Saga-Yonsei Joint Workshop XIX

Intro to Effective Field Theory

Seung J. Lee
Korea University

cosmological constant

Higgs mass: naturalness problem

renormalizable QFT

$$\mathcal{L}_{\rm EFT} = c^{(0)} M^4 + C^{(2)} M^2 O^{(d=2)} + \sum_i c_i^{(4)} O_i^{(d=4)}$$

$$+ \frac{1}{M} \sum_i c_i^{(5)} O_i^{(d=5)} + \frac{1}{M^2} \sum_i c_i^{(6)} O_i^{(d=6)} + \cdots$$

neutrino mass (see-saw mechanism)

New Physics effect might be important here, including flavor physics, etc

January 17, 2023

Effective Field Theory

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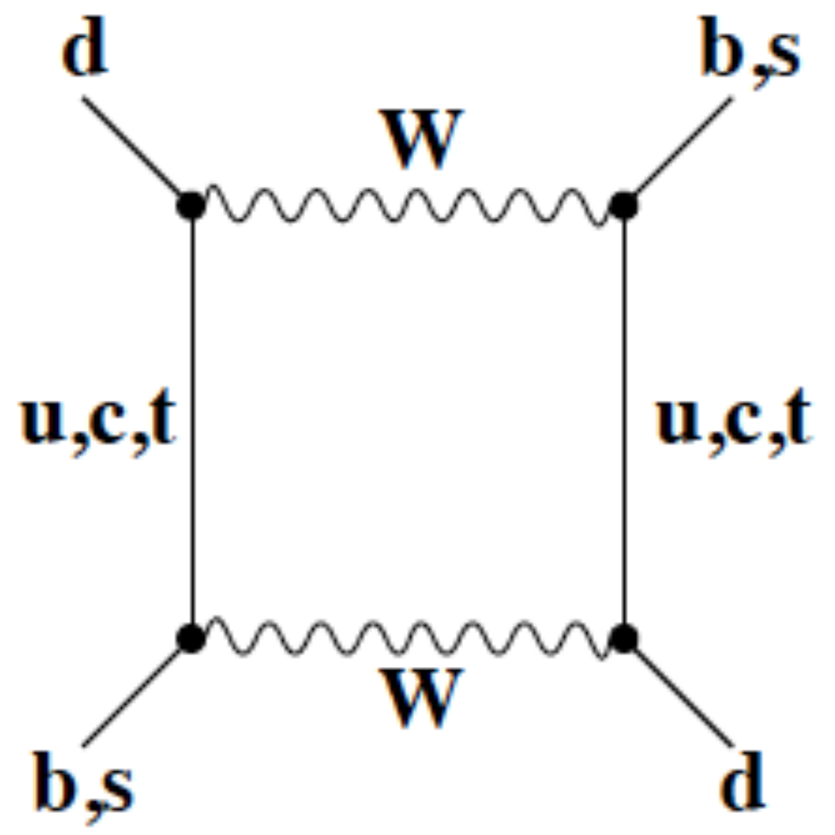
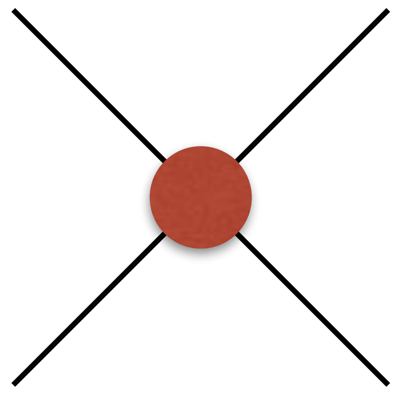


exchange of heavy, virtual particles
between light SM particles



$$C_{Q(\Delta S=2)} Q(\Delta S = 2)$$

$$Q(\Delta S = 2) = (\bar{s}d)_{V-A}(\bar{s}d)_{V-A}$$

induced effective local interactions
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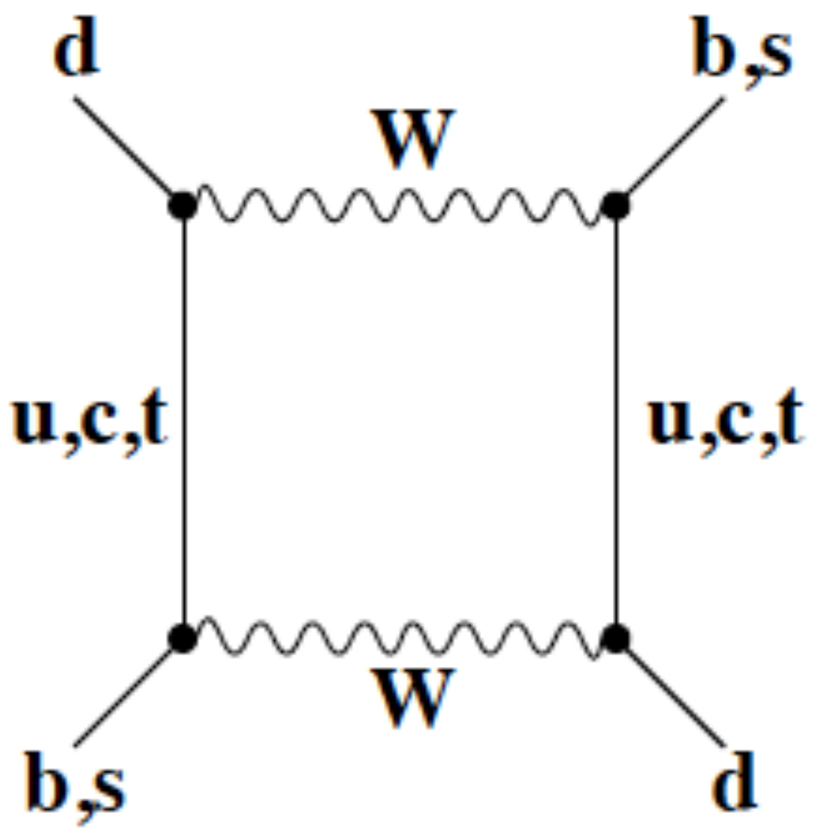

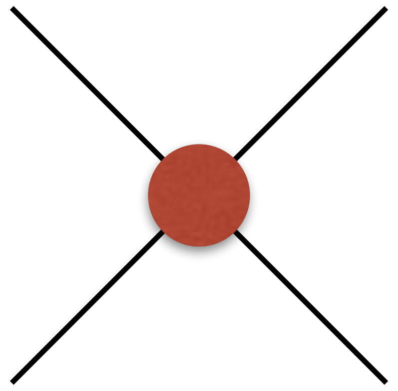

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Basic Idea: when there is a QFT with a high scale M:

- interested in physics @ E << M
can we do Taylor expand in E/M?

Effective Field Theory

- Effective field theory offers a systematic description of virtual heavy-particle effects (more generally, effects of modes with large virtualities) through an expansion in local operators

- It’s possible to construct effective field theory even if fundamental theory is unknown or strongly coupled (non-perturbative)

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systematic way to analyze the situation when there are multiple physics scales exist

* It's possible to construct effective field theory even if fundamental theory is unknown or strongly coupled (non-perturbative)

"Theorem of modesty":
All physical theories are effective (field) theories

Effective Field Theory

- Standard Model is most successful effective field theory to date, even though it leaves some open questions:

$$\mathcal{L}_{\rm EFT} = c^{(0)} M^4 + C^{(2)} M^2 O^{(d=2)} + \sum_i c_i^{(4)} O_i^{(d=4)}$$
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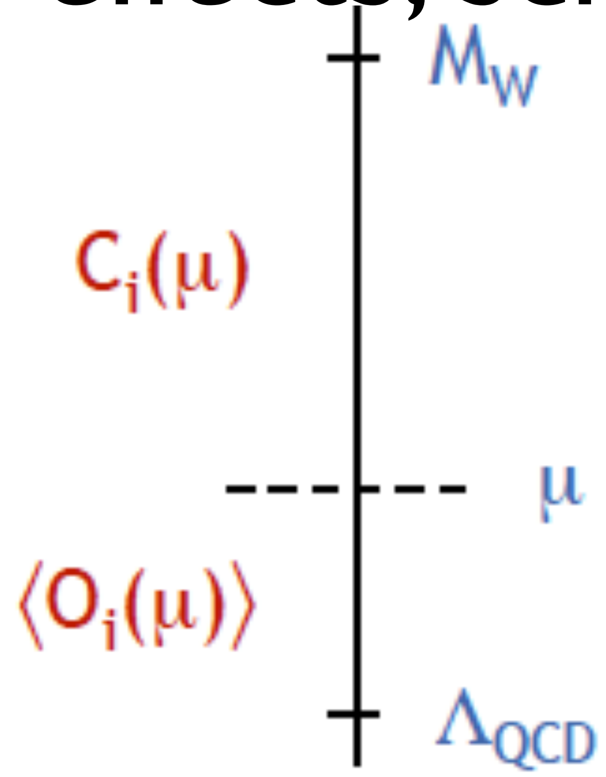
Effective Field Theory

✱ Basic idea of effective field theory: Consider Effective Weak Interaction

- Separation of short- and long-distance effects; schematically:

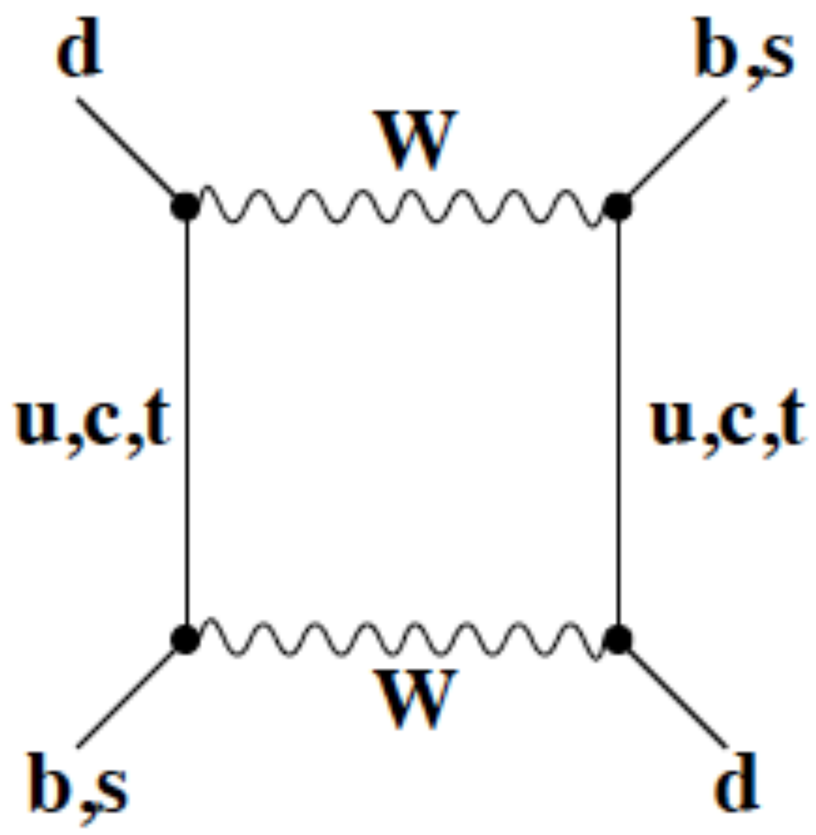
$$\int_{-p^2}^{M_W^2} \frac{dk^2}{k^2} = \int_{\mu^2}^{M_W^2} \frac{dk^2}{k^2} + \int_{-p^2}^{\mu^2} \frac{dk^2}{k^2}$$



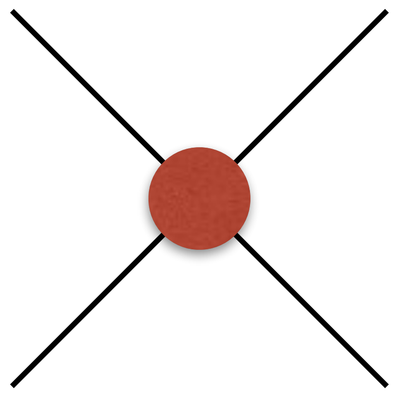
- Short-distance effects ($p \sim M_W$) are perturbatively calculable
- Long-distance effects must be treated using non-perturbative methods
- Dependence on arbitrary separation scale μ, controlled by RG equations

Effective Field Theory

- At low energies, the exchange of heavy, virtual particles (M»E) leads into quasi-local effective interactions



Integrate out heavy particles



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induced effective local interactions at low energy

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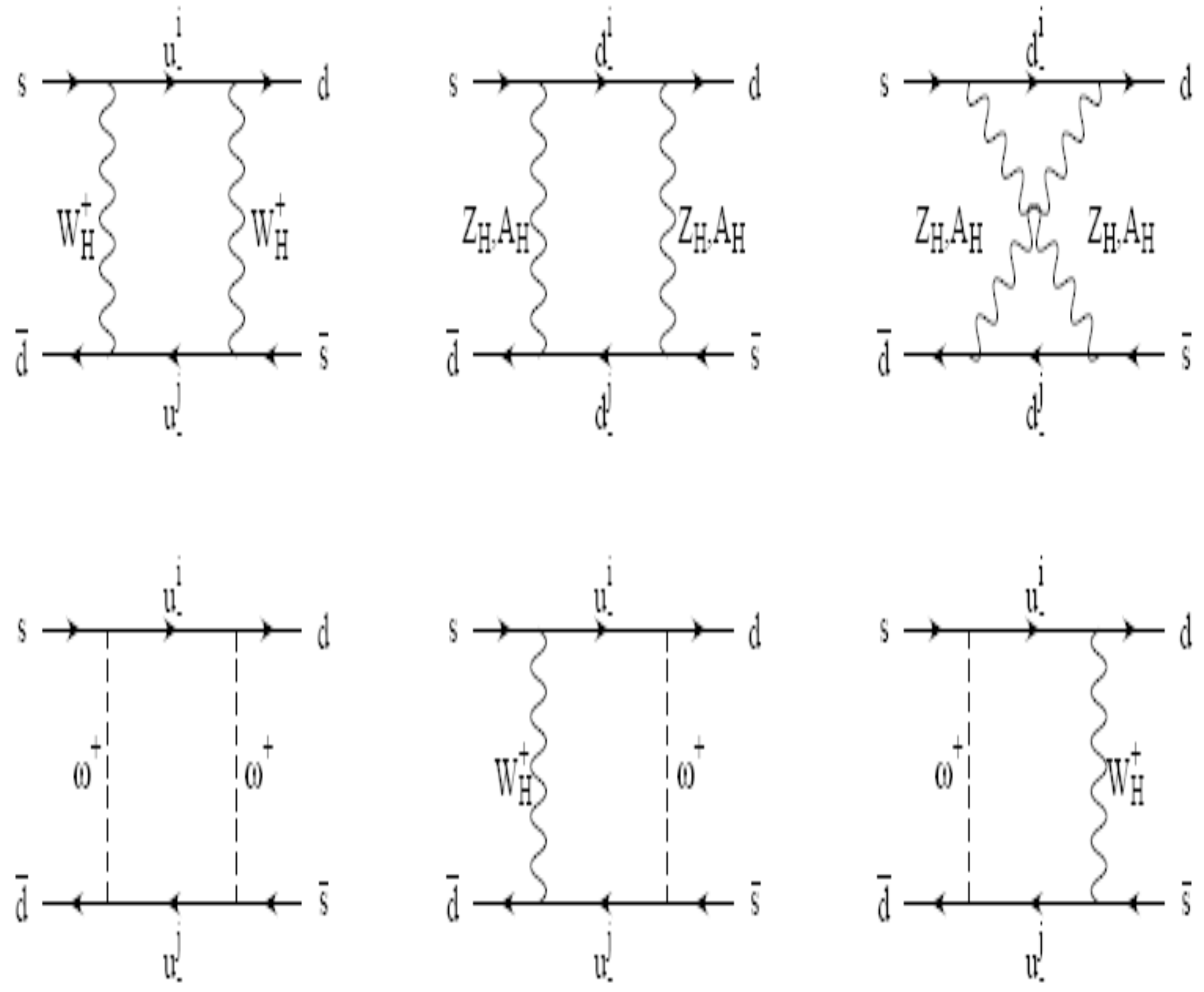

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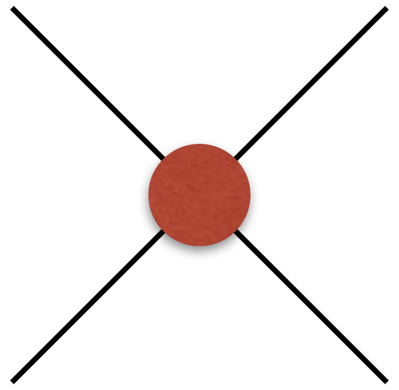

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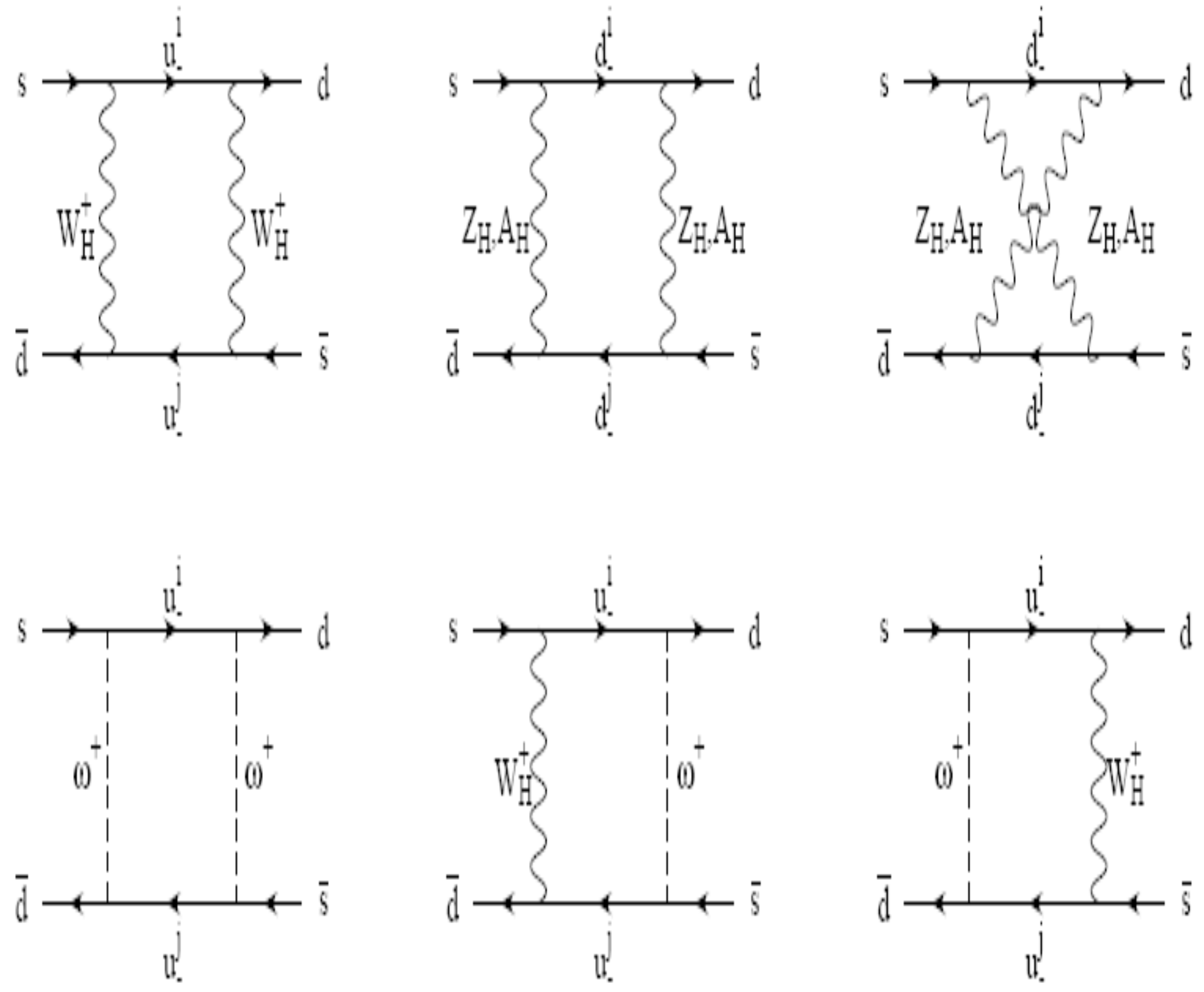

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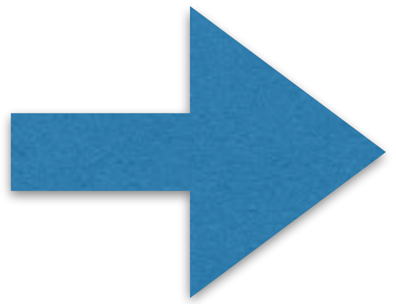

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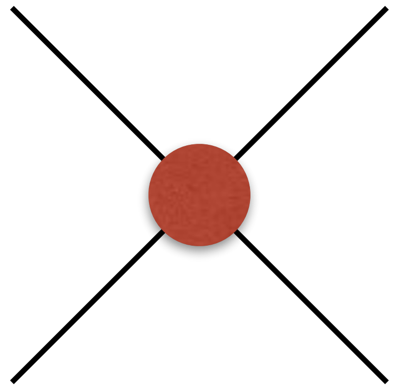

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$$\mathcal{A}_n = \langle f_n | \mathcal{L}_{\rm SM} | i_n \rangle = \sum_i C_i(\mu) \langle f_n | Q_i | i_n \rangle_\mu$$

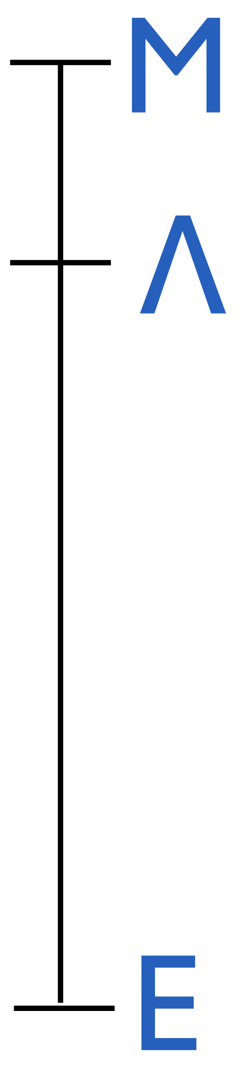
Effective Field Theory

Basic idea of effective field theory

- Step 1: choose cutoff $\Lambda \lesssim M$ and divide field into $\phi=\phi_H+\phi_L$

(M=some fundamental scale)

$\phi=\phi_H+\phi_L$



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$\omega>\Lambda$ $\quad$ $\omega<\Lambda$

