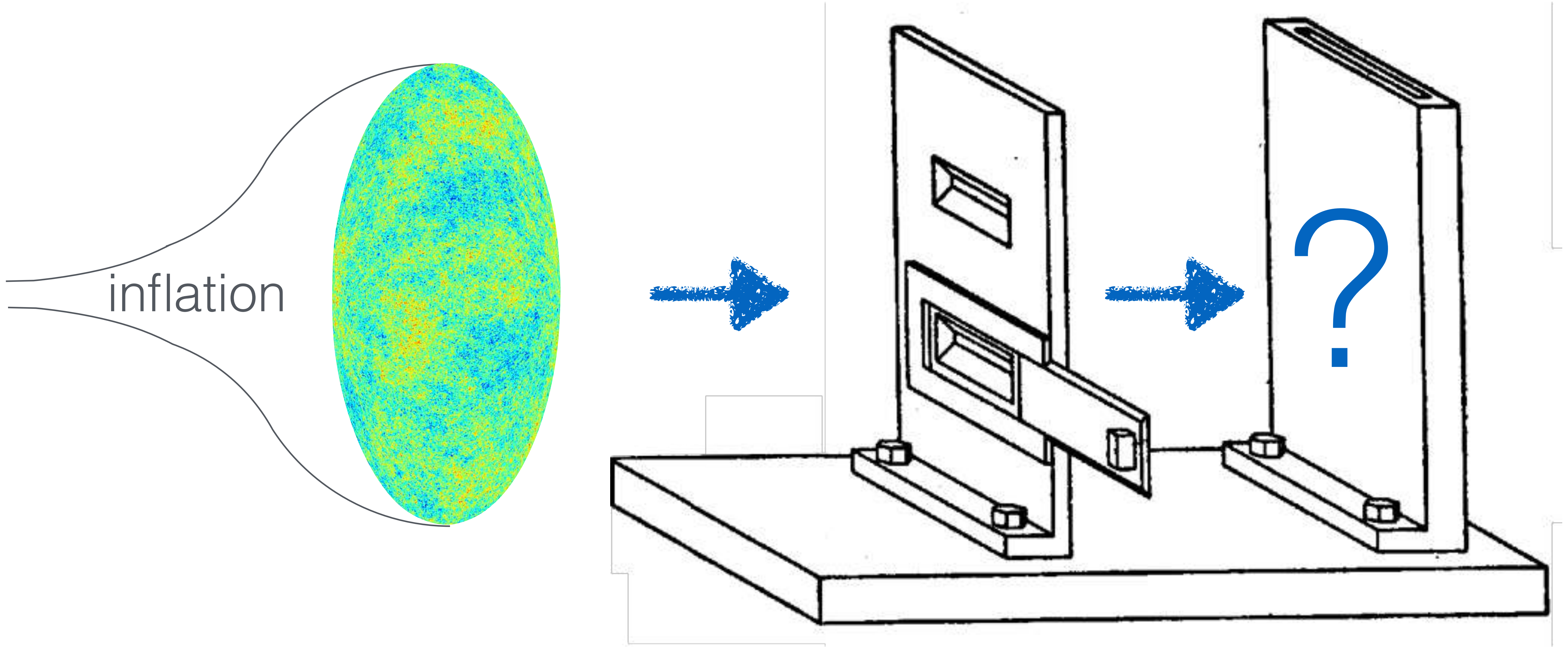
Can we prove that cosmic structures are of quantum-mechanical origins?



Vincent Vennin

Yonsei-Saga Joint Workshop XVIII
27 January 2022

The universe as a quantum lab

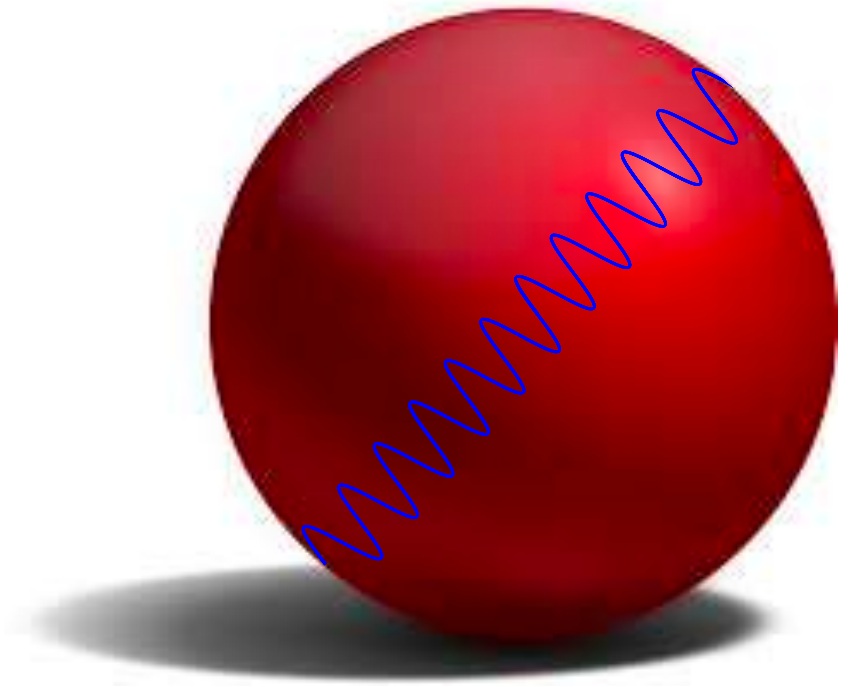
Credit: ESO/M. Kornmesser

Cosmic Inflation

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\vec{x}^2$$

Hubble parameter $H = \dot{a}/a$

H^{-1} : characteristic time scale, or length scale ($c = 1$), of the expansion



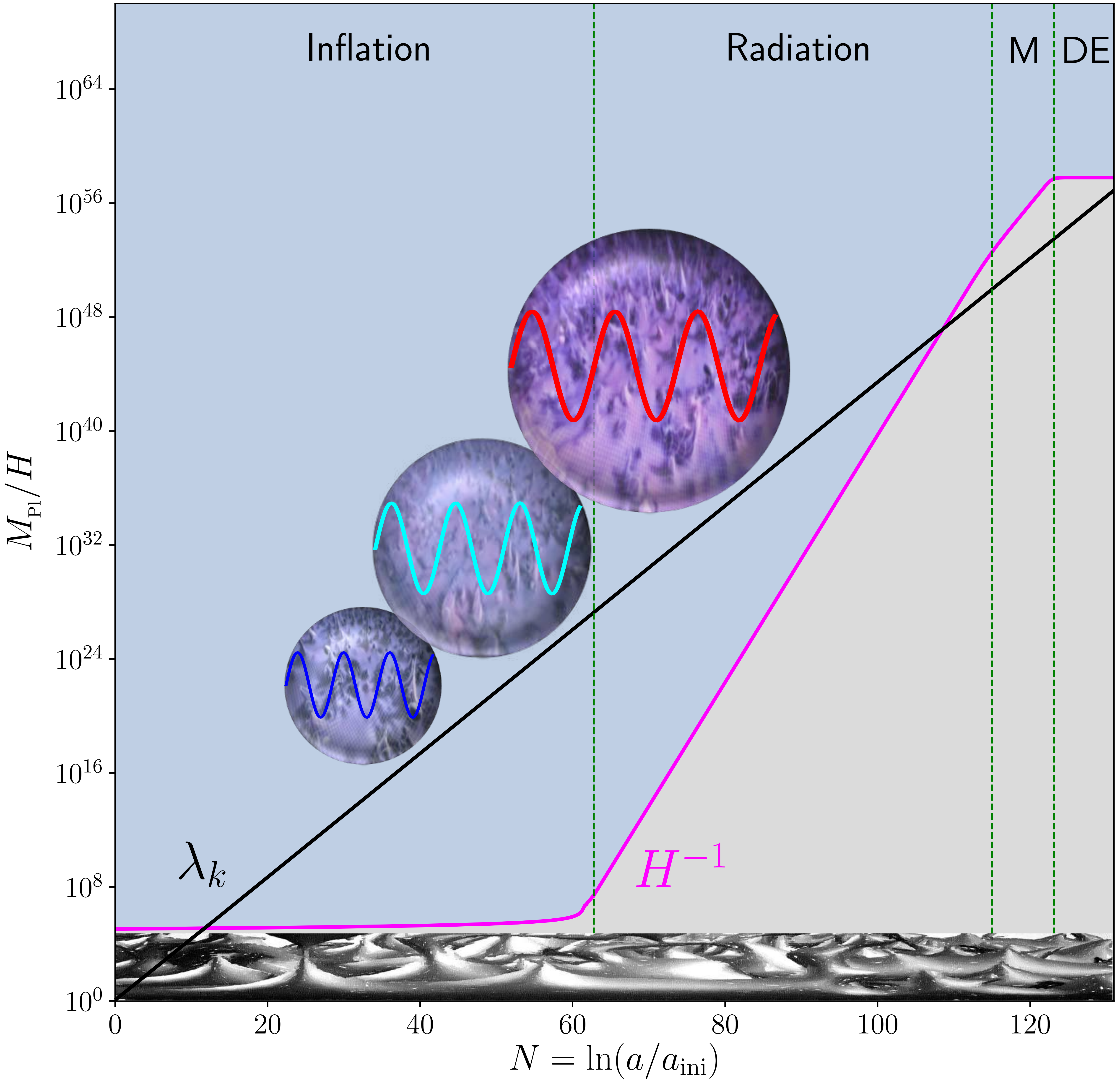
$\lambda \ll H^{-1}$

Insensitive to space-time curvature

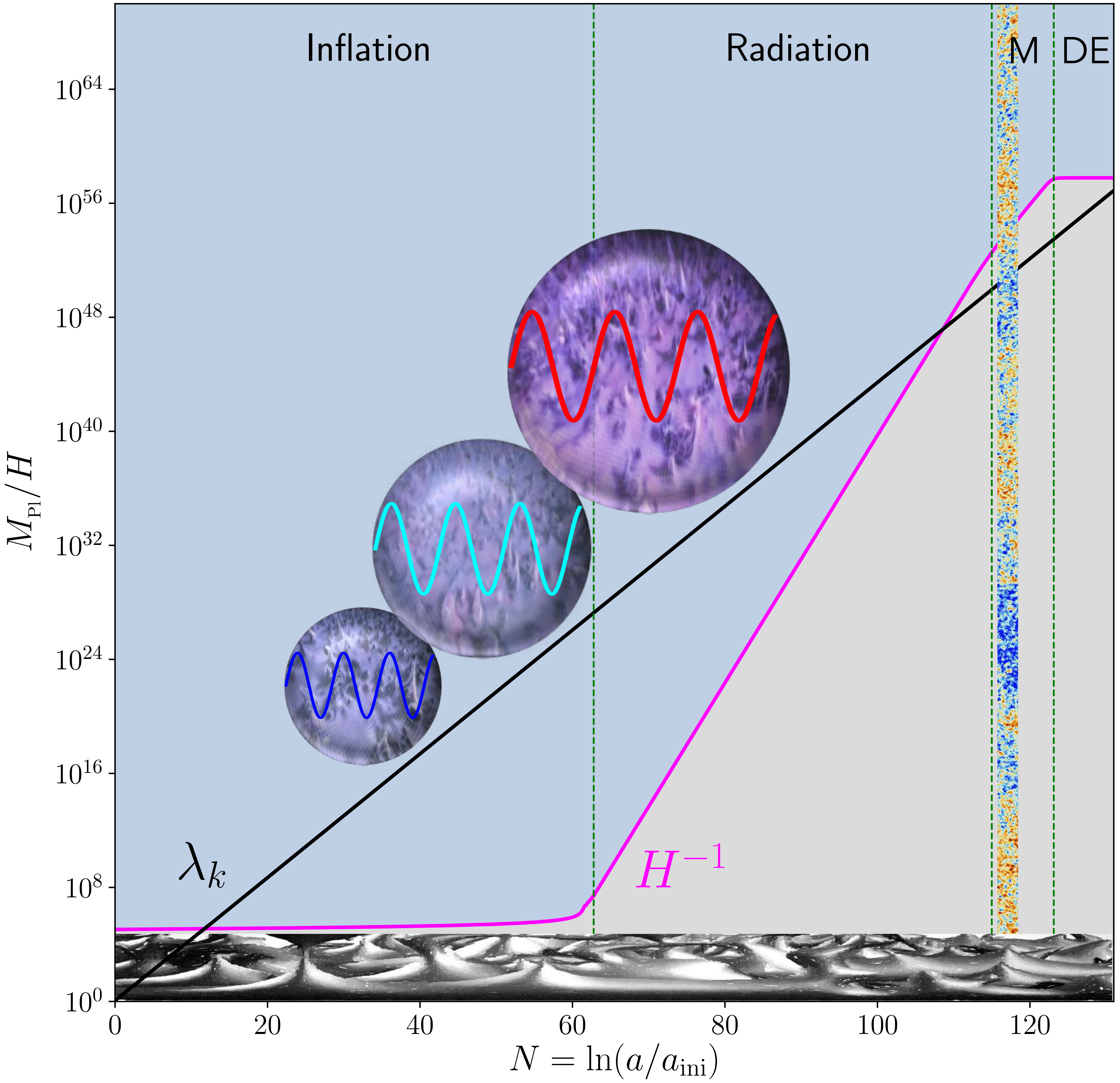
$\lambda \gtrsim H^{-1}$

Feels space-time curvature

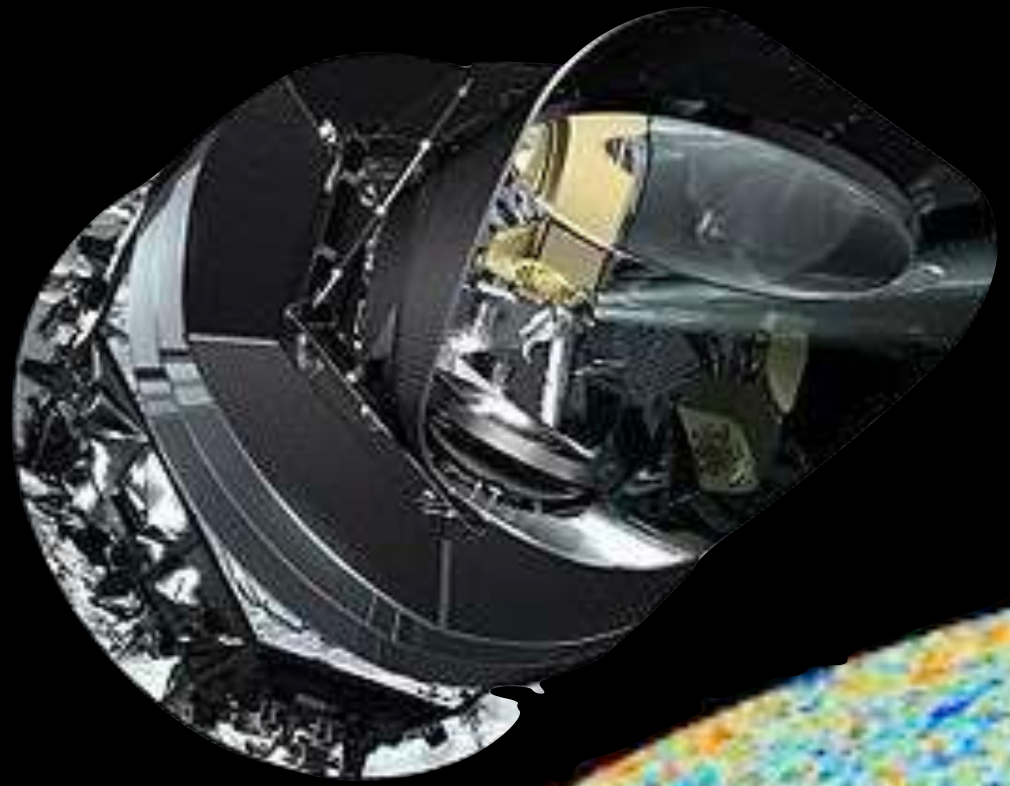
Cosmic Inflation

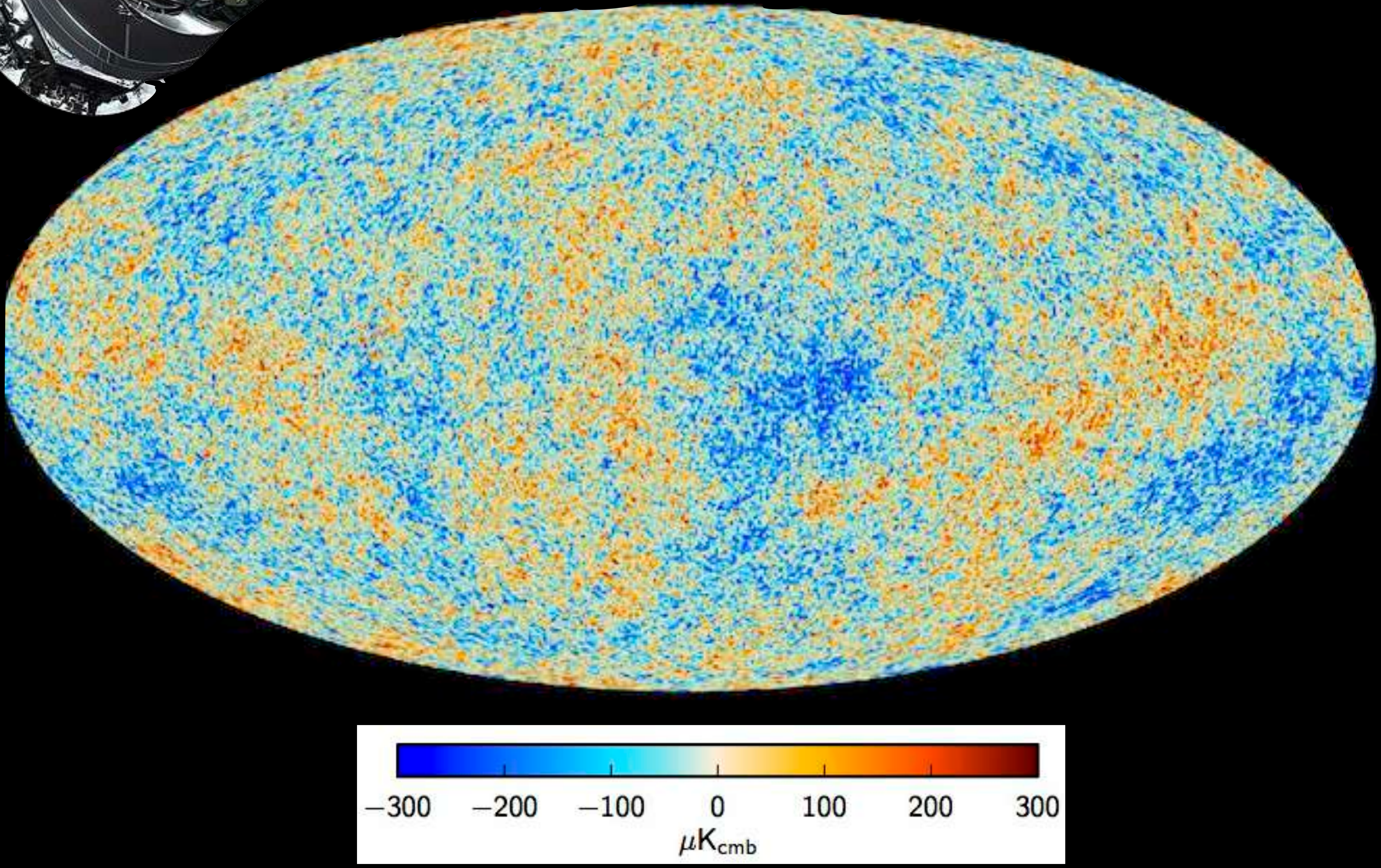


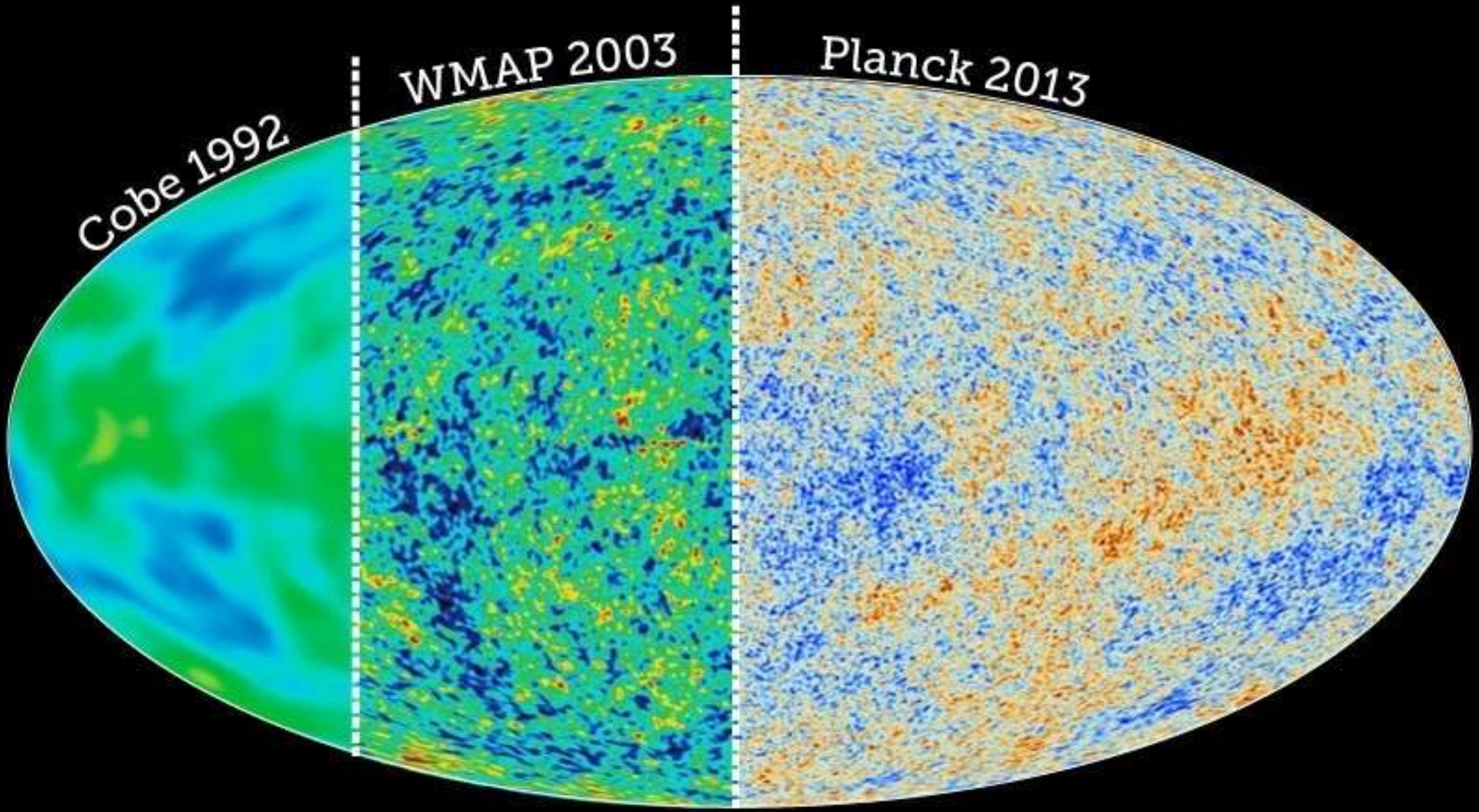
Cosmic Inflation



Planck satellite





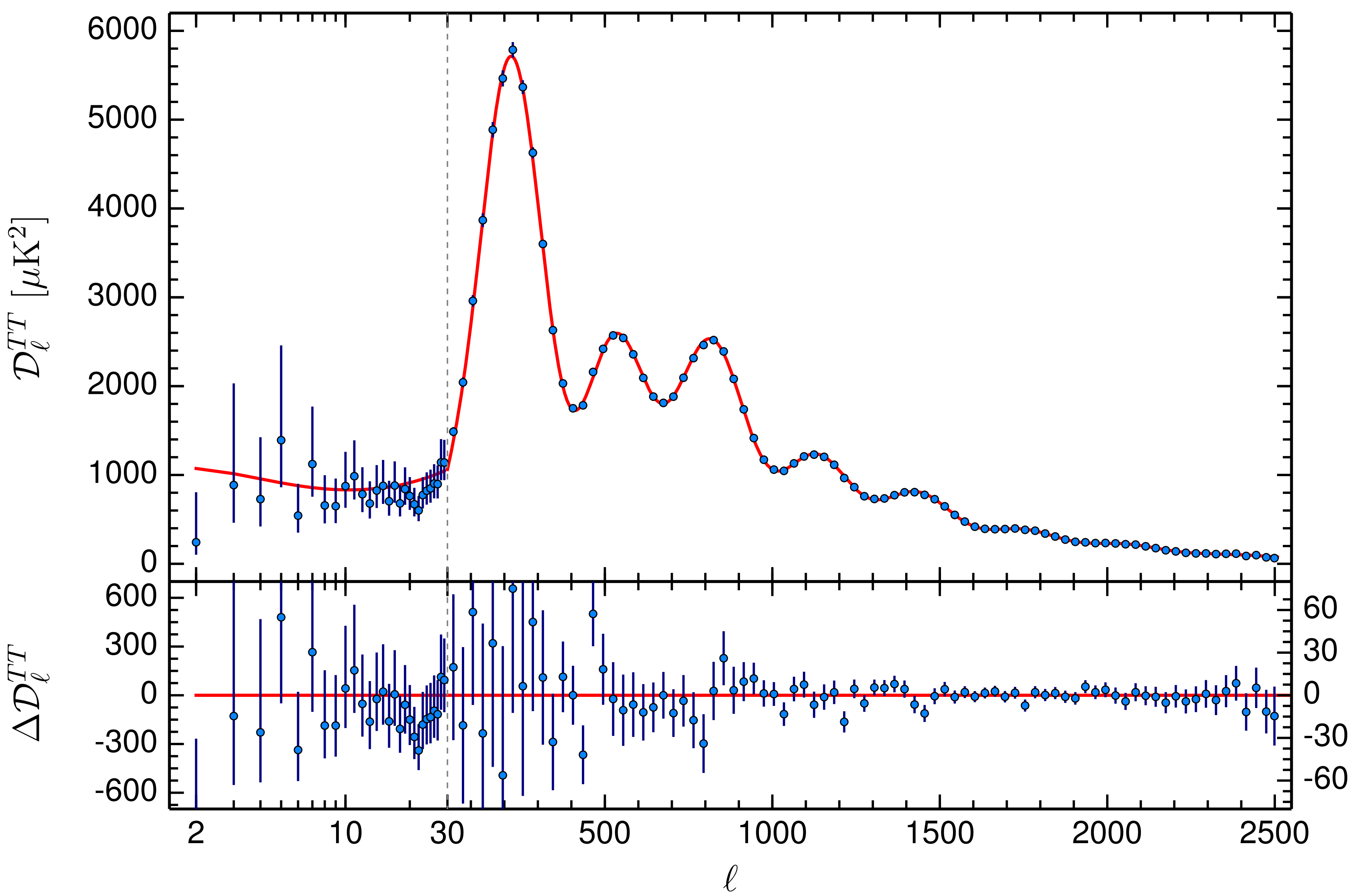

Cobe 1992
WMAP 2003
Planck 2013

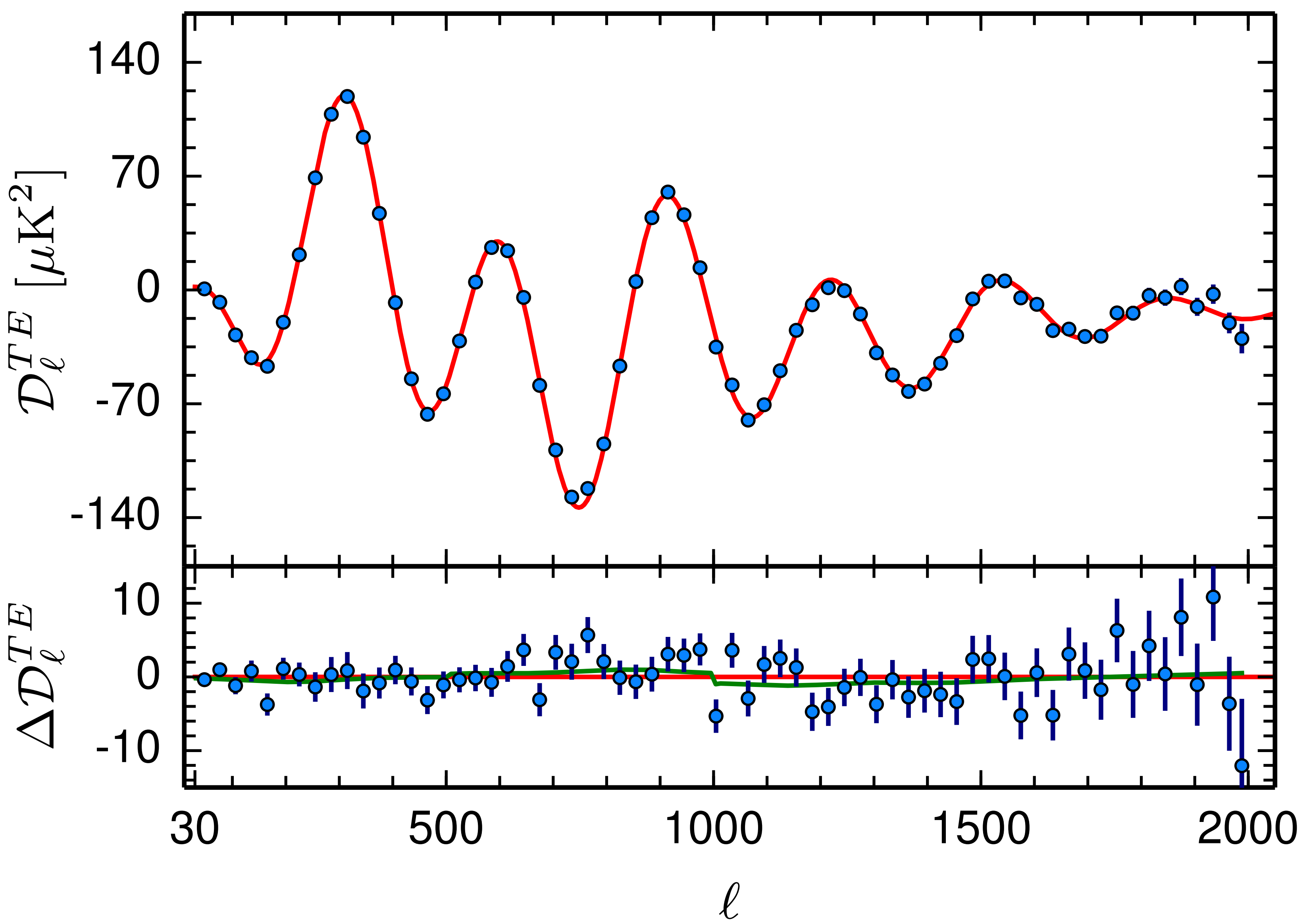
Cosmological perturbations

Single scalar gauge-invariant degree of freedom $v \ni \delta\phi, \delta g_{\mu\nu} \propto \delta T/T$

