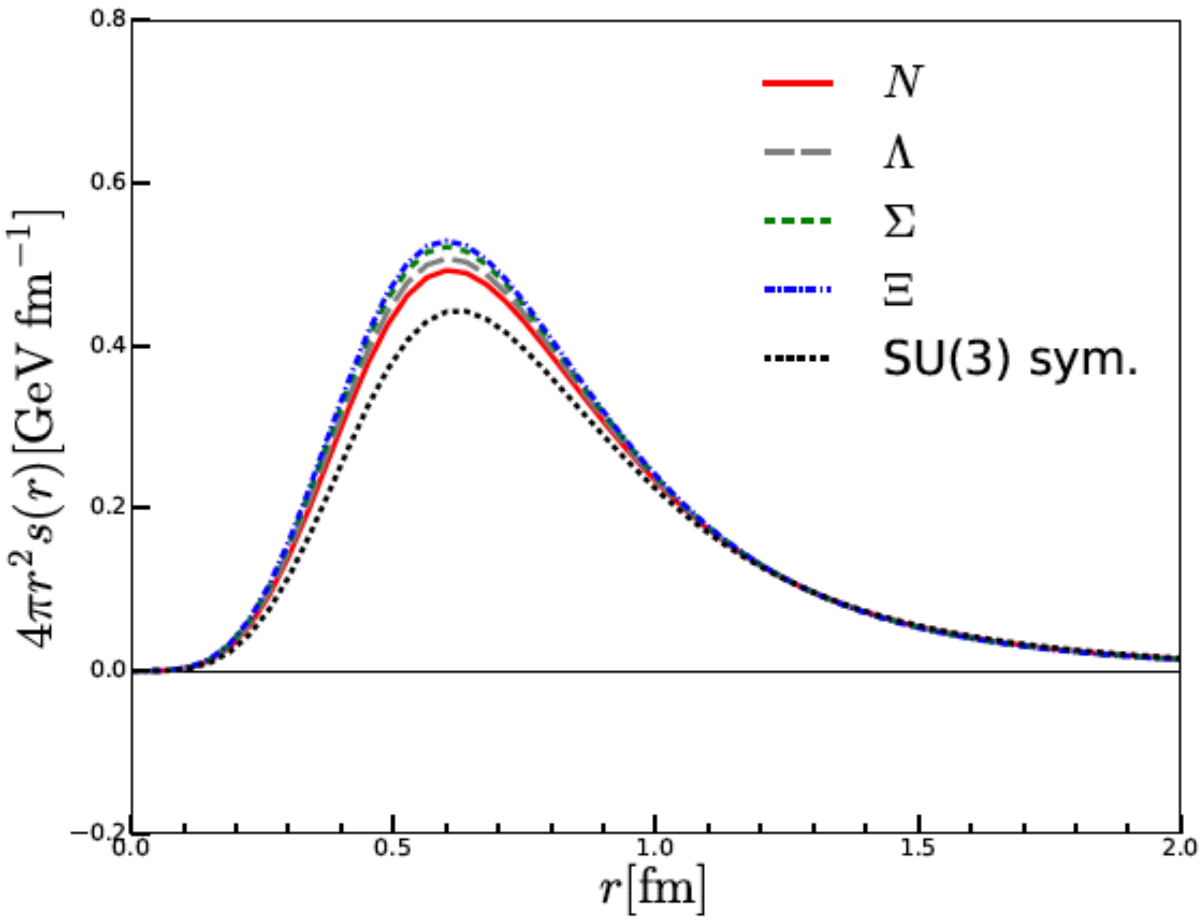
# The 3D & 2D pressure & shear-force densities