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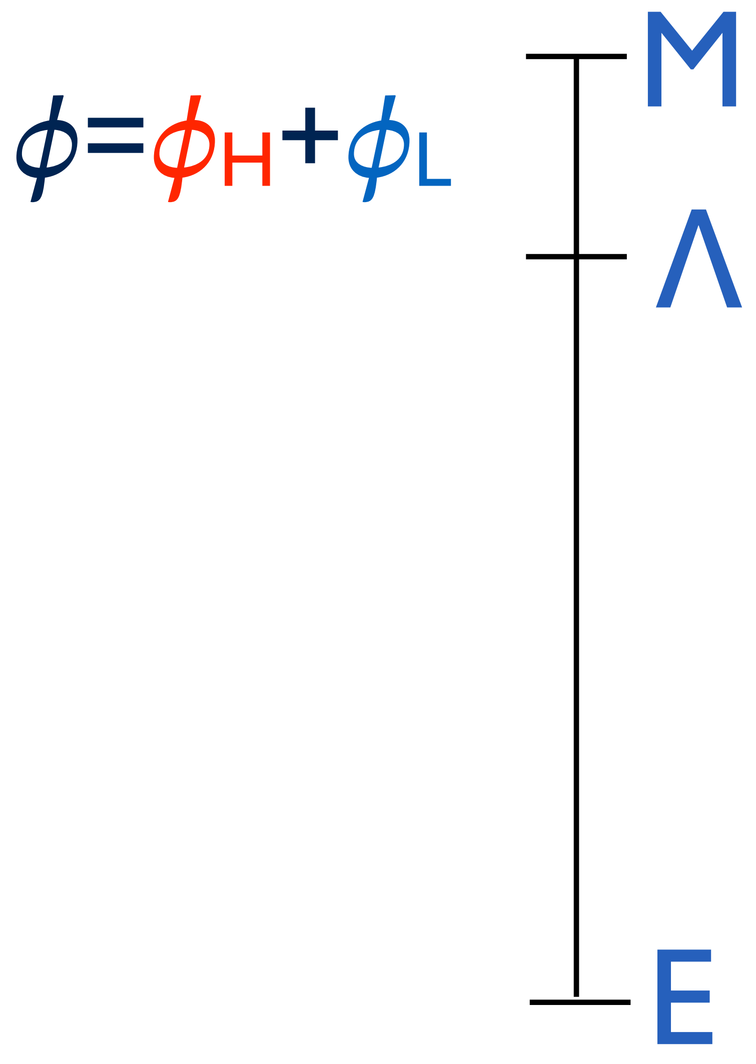

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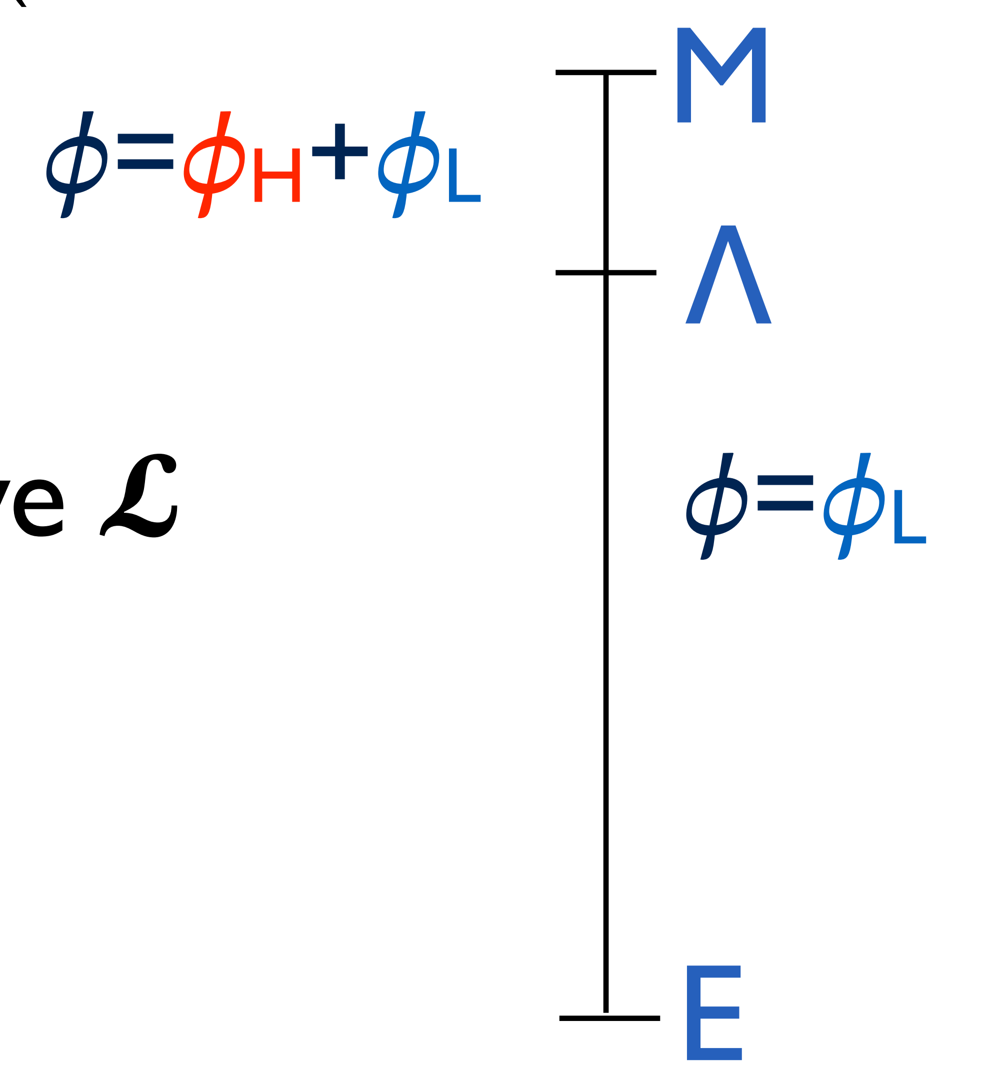

- Step 2: integrate out ϕ_H below Λ to get effective $\mathcal{L}$

Wilsonian effective action

$$e^{iS^{\text{eff}}_{\Lambda}(\phi_L)} = \int \mathcal{D}\phi_H \, e^{iS(\phi_L, \phi_H)}$$

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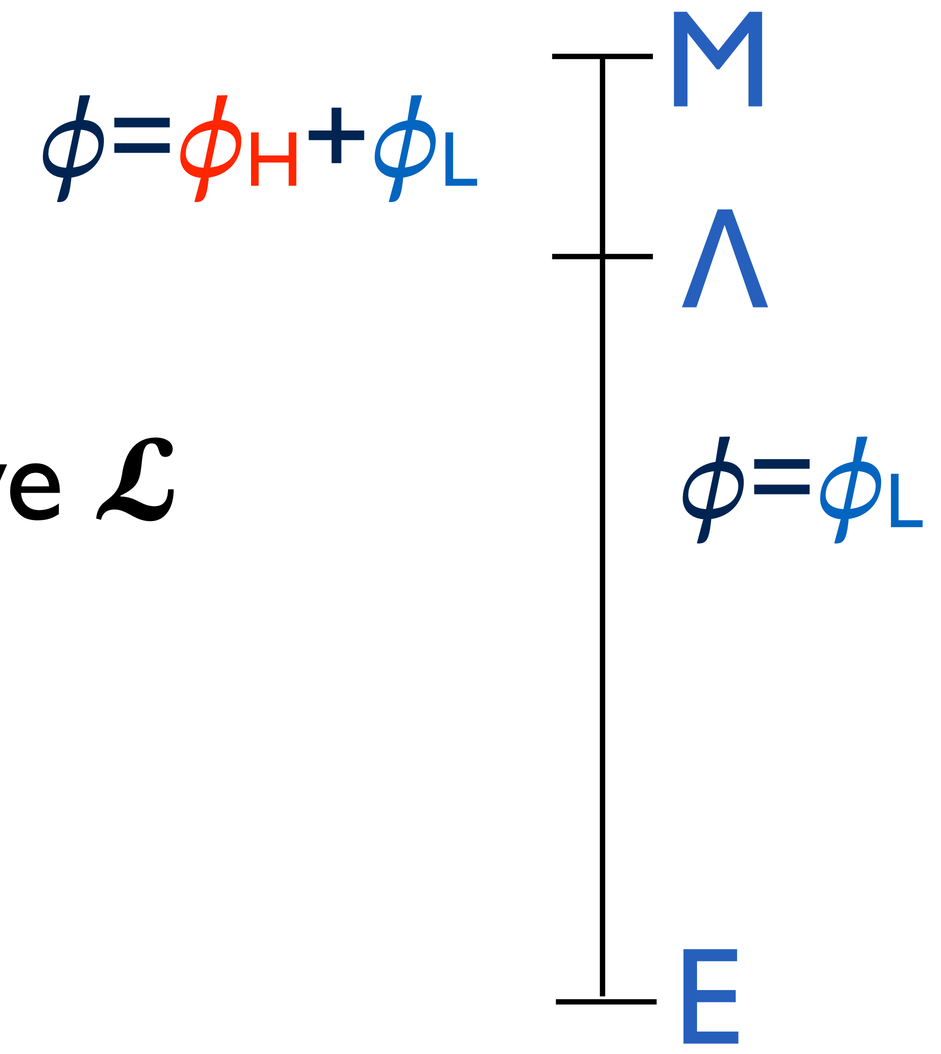

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$$Z[J_L] = \int \mathcal{D}\phi_L \, \mathcal{D}\phi_H \, e^{iS(\phi_L,\phi_H) + i\int d^D x \, J_L(x)\, \phi_L(x)}$$

$$Z[J_L] \equiv \int \mathcal{D}\phi_L \, e^{iS^{\text{eff}}_{\Lambda}(\phi_L) + i\int d^D x \, J_L(x)\, \phi_L(x)}$$

$$\langle 0 | \, T\{\phi_L(x_1) \ldots \phi_L(x_n)\} \, 0 \rangle = \frac{1}{Z[0]} \left(-i \frac{\delta}{\delta J_L(x_1)} \right) \ldots \left(-i \frac{\delta}{\delta J_L(x_n)} \right) Z[J_L] \Big|_{J_L = 0}$$

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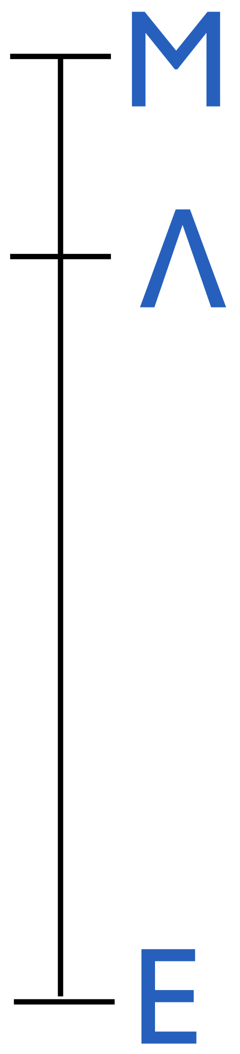

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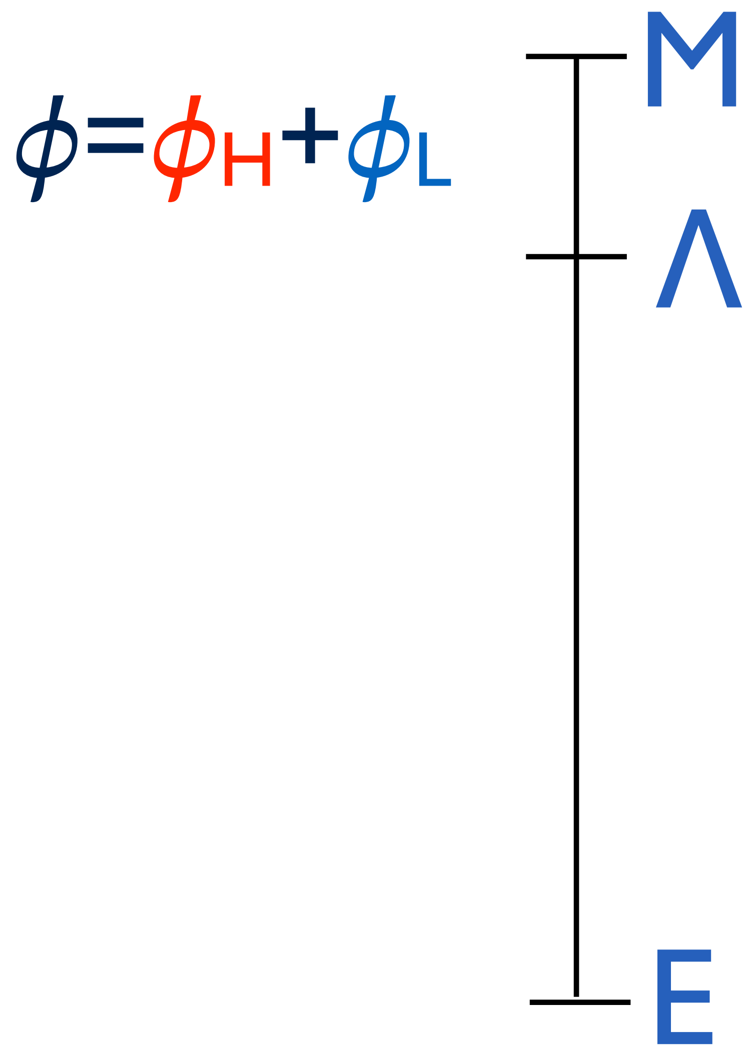

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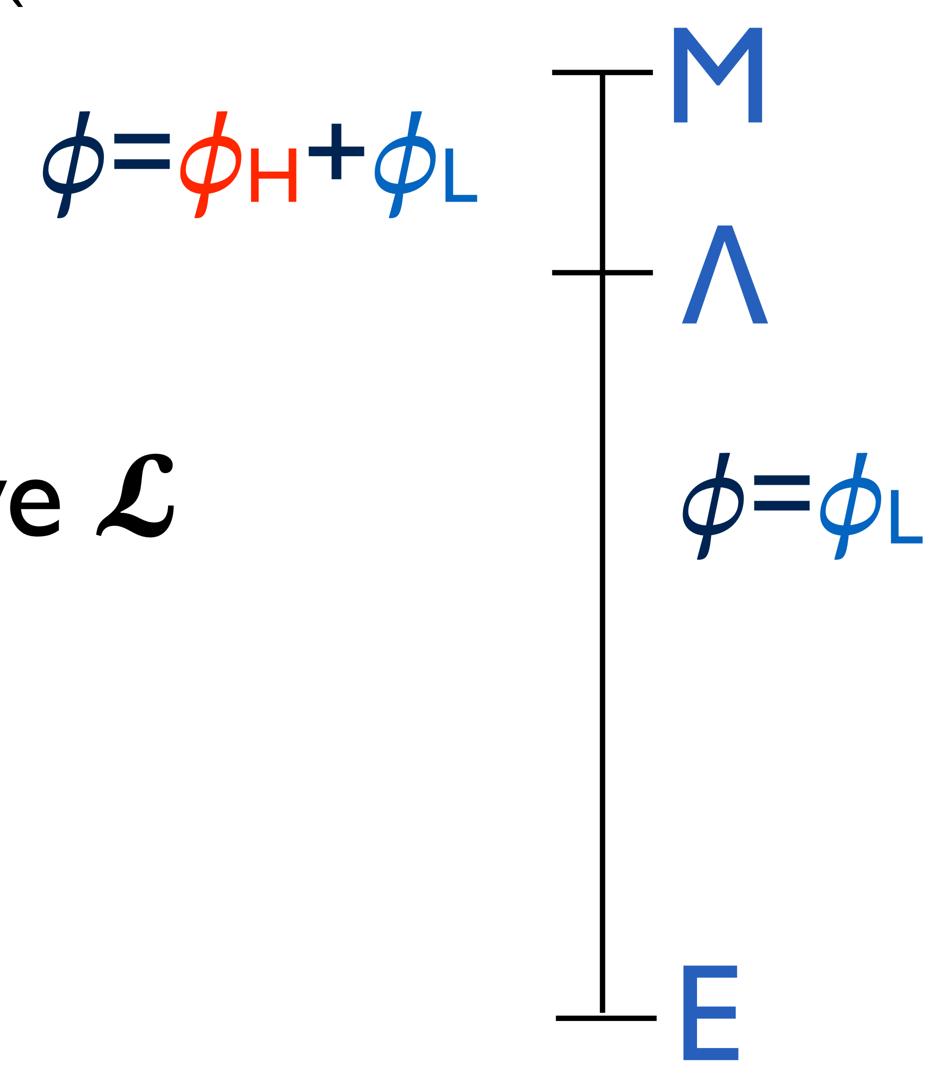

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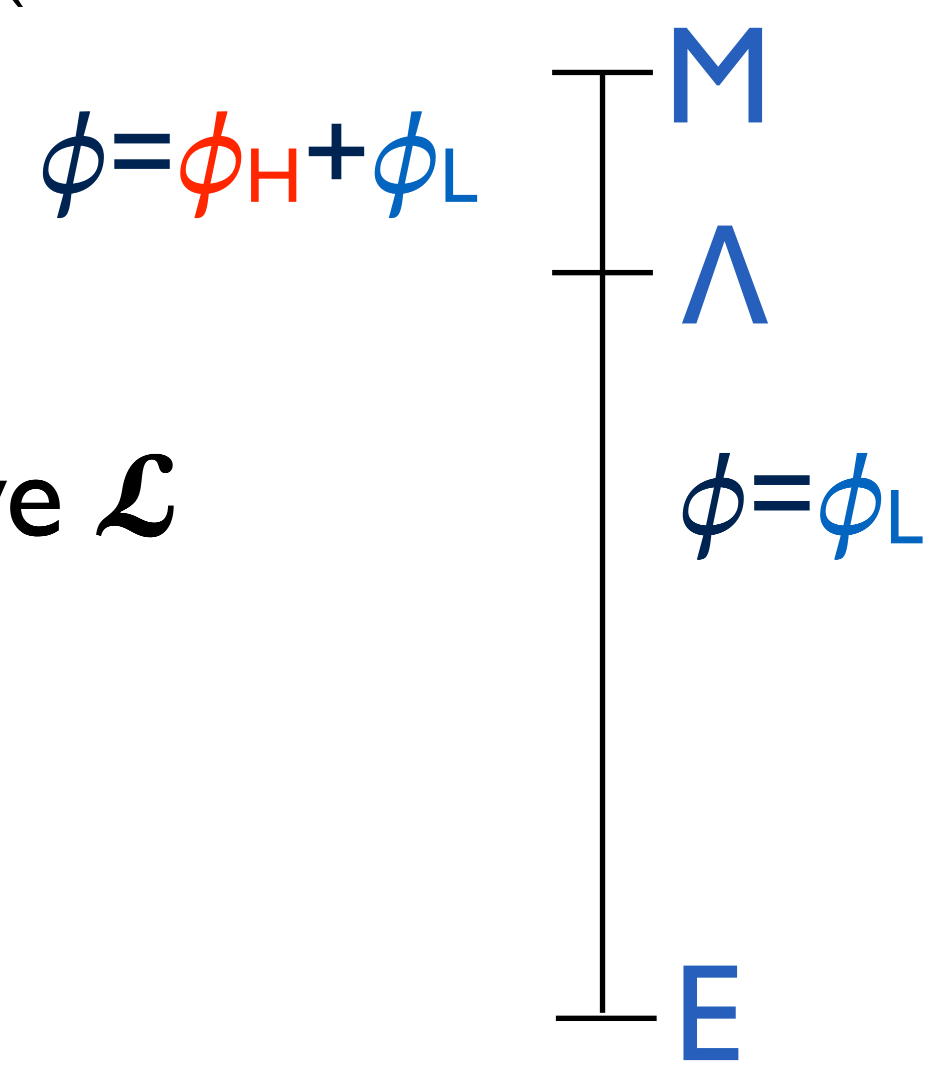

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non-local, but Δx^{μ} is small $\sim 1/\Lambda$

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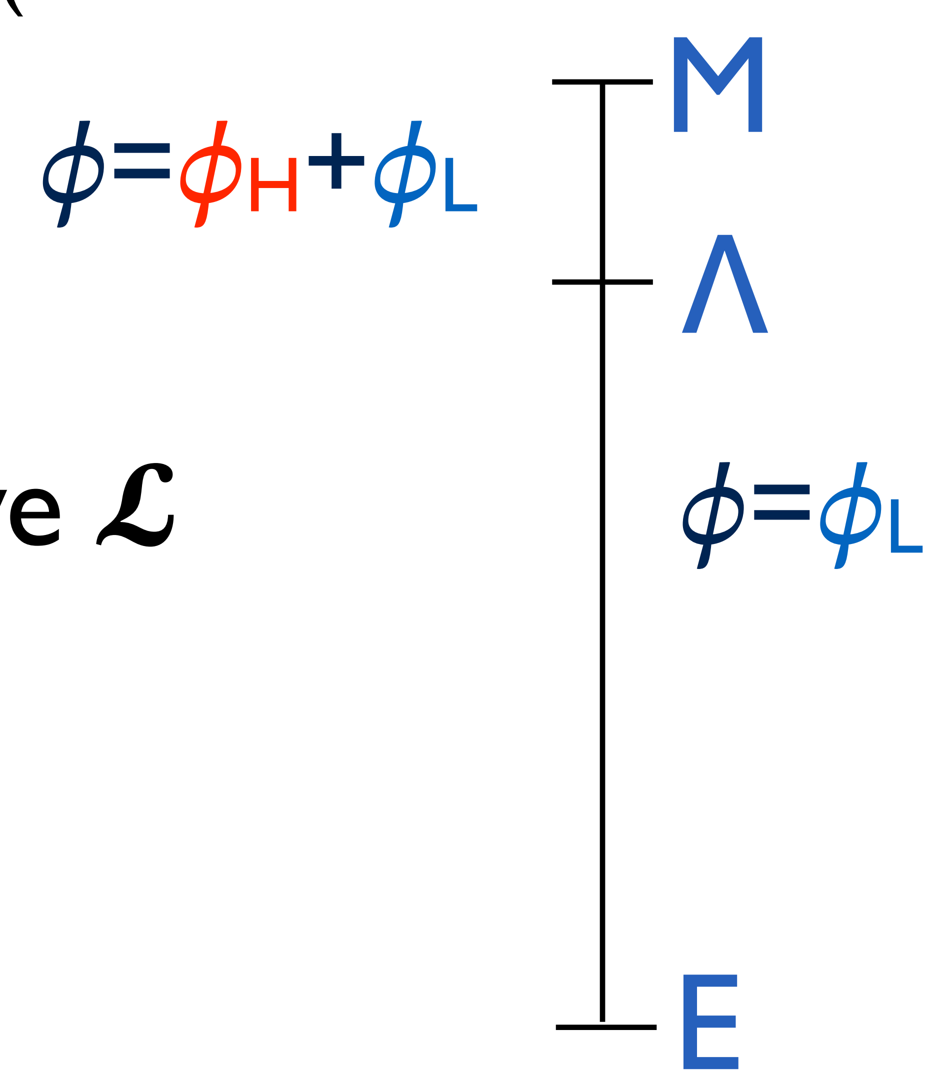

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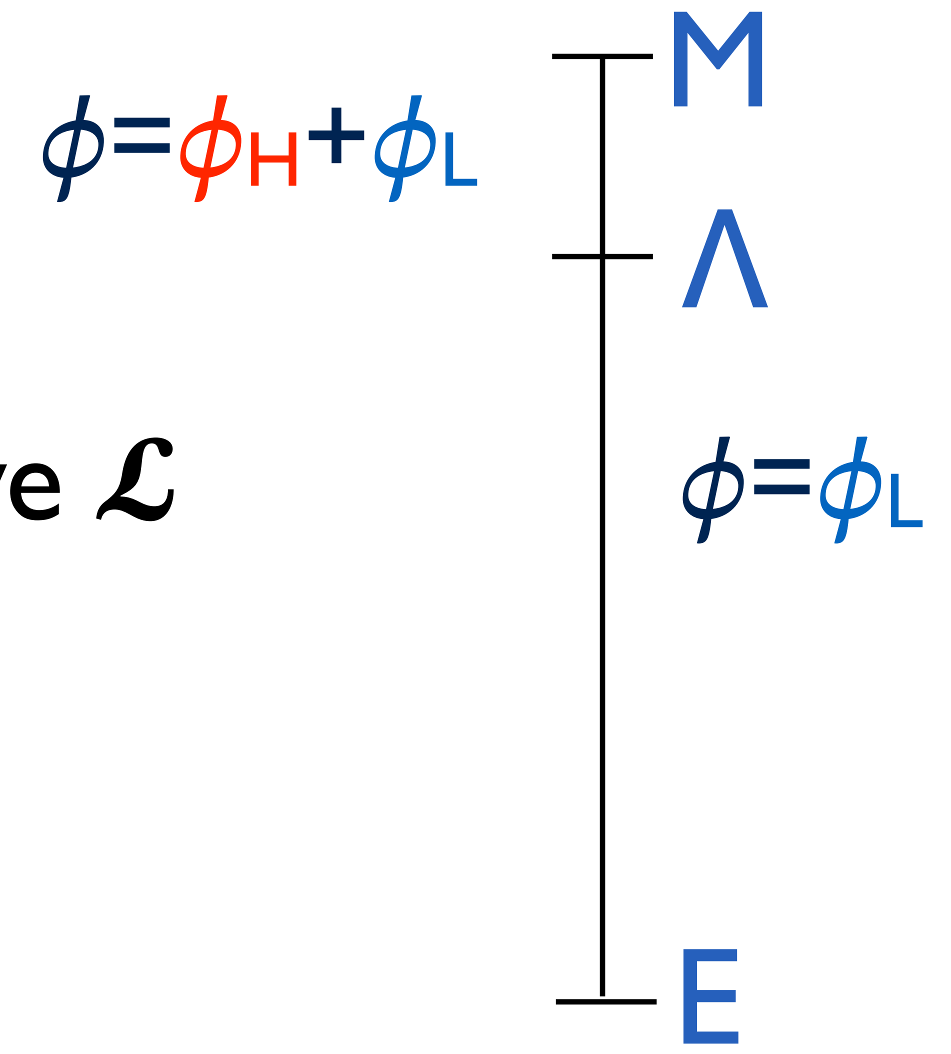

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non-local, but Δx^{μ} is small ~1/Λ

- Step 3: expand non-local action functional in local OPE (E≪ Λ)

$$S^{\text{eff}}_{\Lambda}(\phi_L) = \int d^D x \underbrace{\sum_i g_i \, Q_i(\phi_L(x))}_{\mathcal{L}^{\text{eff}}_{\Lambda}(x)}$$

$Q_i = $ local operators

Effective Field Theory

- Basic idea of effective field theory

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- Näive dimensional analysis (NDA): $[m]=[E]=[p]=[x^{-1}]=[t^{-1}]$
  $[g_i]=-\gamma_i$ "mass dimension"

$\Longrightarrow g_i = c_i\, M^{-\gamma_i}$; c_i is "naturally" $\mathcal{O}(1)$

c_i: dimensionless number

$\delta_i = [\mathbf{Q}_i] \Longrightarrow \gamma_i = \delta_i - D$

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- Contribution of a given operator Q_i in $\mathcal{L}_{\text{eff}}$ to a dimensionless observable would be:

$$c_i \left(\frac{E}{M}\right)^{\gamma_i} = \begin{cases} O(1)\,; & \text{if } \gamma_i = 0, \\ \ll 1\,; & \text{if } \gamma_i > 0, \\ \gg 1\,; & \text{if } \gamma_i < 0. \end{cases}$$

$E \ll \Lambda < M$

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at low energy, make OPE useful!