Quantised starting from Bunch-Davies vacuum $v \longrightarrow \hat{v}$

Quantum mean values compared with statistical averages in the sky





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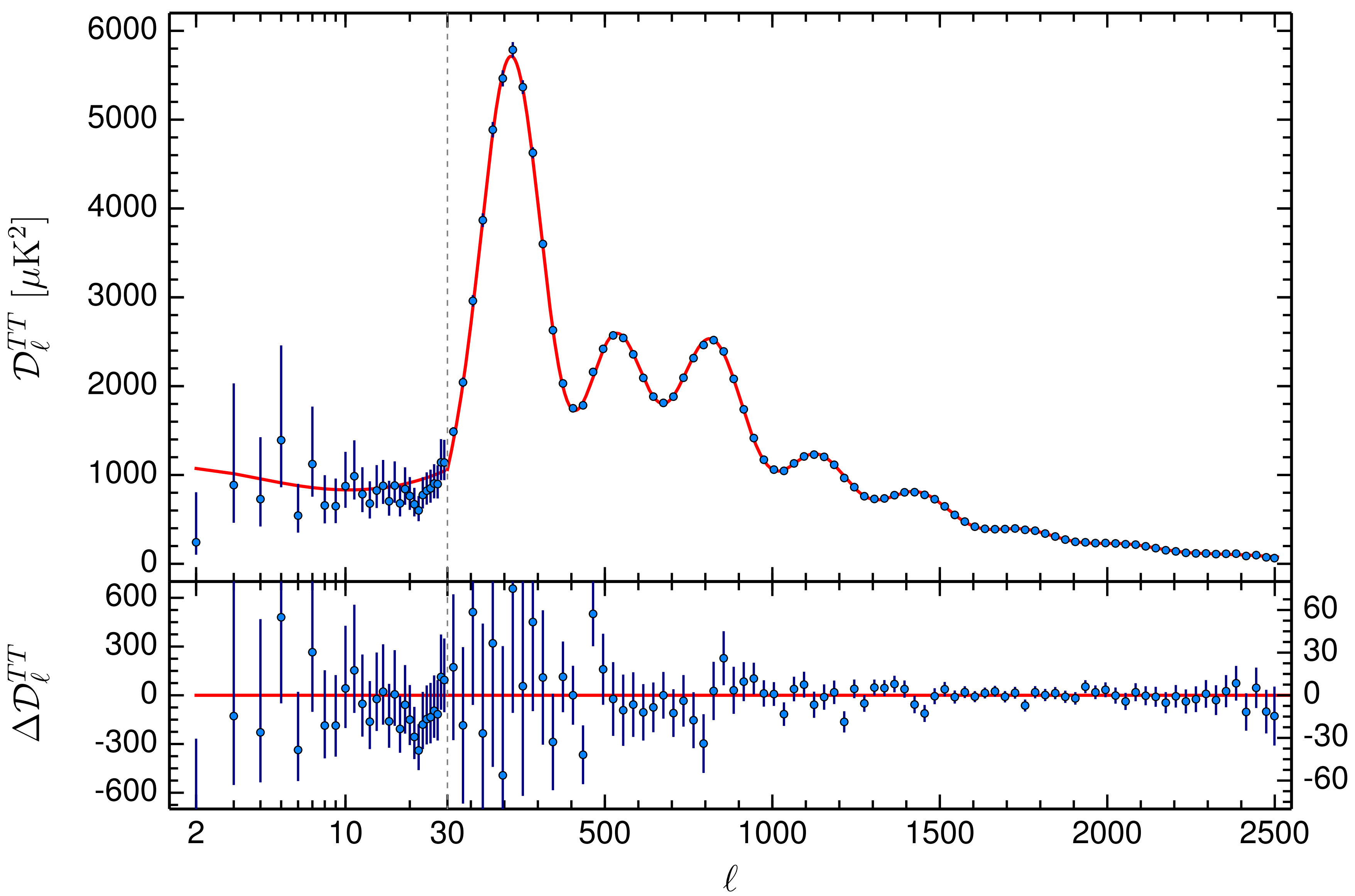
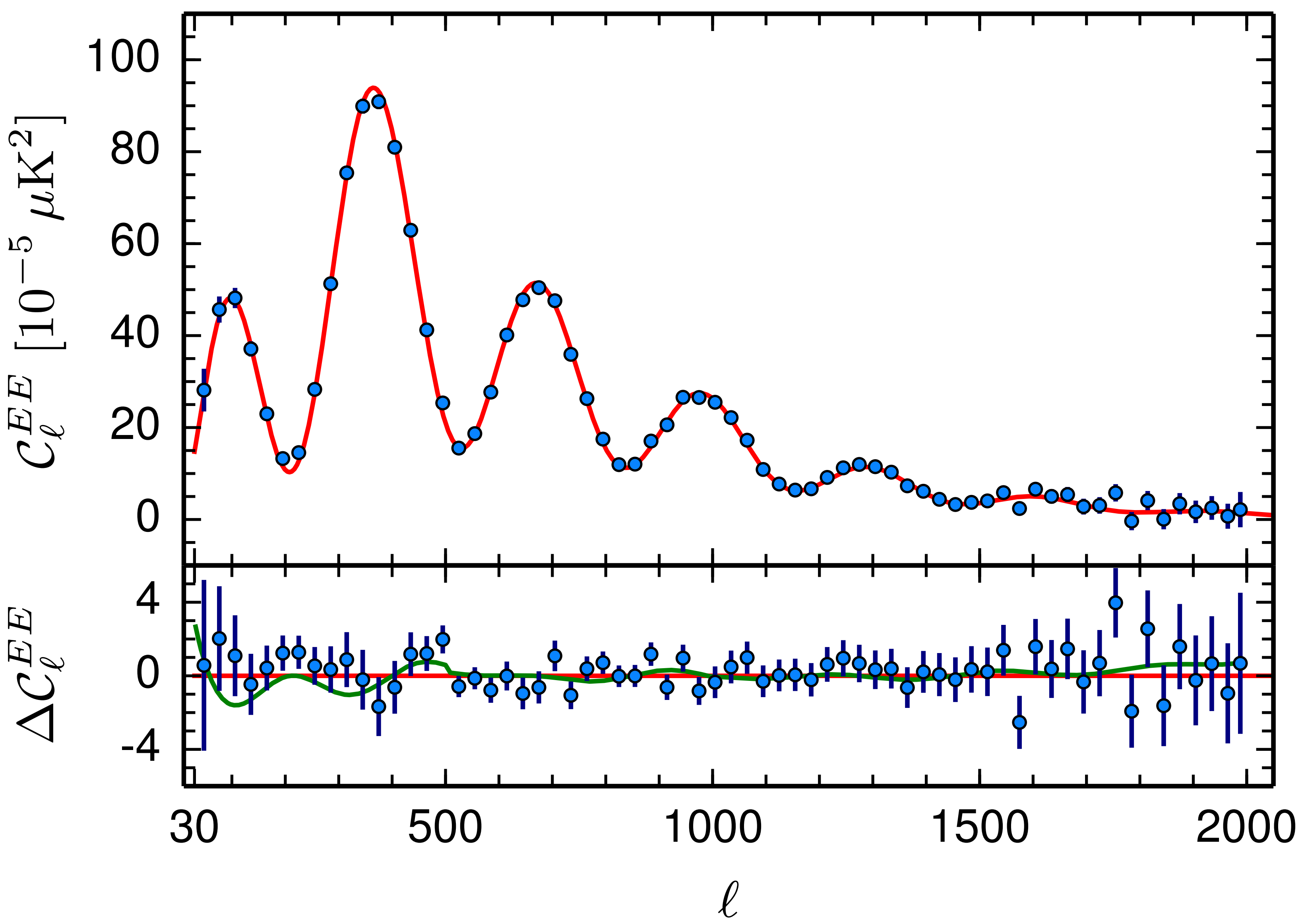
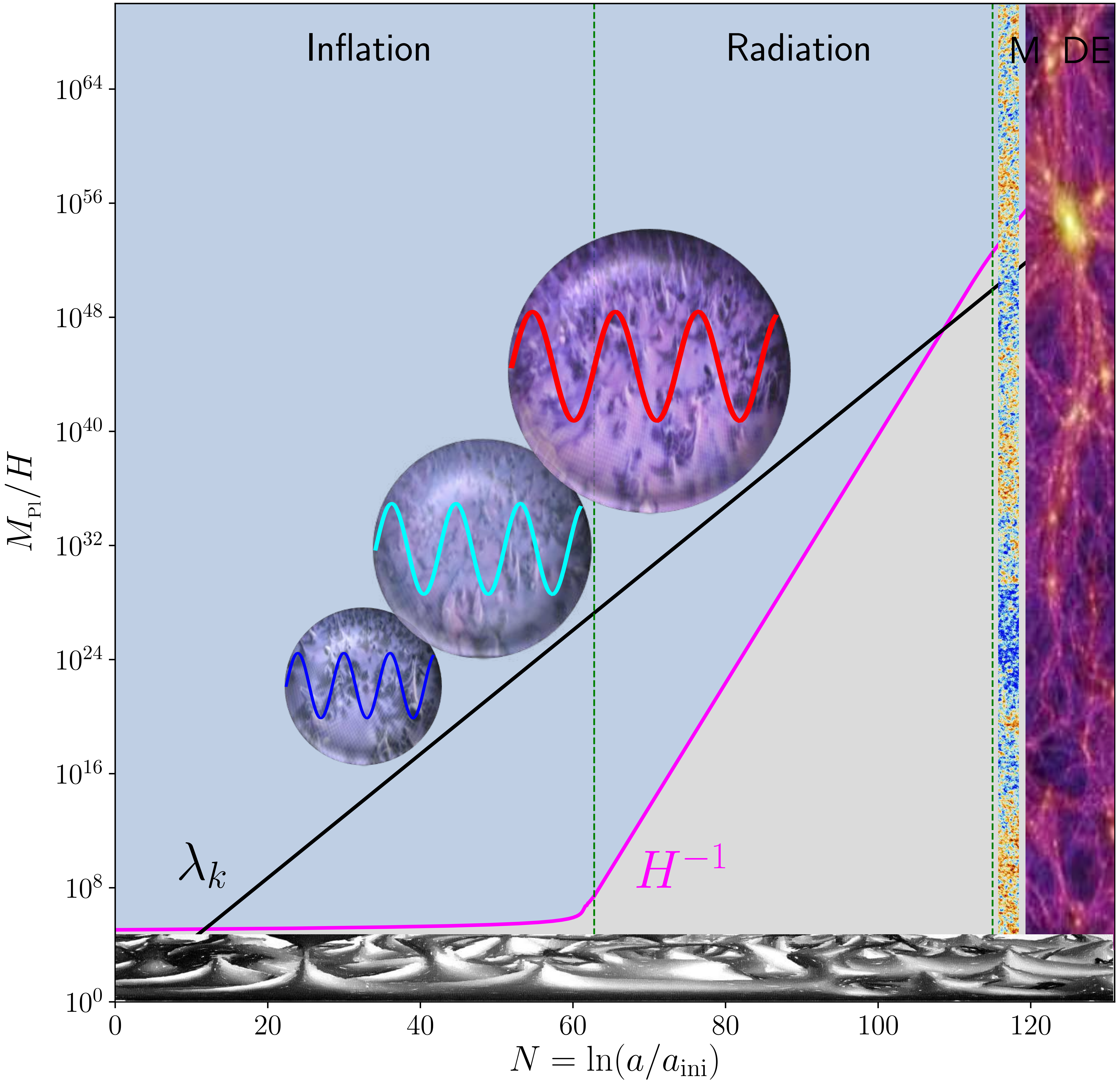




Cosmic Inflation



Structure formation by the gravitational amplification of quantum fluctuations

Quantum mechanics on cosmological scales!

- Can we trust QM at those scales?
- Is it legit to quantise metric fluctuations?
- What about the QM measurement problem?

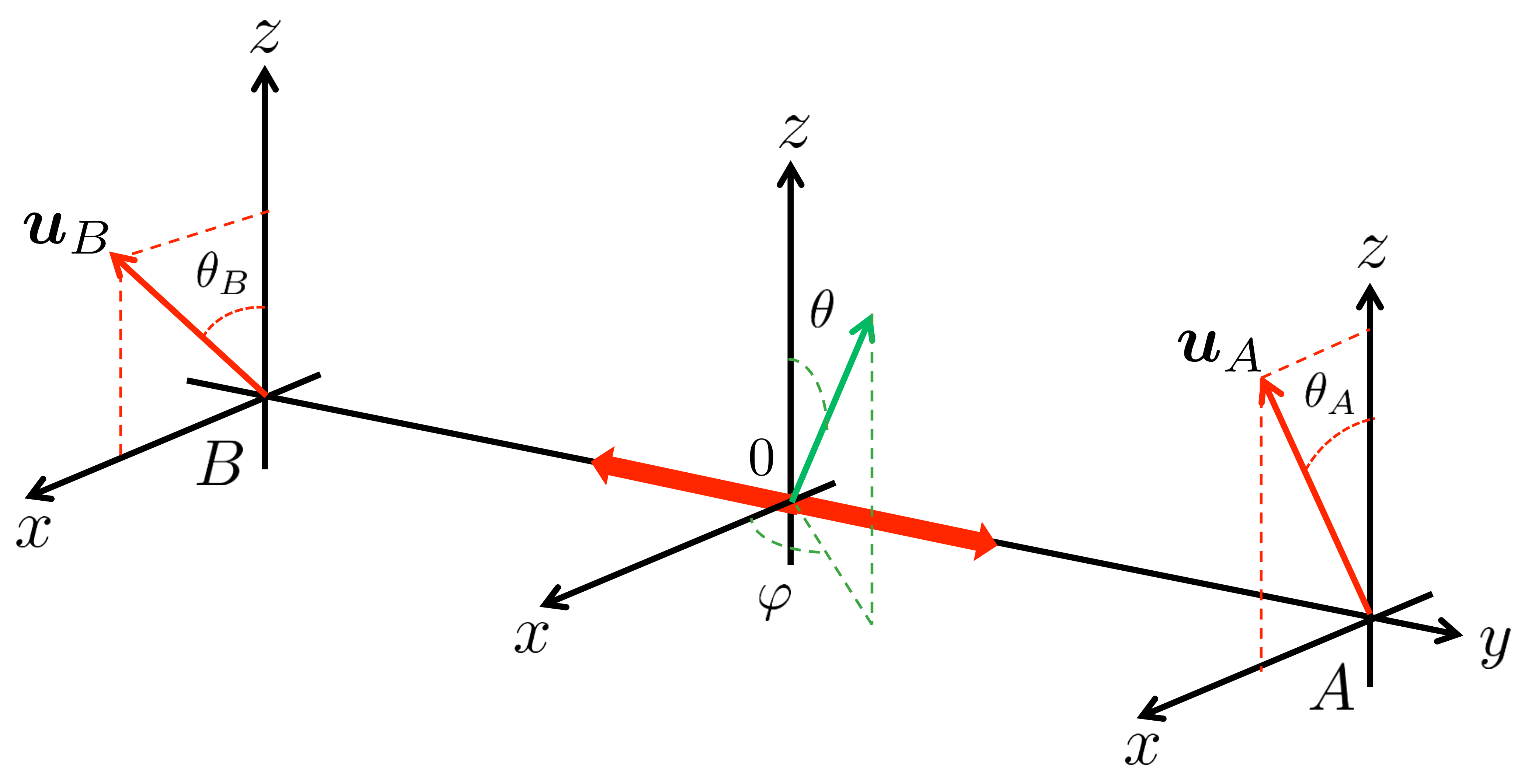
True for inflation, but also for most alternatives (such as contracting cosmologies)

- Strong statement (extraordinary statement requires extraordinary evidence)
- The consequences that can be inferred from this idea are consistent with observations
- This gives an indirect confirmation that cosmological structures have an quantum-mechanical origin

Any direct evidence?

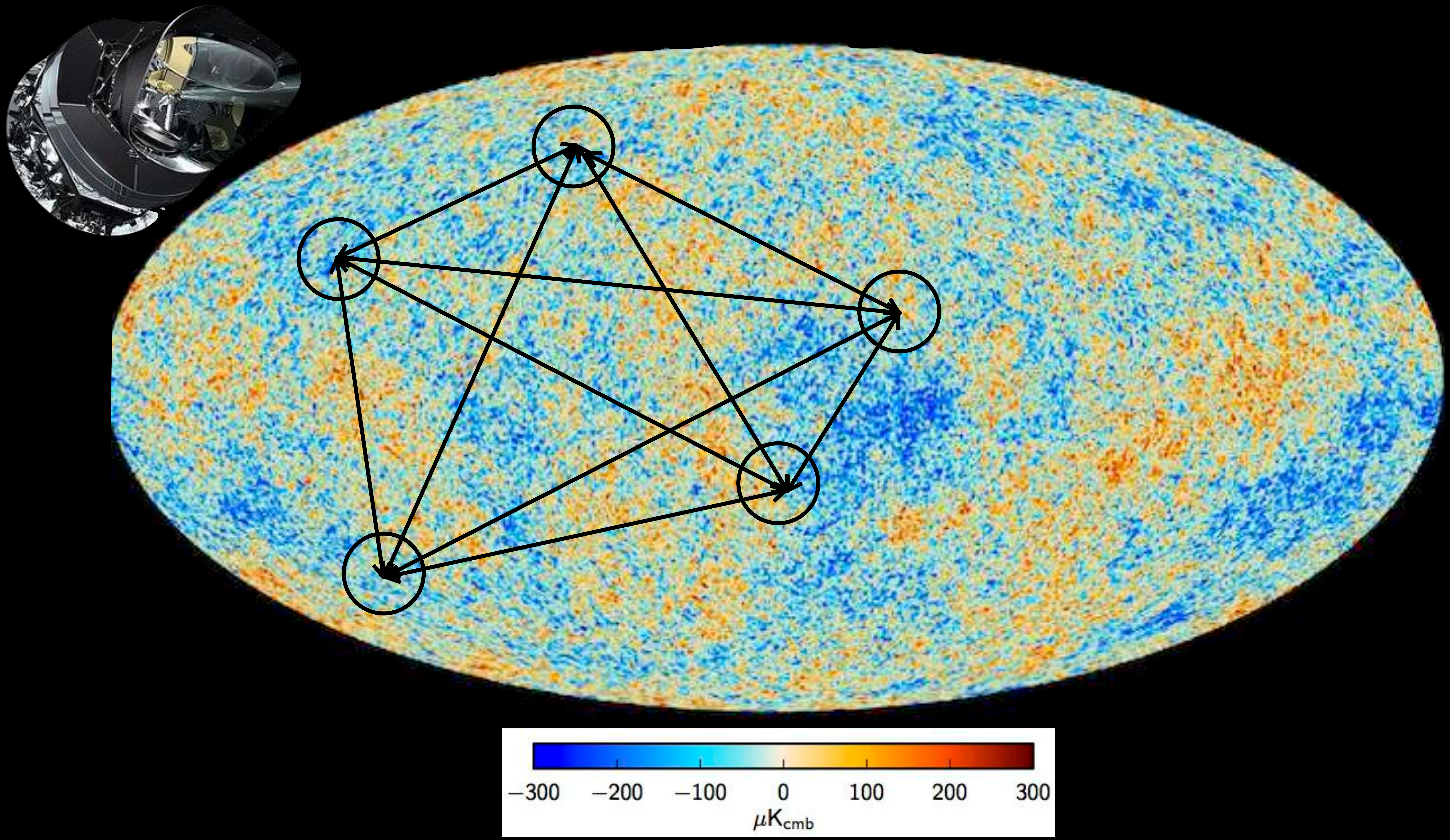
Role of the decaying mode: Lesgourgues, Kiefer, Polarski, Starobinsky (role of decaying mode)
Bell inequalities: Campo, Parentani // Maldacena // Martin, Vennin // Kanno, Shock, Soda // Choudhury, Panda, Singh
Entanglement entropy, Quantum discord: Lim // Martin, Vennin // Hollowood, Mc Donald // Espinosa, Garcia-Bellido
Higher-order statistics: Martin, Vennin // Green, Porto

The quantum and the classical worlds differ by the nature of the correlations they allow for



Are there some quantum correlations in the primordial density field?

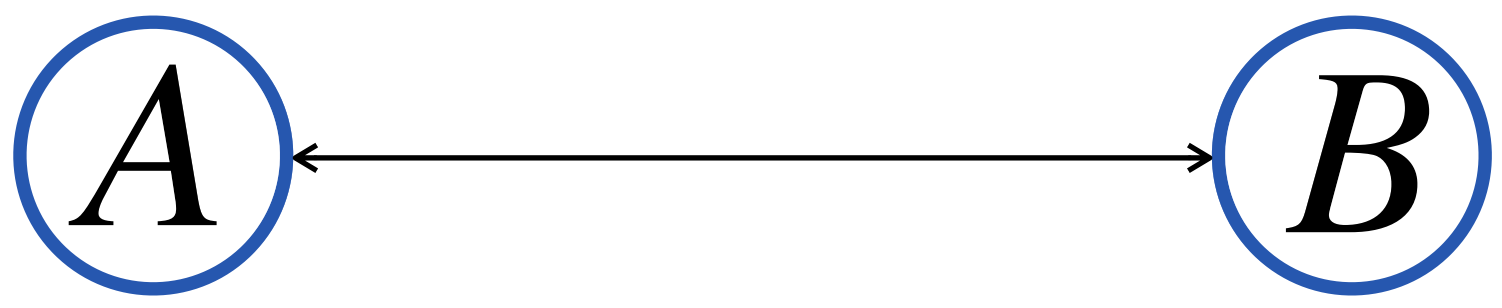
Can we detect them?


−300 −200 −100 0 100 200 300
μK_{cmb}

Quantum discord

Henderson and Vedral 2001; Ollivier and Zurek 2001

How to characterise the presence of quantum correlations?



CLASSICAL MUTUAL INFORMATION

Subsystem *A:* configurations $\{a_i\}$

Subsystem *B:* configurations $\{b_j\}$

Von-Neumann entropy: $S(A) = -\sum_i p(a_i) \ln p(a_i)$

$S(A) = 0$ ⟶ The system is entirely determined

Mutual information: $\mathcal{I} = S(A) + S(B) - S(A,B)$

$\mathcal{I} = 0$ ⟶ *A* and *B* are uncorrelated

Uncorrelated subsystems: $p_{i,j} = p_i p_j$

Shorthand notation: $p_i = p(a_i)$

$p_j = p(b_j)$

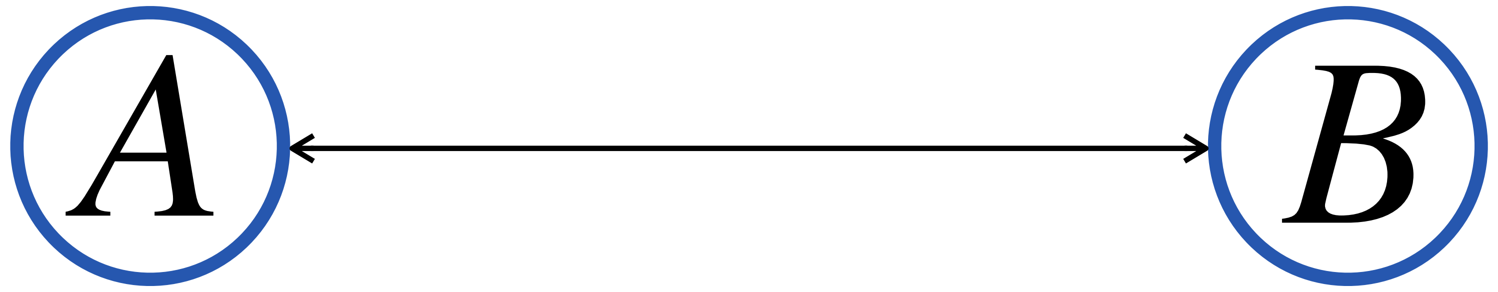
$p_{i,j} = p(a_i, b_j)$

$$\begin{aligned}
\mathcal{I} &= S(A) + S(B) - S(A,B) \\
&= -\sum_i p_i \ln p_i - \sum_j p_j \ln p_j + \sum_{i,j} p_{i,j} \ln p_{i,j} \\
&= -\sum_i p_i \ln p_i - \sum_j p_j \ln p_j + \sum_{i,j} p_i p_j \ln(p_i p_j) \\
&= -\sum_i p_i \ln p_i \left(1 - \sum_j p_j\right) + \sum_j p_j \ln p_j \left(1 - \sum_i p_i\right) = 0
\end{aligned}$$

Quantum discord

Henderson and Vedral 2001; Ollivier and Zurek 2001

How to characterise the presence of quantum correlations?