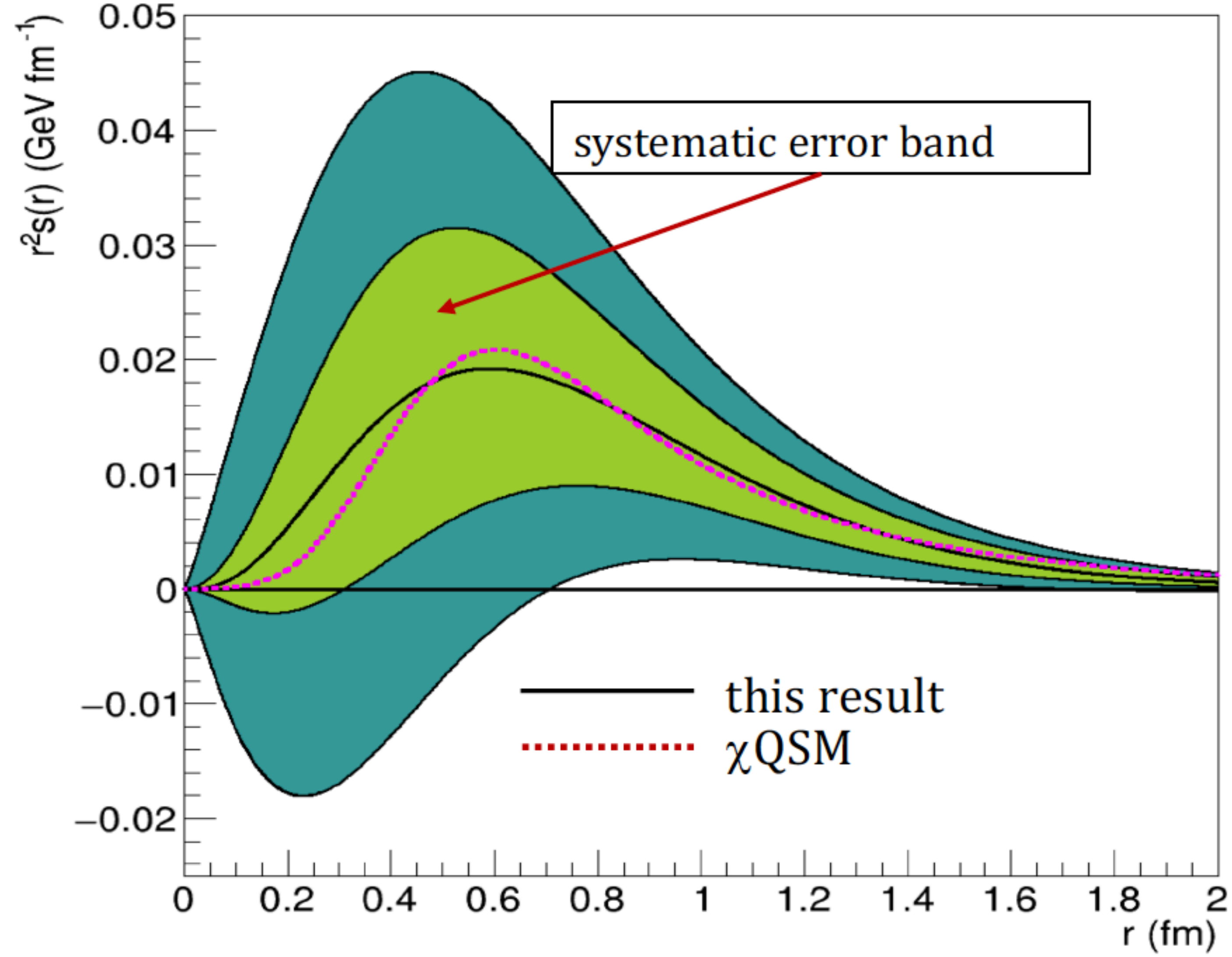

Features of the 3D densities are kept also in the 2D case.