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$E \ll \Lambda < M$

⇒ Given a precision goal, can truncate the series $\sum_i g_i\, Q_i(\phi_L(x))$ in given order in E/M => keep only finite Q_i

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Dimension	Importance for $E \to 0$	Terminology
$\delta_i < D,\ \gamma_i < 0$	grows	relevant operators (super-renormalizable)
$\delta_i = D,\ \gamma_i = 0$	constant	marginal operators (renormalizable)
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usually unimportant (forbidden by symmetry) ← relevant operators

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Dimension	Importance for $E \to 0$	Terminology
$\delta_i < D,\ \gamma_i < 0$	grows	relevant operators (super-renormalizable)
$\delta_i = D,\ \gamma_i = 0$	constant	marginal operators (renormalizable)
$\delta_i > D,\ \gamma_i > 0$	falls	irrelevant operators (non-renormalizable)

relevant operators: usually unimportant (forbidden by symmetry)

marginal operators: renomalizable QFT

Effective Field Theory

- Basic idea of effective field theory

$$S^{\text{eff}}_{\Lambda}(\phi_L) = \int d^D x \underbrace{\sum_i g_i Q_i(\phi_L(x))}_{\mathcal{L}^{\text{eff}}_{\Lambda}(x)}$$

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irrelevant operators: really interesting, sensitive to "fundamental" scale M

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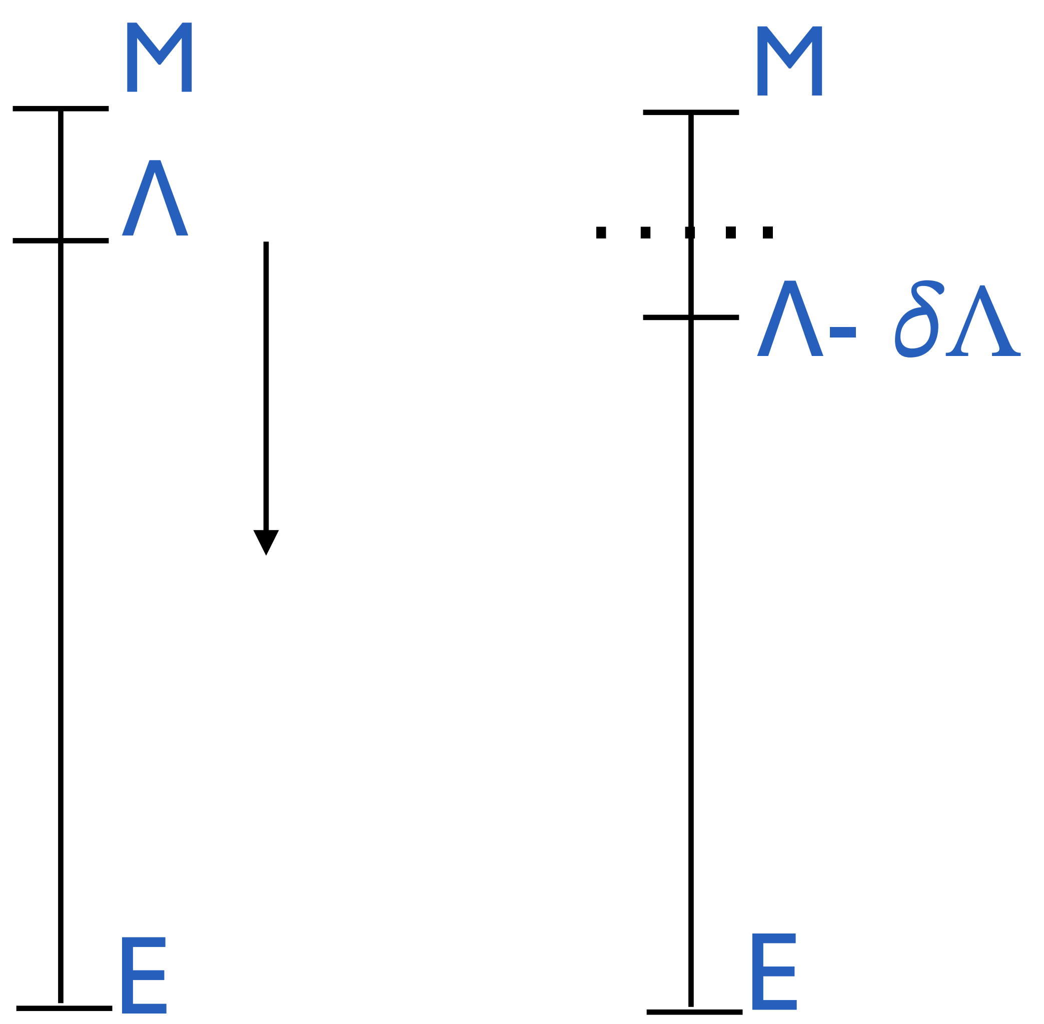
Renormalization & Running coupling

what happens when lower the cutoff?



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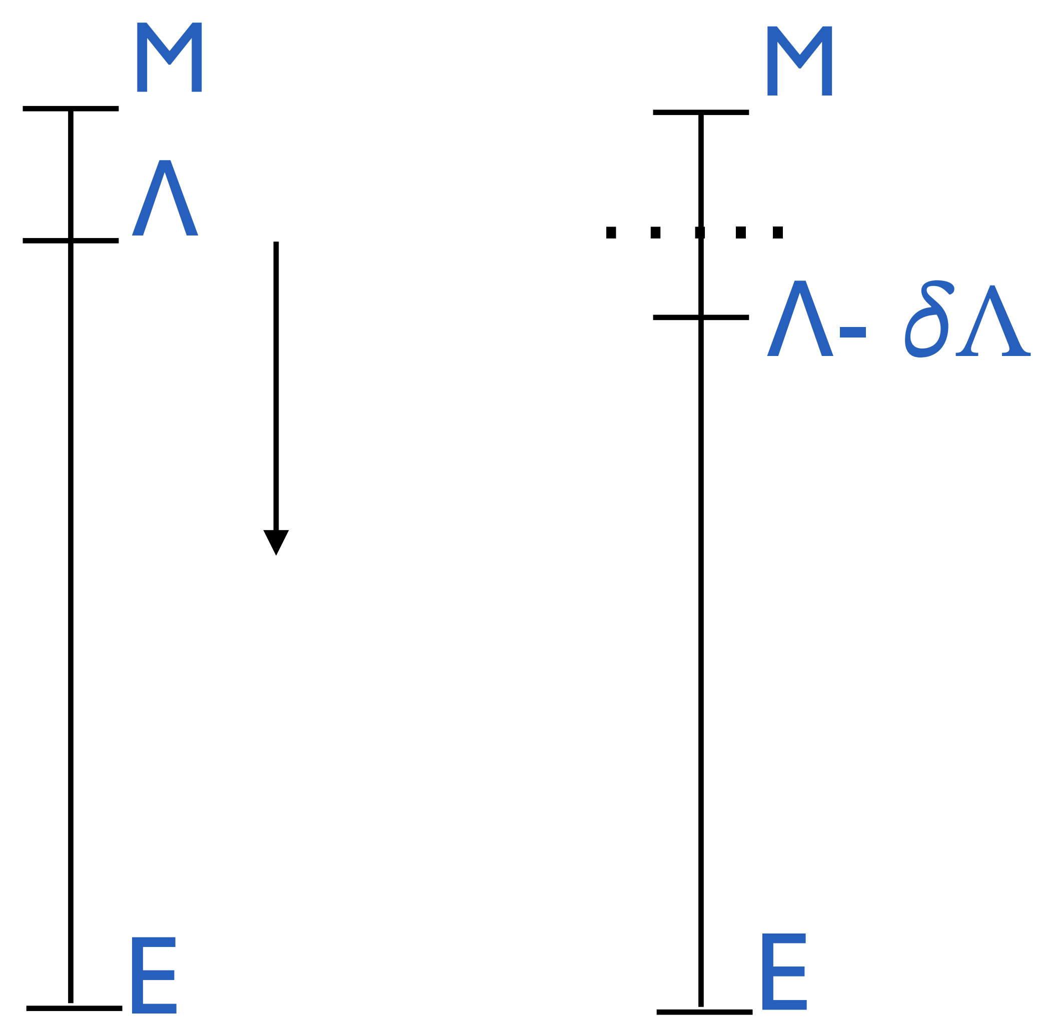

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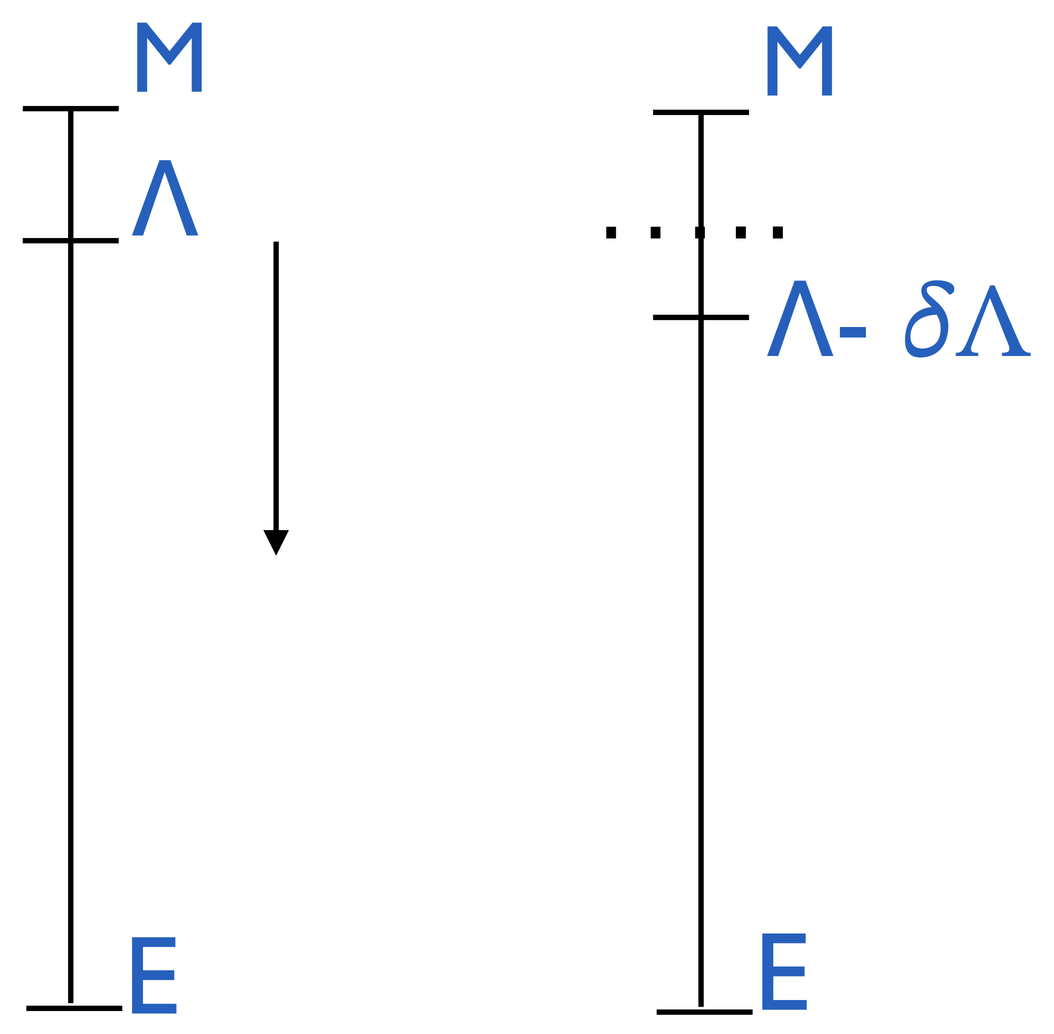

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⇒ intuititive understanding of running coupling

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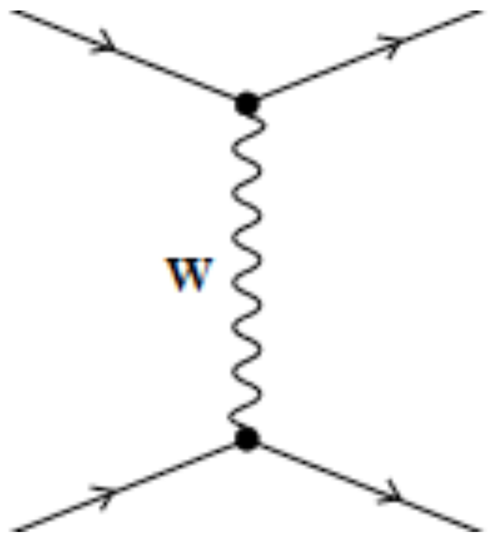
* Effective weak interaction



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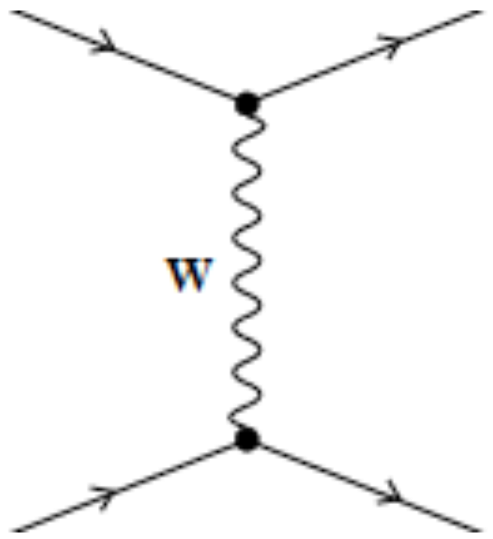

Integrate out heavy particles (W)

Effective Field Theory

- Effective weak interaction

$$\mathcal{L}^{\text{eff}}_{\text{weak}} = -\frac{g^2}{8M_W^2}\left[J^-_\mu\, J^{+\mu} + \frac{1}{M_W^2}\, J^-_\mu\,(\partial^\mu\partial^\nu - g^{\mu\nu}\,\Box)\, J^+_\nu + \dots\right]$$

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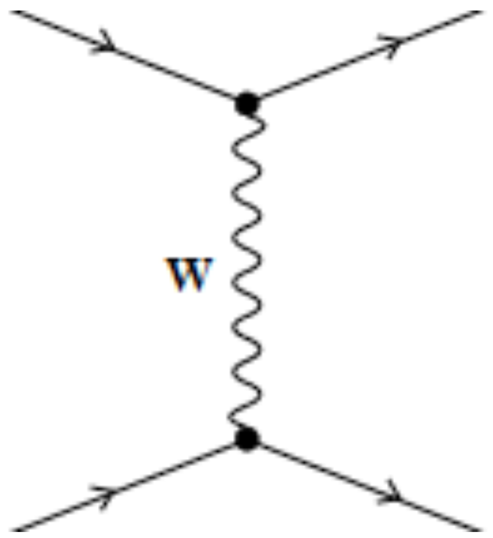

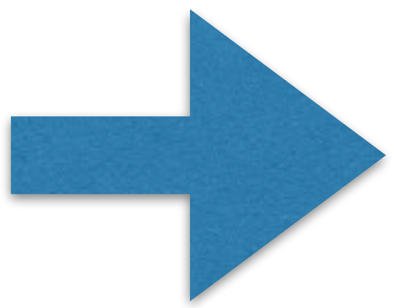

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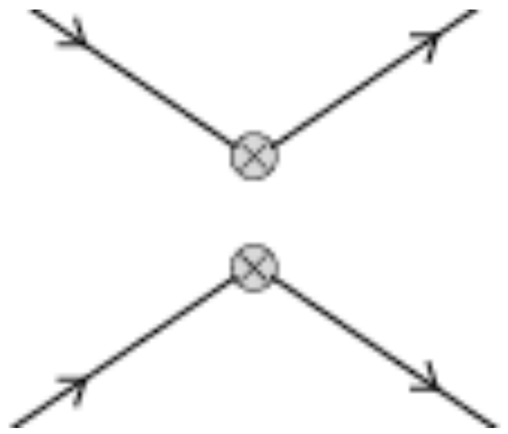

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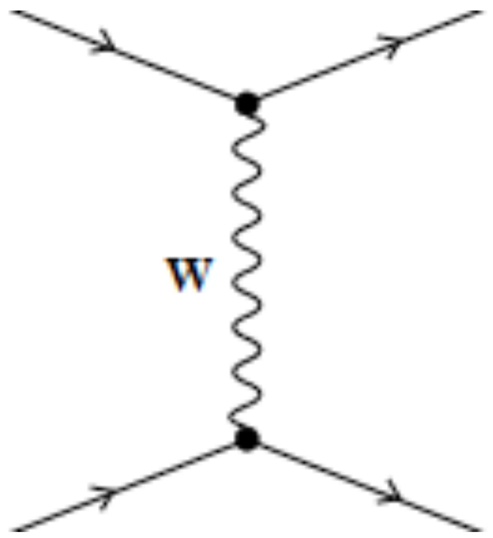

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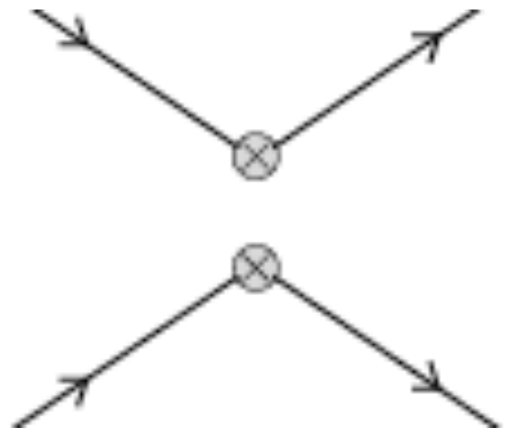
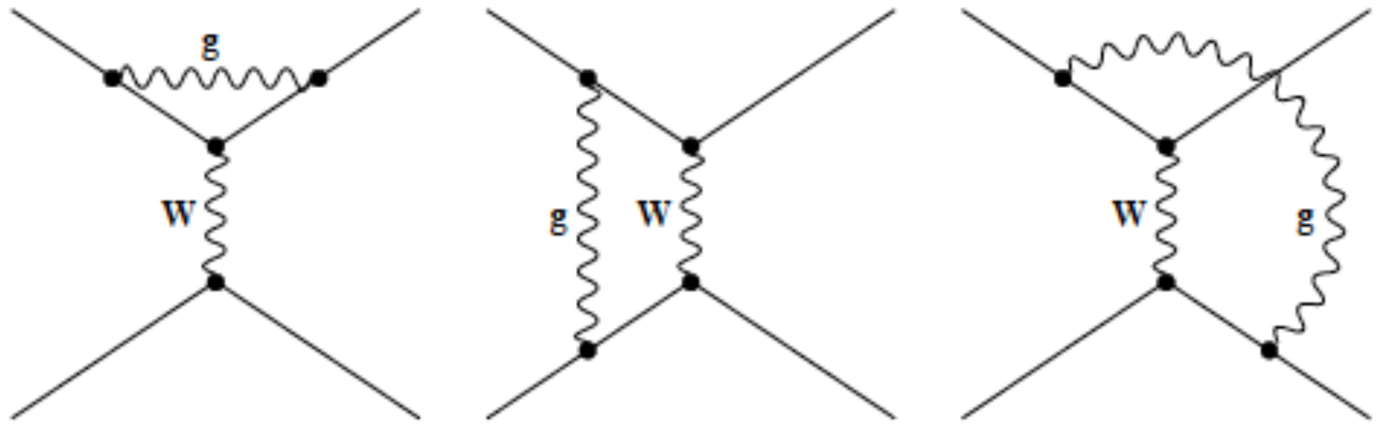
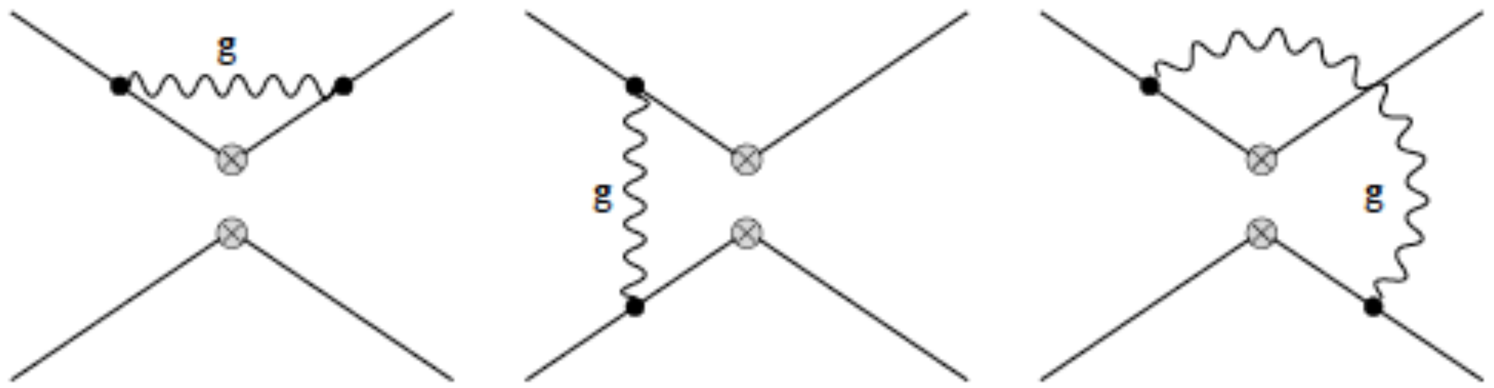


- How about loop corrections?