QUANTUM MUTUAL INFORMATION

Subsystem *A:* configurations $\{a_i\}$, state $\rho_A = \mathrm{Tr}_B(\rho_{A,B})$

Subsystem *B:* configurations $\{b_j\}$, state $\rho_B = \mathrm{Tr}_A(\rho_{A,B})$

Von-Neumann entropy: $S(A) = -\,\mathrm{Tr}(\rho_A \ln \rho_A)$

$S(A) = 0$ ⟶ The system is entirely determined

Mutual information: $\mathcal{I} = S(A) + S(B) - S(A,B)$

$\mathcal{I} = 0$ ⟶ *A* and *B* are uncorrelated

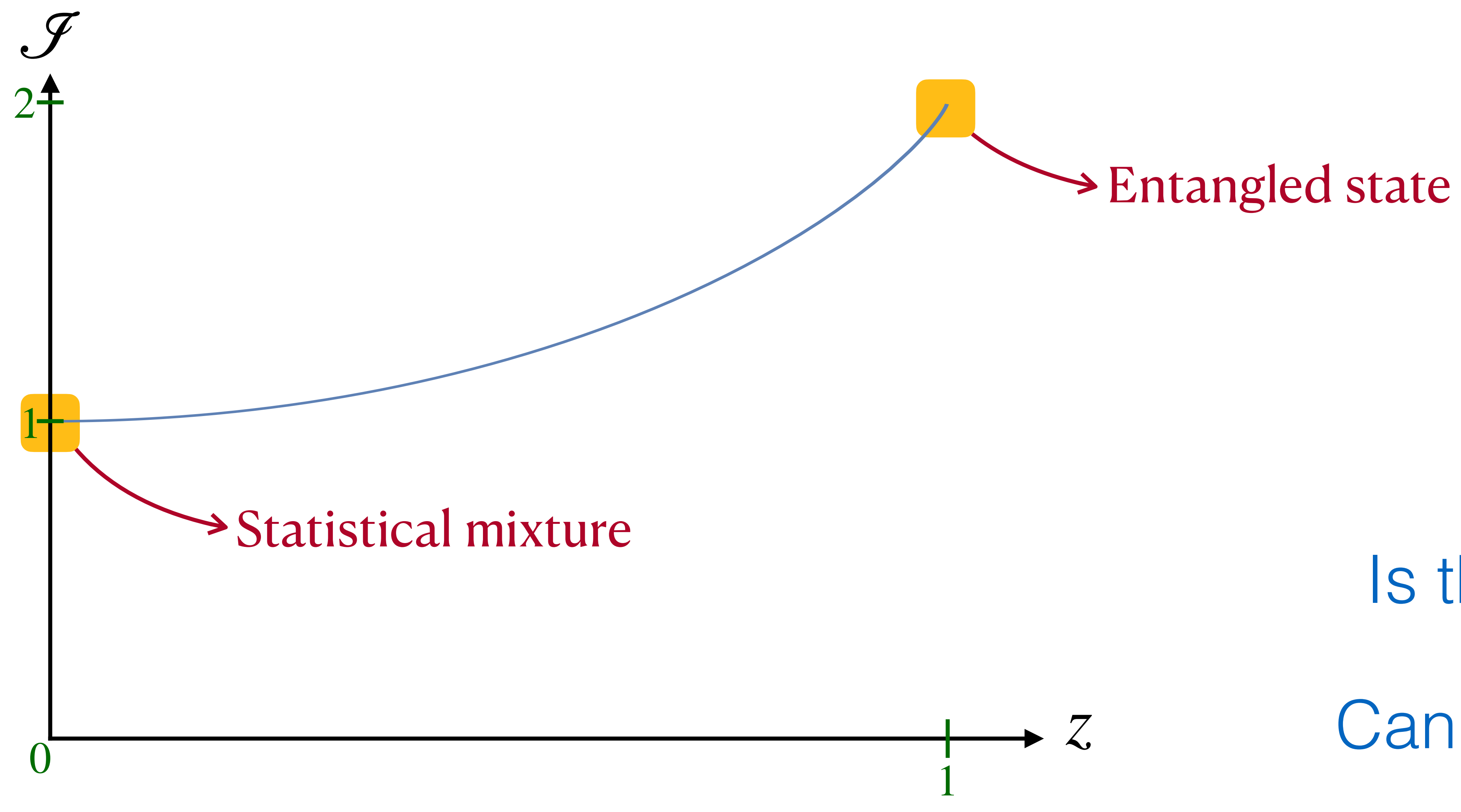
Quantum discord

Henderson and Vedral 2001; Ollivier and Zurek 2001

Example: $|\Psi\rangle = \dfrac{|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle}{\sqrt{2}}$

$$\rho = \frac{1}{2}|\downarrow\downarrow\rangle\langle\downarrow\downarrow| + \frac{1}{2}|\uparrow\uparrow\rangle\langle\uparrow\uparrow| + \frac{z}{2}|\downarrow\downarrow\rangle\langle\uparrow\uparrow| + \frac{z}{2}|\uparrow\uparrow\rangle\langle\downarrow\downarrow|$$

$$\mathscr{I} = 1 + \frac{1-z}{2}\log_2(1-z) + \frac{1+z}{2}\log_2(1+z)$$



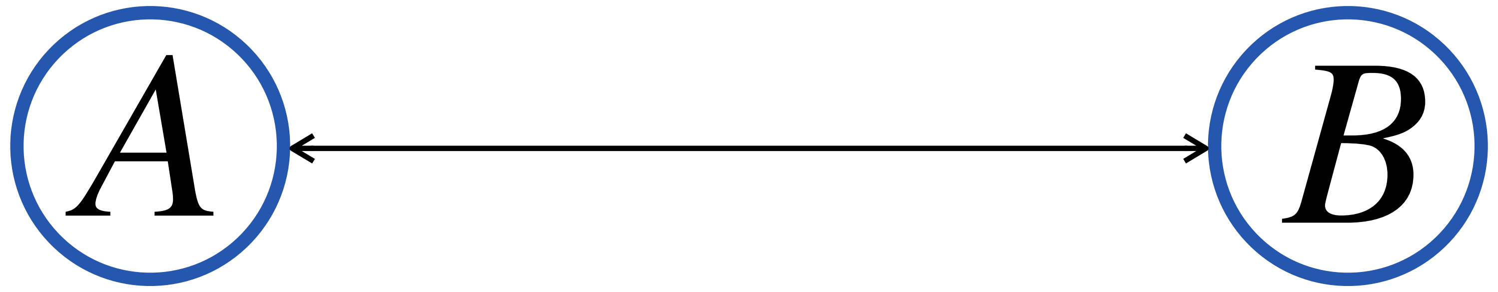
Is this applicable to quantum fields?

Can we isolate **quantum** correlations?

Quantum discord

Henderson and Vedral 2001; Ollivier and Zurek 2001

How to characterise the presence of quantum correlations?



CLASSICAL MUTUAL INFORMATION : ALTERNATIVE use $p_{i,j} = p_j p_{i|j}$

Subsystem *A:* configurations $\{a_i\}$

Subsystem *B:* configurations $\{b_j\}$

$$\mathcal{I} = S(A) + S(B) - S(A,B)$$

$$= -\sum_i p_i \ln p_i - \sum_j p_j \ln p_j + \sum_{i,j} p_{i,j} \ln p_{i,j} = -\sum_i p_i \ln p_i - \sum_j p_j \ln p_j + \sum_{i,j} p_j p_{i|j} \ln \left(p_j p_{i|j}\right)$$

$$= -\sum_i p_i \ln p_i - \sum_j p_j \ln p_j \left(1 - \cancel{\sum_i p_{i|j}}\right) + \sum_{i,j} p_j p_{i|j} \ln p_{i|j} \equiv \mathcal{J}$$

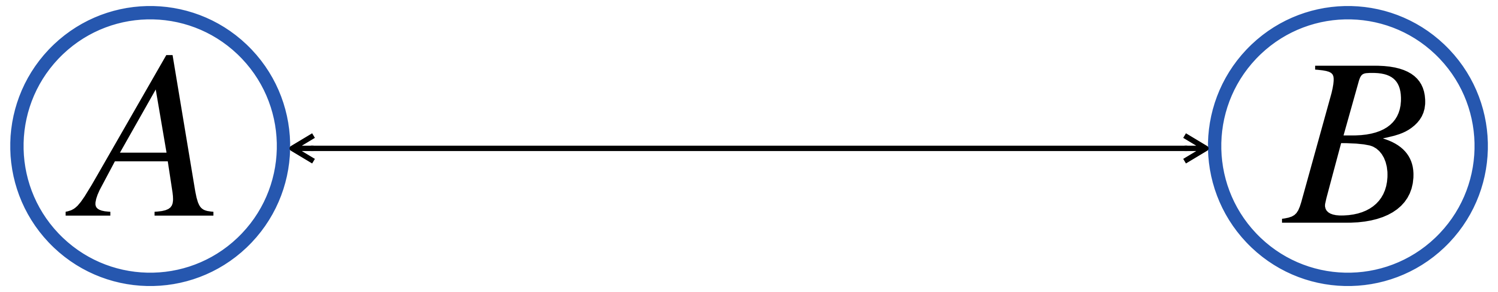
$S(A)$ $\quad$ $-S(A\,|\,B) = -\sum_j p_j S(A\,|\,j)$

Classically: $\mathcal{I} - \mathcal{J} = 0$

Quantum discord

Henderson and Vedral 2001; Ollivier and Zurek 2001

How to characterise the presence of quantum correlations?

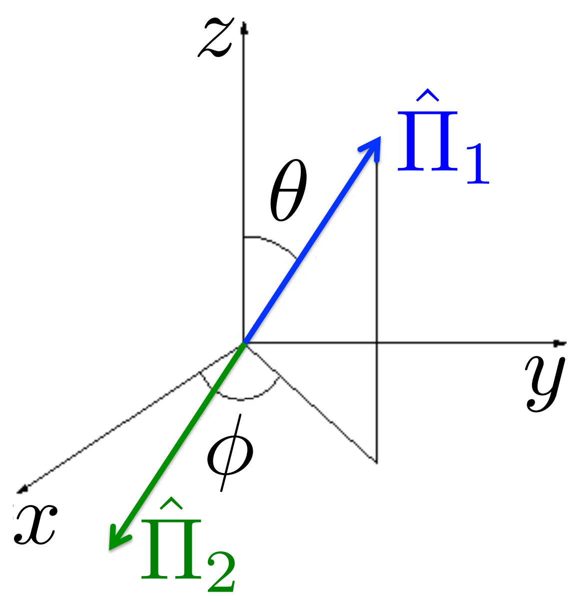


QUANTUM MUTUAL INFORMATION : ALTERNATIVE

$\mathcal{J} = S(A) - S(A\,|\,B)$ where $S_{\text{class}}(A\,|\,B) = \sum_j p_j S(A\,|\,j)$

Subsystem A: configurations $\{a_i\}$, state $\rho_A = \mathrm{Tr}_B(\rho_{A,B})$

Subsystem B: configurations $\{b_j\}$, state $\rho_B = \mathrm{Tr}_A(\rho_{A,B})$



Conditional entropy: with respect to measurements $\hat{\Pi}_j$

$\hat{\Pi}_j$: complete set of projectors defined on $\mathcal{E}_B$

$\hat{\rho} \to \hat{\rho}\hat{\Pi}_j/p_j$ with probability $p_j = \mathrm{Tr}\left(\hat{\rho}\hat{\Pi}_j\right)$ and $\rho_{A;\hat{\Pi}_j} = \mathrm{Tr}_B\left(\rho\hat{\Pi}_j/p_j\right)$

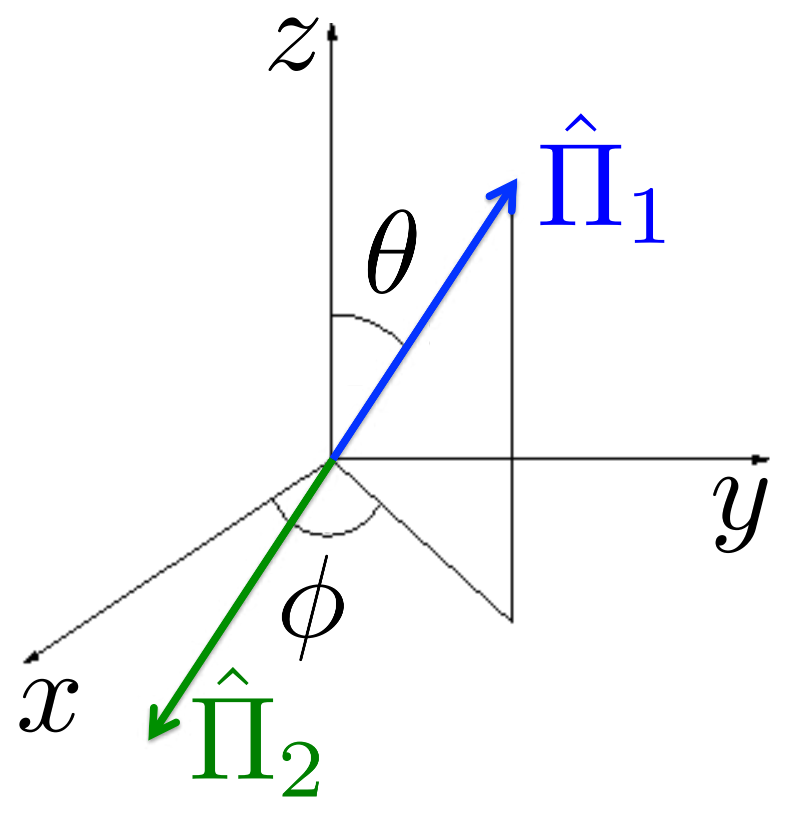
$$S(A|B) = \sum_j p_j S\left(\rho_{A;\hat{\Pi}_j}\right)$$

Quantum-mechanically: $\mathcal{I} - \mathcal{J} \neq 0$ DISCORD →

$$\mathcal{D}(A,B) = \min_{\{\hat{\Pi}_j\}}(\mathcal{I} - \mathcal{J})$$

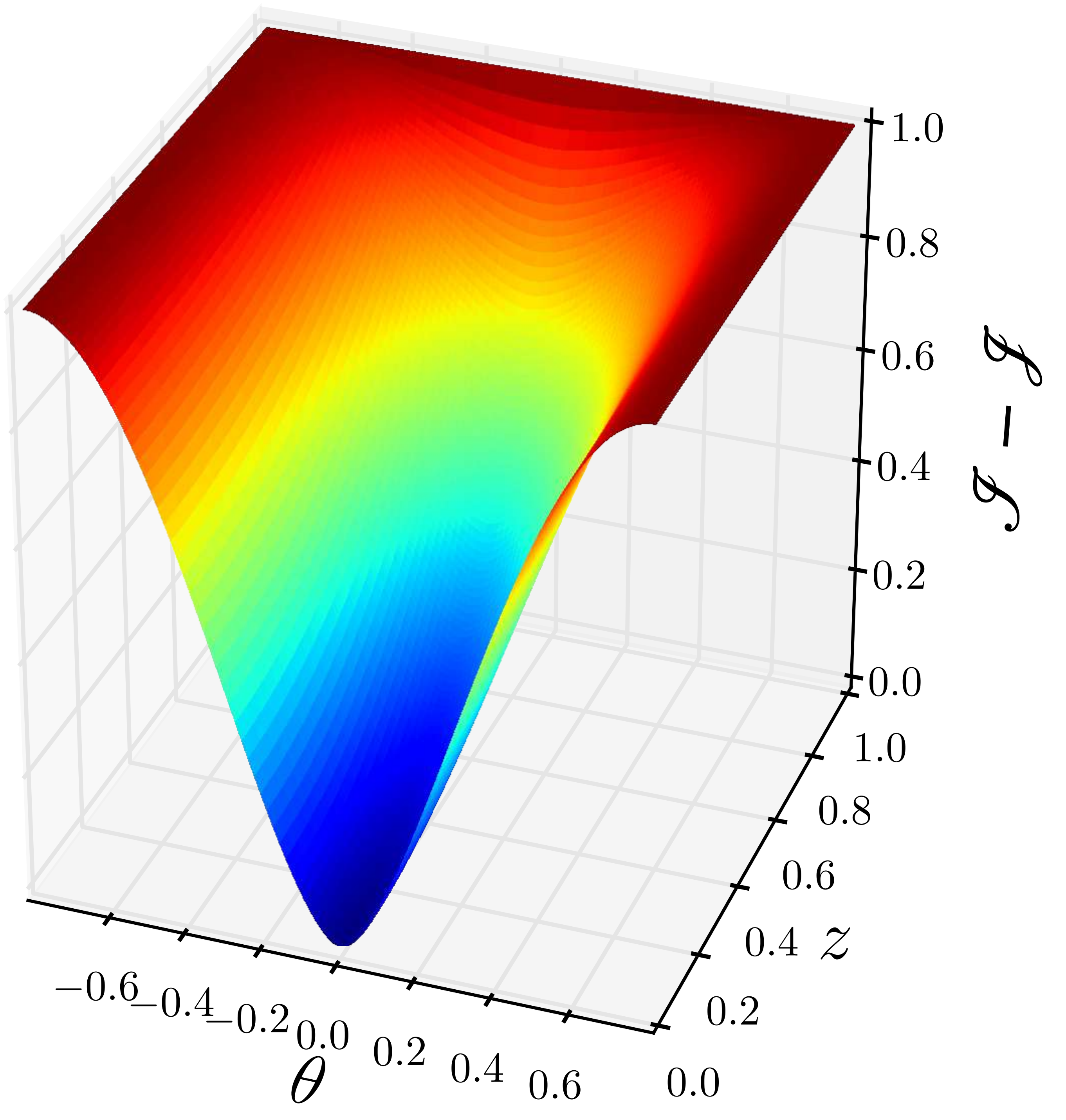
Quantum discord

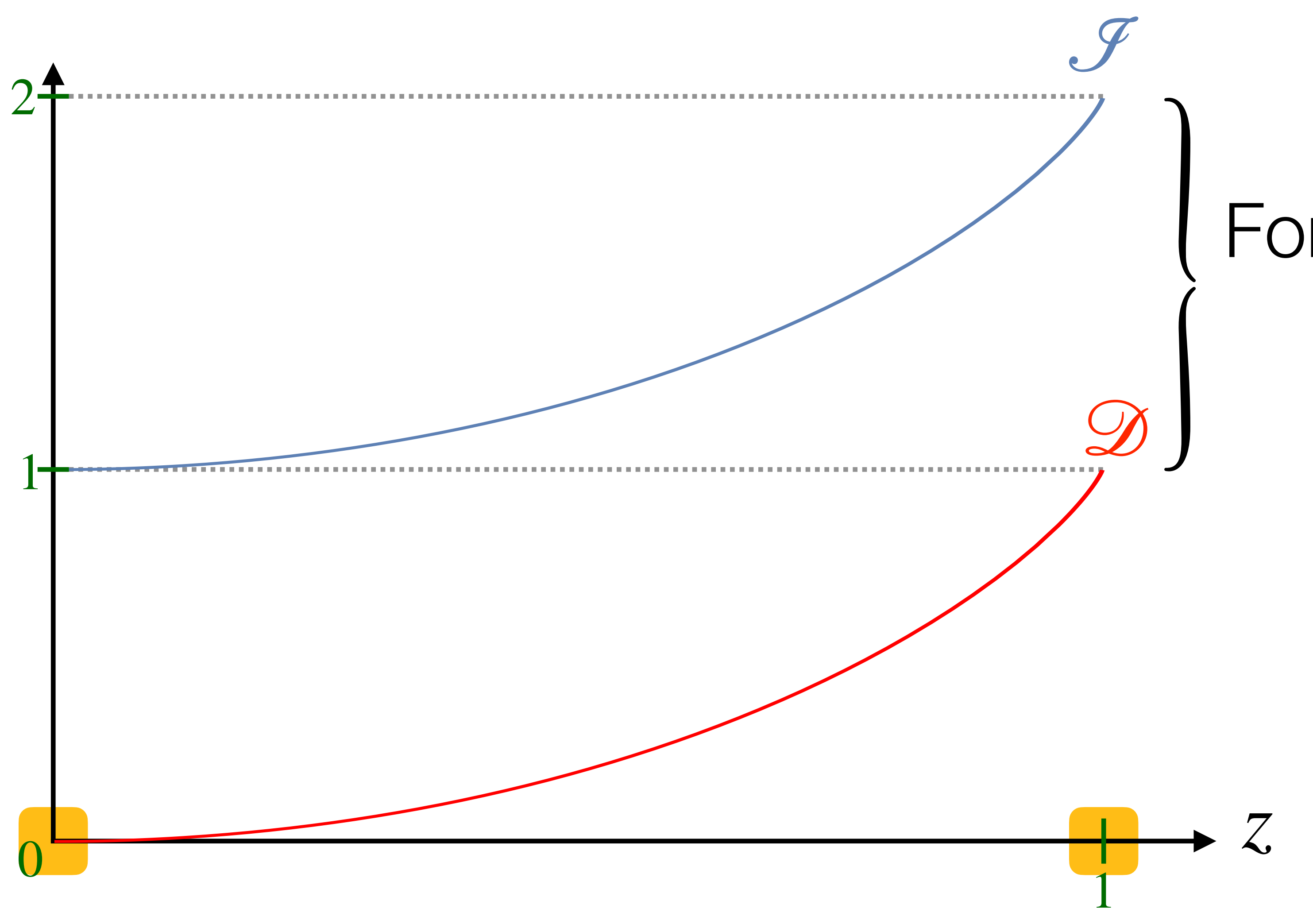
Henderson and Vedral 2001; Ollivier and Zurek 2001



Example: $|\Psi\rangle = \dfrac{|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle}{\sqrt{2}}$

$$\rho = \frac{1}{2}|\downarrow\downarrow\rangle\langle\downarrow\downarrow| + \frac{1}{2}|\uparrow\uparrow\rangle\langle\uparrow\uparrow| + \frac{z}{2}|\downarrow\downarrow\rangle\langle\uparrow\uparrow| + \frac{z}{2}|\uparrow\uparrow\rangle\langle\downarrow\downarrow|$$





For pure states one always has

$$\mathscr{I} = 2\,\mathscr{D}$$

Cosmological perturbations

$$g_{\mu\nu} = \bar{g}_{\mu\nu}(t) + \widehat{\delta g}_{\mu\nu}(t, \boldsymbol{x})$$

$$\phi = \bar{\phi}(t) + \widehat{\delta\phi}(t, \boldsymbol{x})$$

Scalar perturbations are described by a single combination of metric and field fluctuations that directly determines CMB temperature anisotropies → $\hat{\zeta}(t, \boldsymbol{x})$

Expansion of Einstein-Hilbert + scalar field action at second order: independent parametric oscillators, one for each $k \in \mathbb{R}^{3+}$

Free term

Interaction term between the quantum fluctuations and the classical background

$$\hat{H} = \int \mathrm{d}^3\boldsymbol{k} \left[\frac{k}{2}\left(\hat{c}_{\boldsymbol{k}}\hat{c}^{\dagger}_{\boldsymbol{k}} + \hat{c}_{-\boldsymbol{k}}\hat{c}^{\dagger}_{-\boldsymbol{k}}\right) - \frac{i}{2}\frac{\left(a\sqrt{\epsilon_1}\right)'}{a\sqrt{\epsilon_1}}\left(\hat{c}_{\boldsymbol{k}}\hat{c}_{-\boldsymbol{k}} - \hat{c}^{\dagger}_{-\boldsymbol{k}}\hat{c}^{\dagger}_{\boldsymbol{k}}\right)\right]$$

Creation / annihilation of pairs of particles

Pump field: time-dependent coupling constant
Depends only on the scale factor and its derivative
Vanishes if *a* is constant

$$\epsilon_1 = -\dot{H}/H^2$$

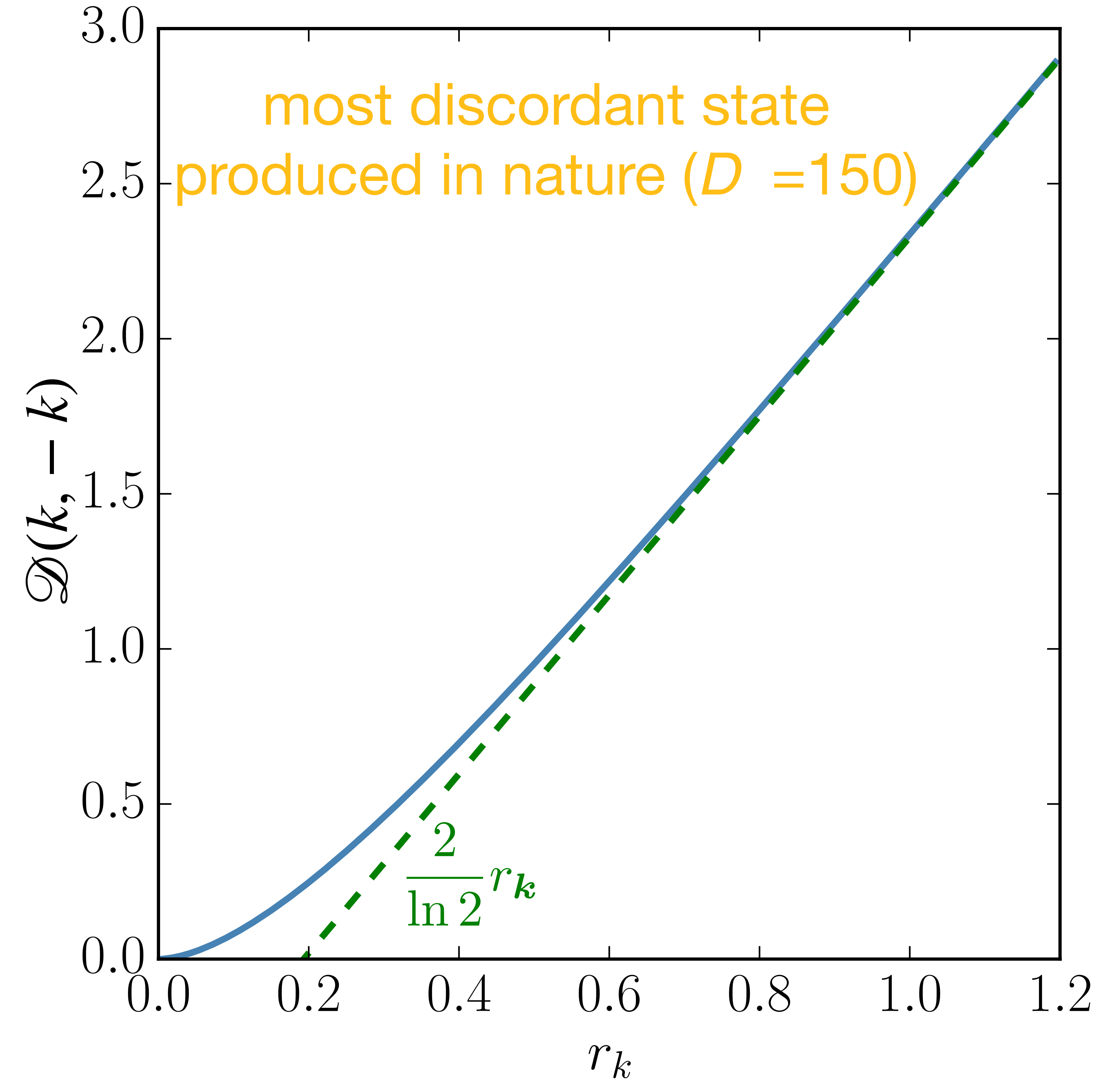
Cosmological perturbations

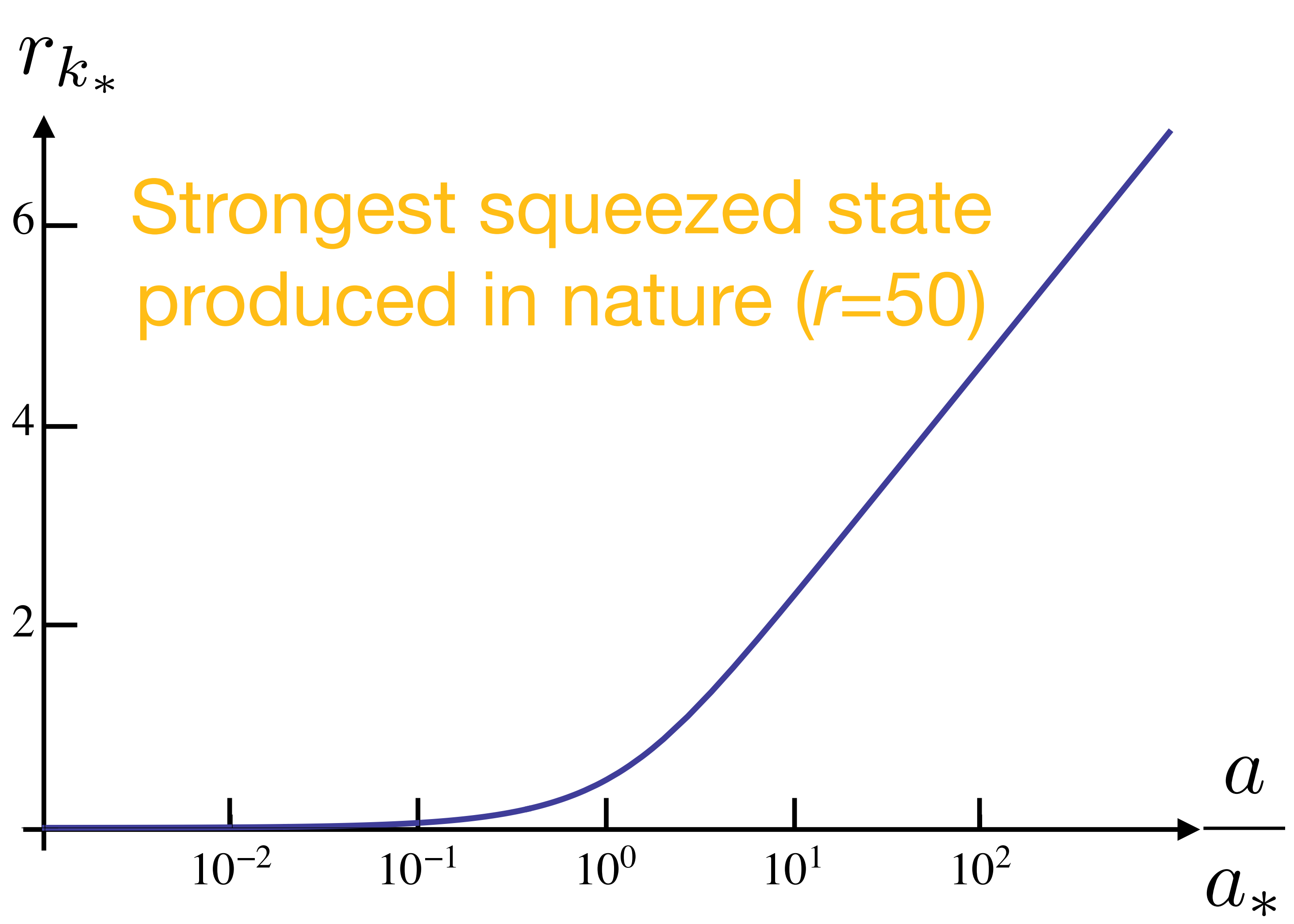
Two-mode squeezed sate $\quad |\Psi_{\mathrm{CMB}}\rangle = \bigotimes_{\boldsymbol{k}\in\mathbb{R}^{3+}} |\Psi_{\boldsymbol{k}}\rangle \quad$ with $\quad |\Psi_{\boldsymbol{k}}\rangle = \frac{1}{\cosh r_k}\sum_{n=0}^{\infty} e^{2in\varphi_k}(-1)^n \tanh^n r_k |n_{\boldsymbol{k}}, n_{-\boldsymbol{k}}\rangle$