Kim et al., PRD 104 (2021)

# Shear-force densities




H.W. Won, J.-Y. Kim, HChK, PRD 106 (2022) 114009



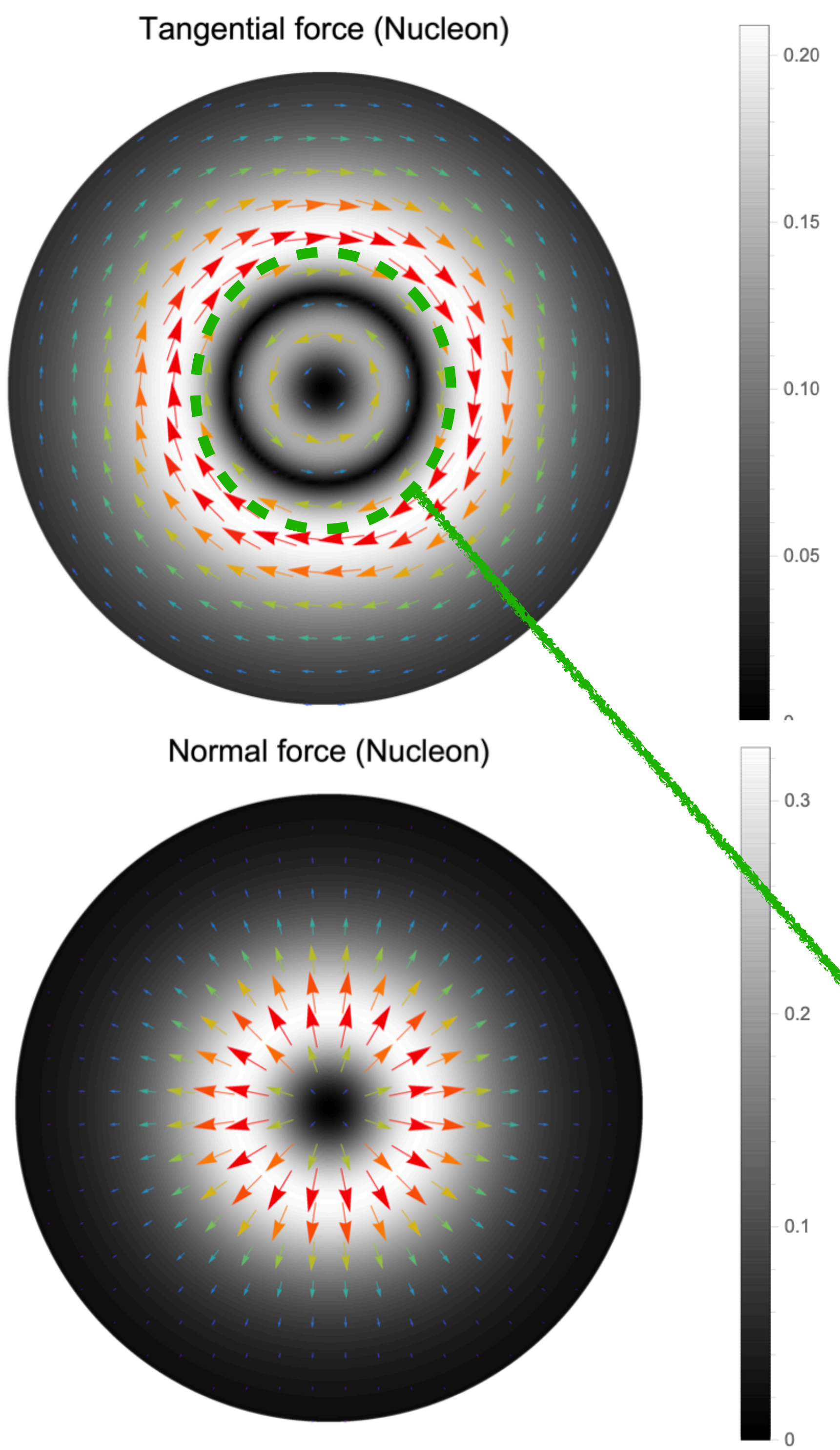

V. Burkert et al., ArXiv:2104.02031

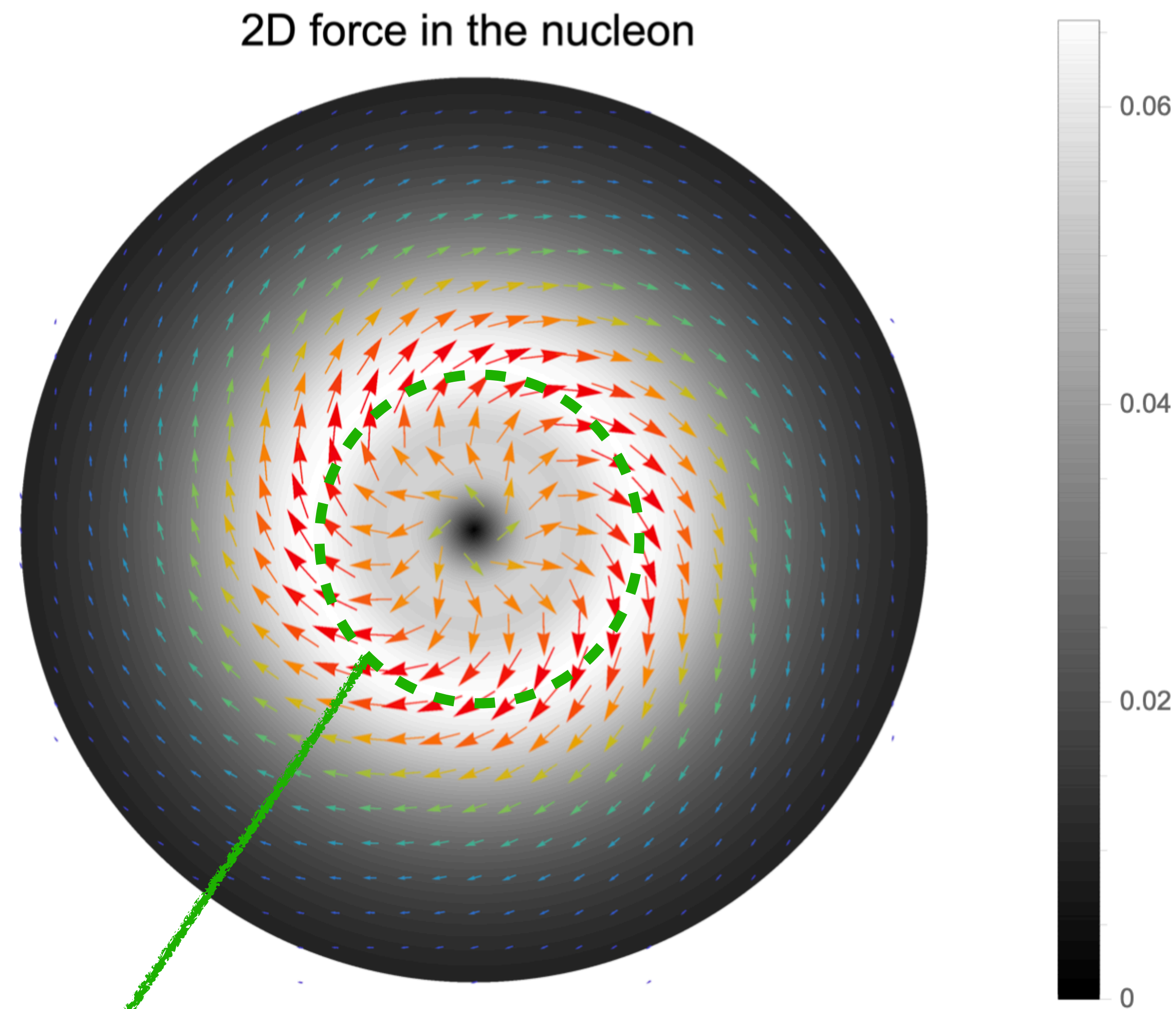
$$\longrightarrow \quad 4\pi r^2 s(r) \approx 0.45\,\mathrm{GeV}\cdot\mathrm{fm}^{-1} = 72\times 10^3\,\mathrm{N}\,!$$

# Force fields

3D force field in the BF

2D force field in the LF




We can see that there is at least one nodal point.