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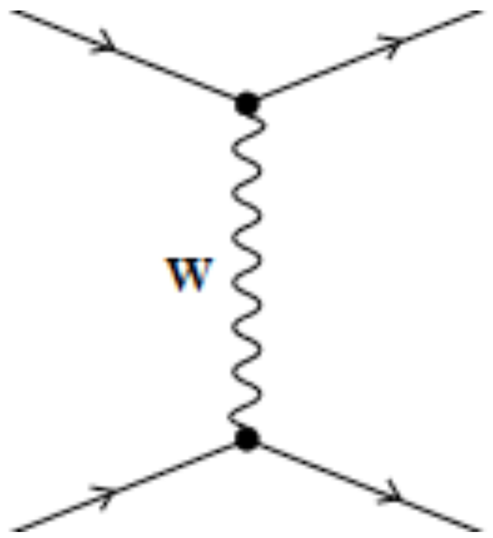

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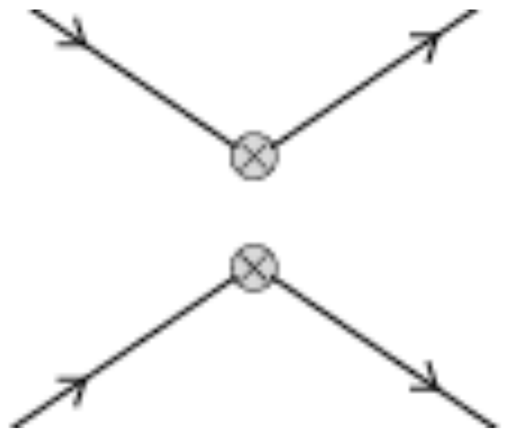

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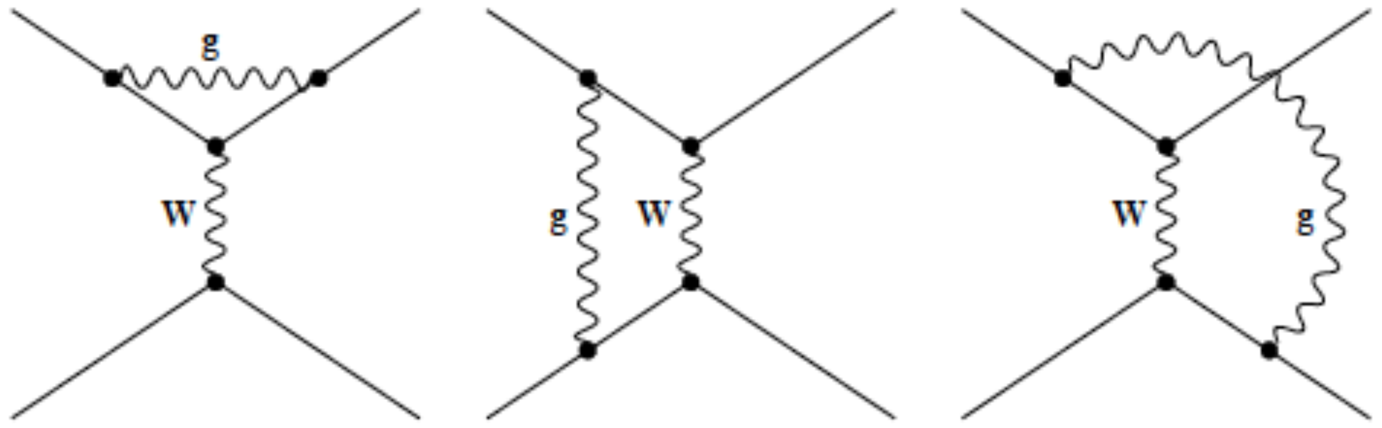

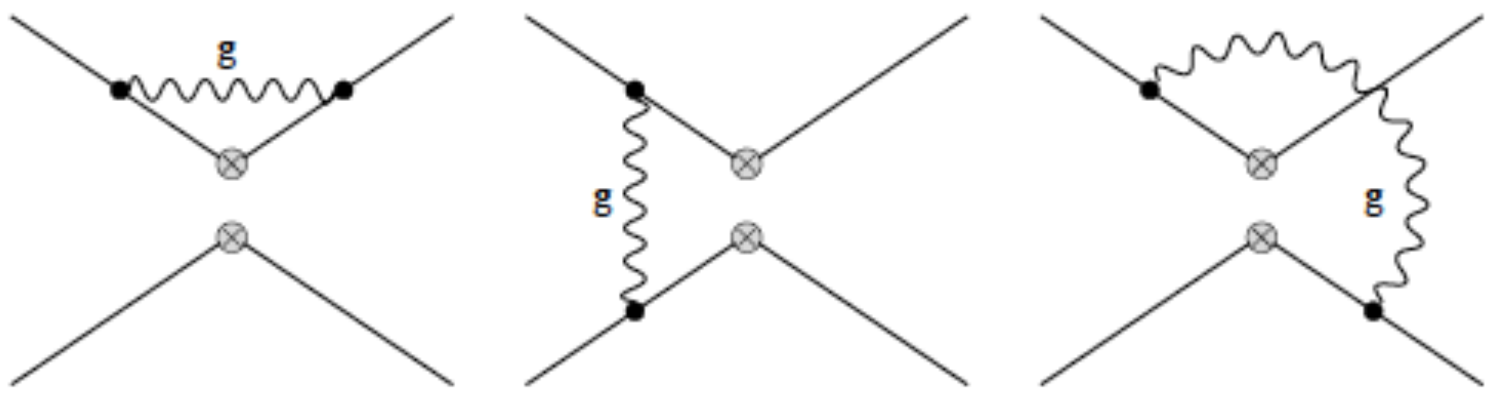

$$\int d^D p \,\frac{1}{M_W^2 - p^2}\, f(p) \neq \frac{1}{M_W^2}\int d^D p \left(1 + \frac{p^2}{M_W^2} + \ldots\right) f(p)$$

For large loop momenta ($p^2 \sim M_W^2$),
two operations – expansion of the W propagator
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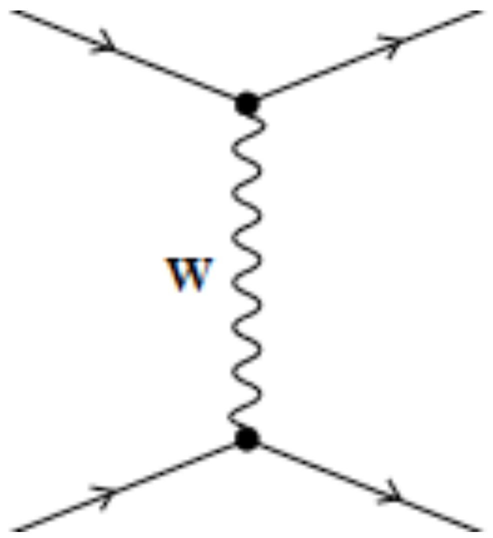

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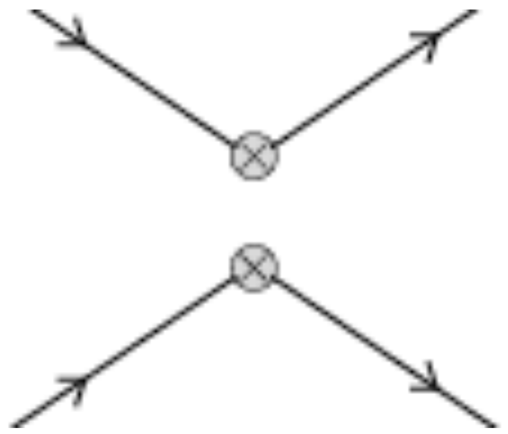

For large momenta QCD is weakly coupled: $\alpha_s(M_W)$ is small and perturbation theory works => Wilson coefficient taken into account of the loop corrections for large momenta can be calculated

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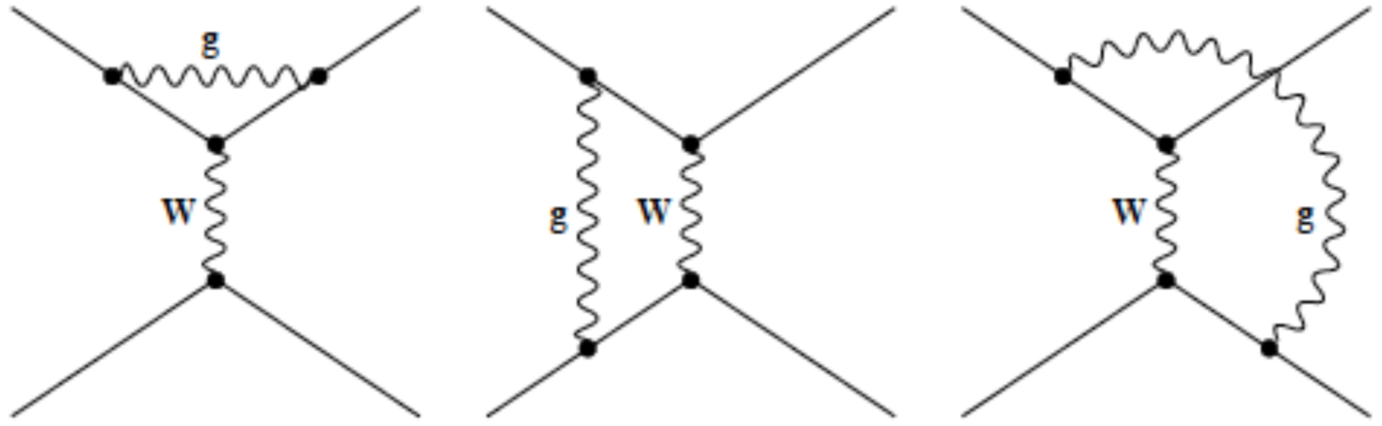

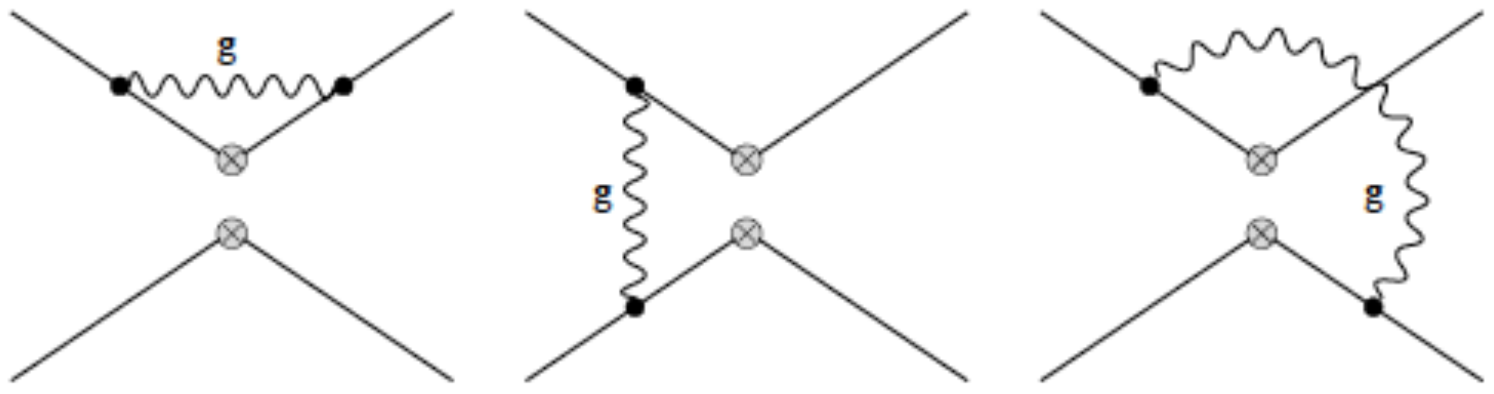

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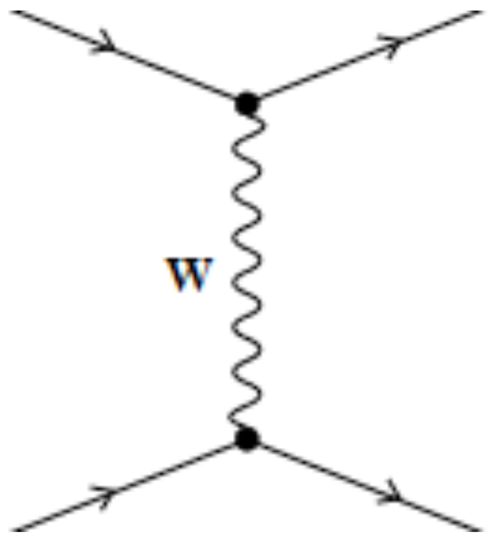

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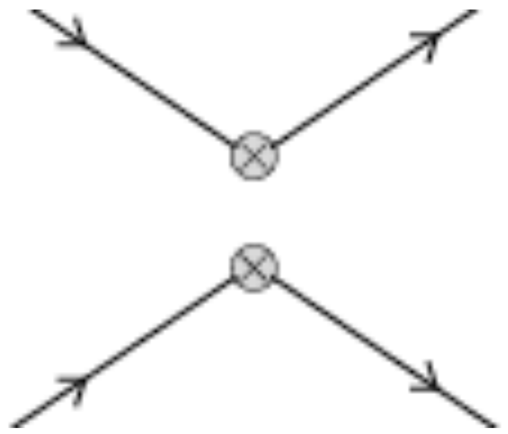

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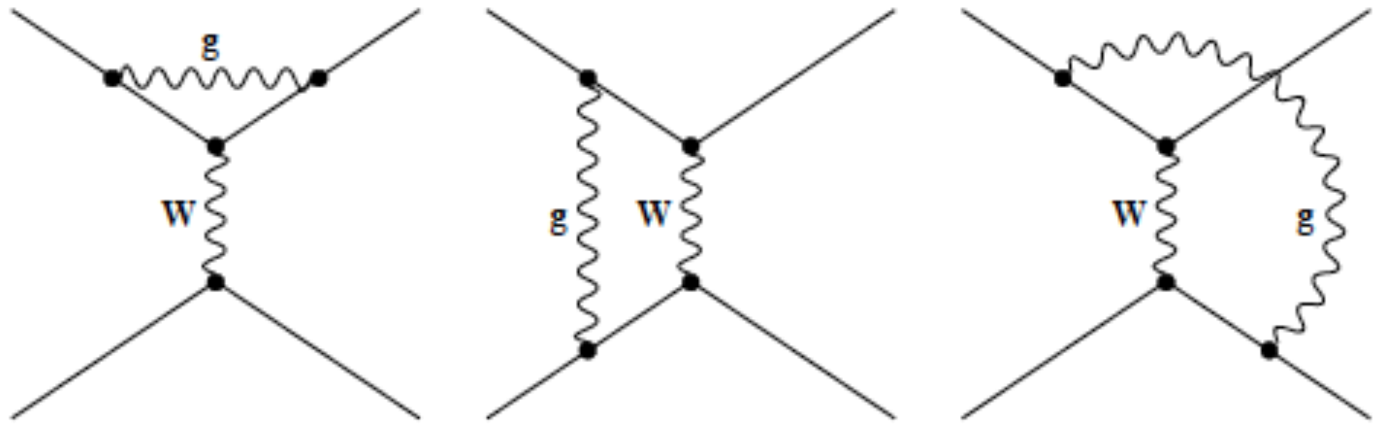

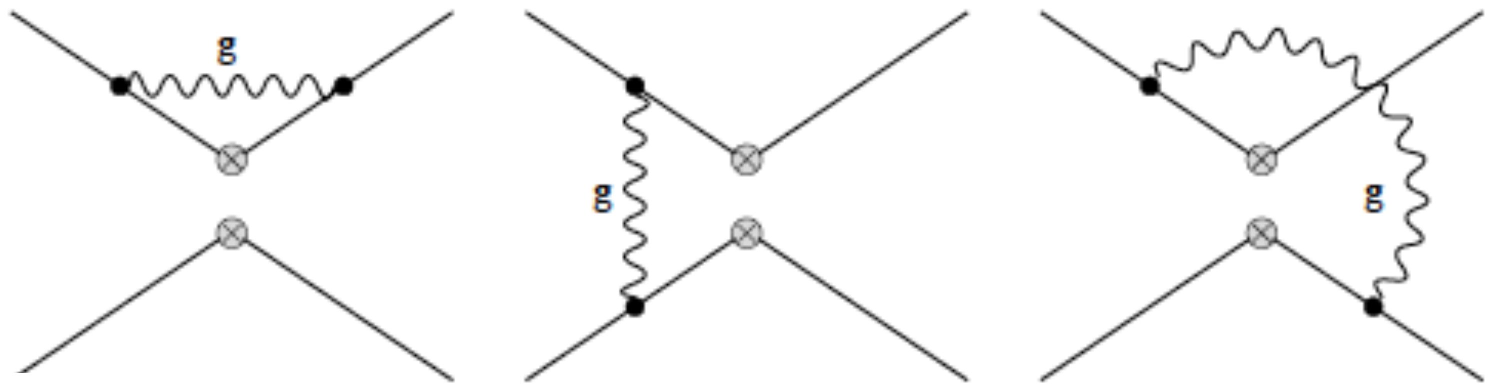

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$\bar{B}^0 \rightarrow \pi^+ D_s^-$:

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matching

$$C_1(\mu) = 1 + \frac{3}{N_c}\frac{\alpha_s(\mu)}{4\pi}\left(\ln\frac{M_W^2}{\mu^2} - \frac{11}{6}\right) + O(\alpha_s^2)\,,$$

$$C_2(\mu) = -3\,\frac{\alpha_s(\mu)}{4\pi}\left(\ln\frac{M_W^2}{\mu^2} - \frac{11}{6}\right) + O(\alpha_s^2)\,.$$

Effective Field Theory

Matching

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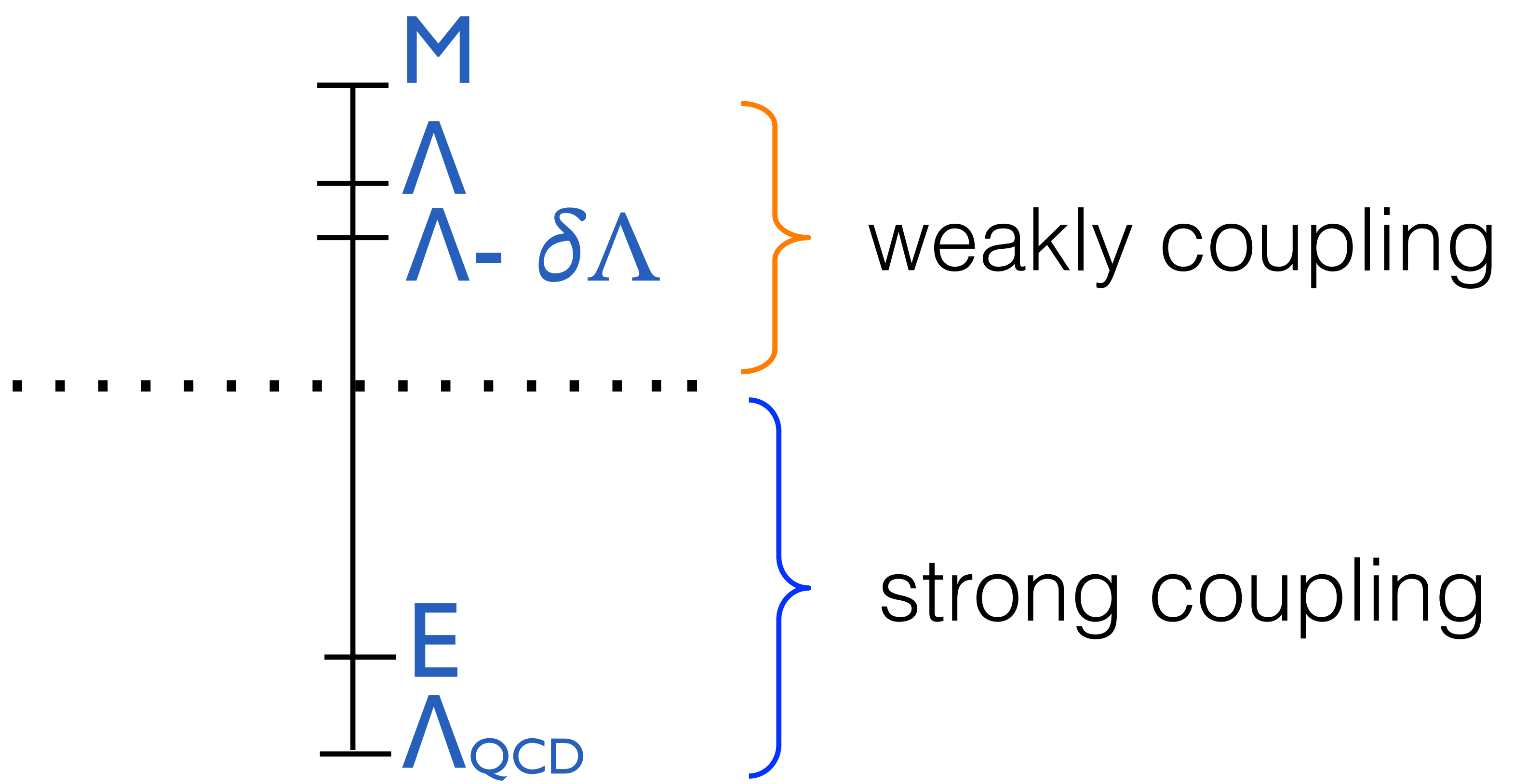
$$\sum_i C_i(\Lambda) O_i(\Lambda)$$