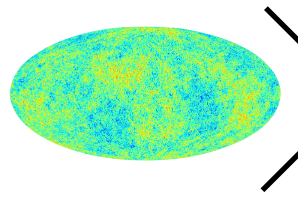
Entangled state
(correlations between modes ***k*** and -***k***)

Martin, Vennin 2015

$$\mathscr{D}(k,-k) = \cosh^2 r_k \log_2(\cosh^2 r_k) - \sinh^2 r_k \log_2(\sinh^2 r_k)$$

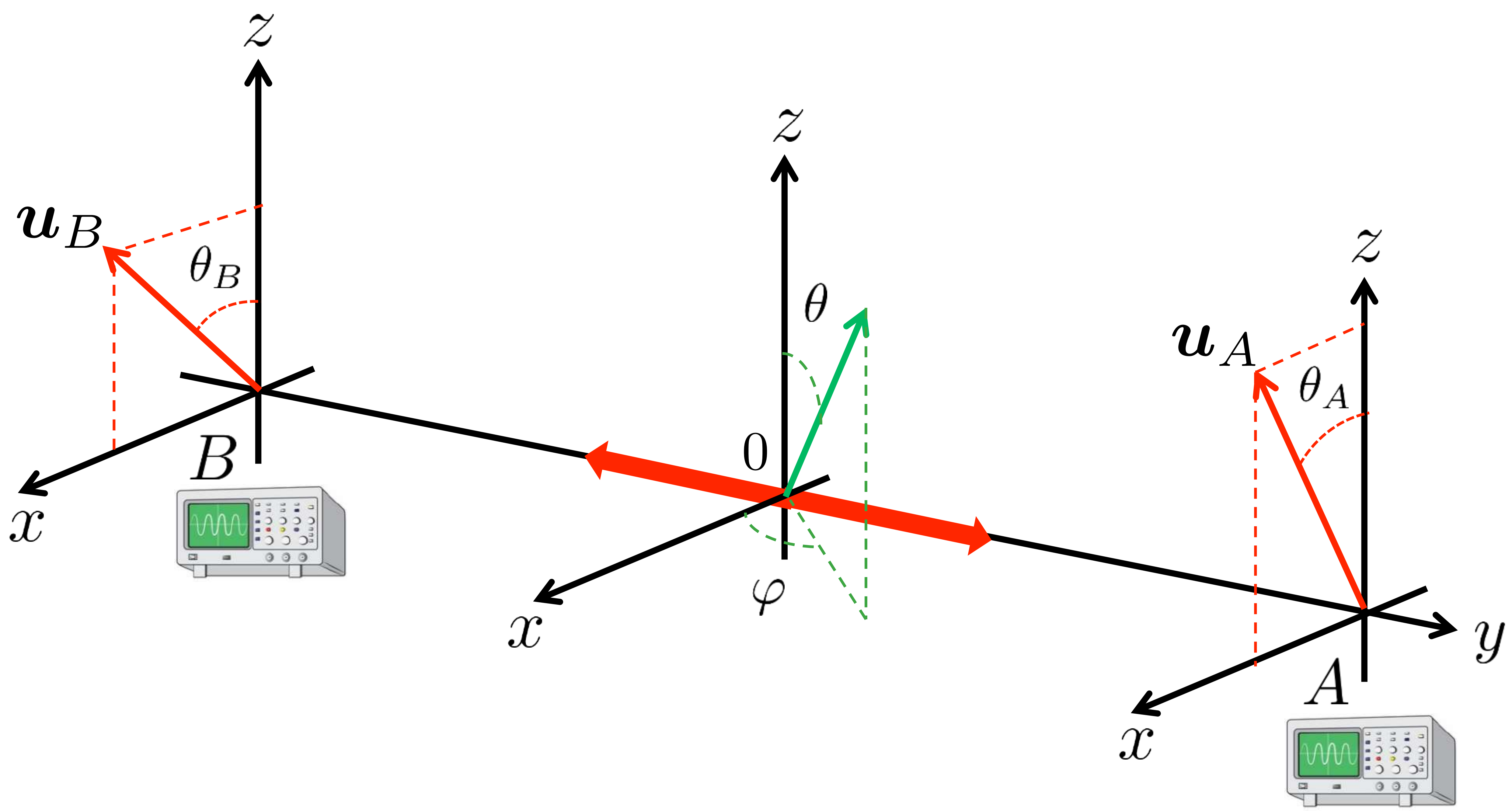




|  ⟩ = highly-non classical state? ⟶ Can we violate Bell's inequalities with the CMB?

Bell inequalities

$$\hat{B} = \left(\boldsymbol{u}_A \cdot \hat{\boldsymbol{S}}_A\right) \otimes \left(\boldsymbol{u}_B \cdot \hat{\boldsymbol{S}}_B\right) + \left(\boldsymbol{u}_A \cdot \hat{\boldsymbol{S}}_A\right) \otimes \left(\boldsymbol{u}'_B \cdot \hat{\boldsymbol{S}}_B\right) + \left(\boldsymbol{u}'_A \cdot \hat{\boldsymbol{S}}_A\right) \otimes \left(\boldsymbol{u}_B \cdot \hat{\boldsymbol{S}}_B\right) - \left(\boldsymbol{u}'_A \cdot \hat{\boldsymbol{S}}_A\right) \otimes \left(\boldsymbol{u}'_B \cdot \hat{\boldsymbol{S}}_B\right)$$



Classically: $\left\langle \hat{B} \right\rangle < 2$

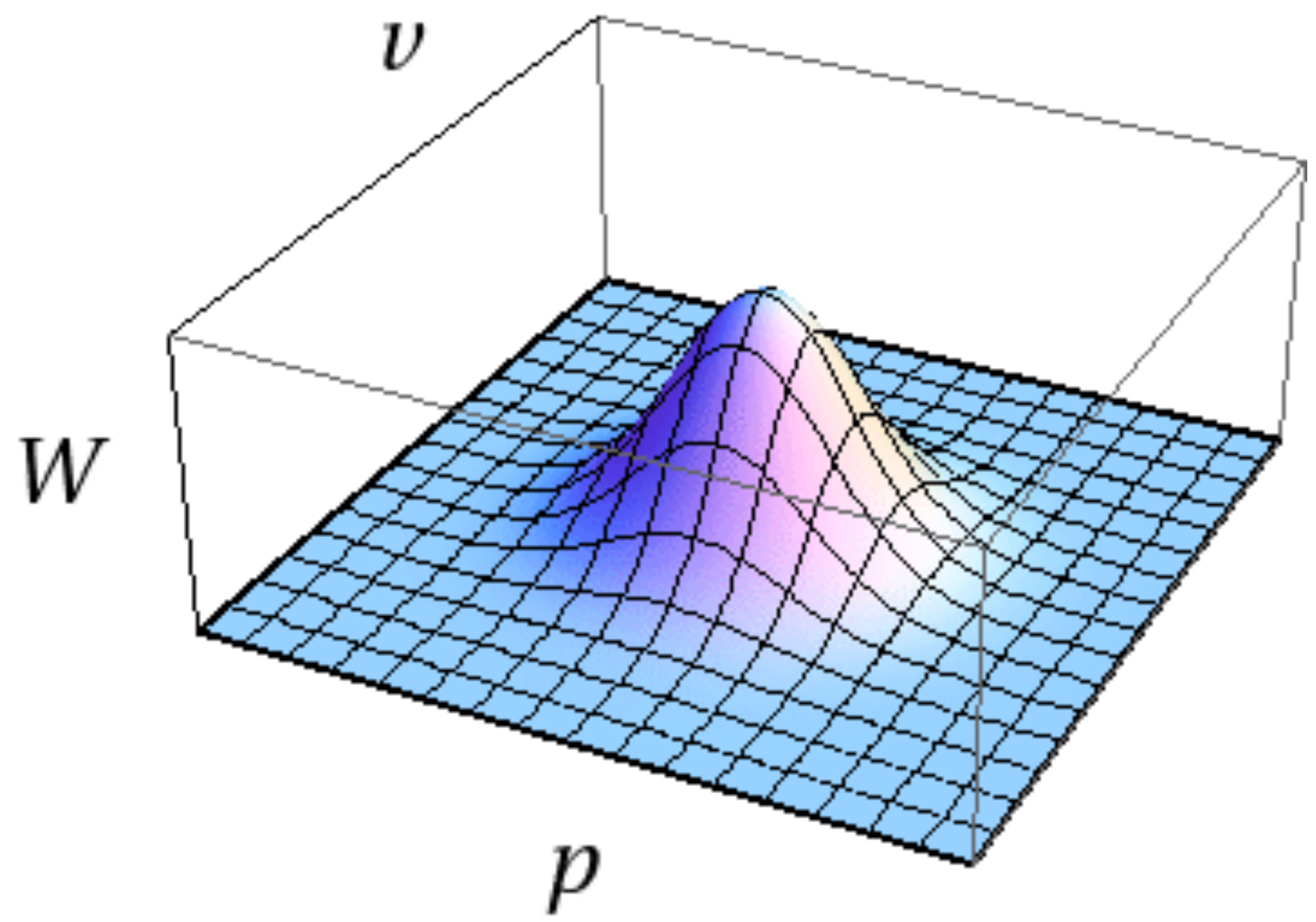
- Bipartite system: *k* and -*k*
- Entangled system: two-mode squeezed state
- <u>improper</u>, spin-like operators (Revzen 2006)

Classicality in the Wigner approach

Wigner function $$W\left(q,p\right)=\int\Psi^{*}\left(q-\frac{u}{2}\right)e^{-ipu}\Psi\left(q+\frac{u}{2}\right)\frac{\mathrm{d}u}{2\pi}$$

Evolution equation: $$\frac{\partial}{\partial t}W\left(q,p,t\right)=-\left\{W(q,p,t),H(q,p,t)\right\}_{\mathrm{Poisson\ bracket}}$$

for quadratic Hamiltonians



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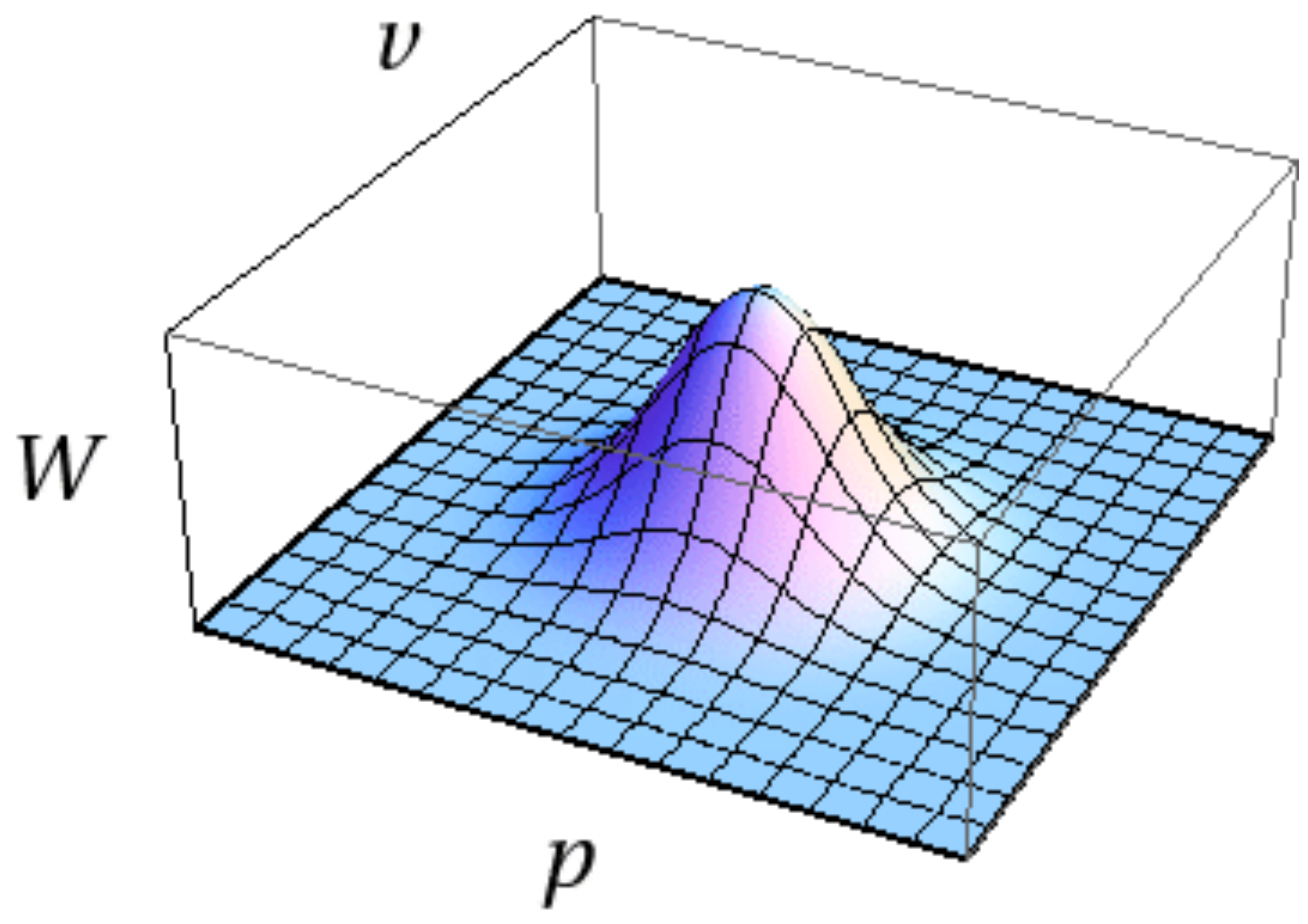

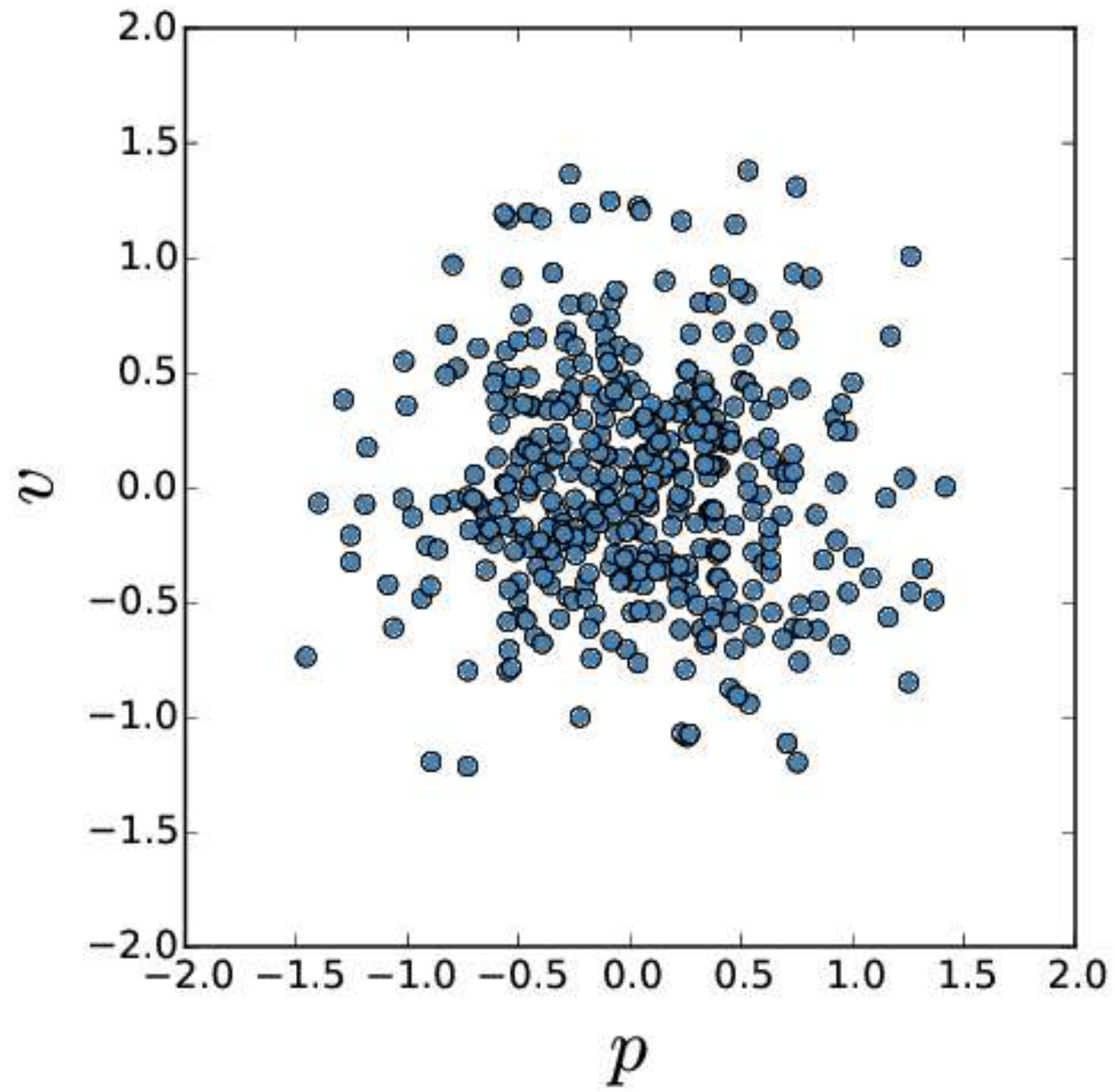

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Evolution equation: $$\frac{\partial}{\partial t}W(q,p,t) = -\{W(q,p,t), H(q,p,t)\}_{\text{Poisson bracket}}$$

for quadratic Hamiltonians

Weyl Transform $$\widetilde{A}(q,p) = \int \mathrm{d}u e^{-ipu}\left\langle q+\frac{u}{2}\right|\hat{A}\left|q-\frac{u}{2}\right\rangle$$

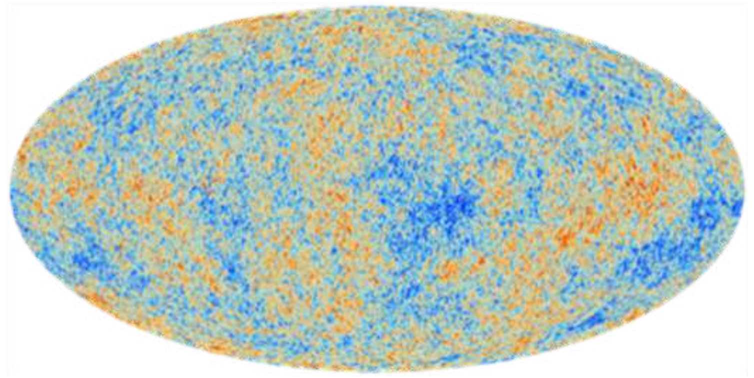
(with this definition: $W = \dfrac{\widetilde{\rho}}{2\pi}$)

Expectation value of quantum operators $$\langle\hat{A}\rangle = \int \widetilde{A}(q,p)\,W(q,p)\,\mathrm{d}q\mathrm{d}p$$

$W > 0$ ⟶ "quasi-probability distribution"

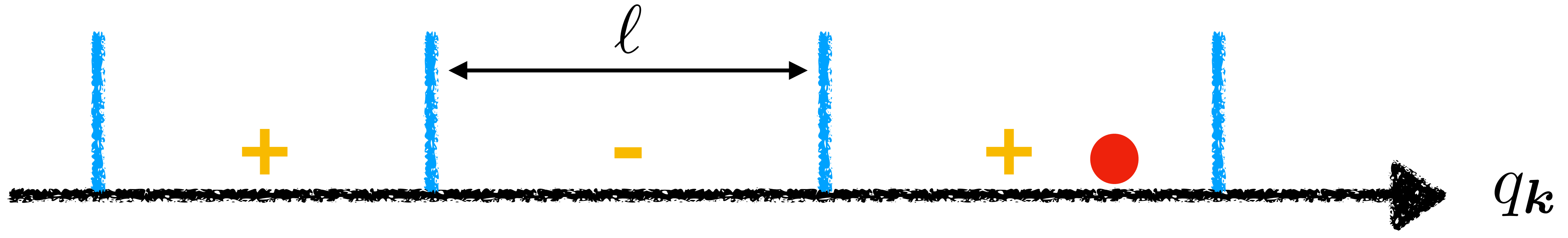
John Bell 1986, *EPR correlations and EPW distributions*:
"**Bell inequality violation requires non-positive Wigner function**"

Bell inequalities



continuous variable

$$\hat{q}_{\boldsymbol{k}} = \frac{\hat{c}_{\boldsymbol{k}} + \hat{c}^{\dagger}_{\boldsymbol{k}}}{\sqrt{2k}} = \hat{q}^{\dagger}_{\boldsymbol{k}}$$



continuous spectrum

- Divide the real axis into intervals $[n\ell, (n+1)\ell]$
- Perform a measurement of $q_{\boldsymbol{k}}$
- Return $S_z(\ell) = (-1)^n$

Larsson 2004

$$\hat{S}_z(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n \int_{n\ell}^{(n+1)\ell} \mathrm{d}q_{\boldsymbol{k}} \, |q_{\boldsymbol{k}}\rangle \langle q_{\boldsymbol{k}}| \longrightarrow \hat{S}_z^2(\ell) = 1$$

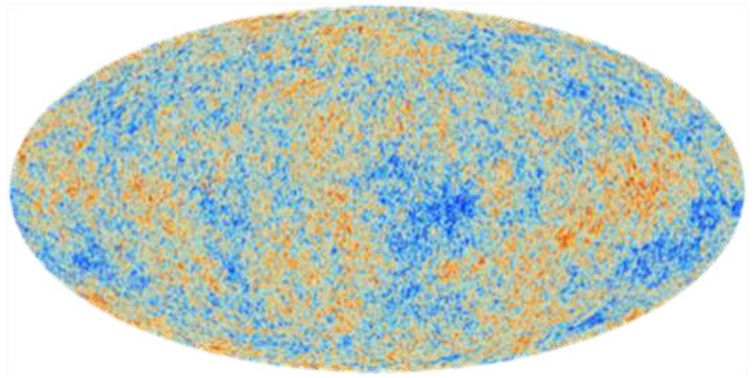
$$\hat{S}_+(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n \int_{2n\ell}^{(2n+1)\ell} \mathrm{d}q_{\boldsymbol{k}} \, |q_{\boldsymbol{k}}\rangle \langle q_{\boldsymbol{k}} + \ell|$$

$$\longrightarrow \hat{S}_x(\ell) = \hat{S}_+(\ell) + \hat{S}_+^{\dagger}(\ell)$$

$$\longrightarrow \hat{S}_y(\ell) = -i\left[\hat{S}_+(\ell) - \hat{S}_+^{\dagger}(\ell)\right]$$

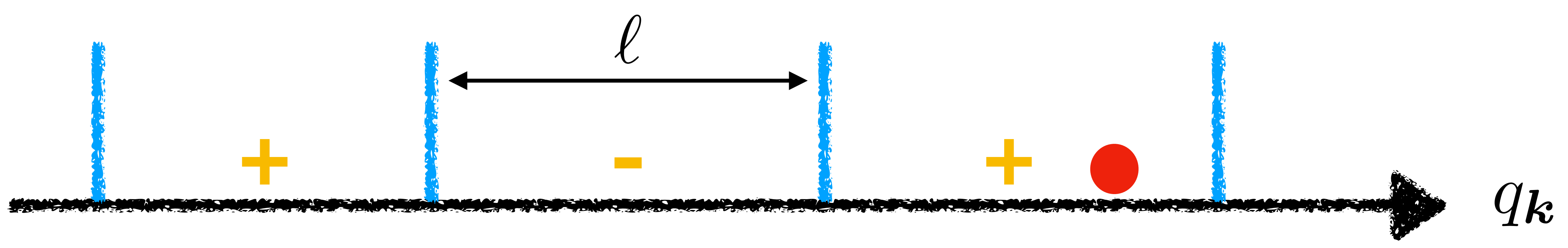
$$\longrightarrow \left[\hat{S}_i(\ell), \hat{S}_j(\ell)\right] = 2i\epsilon_{ijk}\hat{S}_k(\ell)$$ obey spin algebra

Bell inequalities



continuous variable

$$\hat{q}_{\boldsymbol{k}} = \frac{\hat{c}_{\boldsymbol{k}} + \hat{c}^{\dagger}_{\boldsymbol{k}}}{\sqrt{2k}} = \hat{q}^{\dagger}_{\boldsymbol{k}}$$



continuous spectrum

- Divide the real axis into intervals $[n\ell, (n+1)\ell]$
- Perform a measurement of $q_{\boldsymbol{k}}$
- Return $S_z(\ell) = (-1)^n$

Weyl transform:

improper ←

$$\widetilde{S}_x(q_{\boldsymbol{k}}, \pi_{\boldsymbol{k}}) = 2\sum_{n=-\infty}^{+\infty} \cos(\pi_{\boldsymbol{k}}\ell)\left[\Theta\left(q_{\boldsymbol{k}} - n\ell - \frac{\ell}{2}\right) - \Theta\left(q_{\boldsymbol{k}} - n\ell - \frac{3}{2}\ell\right)\right]$$

$$\widetilde{S}_y(q_{\boldsymbol{k}}, \pi_{\boldsymbol{k}}) = 2\sum_{n=-\infty}^{+\infty} \sin(\pi_{\boldsymbol{k}}\ell)\left[\Theta\left(q_{\boldsymbol{k}} - n\ell - \frac{\ell}{2}\right) - \Theta\left(q_{\boldsymbol{k}} - n\ell - \frac{3}{2}\ell\right)\right]$$