Kim et al., PRD 103 (2021); PRD 104 (2021)

# Mechanical radius

$$\langle r^2 \rangle_{\text{mech}} = \frac{\int d^3r\, r^2 \left[\frac{2}{3}s(r) + p(r)\right]}{\int d^3r \left[\frac{2}{3}s(r) + p(r)\right]} = \frac{6D}{\int_{-\infty}^{0} dt\, D(t)}$$

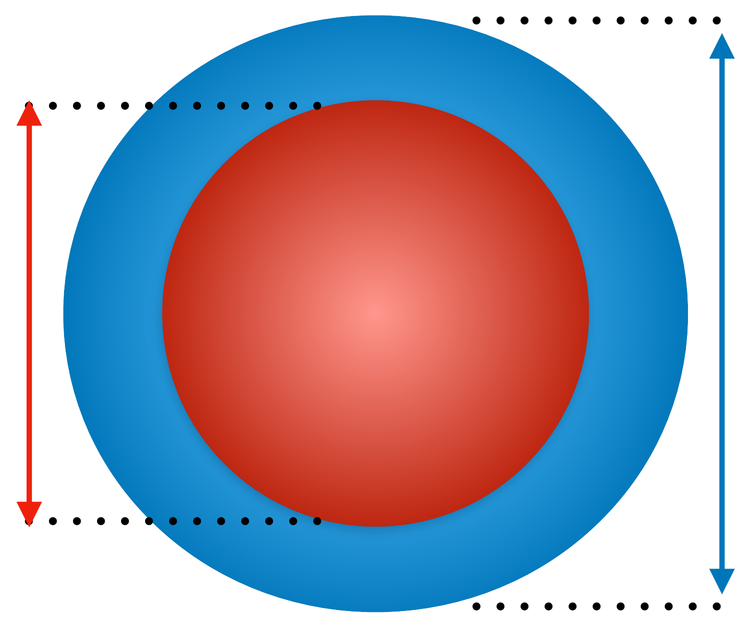
$$\sqrt{\langle r^2 \rangle}_{\text{mech}} = 0.53\,\text{fm}$$

$$\sqrt{\langle r^2 \rangle}_{\text{mech}} = (0.63 \pm 0.06 \pm 0.13)\,\text{fm}$$

V. Burkert et al. (2022)

$$\langle r_\varepsilon^2 \rangle_N < \langle r_{\text{mech}}^2 \rangle_N < \langle r_{\text{charge}}^2 \rangle_N$$

Mechanical radius $\sim 0.6\,\text{fm}$



Charge radius $\sim 0.8\,\text{fm}$

H.W. Won, J.-Y. Kim, HChK, PRD 106 (2022) 114009

# Ordering of the mass radii

Heavier hyperons are more compact than the lighter ones.

$$\langle r_{\varepsilon}^2 \rangle_N > \langle r_{\varepsilon}^2 \rangle_\Lambda > \langle r_{\varepsilon}^2 \rangle_\Sigma > \langle r_{\varepsilon}^2 \rangle_\Xi,$$

$$\langle x_{\perp\mathcal{E}}^2 \rangle_N > \langle x_{\perp\mathcal{E}}^2 \rangle_\Lambda > \langle x_{\perp\mathcal{E}}^2 \rangle_\Sigma > \langle x_{\perp\mathcal{E}}^2 \rangle_\Xi$$

H.W. Won, J.-Y. Kim, HChK, PRD 106 (2022) 114009

# Summary & Conclusions

## Mechanical structure of the Nucleon

- The nucleon is *per se* a relativistic particle
- The 3D BF distributions have a quasi-probabilistic meaning.
- Abel transform makes 3D BF densities equivalent to 2D LF ones.

  (In 2D, we restore a quantum mechanically probabilistic meaning of the densities.)
- The 3D global & local stability conditions are all conveyed to the 2D ones!
- Higher-spin baryons are under investigation by using the Radon transform.

➡ Investigation on Baryon tomography is at work intensively, now.

# Though this be madness, yet there is method in it.

Hamlet Act 2, Scene 2

by Shakespeare

Thank you very much for the attention!