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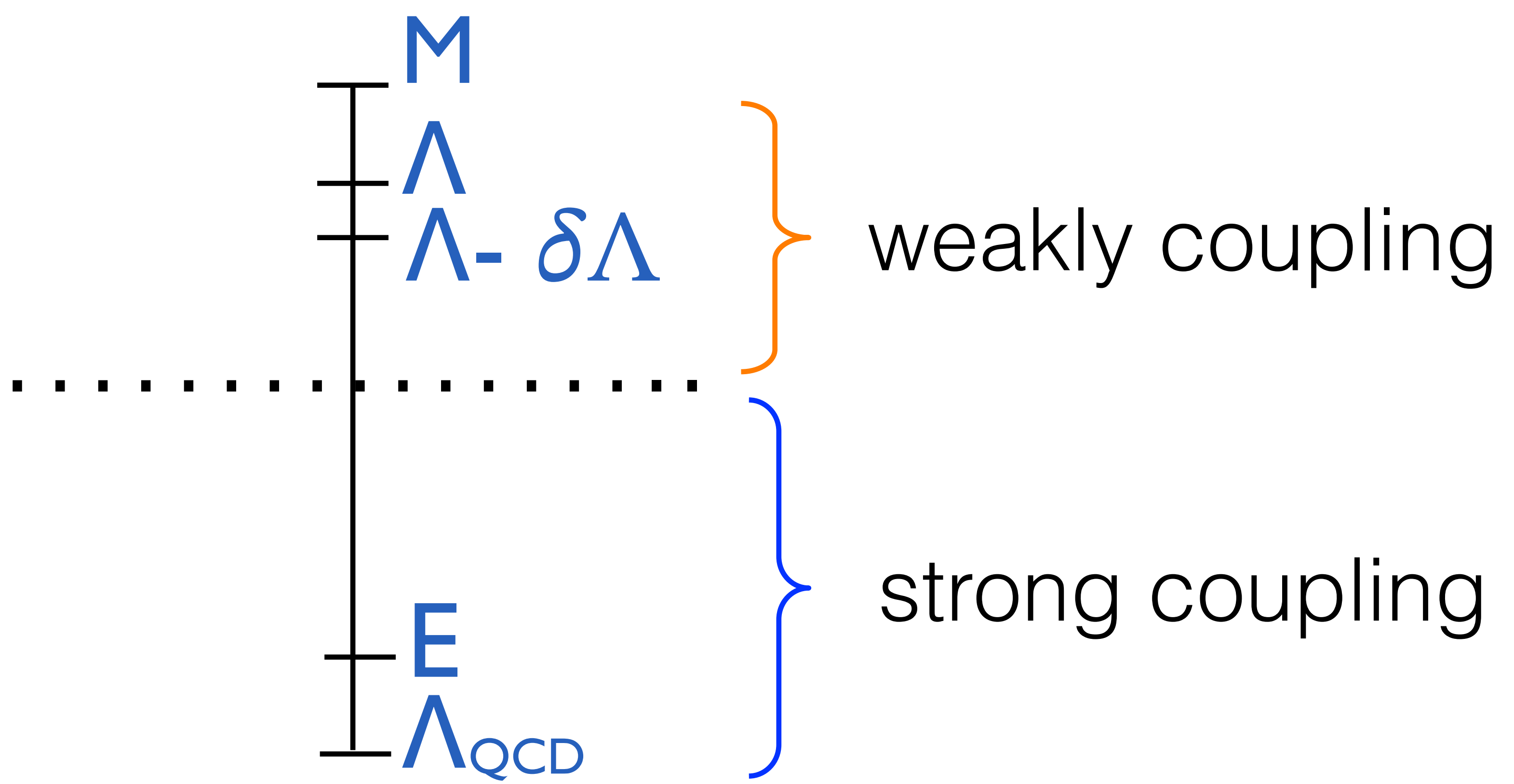

$$\sum_i \underbrace{C_i(\Lambda)}O_i(\Lambda)$$

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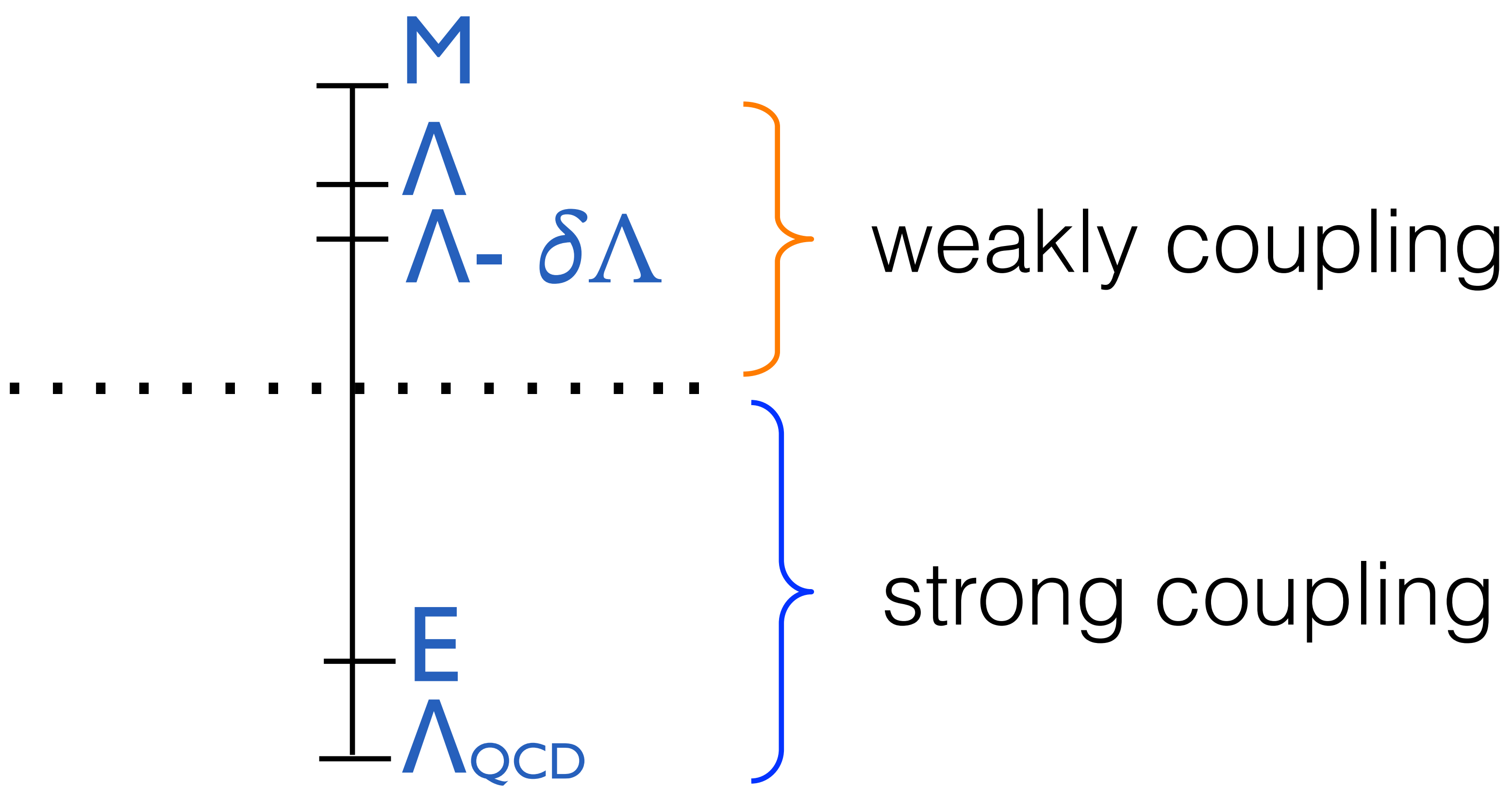

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contains modes with frequency > Λ

EFT=full theory bellow Λ

Effective Field Theory

- Systematically factorize short-distance physics and long-distance physics

- Any sensitivity to high scales (including to physics beyond the Standard Model) can be treated using perturbative methods:

$$C_i(\mu) = C_i^{\mathrm{SM}}(M_W, m_t, \mu) + C_i^{\mathrm{NP}}(M_{\mathrm{NP}}, g_{\mathrm{NP}}, \mu)$$

- Nonperturbative methods (operator product expansion, lattice gauge theory, ...) usually only work at low scales (typically μ~few GeV)

Operator Basis (Effective Weak Interaction)

- The effective weak Hamiltonian for FCNC (hadronic)

$$(\bar{q}_1 q_2)_{V\pm A} \equiv \bar{q}_1 \gamma^\mu (1 \pm \gamma_5) q_2$$

$$\lambda_p = V_{pb} V_{ps}^* \text{ or } \lambda'_p = V_{pb} V_{pd}^*$$

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \Big(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3,\dots,10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \Big) + \text{h.c.}\,,$$

$$Q_1^p = (\bar{p}b)_{V-A} (\bar{s}p)_{V-A}\,, \qquad Q_2^p = (\bar{p}_i b_j)_{V-A} (\bar{s}_j p_i)_{V-A}\,,$$

$$Q_3 = (\bar{s}b)_{V-A} \sum\nolimits_q (\bar{q}q)_{V-A}\,, \qquad Q_4 = (\bar{s}_i b_j)_{V-A} \sum\nolimits_q (\bar{q}_j q_i)_{V-A}\,,$$

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$$Q_7 = (\bar{s}b)_{V-A} \sum\nolimits_q \tfrac{3}{2} e_q (\bar{q}q)_{V+A}\,, \qquad Q_8 = (\bar{s}_i b_j)_{V-A} \sum\nolimits_q \tfrac{3}{2} e_q (\bar{q}_j q_i)_{V+A}\,,$$

$$Q_9 = (\bar{s}b)_{V-A} \sum\nolimits_q \tfrac{3}{2} e_q (\bar{q}q)_{V-A}\,, \qquad Q_{10} = (\bar{s}_i b_j)_{V-A} \sum\nolimits_q \tfrac{3}{2} e_q (\bar{q}_j q_i)_{V-A}\,,$$

$$Q_{7\gamma} = \frac{-e}{8\pi^2} m_b\, \bar{s}\sigma_{\mu\nu} (1+\gamma_5) F^{\mu\nu} b\,, \qquad Q_{8g} = \frac{-g_s}{8\pi^2} m_b\, \bar{s}\sigma_{\mu\nu} (1+\gamma_5) G^{\mu\nu} b\,,$$

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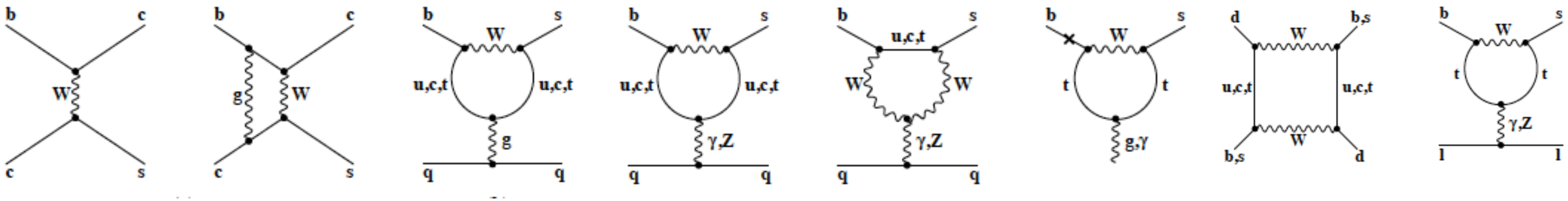
- e.g. Penguins and other loops



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$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \Big(C_1\, Q_1^p + C_2\, Q_2^p + \sum_{i=3,\dots,10} C_i\, Q_i + C_{7\gamma}\, Q_{7\gamma} + C_{8g}\, Q_{8g} \Big) + \text{h.c.}\,,$$

$$Q_1^p = (\bar{p}b)_{V-A}(\bar{s}p)_{V-A}\,, \qquad Q_2^p = (\bar{p}_i b_j)_{V-A}(\bar{s}_j p_i)_{V-A}\,,$$

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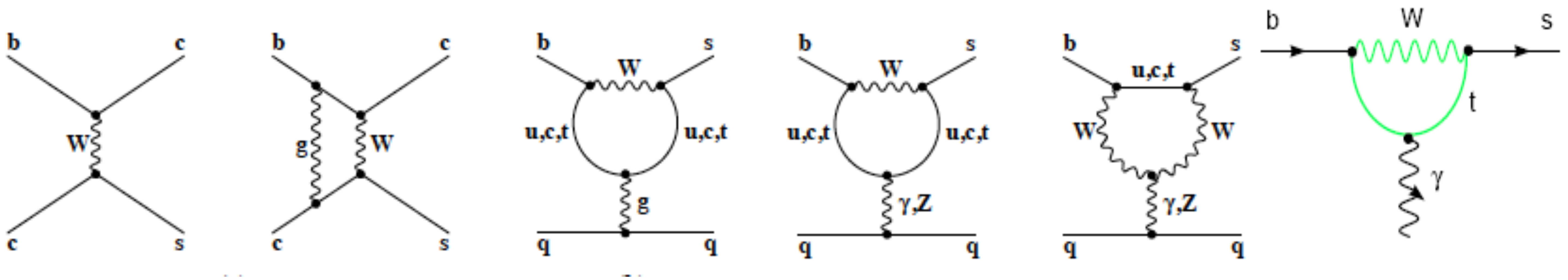
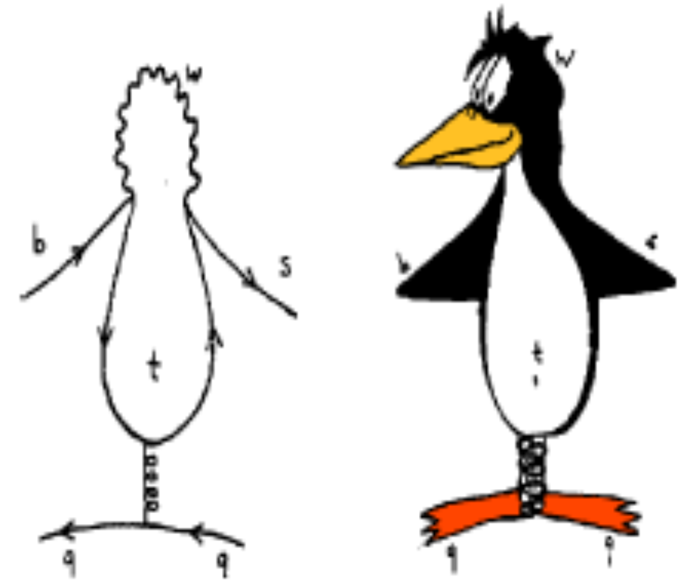
- e.g. Penguins and other loops





Operator Basis (Effective Weak Interaction)

- The effective weak Hamiltonian for FCNC (hadronic)

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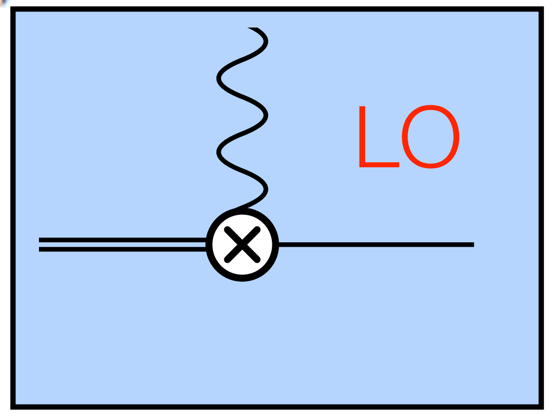

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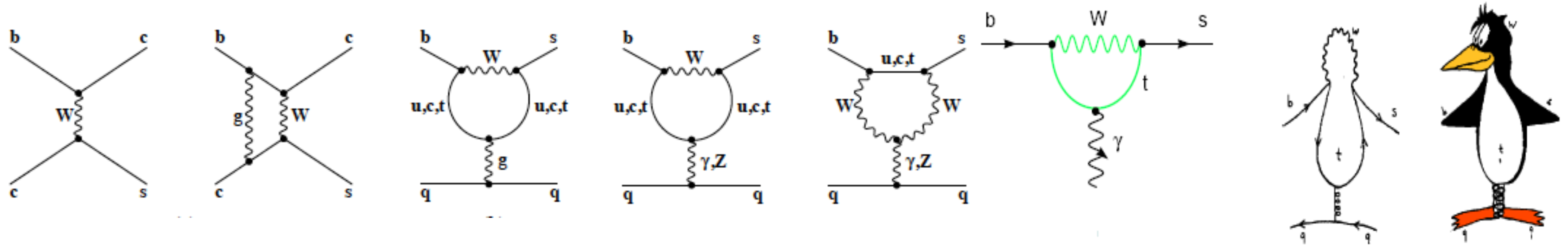

Operator Basis

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$$C_1(M_W) = 1 - \frac{11}{6}\frac{\alpha_s(M_W)}{4\pi},$$

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$$C_3(M_W) = C_5(M_W) = -\frac{1}{6}\widetilde{E}_0\left(\frac{m_t^2}{M_W^2}\right)\frac{\alpha_s(M_W)}{4\pi},$$

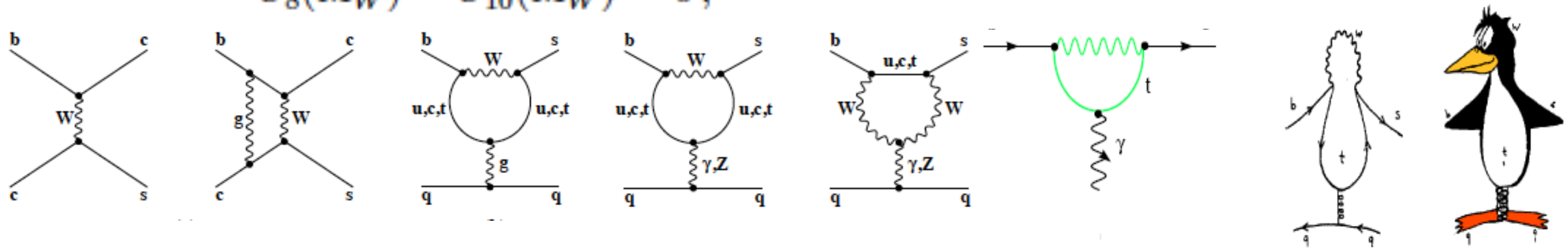
$$C_4(M_W) = C_6(M_W) = \frac{1}{2}\widetilde{E}_0\left(\frac{m_t^2}{M_W^2}\right)\frac{\alpha_s(M_W)}{4\pi},$$

$$C_7(M_W) = f\left(\frac{m_t^2}{M_W^2}\right)\frac{\alpha(M_W)}{6\pi},$$

$$C_9(M_W) = \left[f\left(\frac{m_t^2}{M_W^2}\right) + \frac{1}{\sin^2\theta_W}g\left(\frac{m_t^2}{M_W^2}\right)\right]\frac{\alpha(M_W)}{4\pi},$$

* Penguin

$$C_8(M_W) = C_{10}(M_W) = 0,$$



Effective Theories for Heavy Flavours

* Question: what's there to integrate out, when there is no heavy particle?

  Answer: look for different scales.   e.g. B-physics: $m_b \gg \Lambda_{QCD}$

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- Physics at the scale m_b (short-distance physics) vs. heavy quark related (long-distance) hadronic physics governed at confinement scale Λ_{QCD}.
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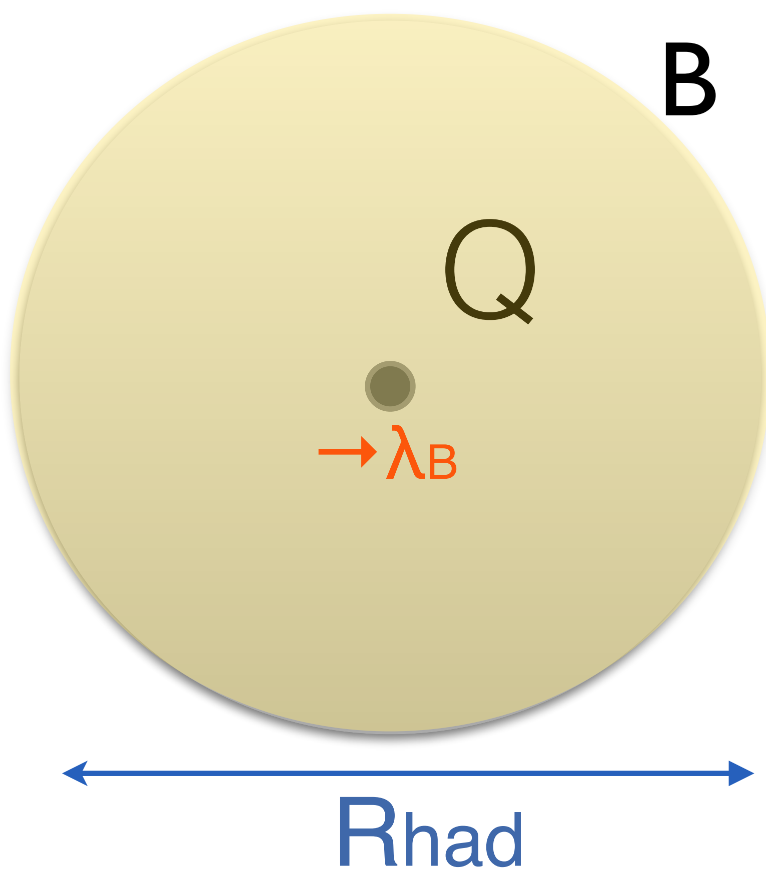
- Prime example: Heavy Quark Effective Field Theory (HQET)

Heavy Quark Effective Field Theory

* What's the physical picture behind?