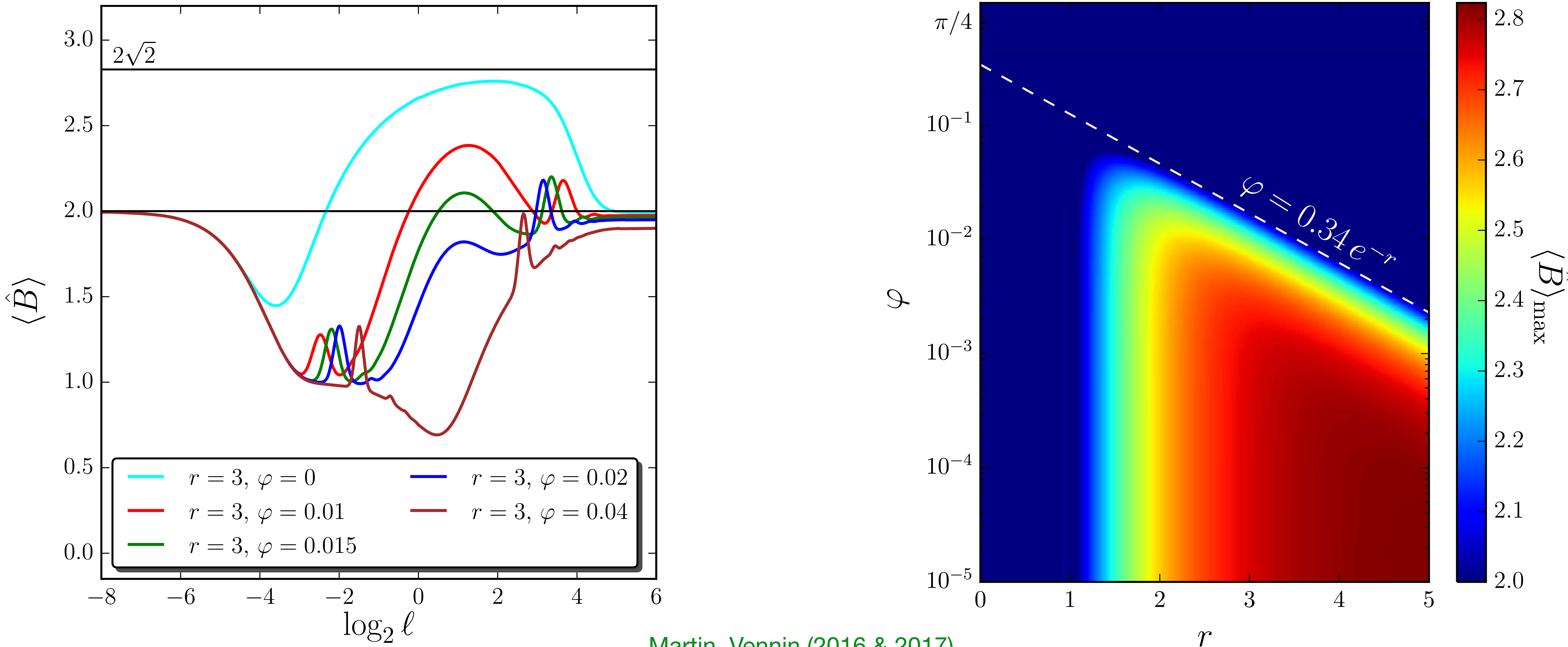
proper ←

$$\widetilde{S}_z(q_{\boldsymbol{k}}, \pi_{\boldsymbol{k}}) = \sum_{n=-\infty}^{+\infty} (-1)^n \left[\Theta(q_{\boldsymbol{k}} - n\ell) - \Theta(q_{\boldsymbol{k}} - n\ell - \ell)\right]$$

Martin, VV 2017

Bell inequalities in the CMB

classically: $\left\langle \hat{B}\left(\ell\right)\right\rangle < 2$

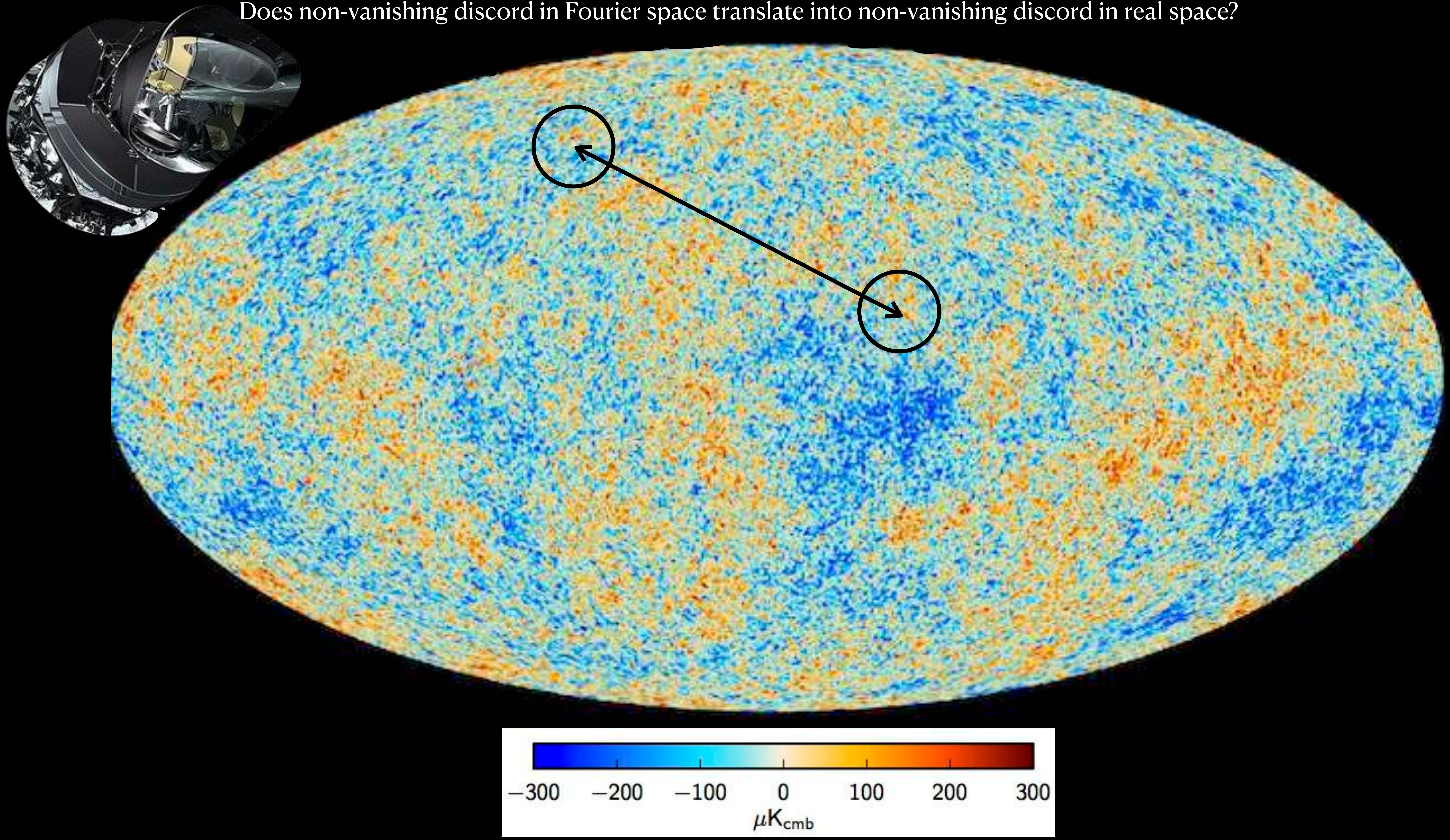


Martin, Vennin (2016 & 2017)

Wait a minute … do we measure correlations in Fourier space?

Does it make sense to test for locality in Fourier space?

Does non-vanishing discord in Fourier space translate into non-vanishing discord in real space?



Real-space entanglement of quantum fields

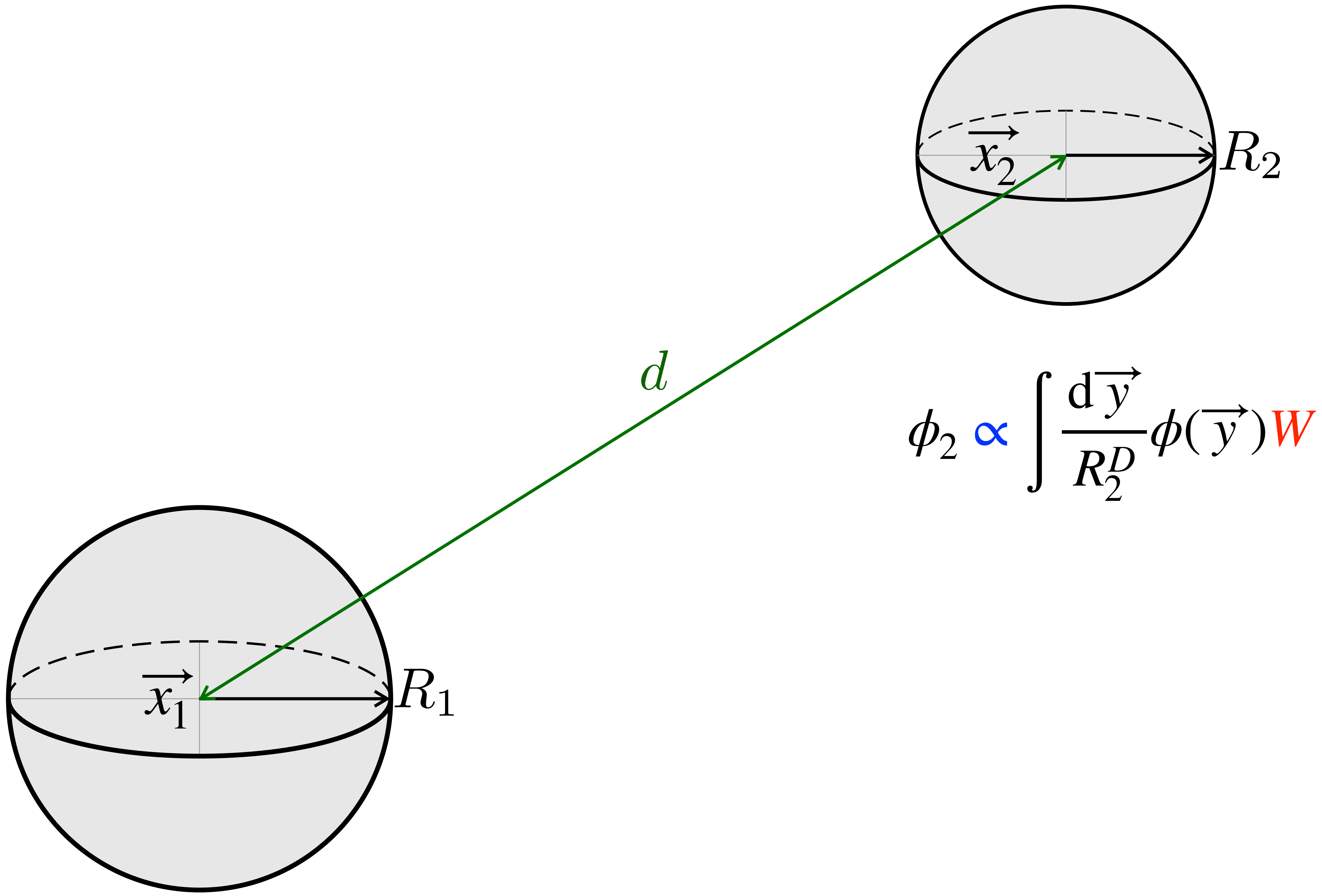
Previous work: Casini, Huerta (2009) // Datta (2009) // Shiba (2012) (for mutual information only, using numerical lattice simulations)

New approach (Martin, Vennin 2021) :

$$[\phi_i, \pi_j] = i\delta_{i,j}$$

The fields need to be rescaled

The support of the window function needs to be compact



$$\phi_2 \propto \int \frac{\mathrm{d}\vec{y}}{R_2^D}\phi(\vec{y})W\left(\frac{|\vec{y}-\vec{x}_2|}{R_2}\right)$$

$$\phi_1 \propto \int \frac{\mathrm{d}\vec{y}}{R_1^D}\phi(\vec{y})W\left(\frac{|\vec{y}-\vec{x}_1|}{R_1}\right)$$

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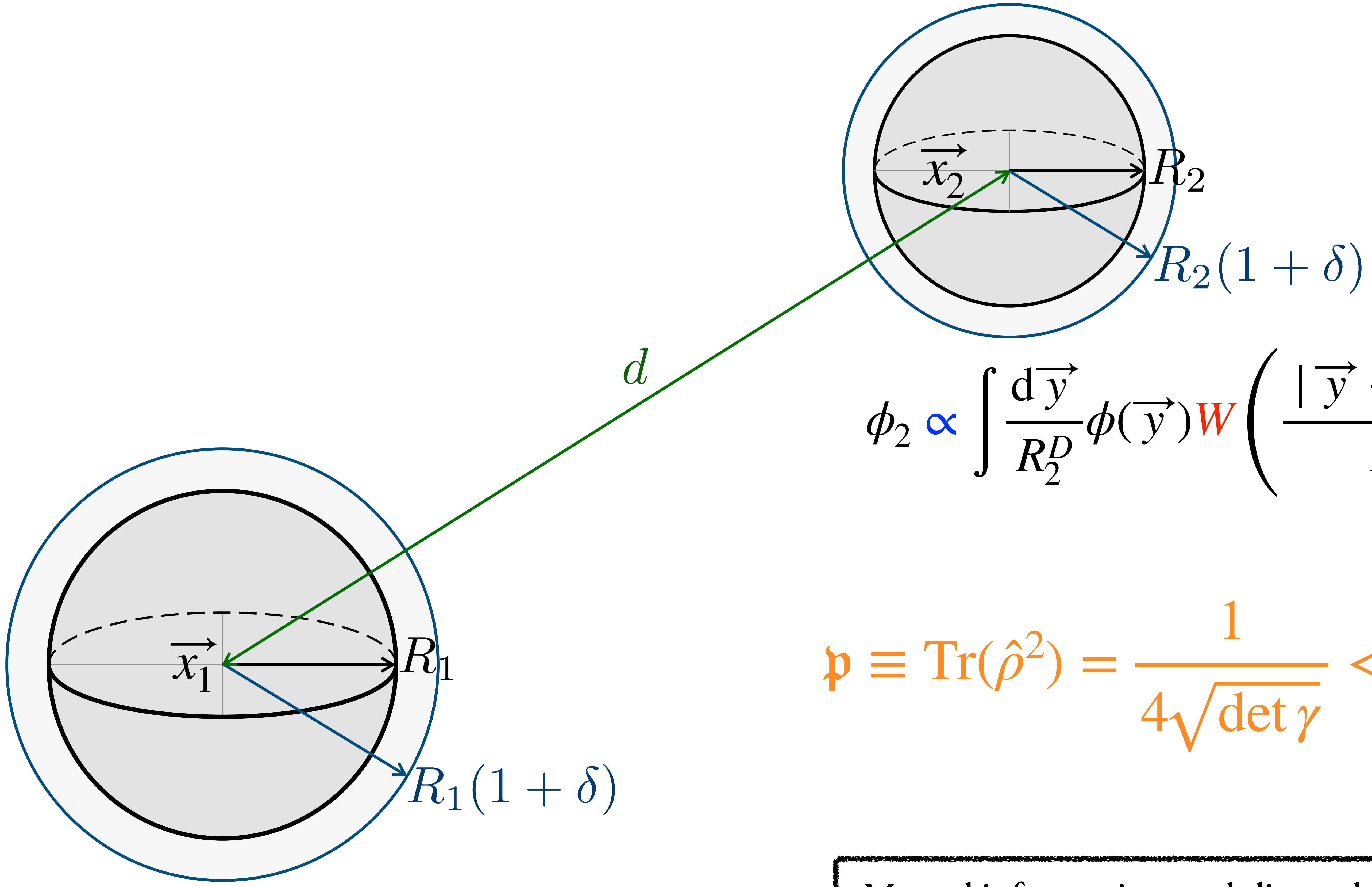


$$\phi_2 \propto \int \frac{\mathrm{d}\vec{y}}{R_2^D}\phi(\vec{y})W\left(\frac{|\vec{y}-\vec{x}_2|}{R_2}\right)$$

$$\phi_1 \propto \int \frac{\mathrm{d}\vec{y}}{R_1^D}\phi(\vec{y})W\left(\frac{|\vec{y}-\vec{x}_1|}{R_1}\right)$$

$$\mathfrak{p} \equiv \mathrm{Tr}(\hat{\rho}^2) = \frac{1}{4\sqrt{\det\gamma}} < 1$$

Mutual information and discord for bipartite Gaussian systems entirely given by the symplectic spectrum of the (sub-blocks of the) covariance matrix [Adesso & Data 2010]

$$[\phi_i, \pi_j] = i\delta_{i,j}$$

The fields need to be rescaled

The support of the window function needs to be compact

↓

Bipartite, "particle-like" system

For Gaussian quantum fields, this is a Gaussian bipartite system with correlation matrix

$$\gamma \propto \int \mathrm{d}\ln \widetilde{W}^2(kR) \begin{pmatrix} \mathcal{P}_{\phi\phi}(k) & \mathcal{P}_{\phi\pi}(k) & \mathcal{P}_{\phi\phi}(k)\,\mathrm{sinc}\,(kd) & \mathcal{P}_{\phi\pi}(k)\,\mathrm{sinc}\,(kd) \\ - & \mathcal{P}_{\pi\pi}(k) & \mathcal{P}_{\phi\phi}(k)\,\mathrm{sinc}\,(kd) & \mathcal{P}_{\pi\pi}(k)\,\mathrm{sinc}\,(kd) \\ - & - & \mathcal{P}_{\phi\phi}(k) & \mathcal{P}_{\phi\pi}(k) \\ - & - & - & \mathcal{P}_{\pi\pi}(k) \end{pmatrix}$$

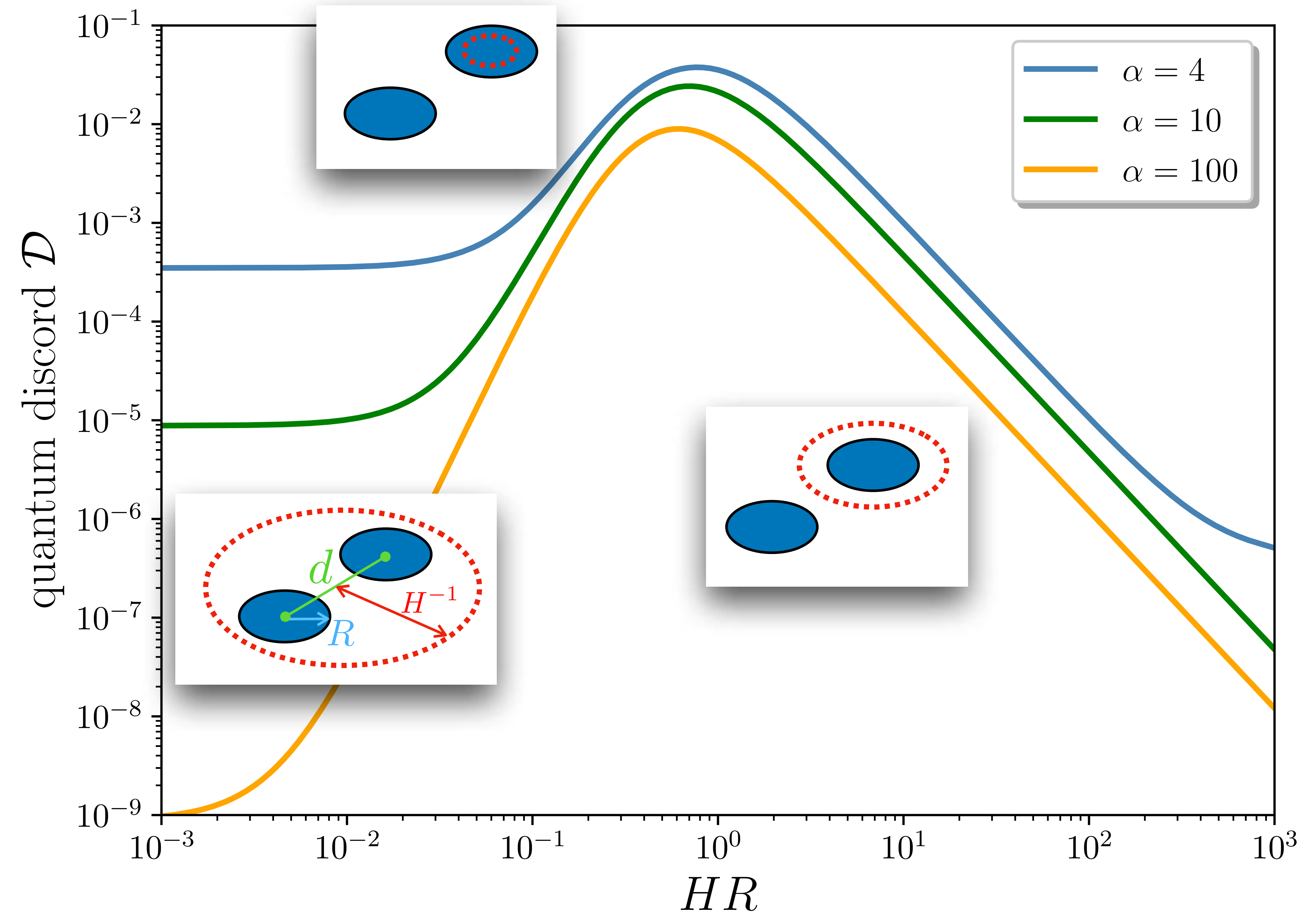
Does not describe a pure state

Real-space entanglement of quantum fields

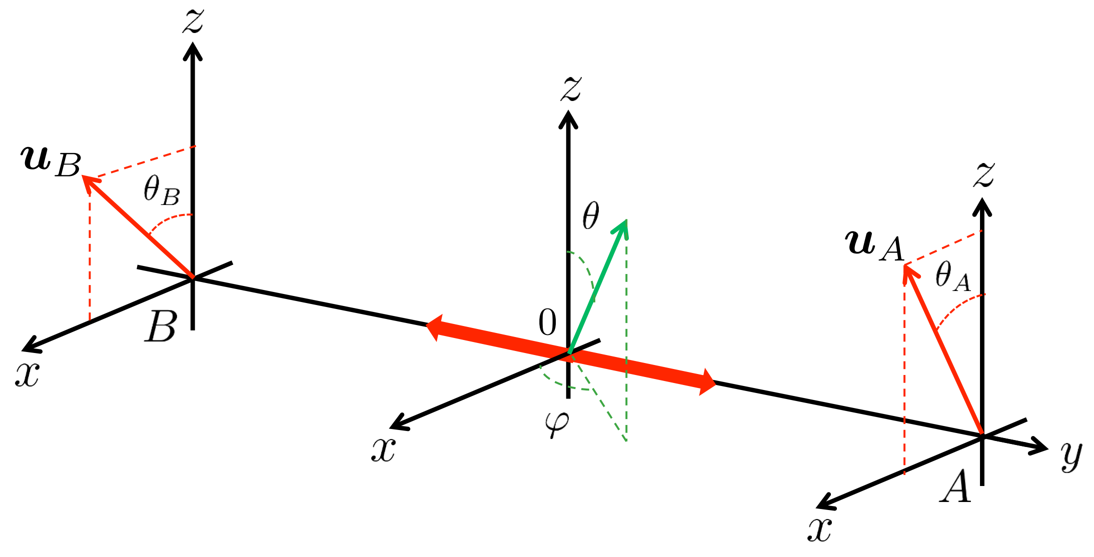
Cosmological perturbations

Martin, Vennin (2021)

$\alpha = d/R$



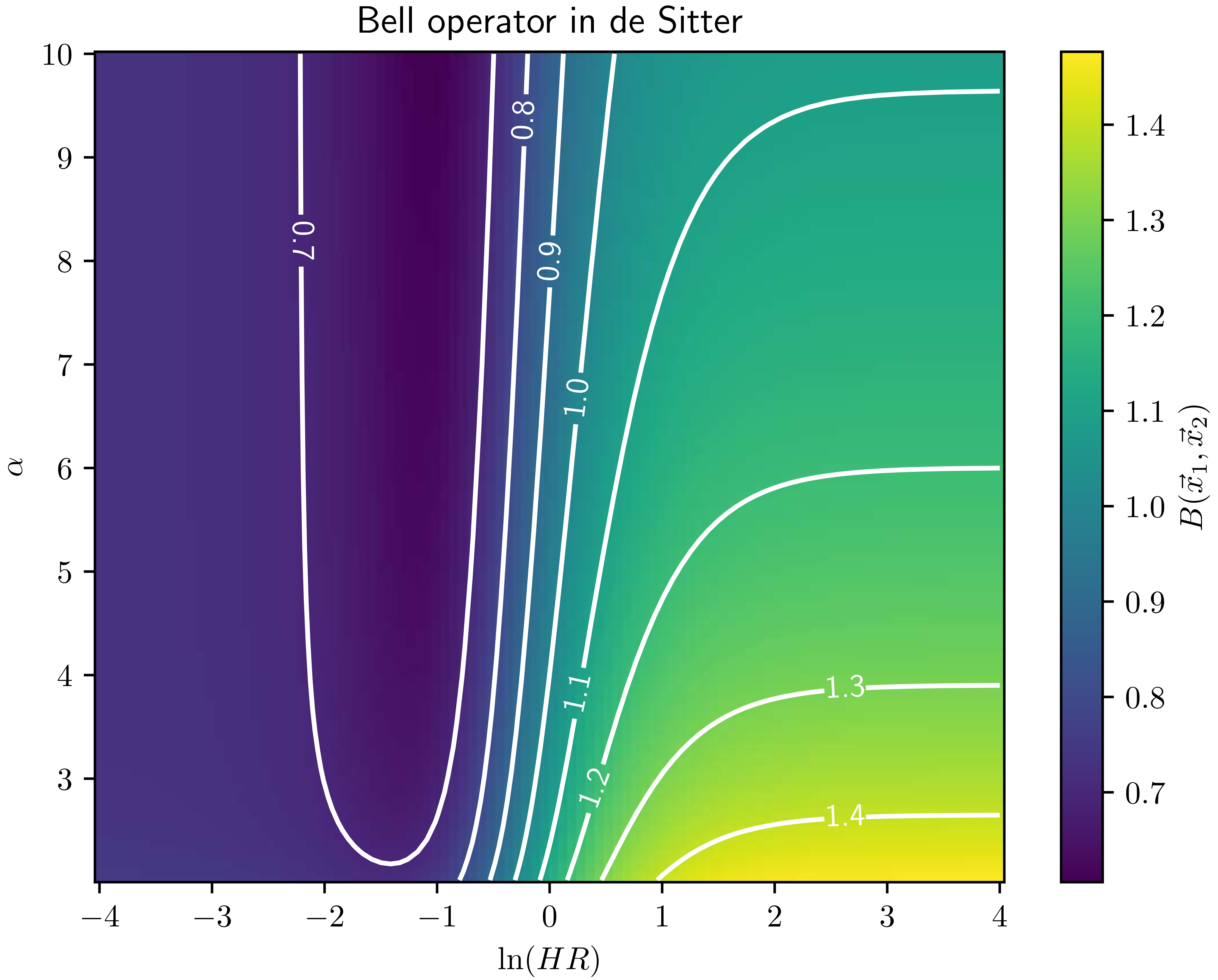
- Flat space results recovered at sub-Hubble scales
- Enhancement at large distance compared to flat space
- Smaller mutual information and discord than in Fourier space (effect of self-decoherence)
- Best place to look for quantum effects: Hubble scale at the end of inflation (compromise between correlations vs "self decoherence", or between particle creation vs decaying mode)

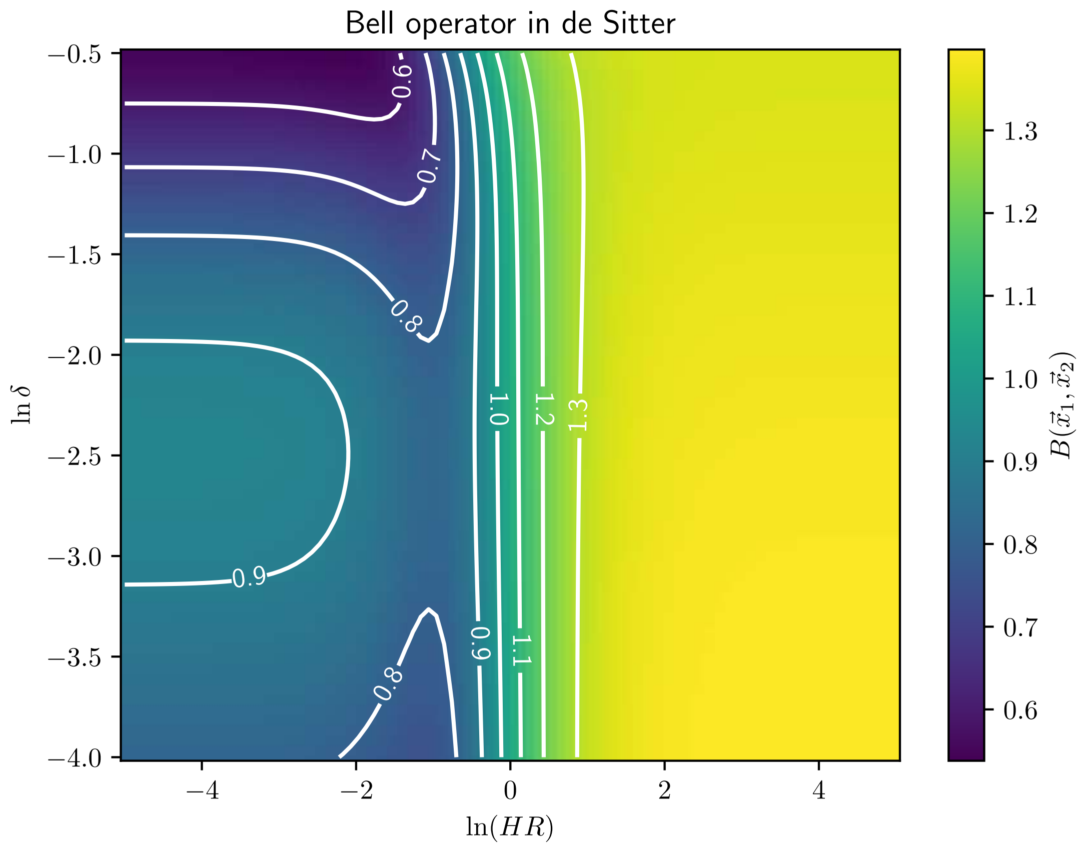


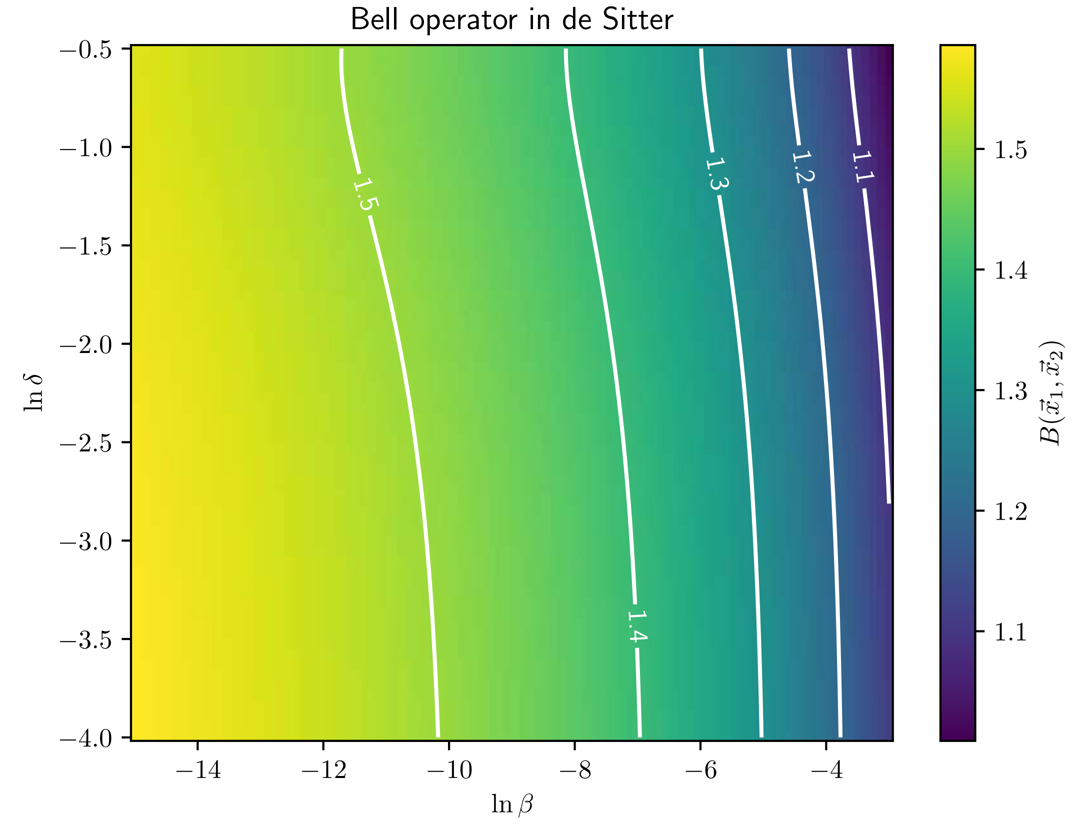
Real-space Bell inequalities

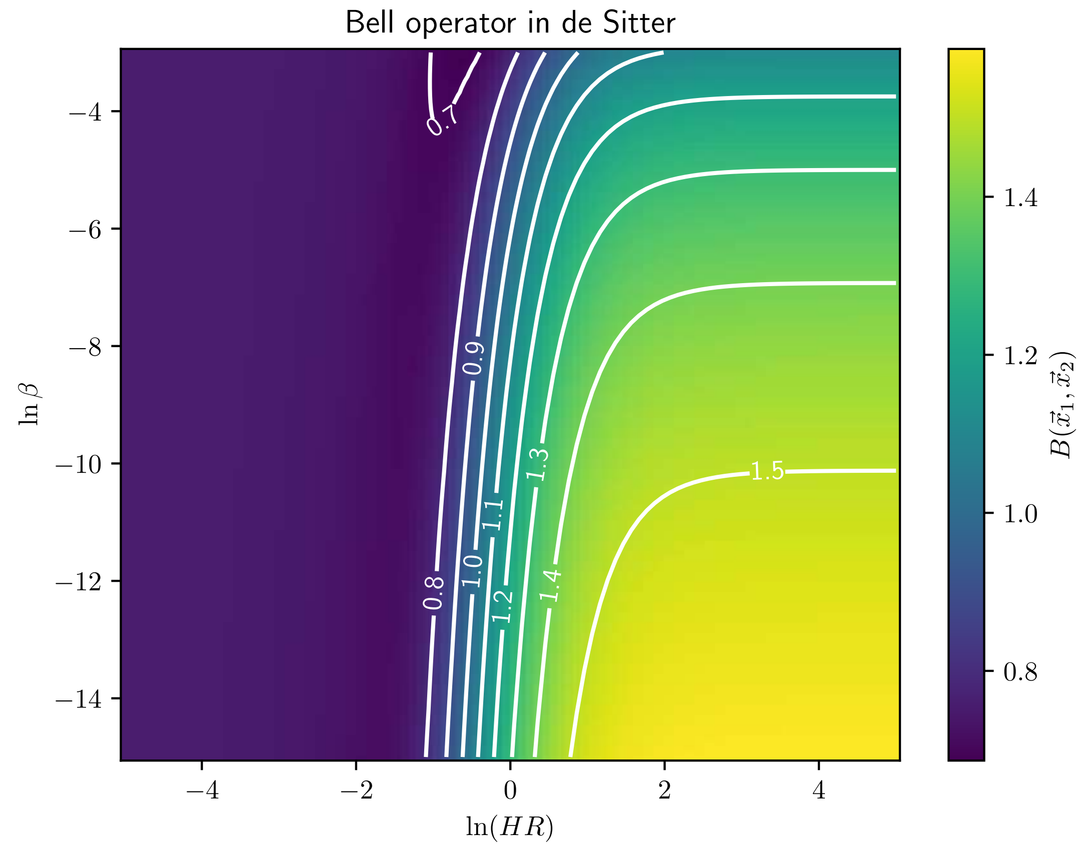
Cosmological perturbations

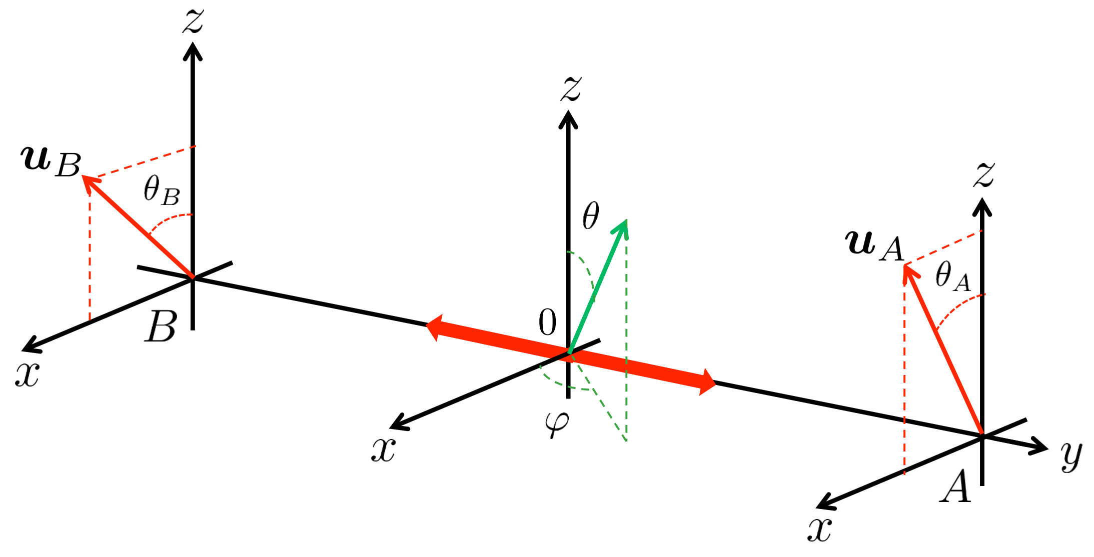
Espinosa-Portales, Vennin (to appear 2022)











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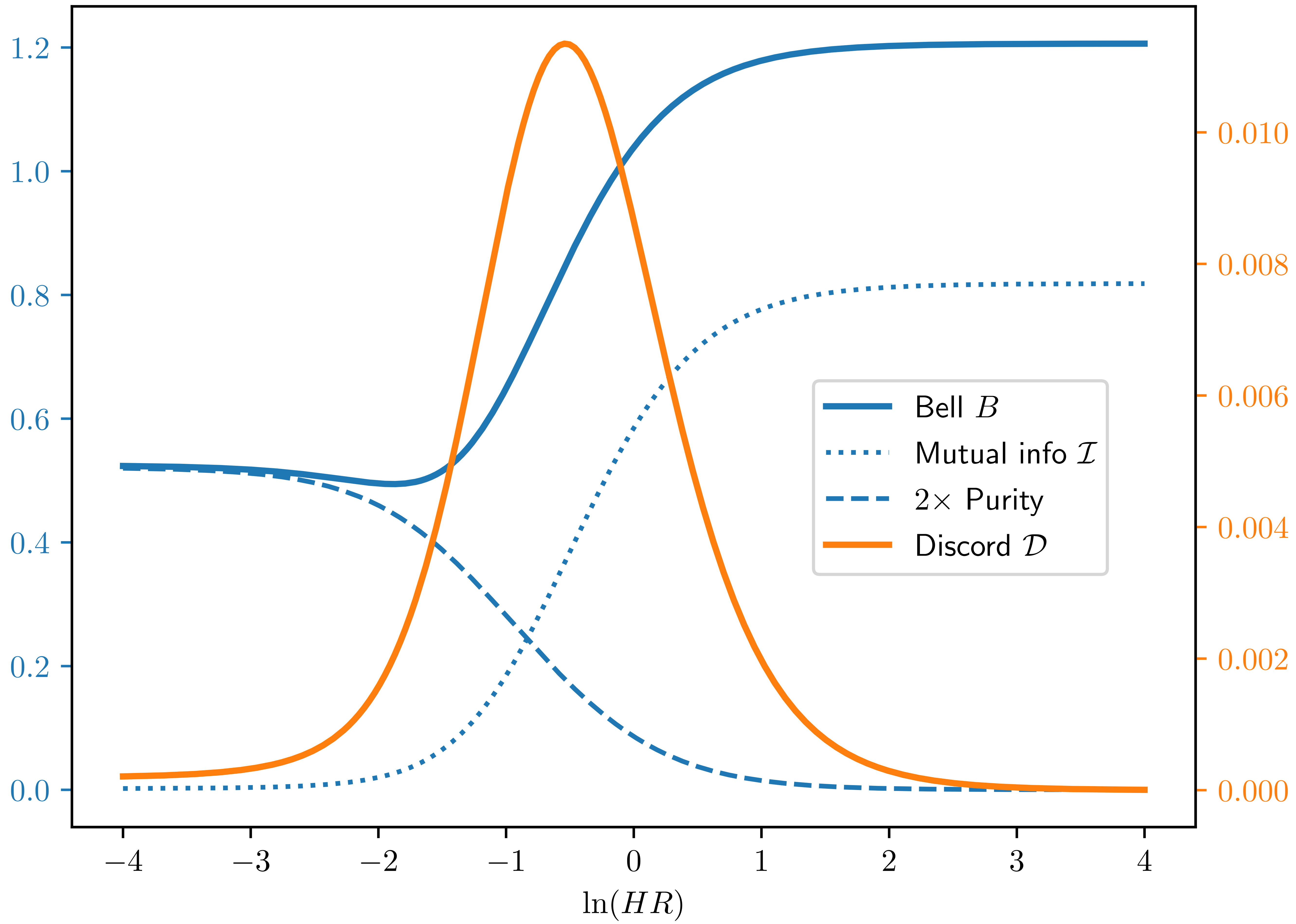


Need to access the decaying mode

How to measure $\hat{S}_+(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n \int_{2n\ell}^{(2n+1)\ell} \mathrm{d}q_{\boldsymbol{k}} \, |q_{\boldsymbol{k}}\rangle \langle q_{\boldsymbol{k}} + \ell|$?

requires to access phase information

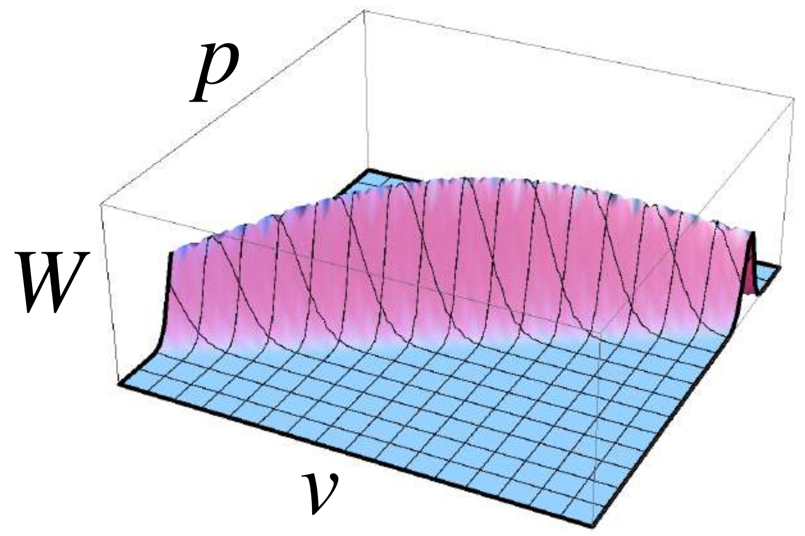
↓

conjugated momentum $\pi_{\boldsymbol{k}}$

↓

decaying mode

$\zeta'_{\boldsymbol{k}} \sim e^{-r_{\boldsymbol{k}}} \zeta_{\boldsymbol{k}}$



$\tilde{A} \to A$ in the large squeezing limit

Example: $O = v_{\boldsymbol{k}} v_{\boldsymbol{k}}^{\dagger} p_{\boldsymbol{k}} p_{\boldsymbol{k}}^{\dagger} + p_{\boldsymbol{k}} p_{\boldsymbol{k}}^{\dagger} v_{\boldsymbol{k}} v_{\boldsymbol{k}}^{\dagger}$

$\tilde{O} = O - 1/4$ with $\langle \hat{O} \rangle = e^{2\left(N - N_{\text{Hubble crossing}}\right)}$

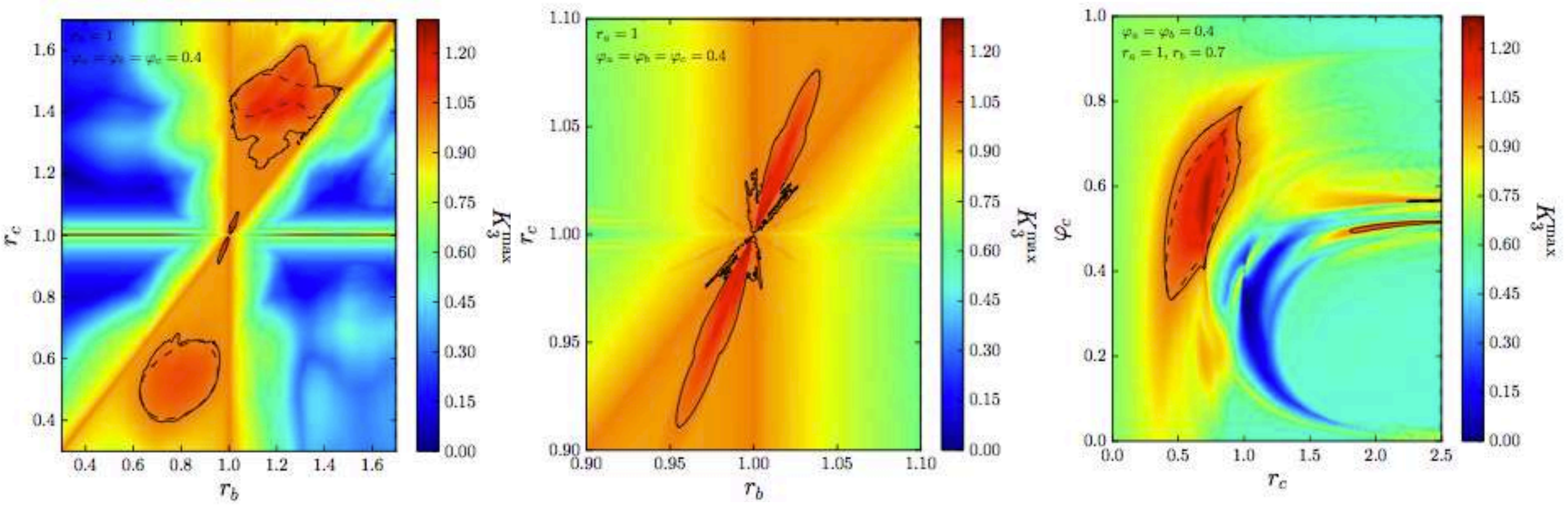
Can we detect quantum correlations using "position" measurements only?

$\widetilde{\zeta_{\boldsymbol{k}}} = \zeta_{\boldsymbol{k}}$ and $\widetilde{f}(\zeta_{\boldsymbol{k}}) = f(\zeta_{\boldsymbol{k}})$ so according to Revzen's theorem: not with Bell inequalities!

Generalised Bell inequalities in the CMB

	Type of inequality	Assumptions	Requires bipartite system	involves single spin measurement only
$\langle \hat{S}_1^a(t)\hat{S}_2^b(t)\rangle$	Spatial Bell	realism and locality	yes	no
$\langle \hat{S}_1^a(t)\hat{S}_1^b(t')\rangle$	Temporal Bell	realism and non-invasiveness	no	no
$\langle \hat{S}_1^a(t)\hat{S}_1^a(t')\hat{S}_1^a(t'')\rangle$	Legget-Garg (≥3 measurement times)	realism and non-invasiveness	no	yes
$\langle \hat{S}_1^a(t)\hat{S}_2^a(t')\rangle$	Bipartite temporal Bell	realism and locality	yes	yes

J. Martin, V.V. (2016) for LGI;



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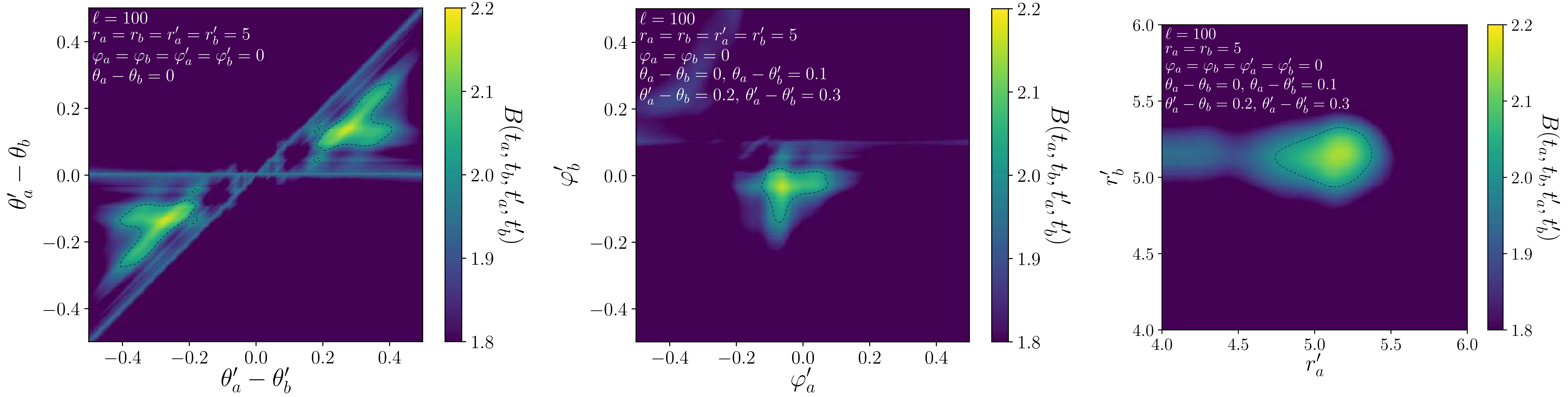
$\langle \hat{S}_1^a(t)\hat{S}_2^a(t')\rangle$	Bipartite temporal Bell	realism and locality	yes	yes

K. Ando, V.V. (2020) for BTBI



But requires to measure zeta at different times … cross correlate measurement at different redshifts?

Conclusions

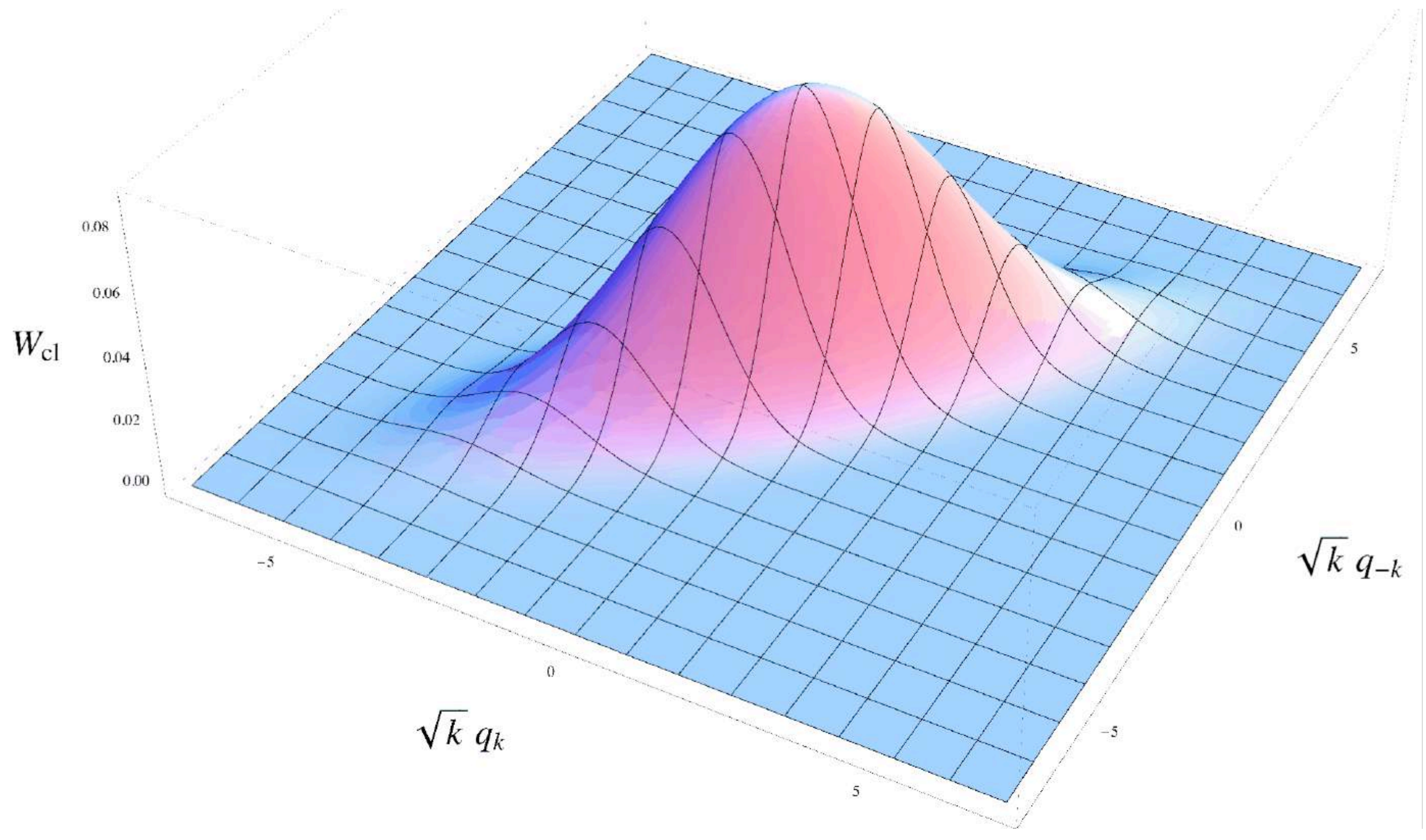
- **Cosmological perturbations** are placed in a two-mode highly **squeezed state** in the very early Universe
- Such a state has a large **quantum discord in Fourier space**, denoting the presence of **large quantum correlations** between particles created with opposite wave momenta
- In **real space**, quantum discord is much more suppressed, and we de not report violations of Bell inequalities.
- Even if we found successful Bell operators, would it require to “measure” somehow the (exponentially suppressed, at least in the standard setup) **decaying mode ?**
- What about **non-Gaussianities?**
- What about **decoherence?**

Non-discordant states

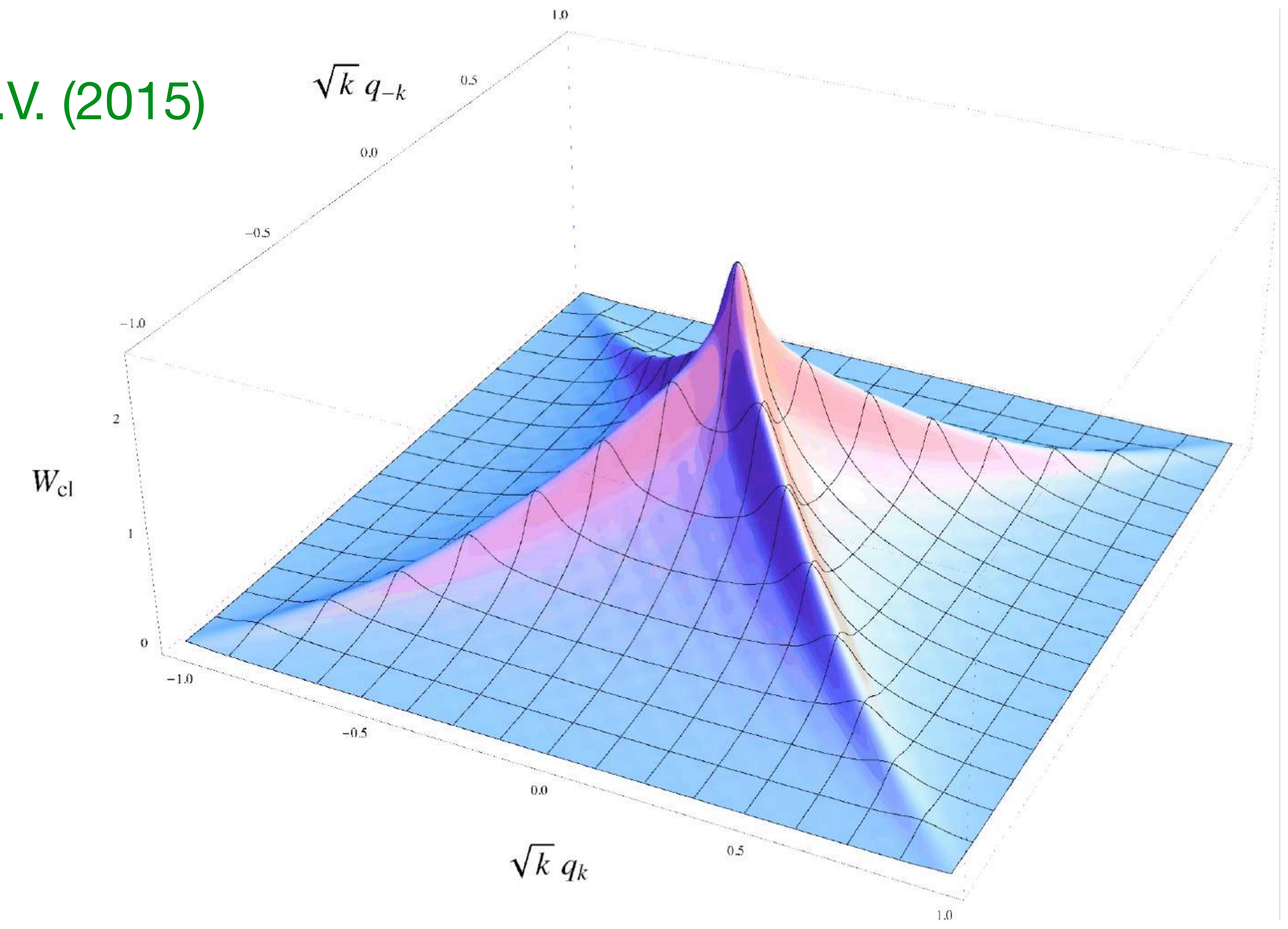
Can we detect quantum correlations using single-time, “position” measurements only?