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- QQ systems is perturbative
- heavy-light bound states (Qq) are not perturbative

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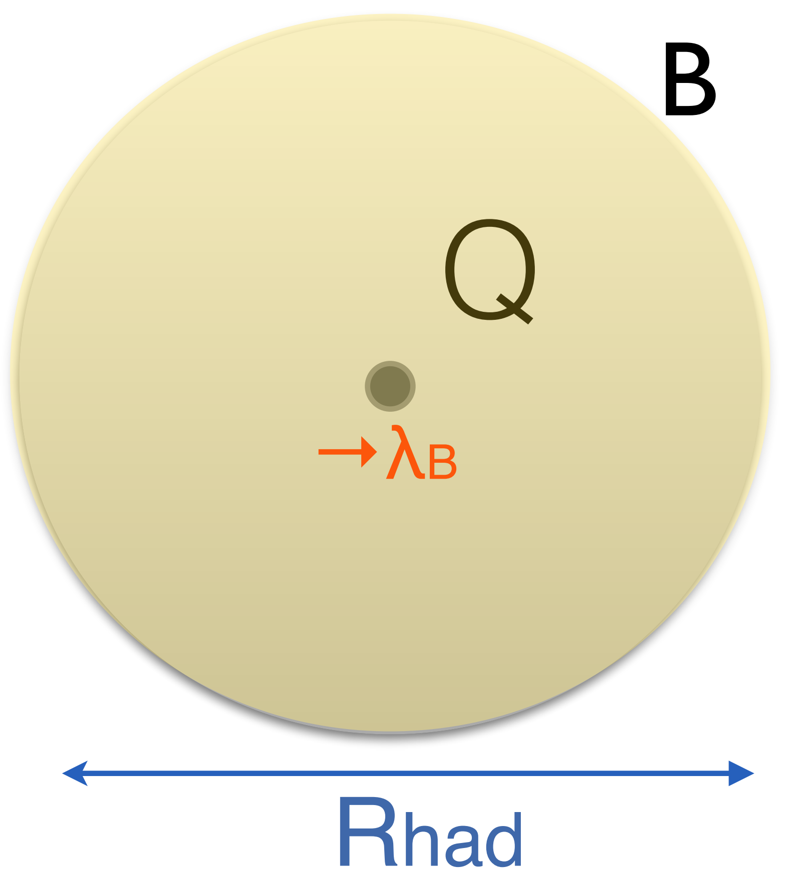

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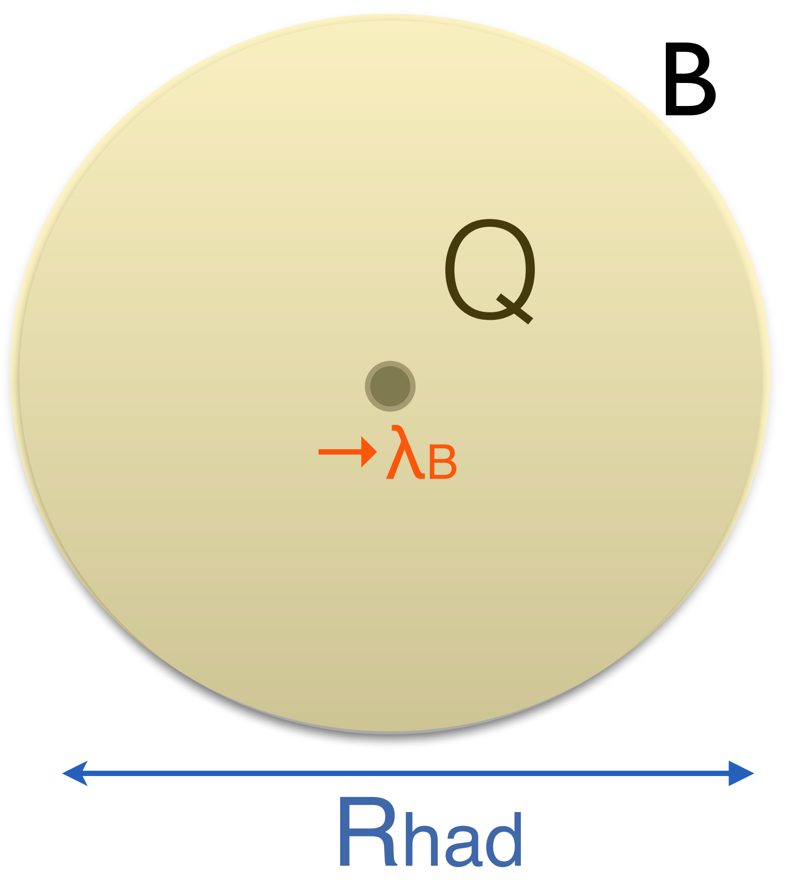

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➡ simplifies physics of hadrons made up of heavy quark

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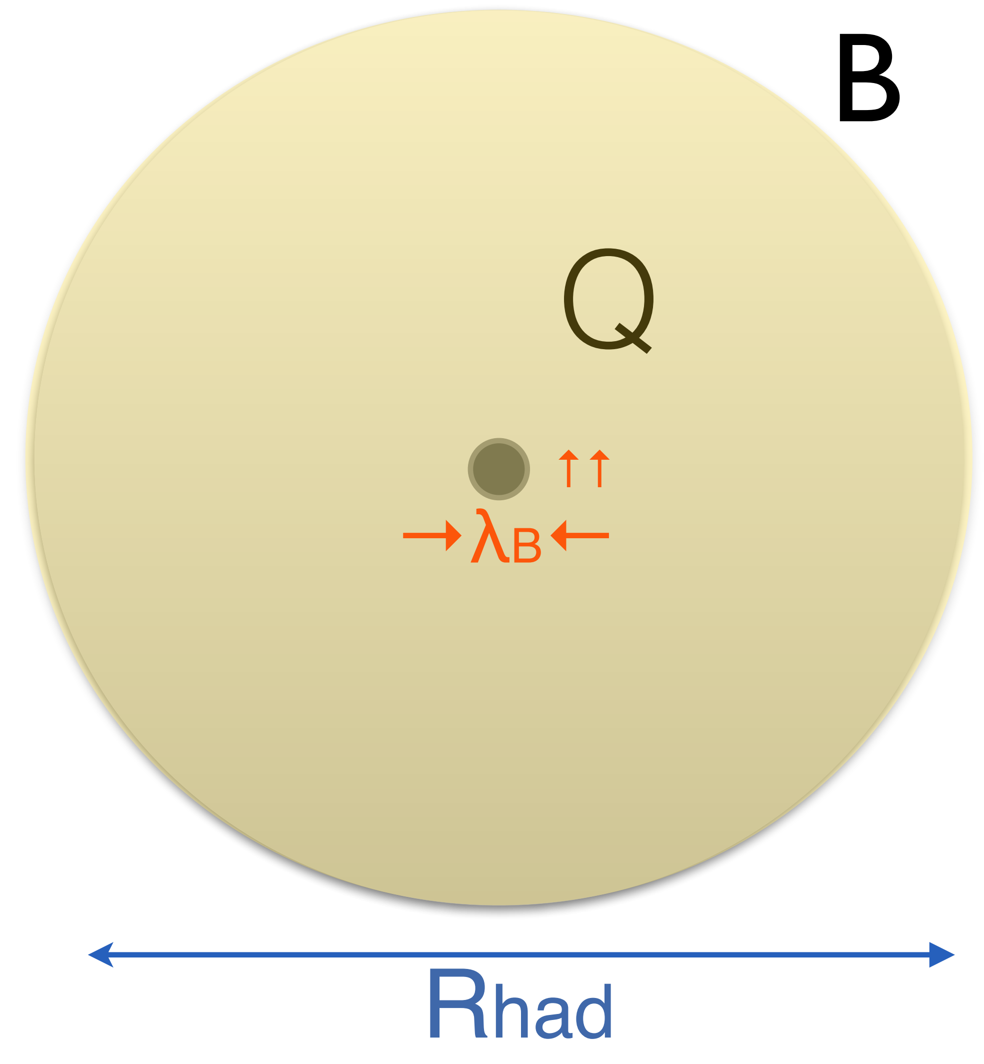

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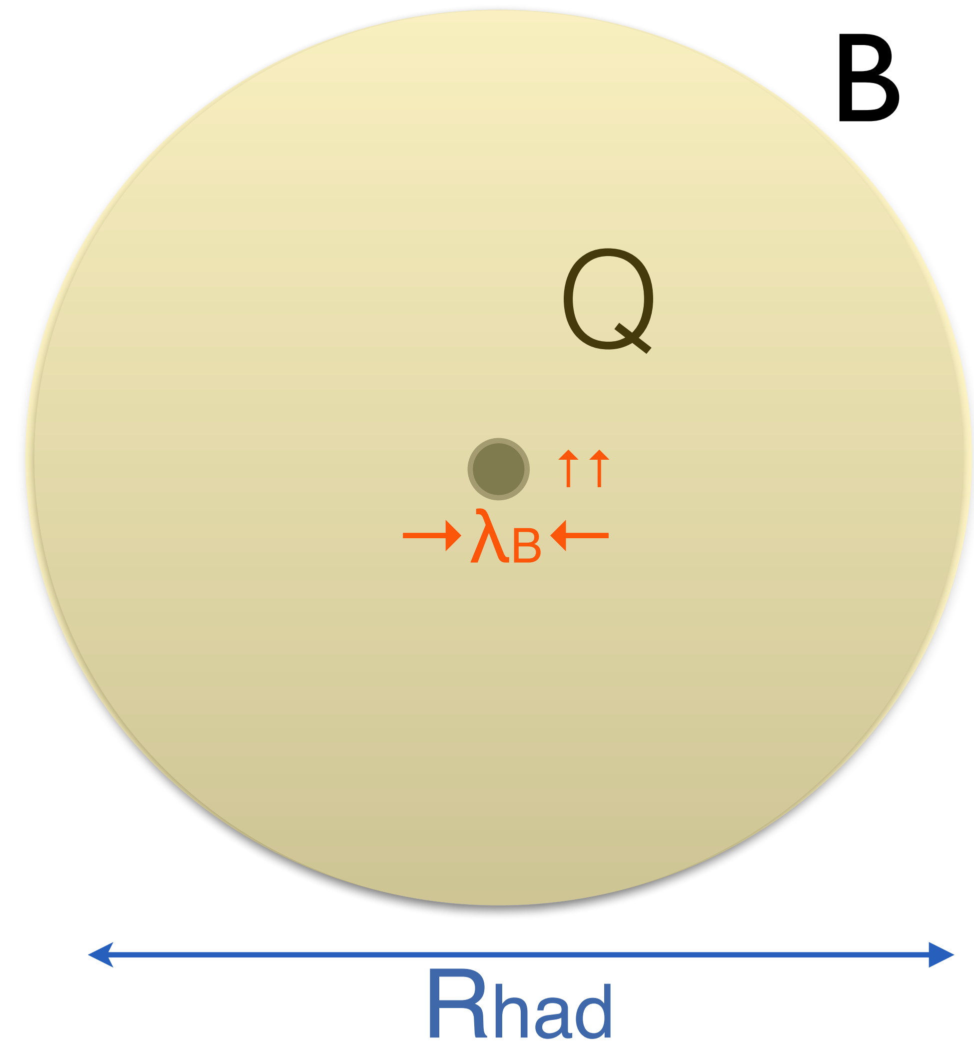
- Flavour Symmetry: Cloud of light degrees of freedom ("antiquark" in a meson) does not feel the mass (flavour) of the heavy quark as $m_Q \to \infty$



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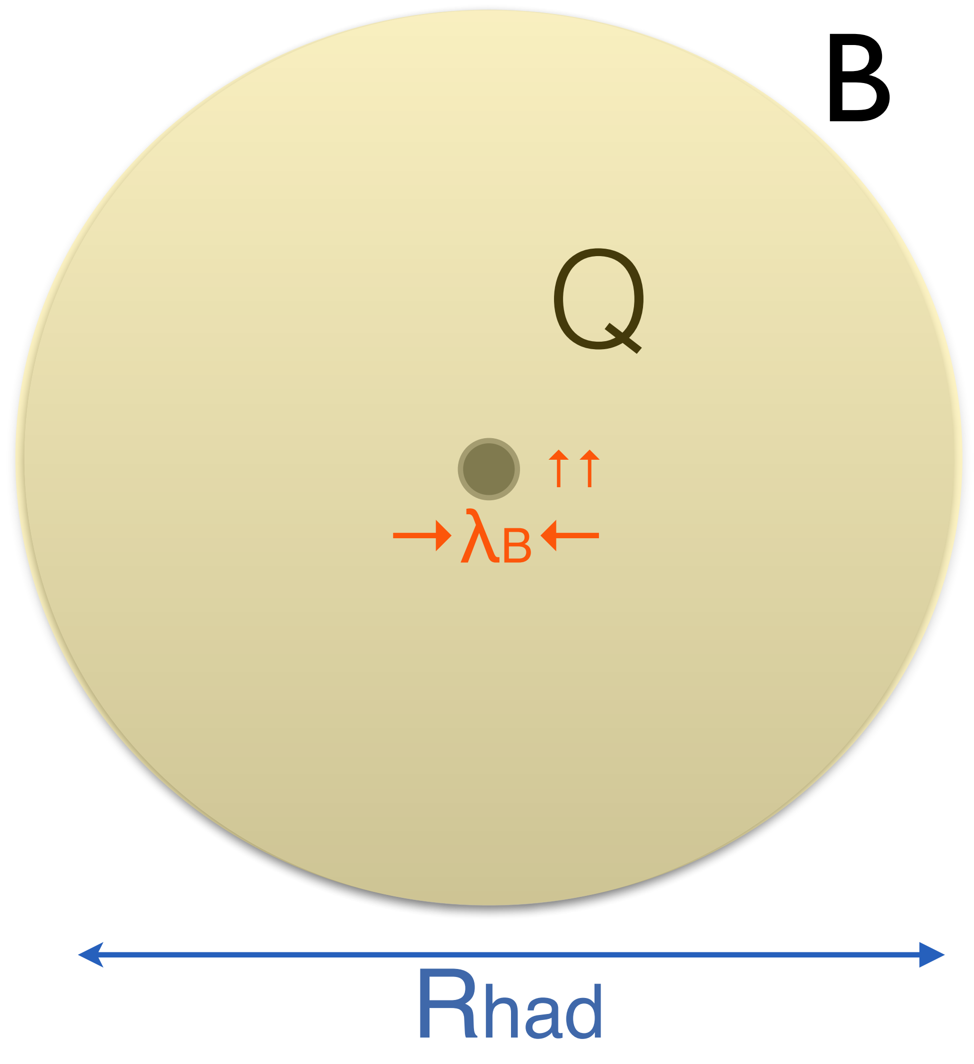

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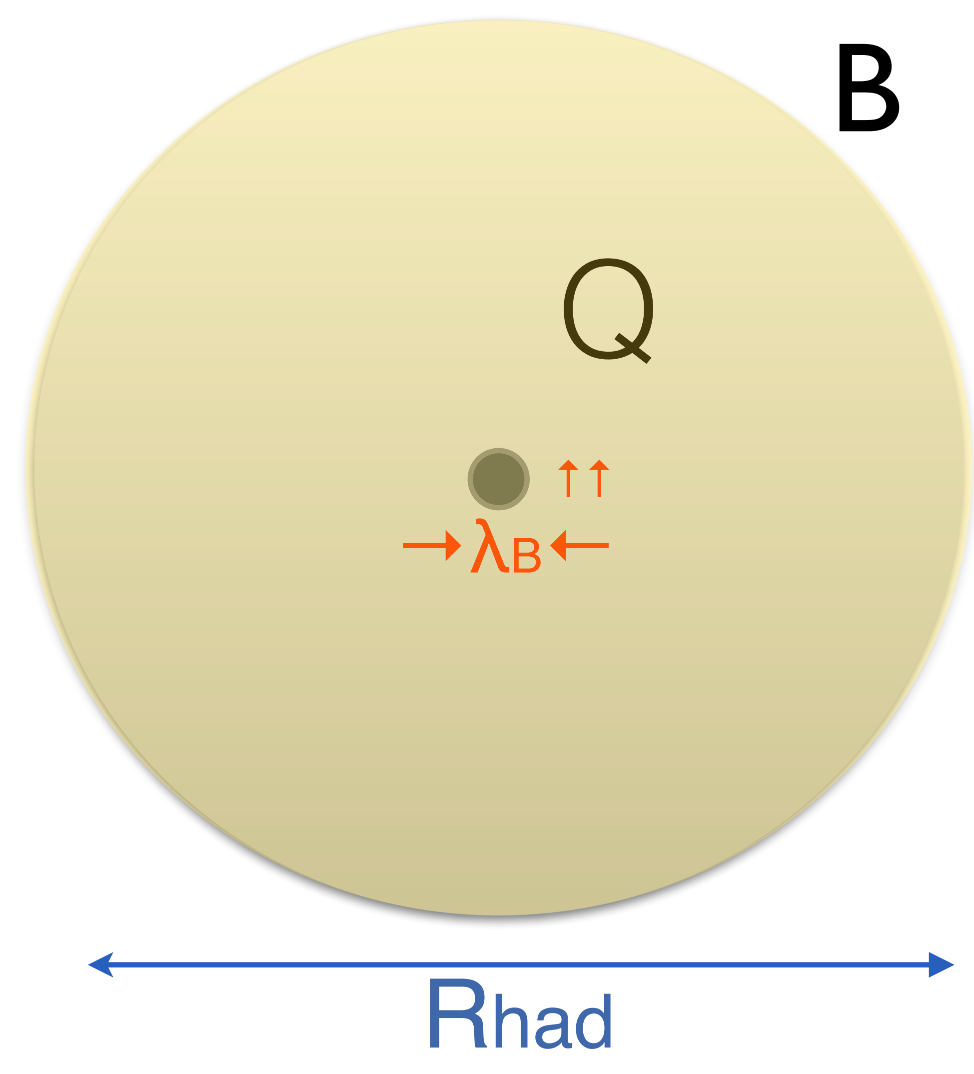
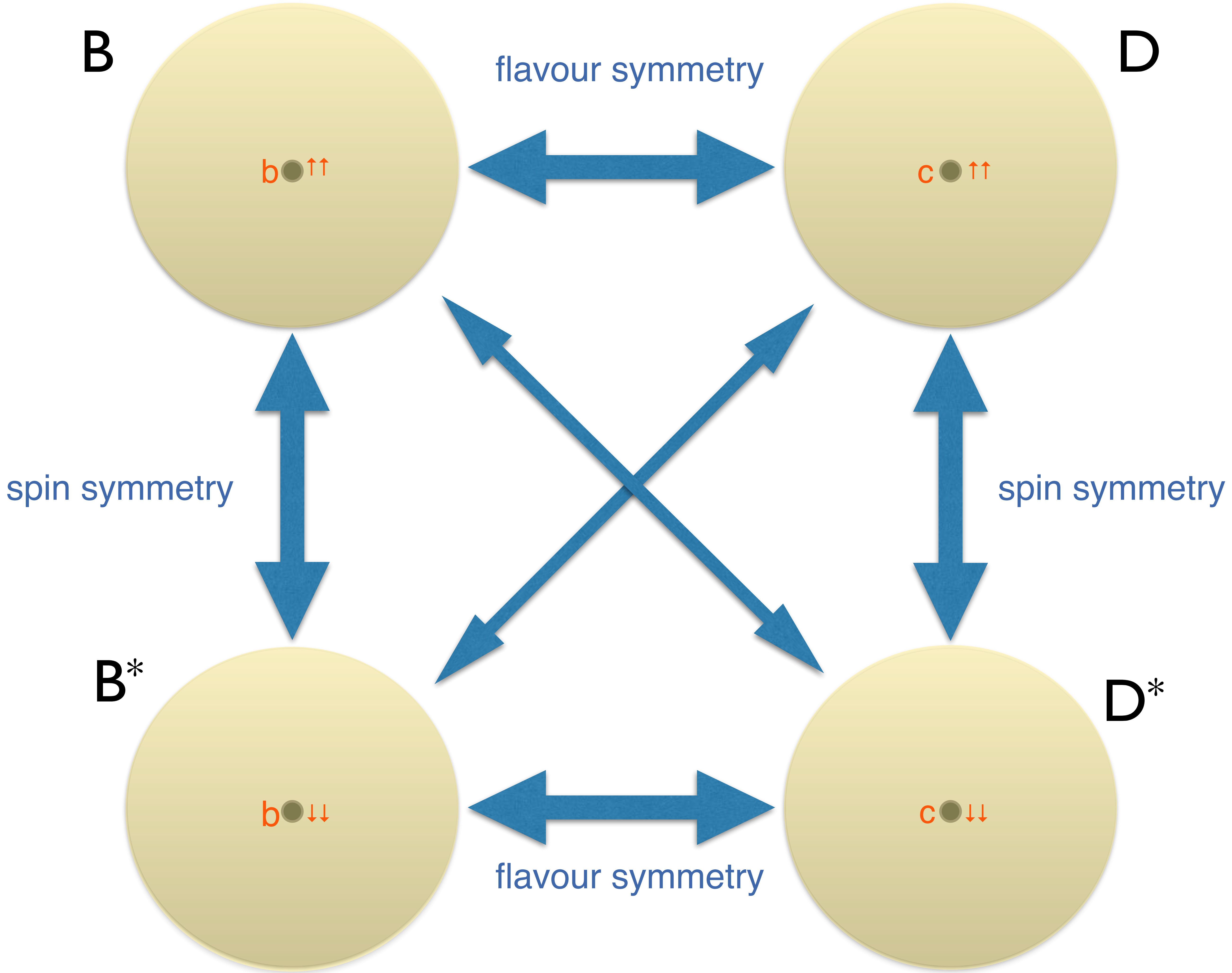


➡ SU($2n_Q$) spin-flavour symmetry: In heavy-quark limit ($m_Q \to \infty$), configuration of light degrees of freedom is independent of the spin and flavor of the heavy quark

Heavy Quark Symmetry

- Relates properties of hadrons:



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✱ What's the physical picture behind?

- Heavy quark carries almost all momentum
- Momentum exchange between heavy quark and light degrees of freedom is predominantly soft (soft gluon exchange):

$$\Delta P_Q = -\Delta P_{light} = O(\Lambda_{QCD}) \Rightarrow \Delta v_Q = O(\Lambda_{QCD}/m_Q)$$

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➡ Georgi: "velocity superselection rule"

Heavy Quark Symmetry

Implications for hadron spectroscopy

- Spin doublets such as (B,B^*) should be degenerate in heavy-quark limit:

$$M_{B^*}-M_B = 46\ \text{MeV} \ll \Lambda_{QCD}$$

- $1/m_Q$ corrections:

$$M_{B^*}-M_B = (c_1-c_0)\ \lambda_2/m_b + O(1/m_b^2)$$

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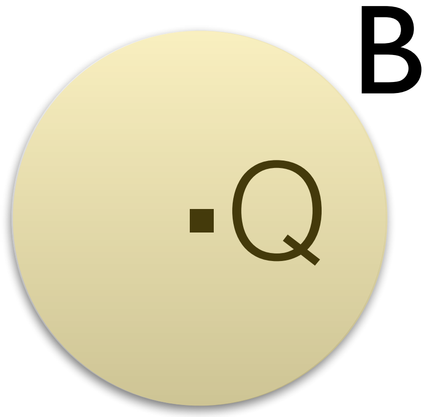
Experimentally: 0.32

Heavy Quark Effective Field Theory

- HQET: simplified description of interaction of heavy quark Q with soft partons (light quark q, and gluon g)



rest frame of B

v^{μ}

small velocity of B

$v^{\mu}=(1,0,0,0)$

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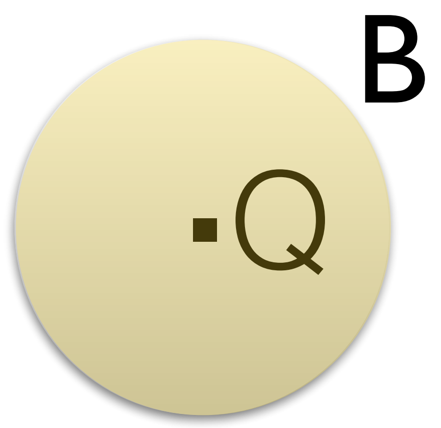

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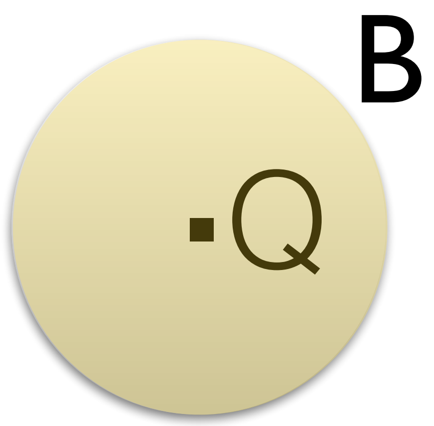

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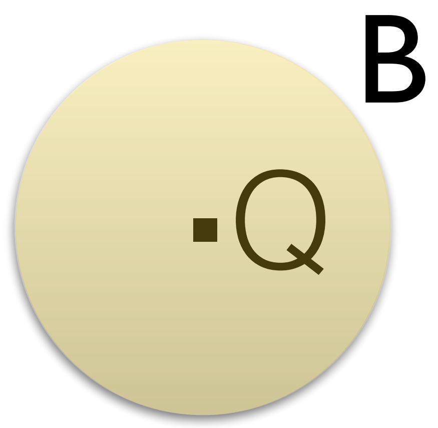

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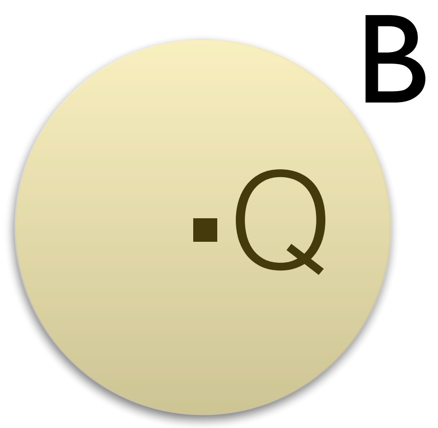

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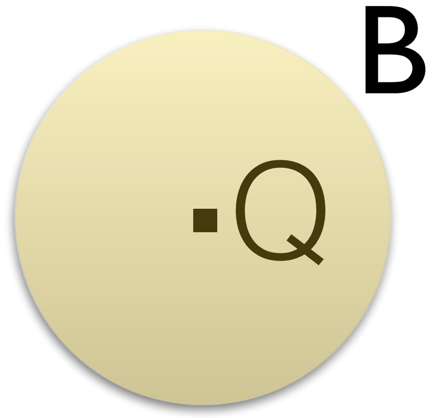

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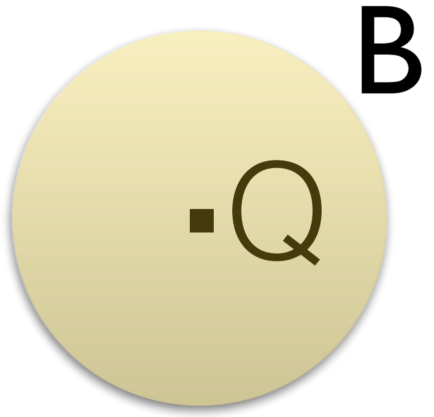

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$$i\vec{D}^{\mu} = iD^{\mu} - v^{\mu}\, iv\cdot D$$

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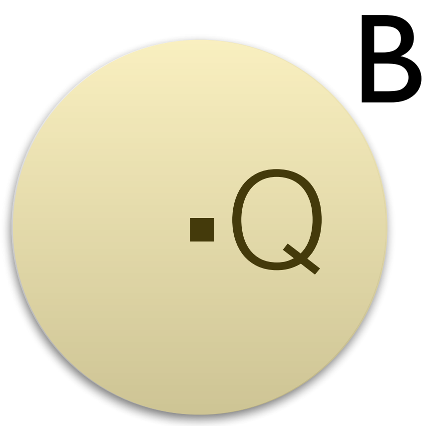

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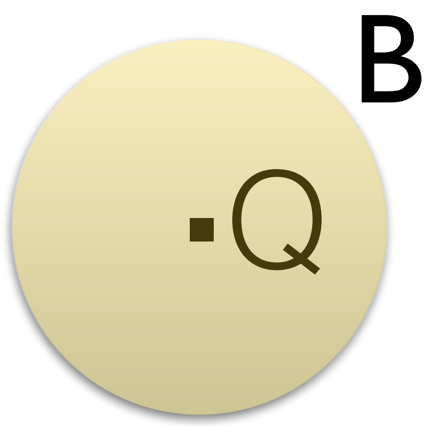
h_V: massless mode
-quantum fluctuation around mass-shell

H_V: massive mode with mass $2m_Q$
-hard quantum fluctuation => integrate out!