Classical states = non-discordant states $\mathscr{D}(k, -k) = 0$

J. Martin, V.V. (2015)



Two-mode squeezed state



Non-discordant state sharing the same two-point functions as the two-mode squeezed state

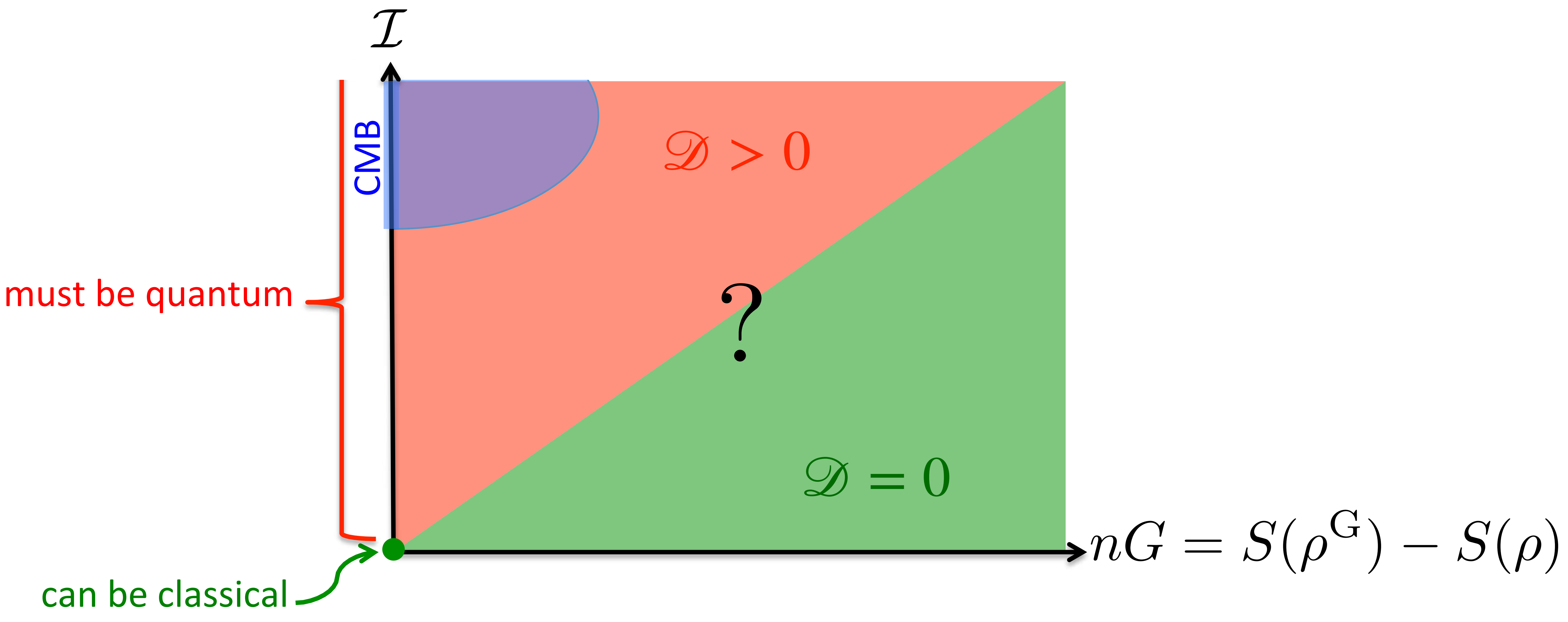
Non-discordant states

Can we detect quantum correlations using single-time, “position” measurements only?

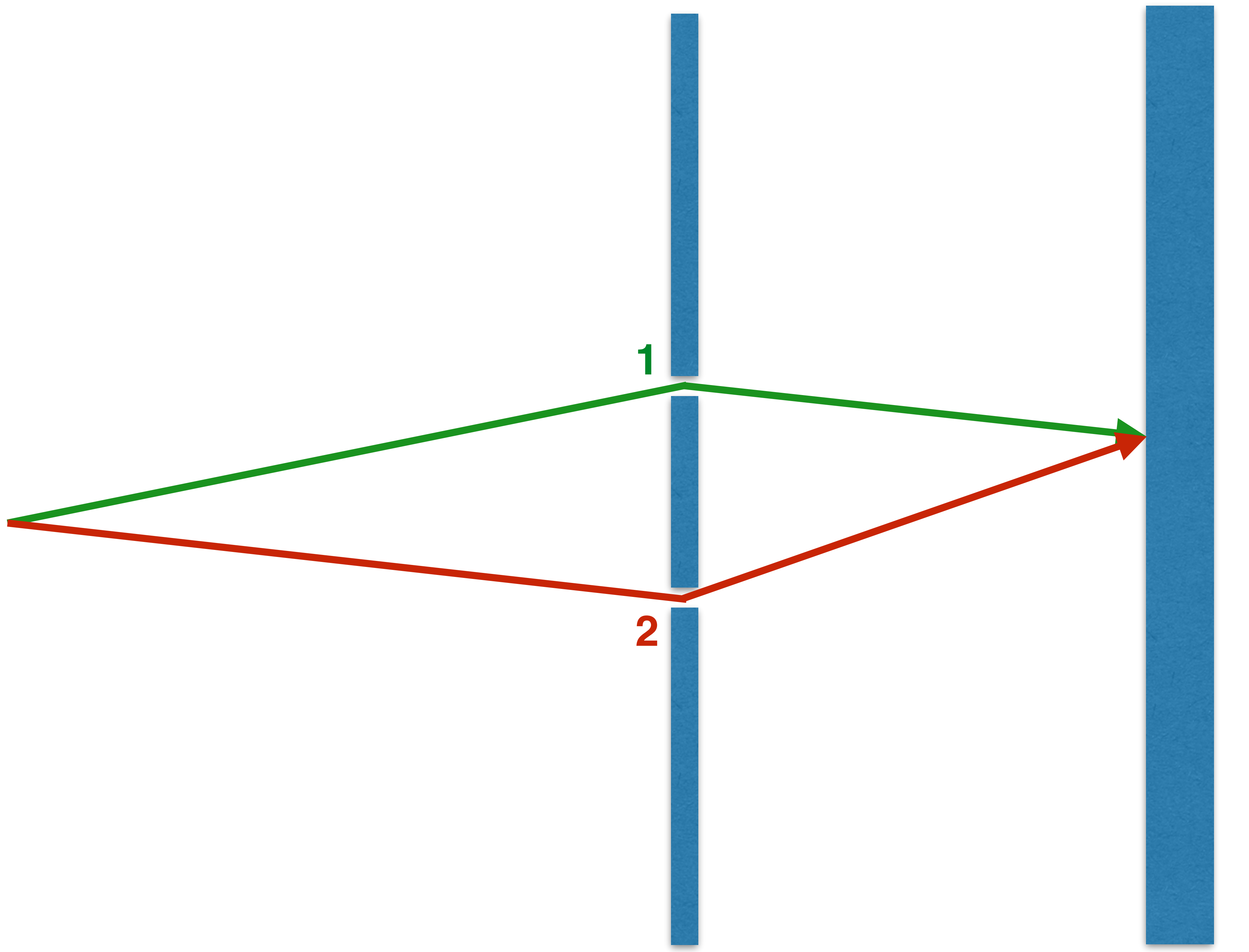
Classical states = non-discordant states $\mathscr{D}(k, -k) = 0$

Theorem: the only classical Gaussian states are product states

Adesso, Datta 2010; Rahimi-Keshari, Calves, Ralph 2013; Mista, Mc Nulty 2014



Quantum Decoherence: The double-slit experiment

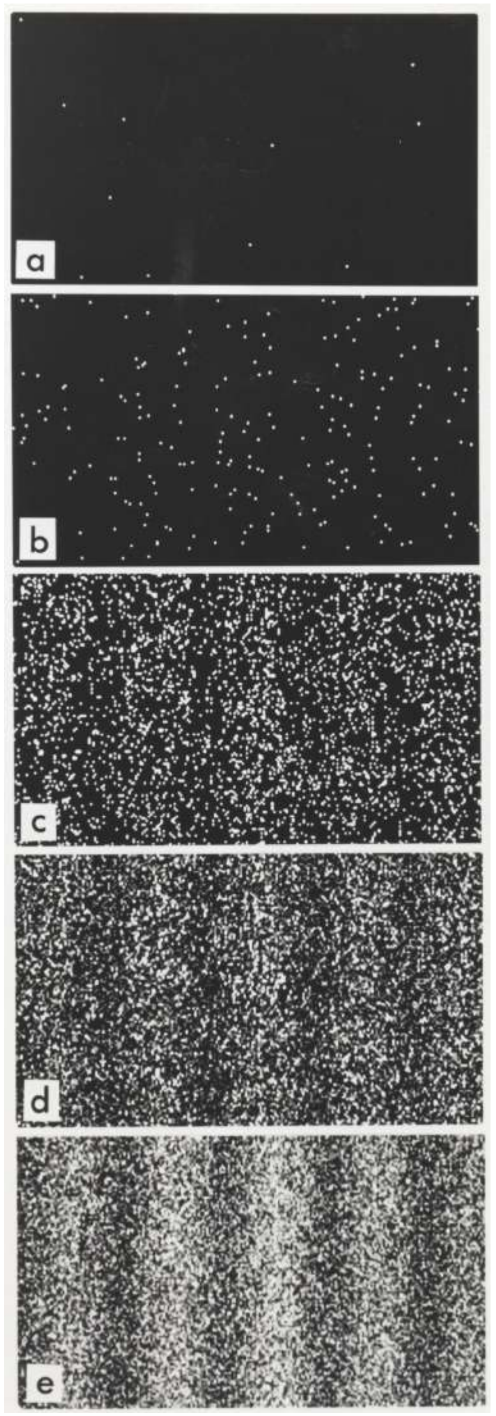


$$|\Psi\rangle = |\Psi_1\rangle + |\Psi_2\rangle$$

interference

↕

phase coherence

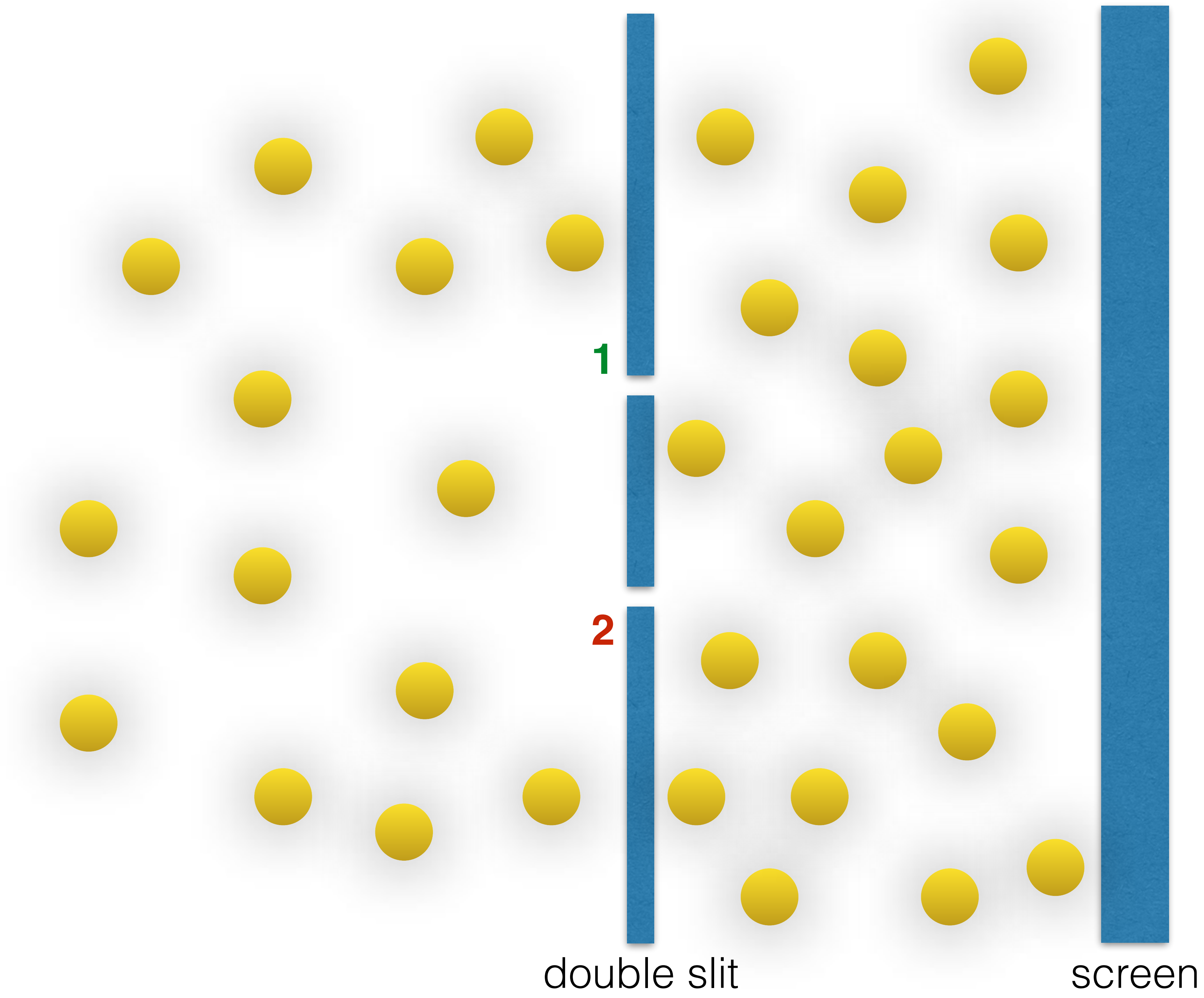


$$\Psi(x) = \Psi_1(x) + \Psi_2(x)$$
$$\propto e^{i\phi_1(x)} + e^{i\phi_2(x)}$$

⟶

$$p(x) = |\Psi(x)|^2$$
$$= |\Psi_1(x)|^2 + |\Psi_2(x)|^2 + \Psi_1(x)\Psi_2^*(x) + \Psi_1^*(x)\Psi_1(x)$$
$$= 2 + 2\cos(\phi_1 - \phi_2)$$

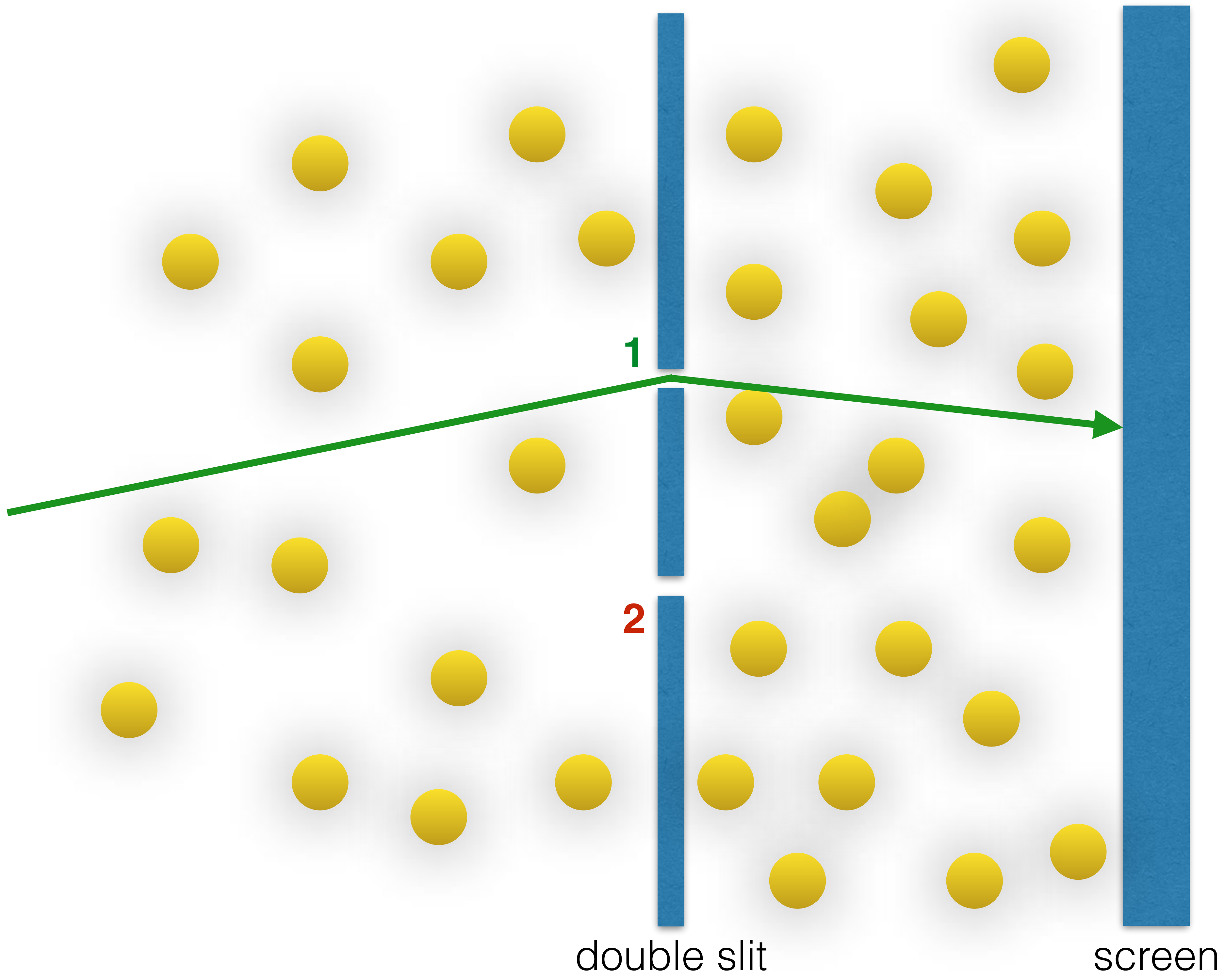
Quantum Decoherence:
The double-slit experiment



Adding the environment

$|\mathcal{E}\rangle$

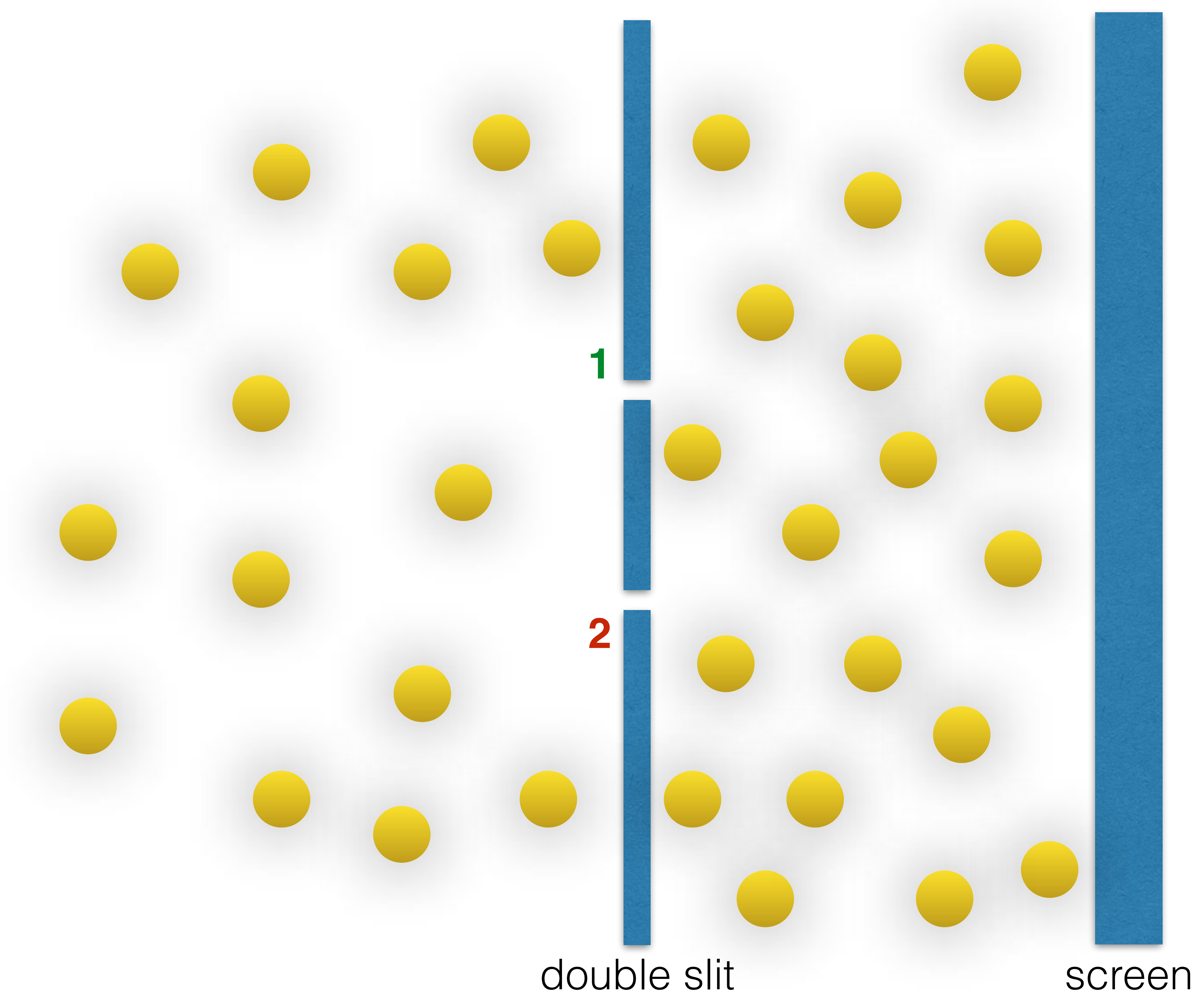
Quantum Decoherence:
The double-slit experiment



Adding the environment

$$|\mathcal{E}\rangle \longrightarrow |\mathcal{E}_1\rangle$$

Quantum Decoherence:
The double-slit experiment



Adding the environment

$|\mathcal{E}\rangle$

Quantum Decoherence:
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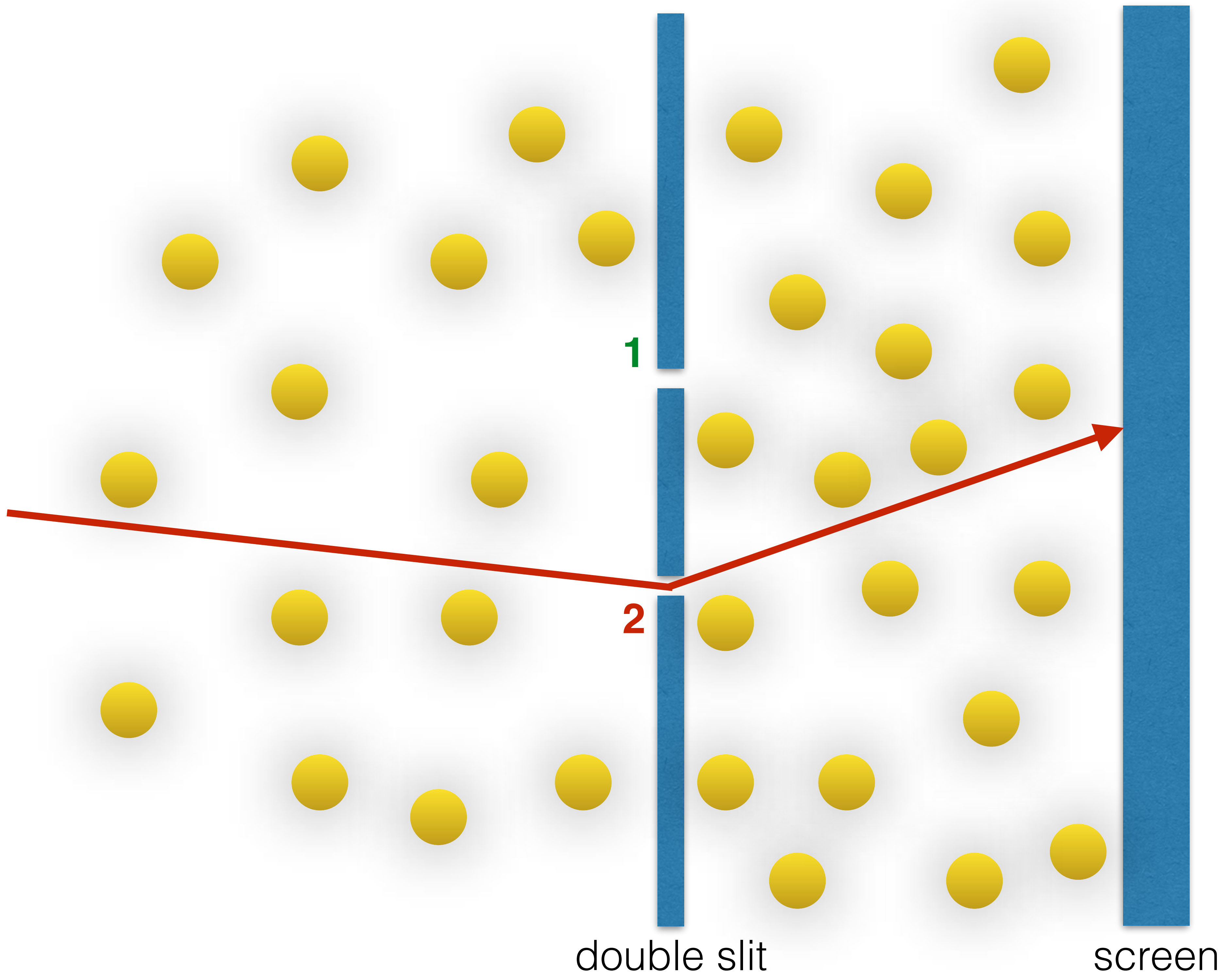
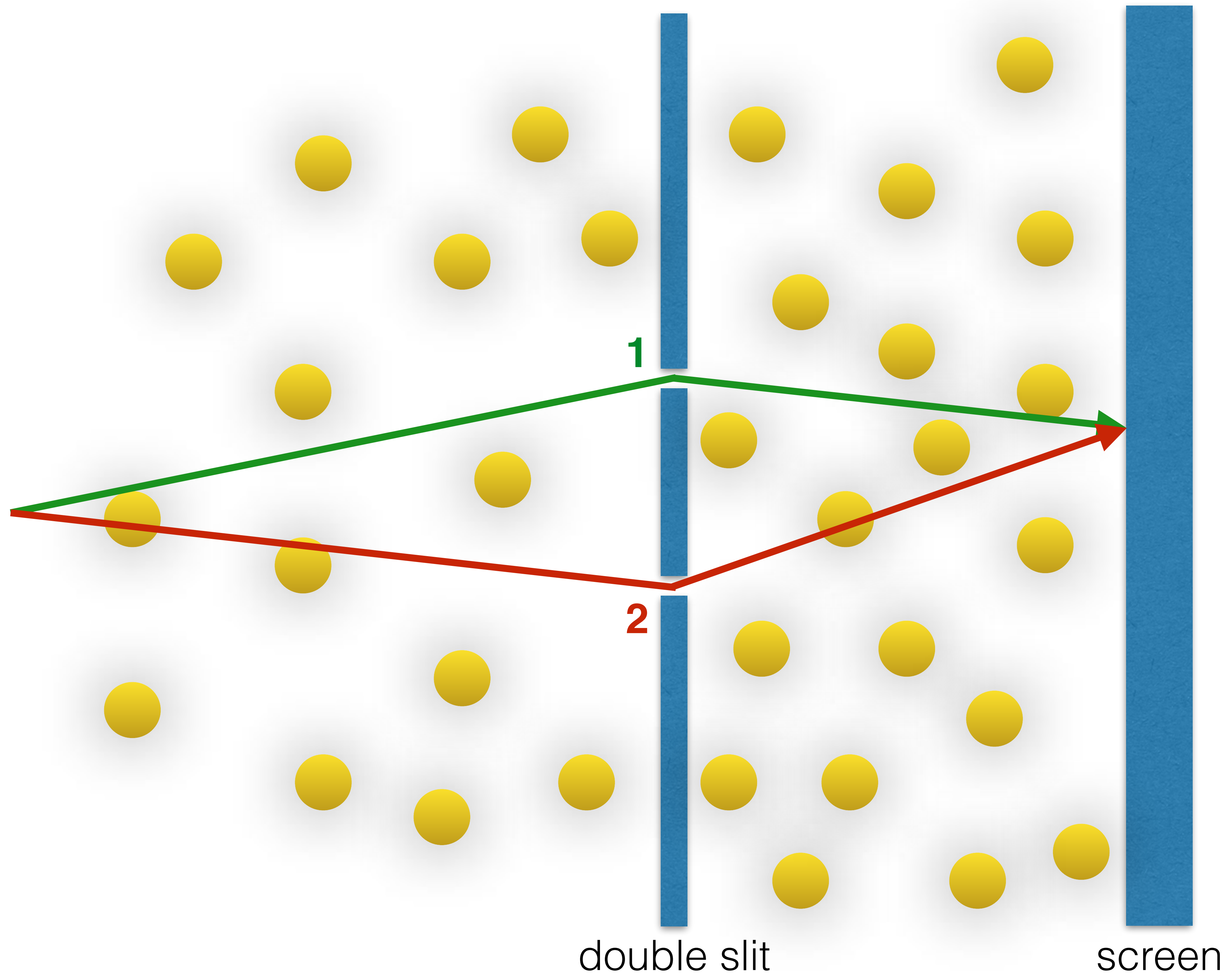