Heavy Quark Effective Field Theory

- HQET: simplified description of interaction of heavy quark Q with soft partons (light quark q, and gluon g)



rest frame of B

V^{μ}

small velocity of B

$v^{\mu}=(1,0,0,0)$

$$
\begin{aligned}
\mathcal{L}_Q &= \bar{Q}\,(i\not{D} - m_Q)\,Q \\
&= \bar{h}_v\, i\not{D}\, h_v + \bar{H}_v\,(i\not{D} - 2m_Q)\,H_v + \bar{h}_v\, i\not{D}\, H_v + \bar{H}_v\, i\not{D}\, h_v \\
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$$

- Integrate out H_v by using classical equation of motion:

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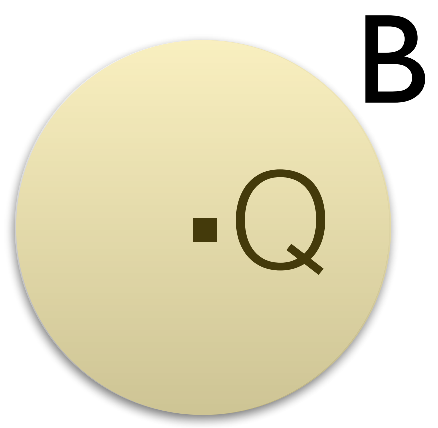

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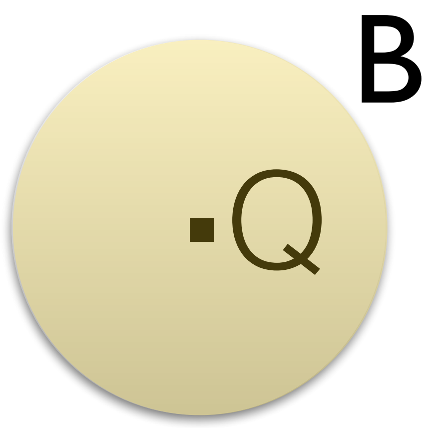

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$$\Rightarrow \quad H_v = \left(\frac{D}{m_Q}\right)h_v \sim \left(\frac{\Lambda_{\rm QCD}}{m_Q}\right)h_v$$

Expansion in local derivative operators justified, since $k \ll m_Q$

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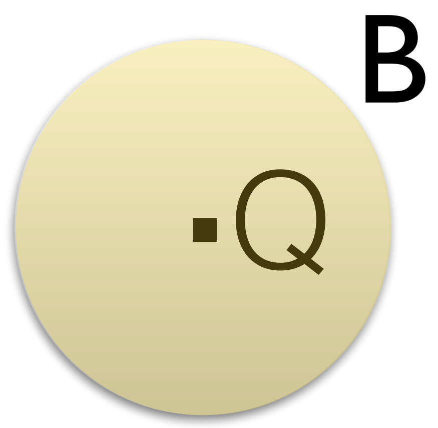

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\mathcal{L}_{\mathrm{HQET}} = \bar{h}_v\, iv\cdot D_s\, h_v + O(1/m_Q)
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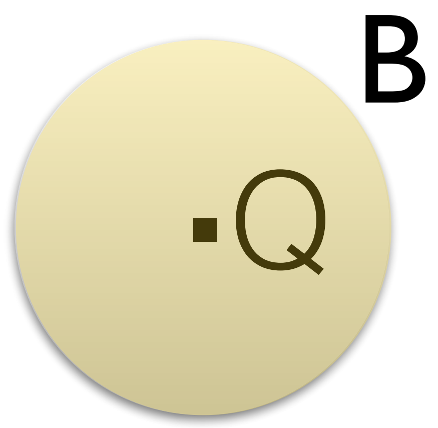

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$$\mathcal{L}_{\rm HQET} = \bar{h}_v\, iv\cdot D_s\, h_v + O(1/m_Q)$$

$$iD_s^\mu = i\partial^\mu + g_s A_s^\mu$$ only soft gluon!

Heavy Quark Effective Field Theory

- What about higher order power corrections?

$$H_v = \frac{1}{2m_Q + iv\cdot D}\, i\vec{\not{D}}\, h_v = \frac{1}{2m_Q}\sum_{n=0}^{\infty}\left(-\frac{iv\cdot D}{2m_Q}\right)^n i\vec{\not{D}}\, h_v$$

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SU($2n_Q$) spin-flavor symmetry

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- Only first terms survives in heavy-quark limit!

Heavy Quark Effective Field Theory

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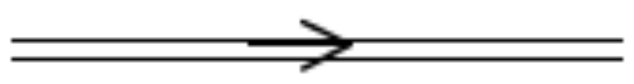
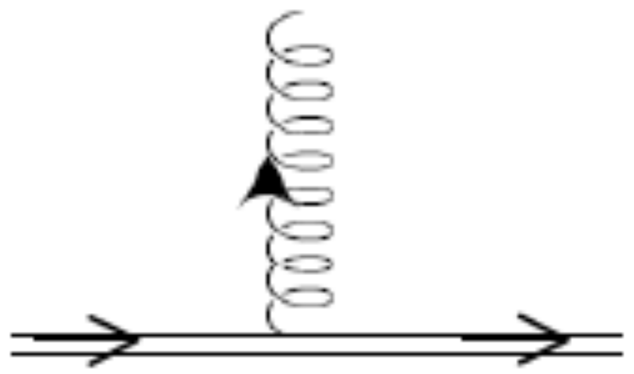
- Feynman rules (HQET):


$= i/v\cdot k$


$= ig_s v^\mu t_a$

Heavy Quark Effective Field Theory

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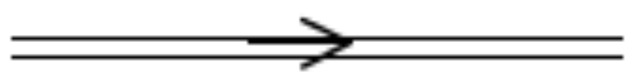

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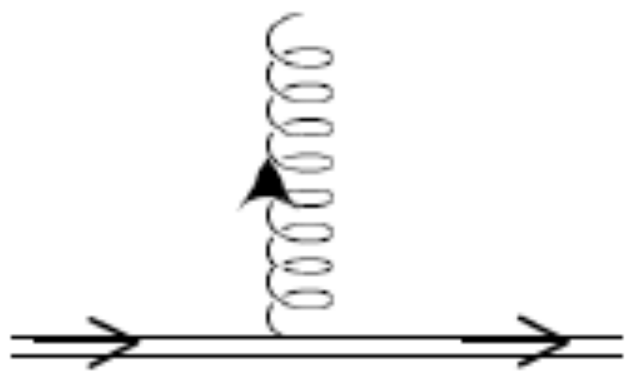

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Much simpler than QCD!!!

Heavy Quark Effective Field Theory

* Symmetries of the effective Lagrangian

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SU(2n_Q) spin-flavor symmetry

Heavy Quark Effective Field Theory

Symmetries of the effective Lagrangian

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SU($2n_Q$) spin-flavor symmetry

- No reference to heavy-quark mass (flavor symmetry)
- Invariance under spin rotations (spin symmetry):

$$h_v \rightarrow (1 + i/2\ \vec{\varepsilon} \cdot \vec{\sigma})\ h_v$$

Heavy Quark Effective Field Theory

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Effective Lagrangian exhibits spin-flavor symmetry at leading order, broken by $O(1/m_Q)$ corrections

Heavy Quark Effective Field Theory

Symmetries of the effective Lagrangian

$$\mathcal{L}_{\rm HQET} = \bar{h}_v \, iv\cdot D_s \, h_v + \frac{1}{2m_Q}\left[\bar{h}_v\,(i\vec{D}_s)^2 h_v + C_{\rm mag}(\mu)\,\frac{g_s}{2}\,\bar{h}_v\,\sigma_{\mu\nu}\,G_s^{\mu\nu} h_v\right] + \ldots .$$

- kinetic energy operator (p^2 / 2m)
- chromo-magnetic interaction

$\sim \vec{\sigma}\cdot\vec{B}$

Heavy Quark Effective Field Theory

Symmetries of the effective Lagrangian

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- kinetic energy operator (p² / 2m)
- chromo-magnetic interaction

$\sim \vec{\sigma} \cdot \vec{B}$

Both operators violate the flavor symmetry, but only the second one breaks the spin symmetry

Heavy Quark Effective Field Theory

Short-distance radiative corrections

- Heavy-quark symmetries also broken by hard gluon exchange

  – effects calculable in perturbation theory

  – in general, they renormalize the coefficients in the effective Lagrangian

- No renormalization at leading order (only WFR)

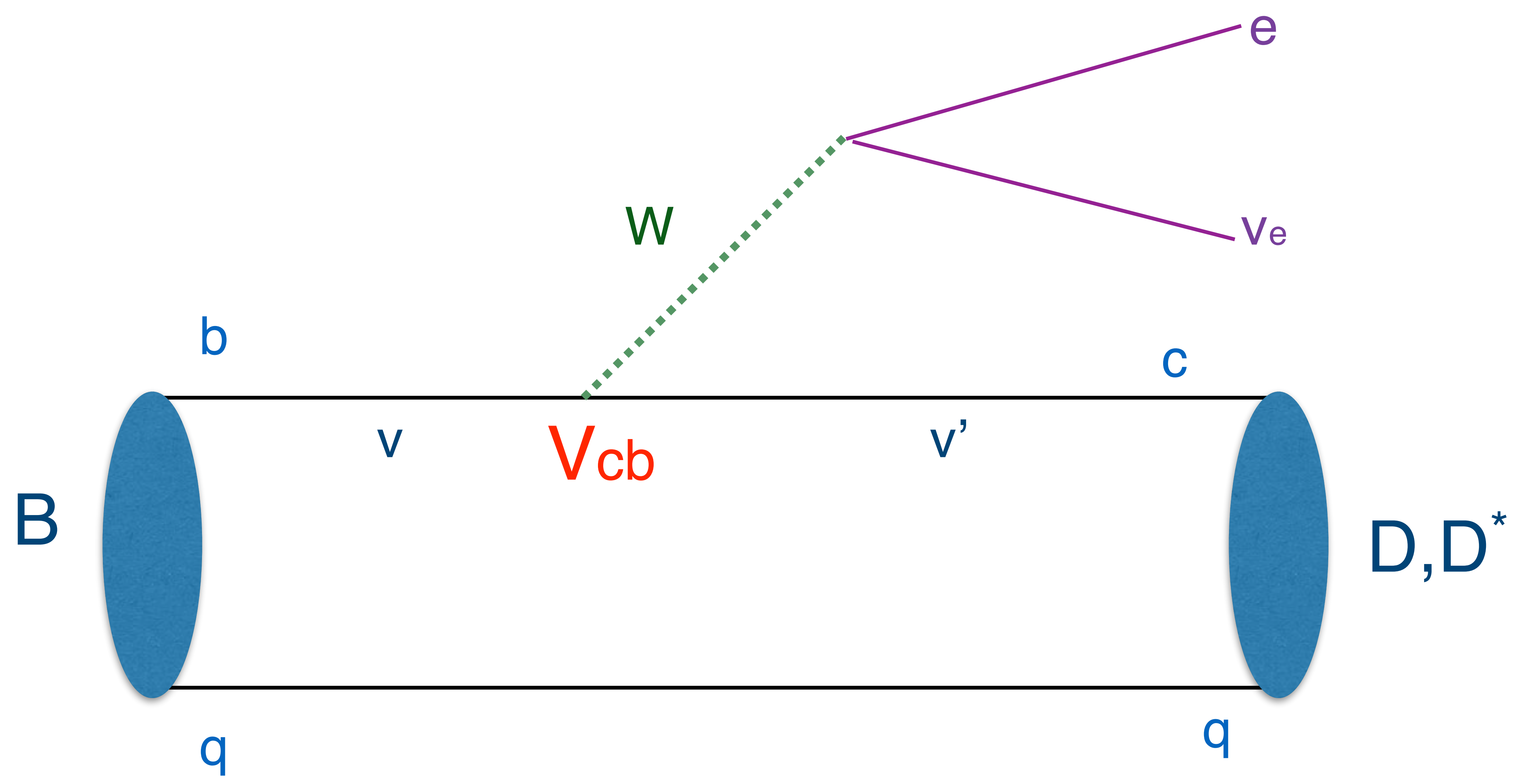
- No renormalization of kinetic operator due to Lorentz invariance ("reparametrization invariance")

- Chromo-magnetic interaction is affected: $C_{mag}(\mu) \neq 1$

Exclusive Semileptonic B Decays: Form factor relations and extraction of $|V_{cb}|$

Form factor relations

- Heavy-quark symmetry allows us to extract the CKM matrix element $|V_{cb}|$ with controlled theoretical uncertainties



- How to deal with confinement effects in this hadronic process?

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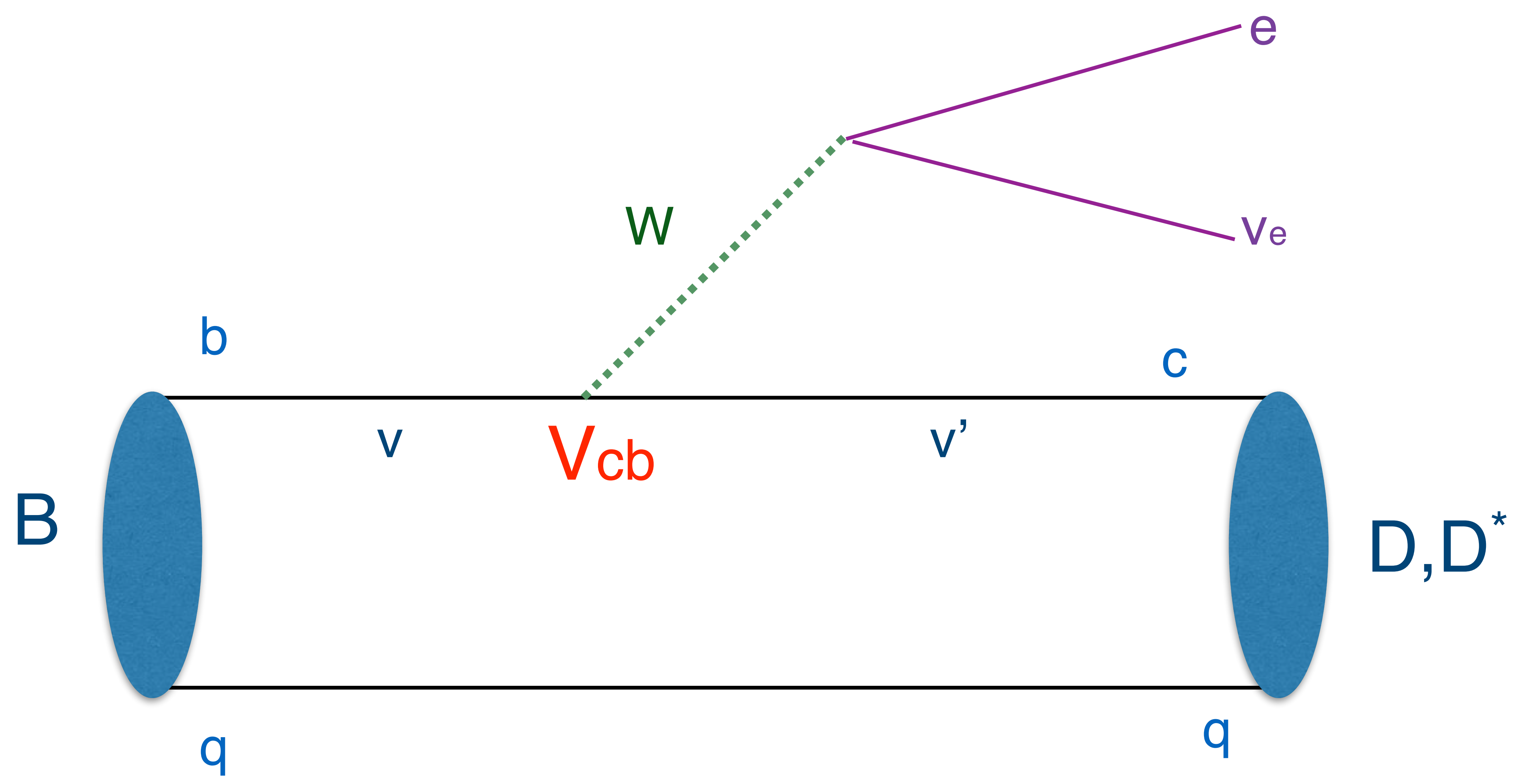


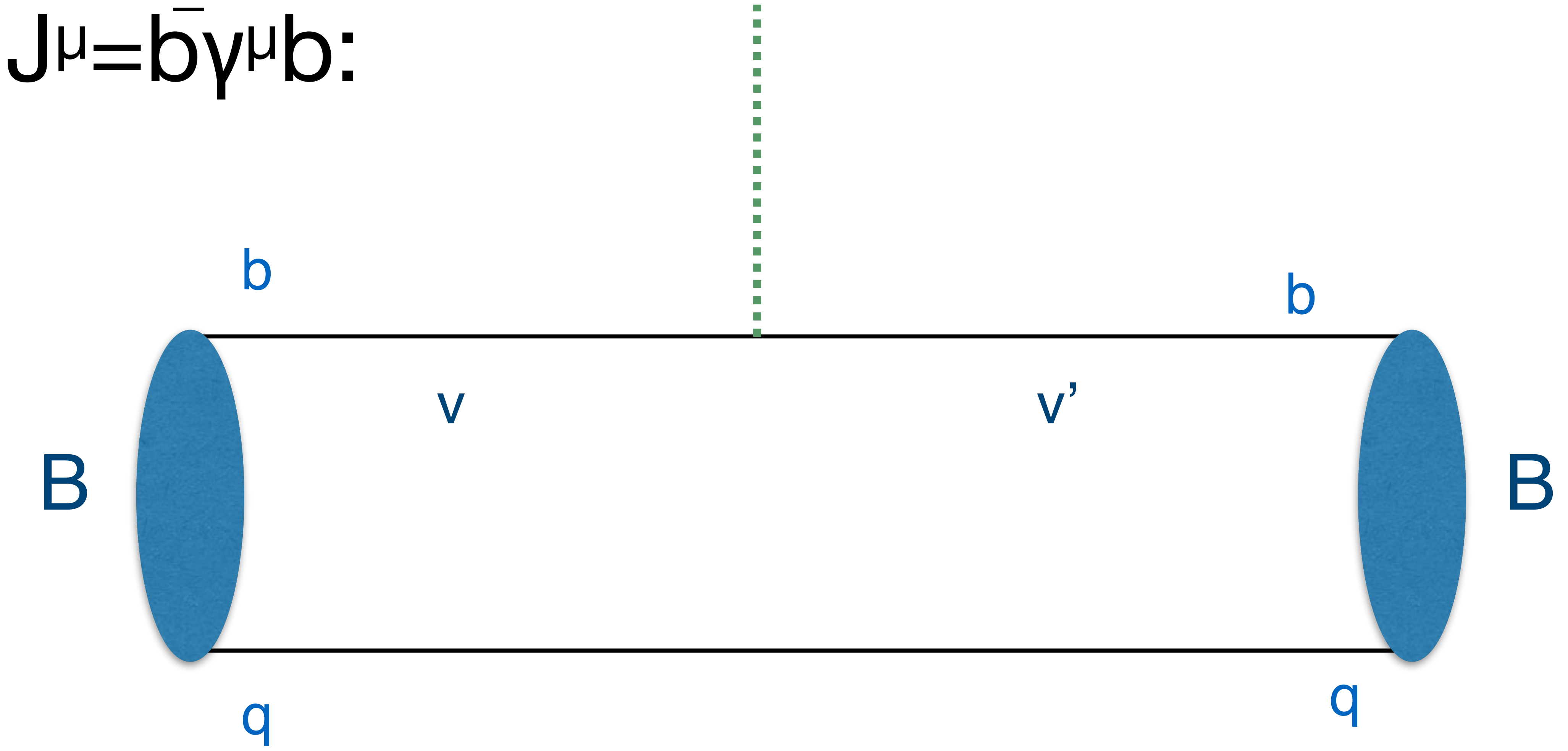
clever use of heavy-quark symmetries allows us to calculate the decay rates at the special kinematic point of maximum momentum transfer to the leptons (v=v')
("zero recoil" point)

- How to deal with confinement effects in this hadronic process?

Exclusive Semileptonic B Decays: Form factor relations and extraction of $|V_{cb}|$

Form factor relations

- Consider elastic scattering of a B meson by the vector current $J^\mu = \bar{b}\gamma^\mu b$:

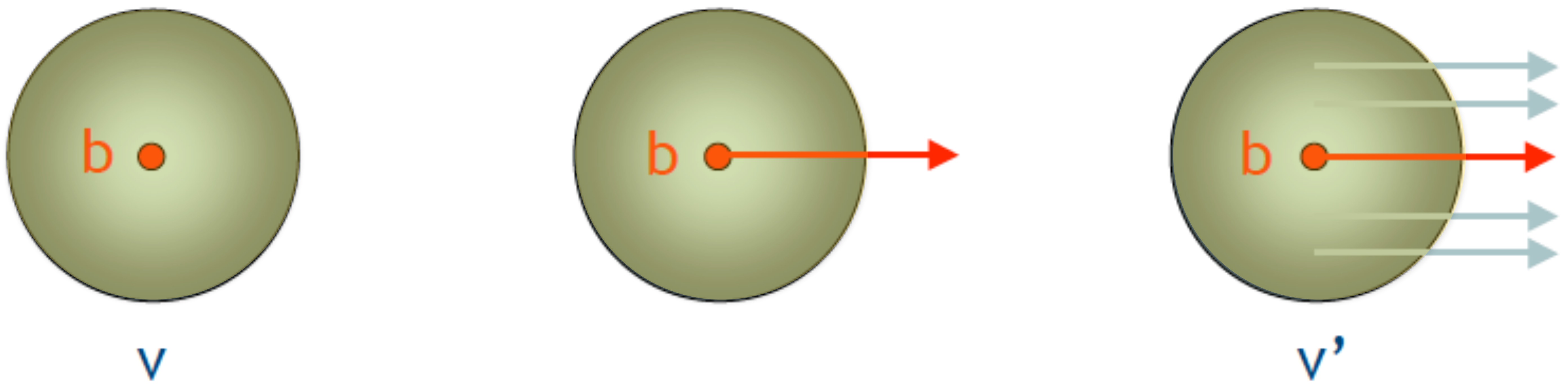


- nothing happens if v=v'; final state remains a B meson with probability 1

- for $v \neq v'$, probability for an elastic transition is less than 1

Exclusive Semileptonic B Decays: Form factor relations and extraction of $|V_{cb}|$

* Form factor relations



- required soft gluon exchange leads to form factor suppression

- for $m_b \rightarrow \infty$, process described by a dimensionless probability function:

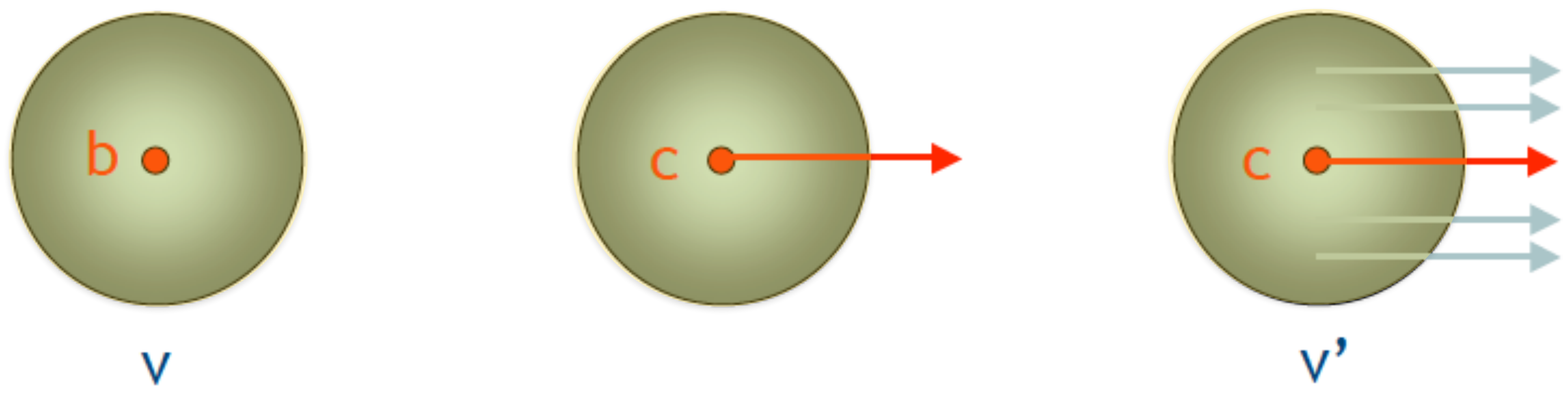
$$\langle P(v')|\bar{h}_{v'}\gamma^\mu h_v|P(v)\rangle = \xi(v\cdot v')(v+v')^\mu$$

with: $\xi(v \cdot v') \leq 1$, with $\xi(1)=1$ (Isgur-Wise function)

Exclusive Semileptonic B Decays: Form factor relations and extraction of $|V_{cb}|$

Form factor relations

- Use flavor symmetry to replace b- by c-quark in final state, thereby obtaining a B→D transition:



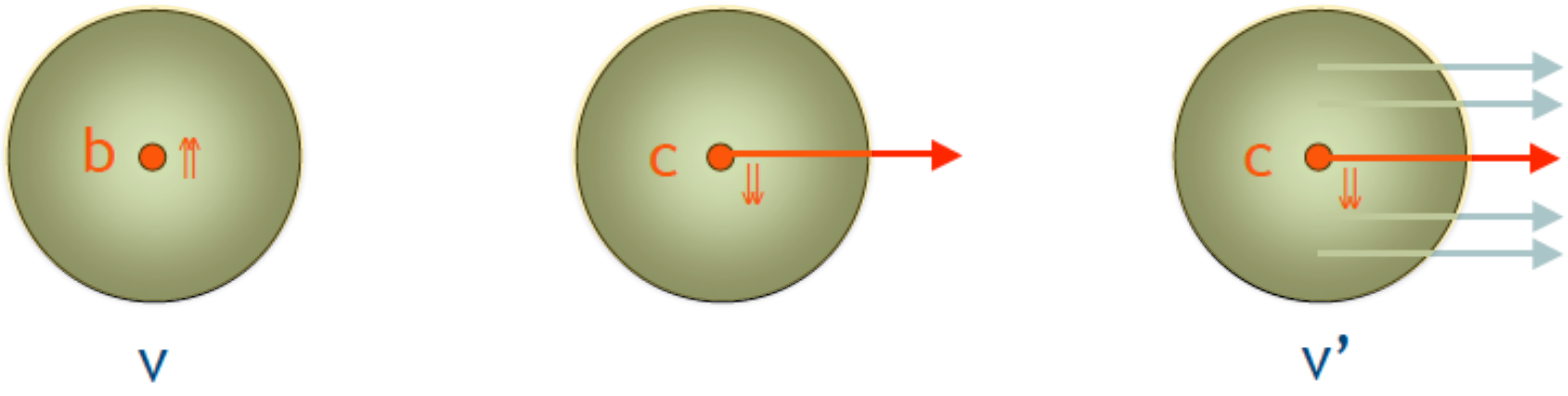
– nothing happens (symmetry in heavy-quark limit)!

$$\langle P'(v')|\bar{h}'_{v'}\gamma^\mu h_v|P(v)\rangle = \xi(v\cdot v')(v+v')^\mu$$

Exclusive Semileptonic B Decays: Form factor relations and extraction of $|V_{cb}|$

Form factor relations

- Next, use spin symmetry to flip spin of c-quark in final state, thereby obtaining a B→D* transition:



– current gets transformed, but else nothing happens:

$$\langle V'(v',\epsilon)|\bar{h}'_{v'}\gamma^{\mu}(1-\gamma_5)h_v|P(v)\rangle = i\epsilon^{\mu\nu\alpha\beta}\epsilon^*_{\nu}v'_{\alpha}v_{\beta}\xi(v\cdot v') - \Big\{\epsilon^{*\mu}(v\cdot v'+1) - v'^{\mu}\epsilon^*\cdot v\Big\}\xi(v\cdot v')$$

Exclusive Semileptonic B Decays: Form factor relations and extraction of $|V_{cb}|$

Form factor relations

- In general, these processes are described by six a priori independent hadronic form factors:

$$\langle D(v') | V_\mu^{cb} | \bar{B}(v) \rangle = \sqrt{m_B m_D} \left\{ \xi_+(v\cdot v')(v+v')_\mu + \xi_-(v\cdot v')(v-v')_\mu \right\}$$

$$\langle D^*(v') | V_\mu^{cb} | \bar{B}(v) \rangle = i\sqrt{m_B m_{D^*}}\, \xi_V(v\cdot v')\, \epsilon_{\mu\nu\alpha\beta}\, \epsilon^{*\nu} v'^\alpha v^\beta ,$$

$$\langle D^*(v') | A_\mu^{cb} | \bar{B}(v) \rangle = \sqrt{m_B m_{D^*}} \left\{ \xi_{A_1}(v\cdot v')(v\cdot v'+1)\, \epsilon_\mu^* - \xi_{A_2}(v\cdot v')\, \epsilon^*\cdot v\, v_\mu - \xi_{A_3}(v\cdot v')\, \epsilon^*\cdot v\, v'_\mu \right\}$$

⇒ all equal in heavy-quark limit: $\xi_+ = \xi_V = \xi_{A_1} = \xi_{A_3} = \xi(v\cdot v')$

$\xi_- = \xi_{A_2} = 0$

Exclusive Semileptonic B Decays: Form factor relations and extraction of $|V_{cb}|$

Semileptonic decay rates

- These form factors describe the semileptonic decays $B \to D^{(*)} l \nu$ entirely; in terms of $y = v \cdot v'$:

$$\frac{d\Gamma}{dy} = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 m_B^2 m_{D^{(*)}}^3 (y^2 - 1)^{1/2} (y+1)^2 \times F(r, y)$$

$r = M_{D^{(*)}} / M_B$

with:

$$F(r,y) = \begin{cases} (1+r)^2 \frac{y-1}{y+1} \left[\xi_+(y) - \frac{1-r}{1+r} \xi_-(y) \right]^2 & ; \bar{B} \to D \, \ell \, \bar{\nu}_\ell \\ 2(1 - 2yr + r^2) \left[\xi_{A_1}^2(y) + \frac{y-1}{y+1} \xi_V^2(y) \right] & ; \bar{B} \to D_T^* \, \ell \, \bar{\nu}_\ell \\ \left[(y-r)\, \xi_{A_1}(y) - (y-1) \left(\xi_{A_3}(y) + r\, \xi_{A_2}(y) \right) \right] & ; \bar{B} \to D_L^* \, \ell \, \bar{\nu}_\ell \end{cases}$$

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➡ rates are absolutely noramlized at y=1 (zero recoil point)!

Exclusive Semileptonic B Decays: Form factor relations and extraction of $|V_{cb}|$

Semileptonic decay rates

- Corrections to heavy-quark limit:
  - perturbative corrections (hard gluons) known to $O(\alpha_s^2)$
  - power corrections estimated to $O(1/m_Q^2)$
- Luke's theorem: $B \to D^* l\nu$ decay rate does not receive first-order 1/mQ corrections at y=1 (y=v·v')
- Results:

$$F_{B\to D^*}(1) = 1 - (0.040\pm0.007)_{pert} - (0.055\pm0.025)_{power}$$
$$= 0.91\pm0.03$$

Exclusive Semileptonic B Decays: Form factor relations and extraction of $|V_{cb}|$

Semileptonic decay rates

- Corrections to heavy-quark limit:
  - perturbative corrections (hard gluons) known to $O(\alpha_s^2)$
  - power corrections estimated to $O(1/m_Q^2)$
- Luke's theorem: $B \to D^* l\nu$ decay rate does not receive first-order $1/m_Q$ corrections at $y=1$ ($y = v \cdot v'$)
- Results:

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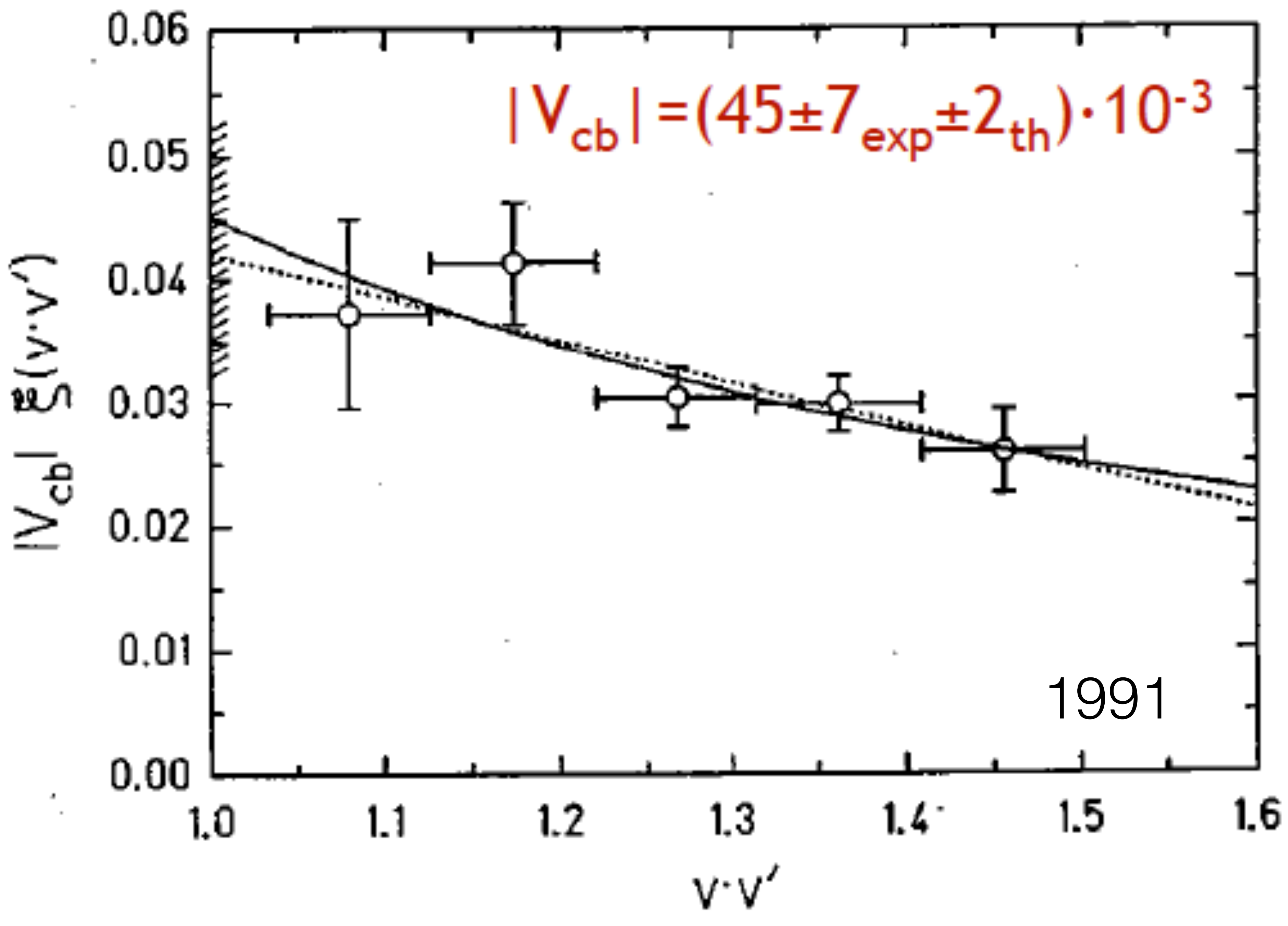
→ allows for measurement of $|V_{cb}|$ with theoretical accuracy of 3%

Exclusive Semileptonic B Decays: Form factor relations and extraction of $|V_{cb}|$

Extraction of $|V_{cb}|$

- Extrapolation of the spectrum $d\Gamma/dy$ measured in $B \to D^* l \nu$ decay to zero-recoil point $y=1$:



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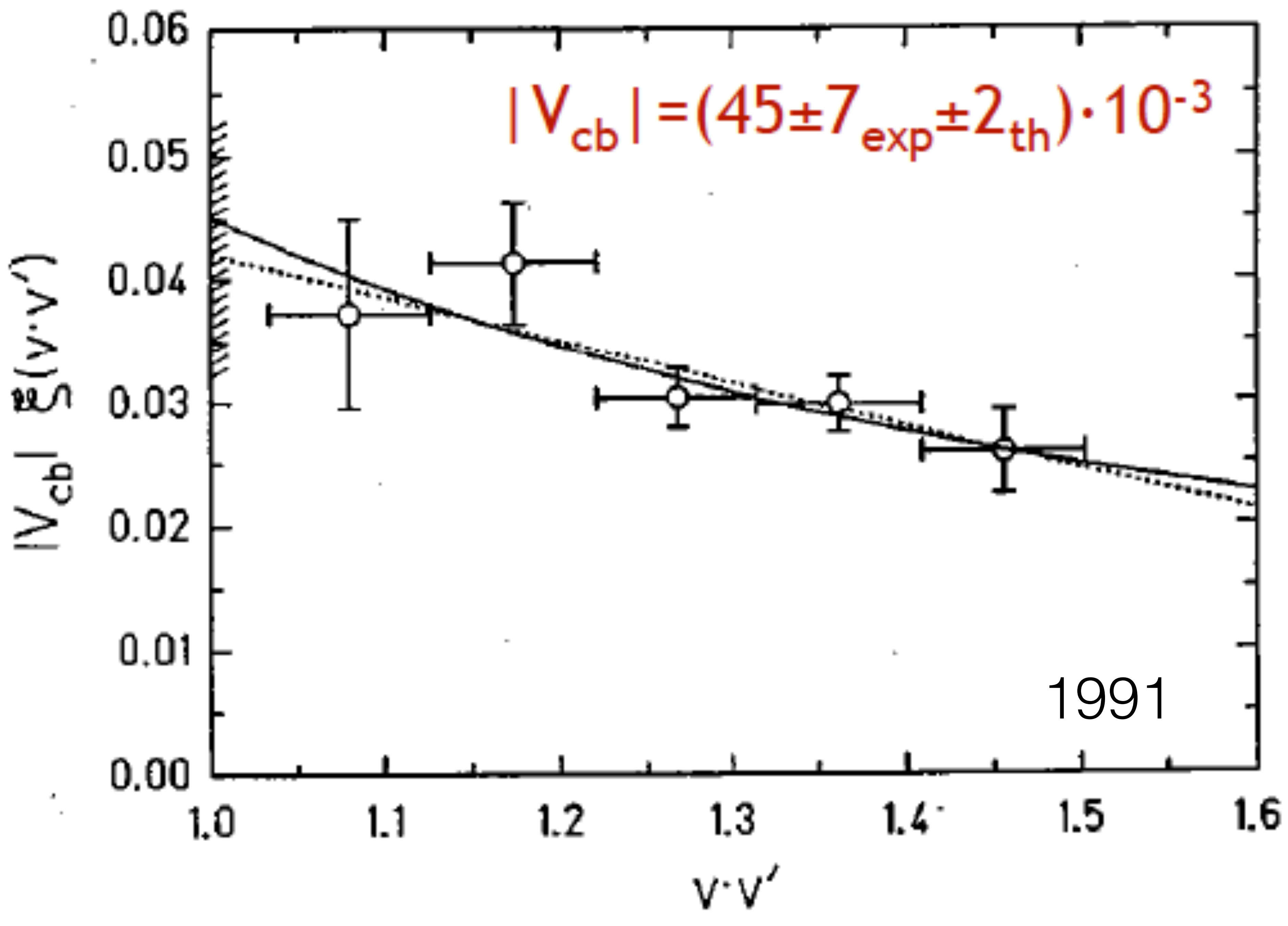

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best known entry in CKM matrix after |Vud| and |Vus|

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SM EFT is defined as a double expansion in $1/\Lambda$ and $1/\Lambda_L$.
The expansion is useful assuming $v \ll \Lambda$ and $v \ll \Lambda_L$

SM EFT: dimension 5 operator

- Unique operators $[O_5]_{IJ} = (\epsilon_{ij} H^i L_I^j)(\epsilon_{kl} H^i L_J^j)$ $I, J = 1, 2, 3$

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* dimension-5 operators in the SM EFT Lagrangian makes them practically unobservable at LHC and foreseeable future colliders!

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D=6 Operators can be probed in the LHC and HL-LHC!

SM EFT: dimension 6 operator

* Bosonic Operators for D=6

Bosonic CP-even		Bosonic CP-odd	
O_H	$(H^\dagger H)^3$		
$O_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$		
O_{HD}	$\left\|H^\dagger D_\mu H\right\|^2$		
O_{HG}	$H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu}$	$O_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}^a_{\mu\nu} G^a_{\mu\nu}$
O_{HW}	$H^\dagger H\, W^i_{\mu\nu} W^i_{\mu\nu}$	$O_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^i_{\mu\nu} W^i_{\mu\nu}$
O_{HB}	$H^\dagger H\, B_{\mu\nu} B_{\mu\nu}$	$O_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B_{\mu\nu}$
O_{HWB}	$H^\dagger \sigma^i H\, W^i_{\mu\nu} B_{\mu\nu}$	$O_{H\widetilde{W}B}$	$H^\dagger \sigma^i H\, \widetilde{W}^i_{\mu\nu} B_{\mu\nu}$
O_W	$\epsilon^{ijk} W^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$	$O_{\widetilde{W}}$	$\epsilon^{ijk} \widetilde{W}^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$
O_G	$f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$	$O_{\widetilde{G}}$	$f^{abc} \widetilde{G}^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$

SM EFT: dimension 6 operator

- Fermionic Operators for D=6

Yukawa

$[O^\dagger_{eH}]_{IJ}$	$H^\dagger H e^c_I H^\dagger \ell_J$
$[O^\dagger_{uH}]_{IJ}$	$H^\dagger H u^c_I \widetilde{H}^\dagger q_J$
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Vertex

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Dipole

$[O^\dagger_{eW}]_{IJ}$	$e^c_I \sigma_{\mu\nu} H^\dagger \sigma^i \ell_J W^i_{\mu\nu}$
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$(\bar{R}R)(\bar{R}R)$

O_{ee}	$\eta(e^c\sigma_\mu \bar{e}^c)(e^c\sigma_\mu \bar{e}^c)$
O_{uu}	$\eta(u^c\sigma_\mu \bar{u}^c)(u^c\sigma_\mu \bar{u}^c)$
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$O^{(8)}_{ud}$	$(u^c\sigma_\mu T^a \bar{u}^c)(d^c\sigma_\mu T^a \bar{d}^c)$

$(\bar{L}L)(\bar{R}R)$

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O'_{qq}	$\eta(\bar{q}\bar{\sigma}_\mu \sigma^i q)(\bar{q}\bar{\sigma}_\mu \sigma^i q)$
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O_{quqd}	$(u^c q^j)\epsilon_{jk}(d^c q^k)$
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Vertex

$[O_{H\ell}]_{IJ}$	$i\bar{\ell}_I \bar{\sigma}_\mu \ell_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O^{(3)}_{H\ell}]_{IJ}$	$i\bar{\ell}_I \sigma^i \bar{\sigma}_\mu \ell_J H^\dagger \sigma^i \overleftrightarrow{D}_\mu H$
$[O_{He}]_{IJ}$	$ie^c_I \sigma_\mu \bar{e}^c_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O_{Hq}]_{IJ}$	$i\bar{q}_I \bar{\sigma}_\mu q_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O^{(3)}_{Hq}]_{IJ}$	$i\bar{q}_I \sigma^i \bar{\sigma}_\mu q_J H^\dagger \sigma^i \overleftrightarrow{D}_\mu H$
$[O_{Hu}]_{IJ}$	$iu^c_I \sigma_\mu \bar{u}^c_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O_{Hd}]_{IJ}$	$id^c_I \sigma_\mu \bar{d}^c_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O_{Hud}]_{IJ}$	$iu^c_I \sigma_\mu \bar{d}^c_J \tilde{H}^\dagger D_\mu H$

$[O^\dagger_{dB}]_{IJ}$	$d^c_I \sigma_{\mu\nu} H^\dagger q_J B_{\mu\nu}$

$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$O'_{\ell q}$	$(\bar{\ell}\bar{\sigma}_\mu \sigma^i \ell)(\bar{q}\bar{\sigma}_\mu \sigma^i q)$	$O_{\ell e d q}$	$(\bar{\ell}\bar{e}^c)(d^c q)$

Warsaw basis => total 2499 independent parameters
(Grzadkowski, Iskrzynski, Misiak, and Rosiek)

SILH basis (Giudice, Grojean, Pomarol, and Rattazzi)

Systematic method for counting # of operators: using conformal group
(Henning, Lu, Melia, and Murayama)

Summary

- Effective field theory allows separation of different scales (separation of calculable parts and nonperturbative parts)
- Any sensitivity to high scales (including to physics beyond the Standard Model) can be treated using perturbative methods
- For Heavy flavor physics, when there is no heavy particle to integrate out, we can integrate out all short-distance fluctuations associated with scales $\gg \Lambda_{QCD}$
- Heavy Quark Symmetry (HQS): SU($2n_Q$) spin-flavour symmetry: In heavy-quark limit ($m_Q \to \infty$), configuration of light degrees of freedom is independent of the spin and flavor of the heavy quark
- Systematic way of using HQS: HQET
- Also other EFT, e.g. NRQCD, SCET,…
- Higher Dimensional Operator Expansion for BSM: SMEFT
- Much more!!