Adding the environment

$$|\mathcal{E}\rangle \longrightarrow |\mathcal{E}_2\rangle$$

Quantum Decoherence: The double-slit experiment



$$|\Psi\rangle = |\Psi_1\rangle \otimes |\mathcal{E}_1\rangle + |\Psi_2\rangle \otimes |\mathcal{E}_2\rangle$$

system becomes entangled with environment

$$\langle \mathcal{E}_i | \mathcal{E}_j \rangle = \delta_{ij}$$

↓

destruction of interferences

↓

loss of phase coherence

↓

"decoherence"

H.D. Zeh 1970

$$\langle \Psi | \hat{O}_{\mathrm{S}} | \Psi \rangle = \langle \Psi_1 | \hat{O}_{\mathrm{S}} | \Psi_1 \rangle \langle \mathcal{E}_1 | \mathcal{E}_1 \rangle + \langle \Psi_2 | \hat{O}_{\mathrm{S}} | \Psi_2 \rangle \langle \mathcal{E}_2 | \mathcal{E}_2 \rangle$$
$$+ \langle \Psi_1 | \hat{O}_{\mathrm{S}} | \Psi_2 \rangle \langle \mathcal{E}_1 | \mathcal{E}_2 \rangle + \langle \Psi_2 | \hat{O}_{\mathrm{S}} | \Psi_1 \rangle \langle \mathcal{E}_2 | \mathcal{E}_1 \rangle$$

Quantum Decoherence:
The double-slit experiment

$$|\Psi\rangle = \big(|\Psi_1\rangle + |\Psi_1\rangle\big) \otimes |\mathcal{E}\rangle \longrightarrow |\Psi\rangle = |\Psi_1\rangle \otimes |\mathcal{E}_1\rangle + |\Psi_2\rangle \otimes |\mathcal{E}_2\rangle$$

$$\langle\Psi|\,\hat{O}_{\mathrm{S}}\,|\Psi\rangle = \langle\Psi_1|\,\hat{O}_{\mathrm{S}}\,|\Psi_1\rangle\,\langle\mathcal{E}_1|\mathcal{E}_1\rangle + \langle\Psi_2|\,\hat{O}_{\mathrm{S}}\,|\Psi_2\rangle\,\langle\mathcal{E}_2|\mathcal{E}_2\rangle$$
$$+ \langle\Psi_1|\,\hat{O}_{\mathrm{S}}\,|\Psi_2\rangle\,\langle\mathcal{E}_1|\mathcal{E}_2\rangle + \langle\Psi_2|\,\hat{O}_{\mathrm{S}}\,|\Psi_1\rangle\,\langle\mathcal{E}_2|\mathcal{E}_1\rangle$$

Quantum Decoherence: The double-slit experiment

$$|\Psi\rangle = \left(|\Psi_1\rangle + |\Psi_1\rangle\right) \otimes |\mathcal{E}\rangle \longrightarrow |\Psi\rangle = |\Psi_1\rangle \otimes |\mathcal{E}_1\rangle + |\Psi_2\rangle \otimes |\mathcal{E}_2\rangle$$

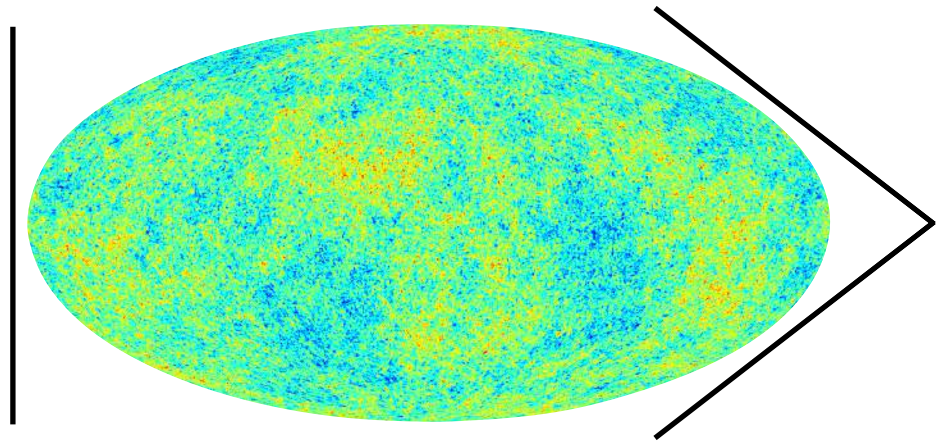
$$\mathrm{Tr}_{\mathcal{E}}\left(\hat{\rho}\right) = \hat{\rho}_{\mathrm{S}} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \longrightarrow \hat{\rho}_{\mathrm{S}} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Decoherence: decay of the off-diagonal elements of the density matrix, in the basis selected by the form of the interaction with the environment

$$\langle\Psi|\,\hat{O}_{\mathrm{S}}\,|\Psi\rangle = \mathrm{Tr}\left(\hat{\rho}\hat{O}_{\mathrm{S}}\right) = \mathrm{Tr}_{\mathrm{S}}\mathrm{Tr}_{\mathcal{E}}\left(\hat{\rho}\hat{O}_{\mathrm{S}}\right) = \mathrm{Tr}_{\mathrm{S}}\left[\mathrm{Tr}_{\mathcal{E}}\left(\hat{\rho}\right)\hat{O}_{\mathrm{S}}\right]$$

Quantum-to-classical transition of primordial fluctuations

$|\mathrm{CMB}\rangle$ =



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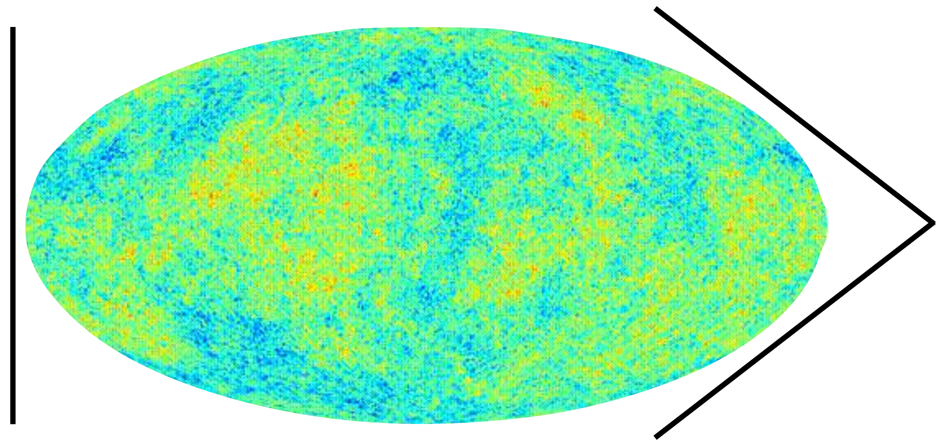
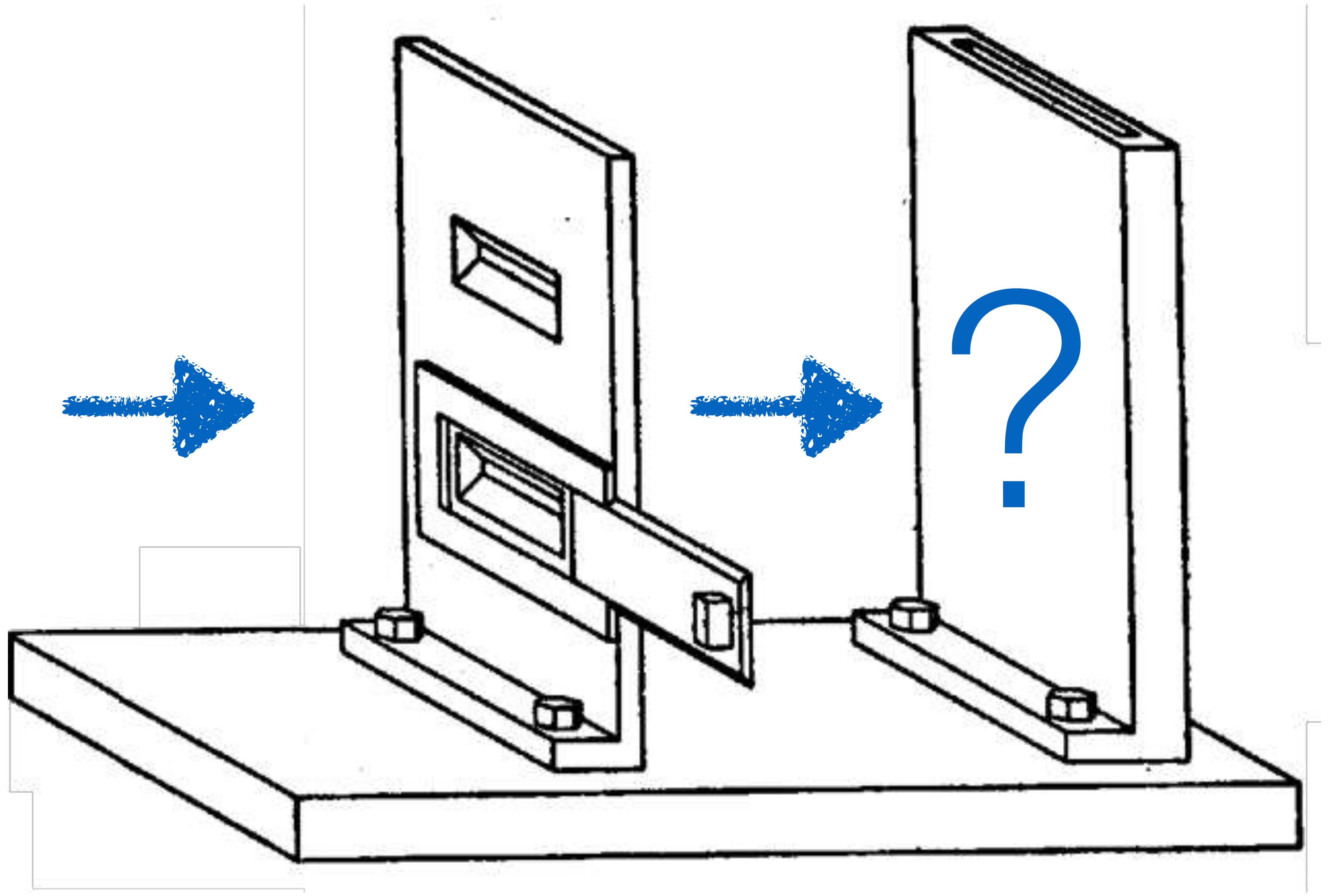
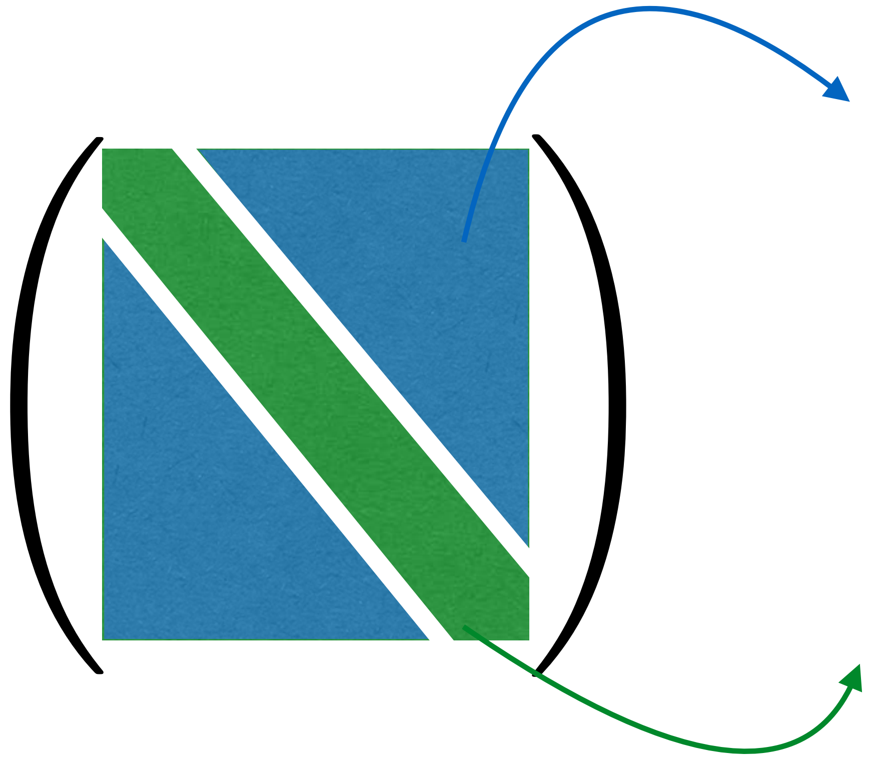


+ ...



$\rho =$



Phase coherence between the different outcomes

Probability of observing the different outcomes

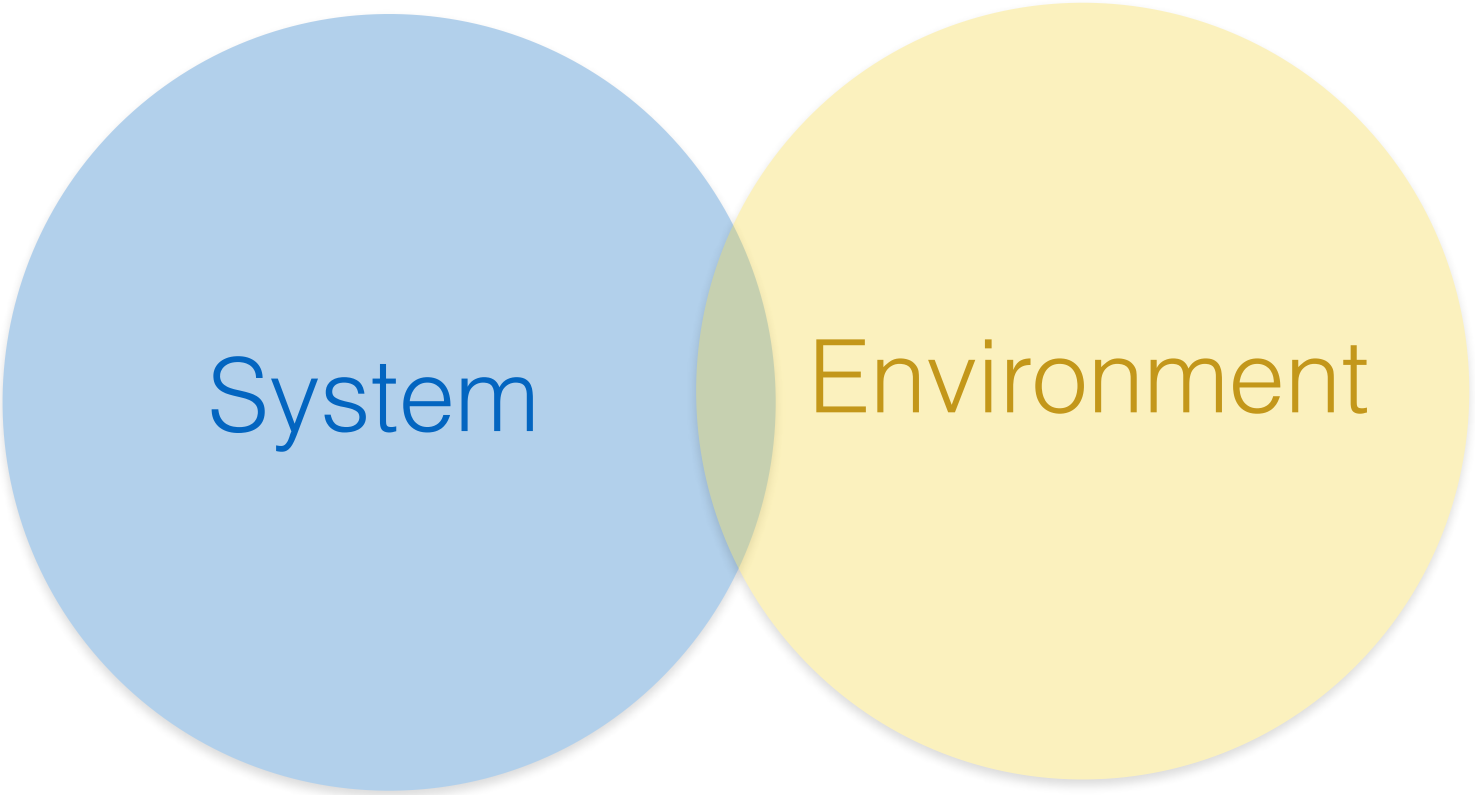
Can we change one without the other?

Interaction with an environment is inevitable anyway!

The Lindblad equation

cosmological perturbations

$$v\left(\eta, \boldsymbol{x}\right)$$



-other fields

-cosmological perturbations outside our causal horizon

-physics beyond the UV or IR cutoffs of the theory

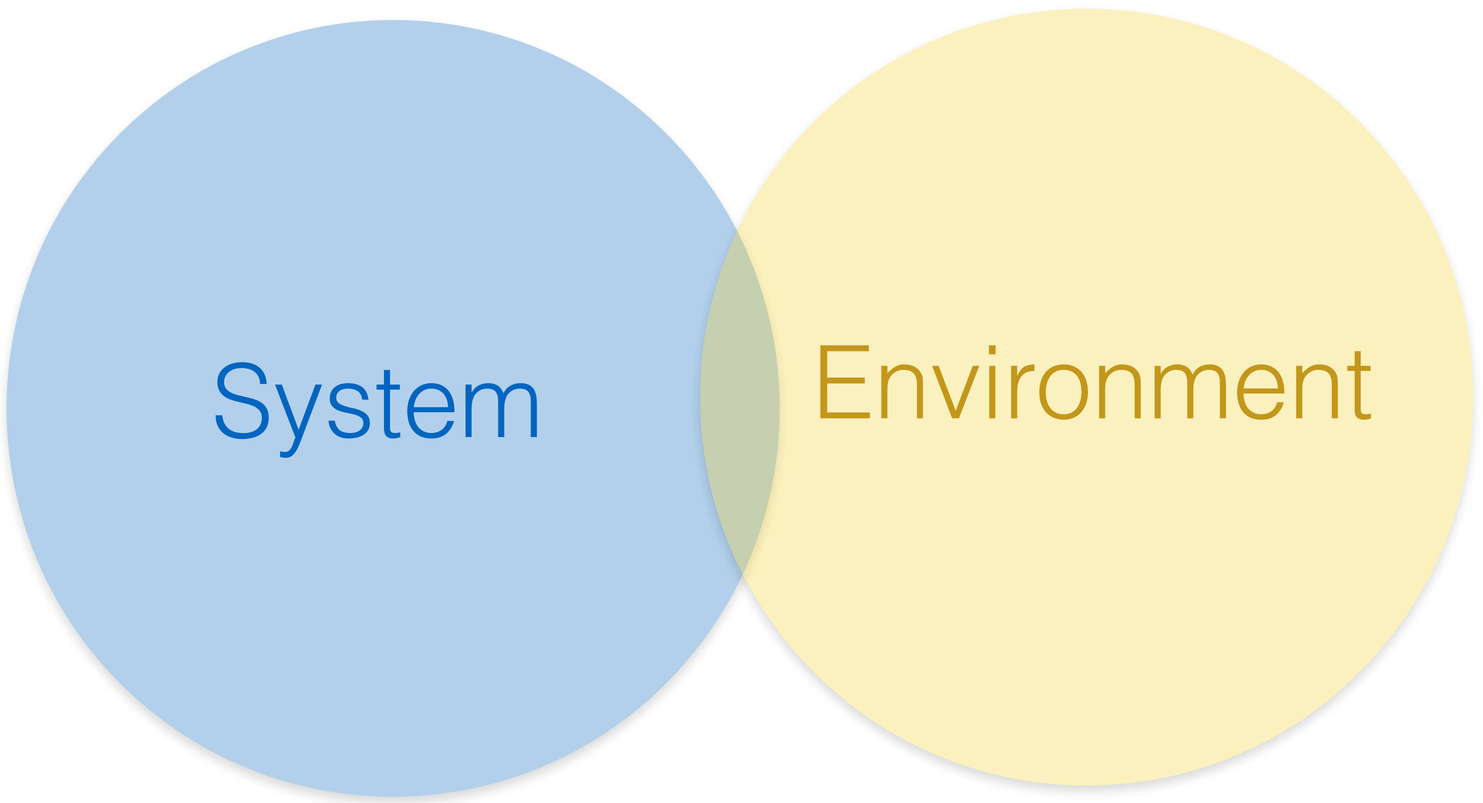
…

$$\hat{H} = \hat{H}_v \otimes \hat{\mathbb{I}}_{\mathrm{env}} + \hat{\mathbb{I}}_v \otimes \hat{H}_{\mathrm{env}} + g\hat{H}_{\mathrm{int}}$$

The Lindblad equation

cosmological perturbations

$v\left(\eta, \boldsymbol{x}\right)$



-other fields

-cosmological perturbations outside our causal horizon

-physics beyond the UV or IR cutoffs of the theory

...

$$\hat{H} = \hat{H}_v \otimes \hat{\mathbb{I}}_{\mathrm{env}} + \hat{\mathbb{I}}_v \otimes \hat{H}_{\mathrm{env}} + g \int \mathrm{d}^3\boldsymbol{x}\, \hat{A}\left(\eta, \boldsymbol{x}\right) \otimes \hat{R}\left(\eta, \boldsymbol{x}\right)$$

$$\frac{\mathrm{d}\hat{\rho}_v}{\mathrm{d}\eta} = -i\left[\hat{H}_v, \hat{\rho}_v\right] - \frac{\gamma}{2}\int \mathrm{d}^3\boldsymbol{x}\,\mathrm{d}^3\boldsymbol{y}\, C_R\left(\boldsymbol{x}, \boldsymbol{y}\right)\left[\hat{A}(\boldsymbol{x}), \left[\hat{A}(\boldsymbol{y}), \hat{\rho}_v\right]\right]$$

$\hat{\rho}_v$: $\mathrm{Tr}_{\mathrm{env}}\left(\left|v, \mathrm{env}\right\rangle\left\langle v, \mathrm{env}\right|\right)$

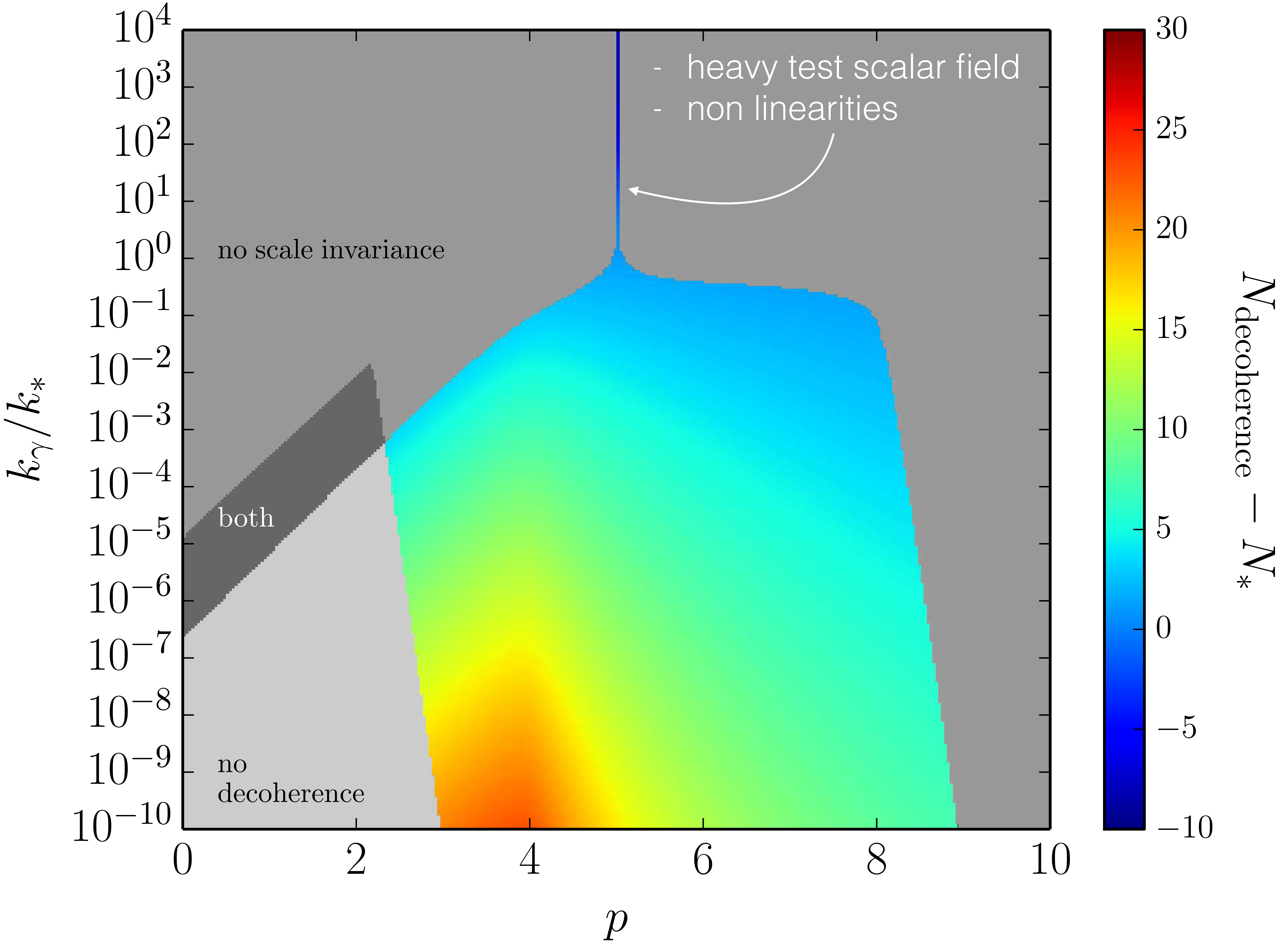
γ: $2g^2\eta_{\mathrm{c}}$

$C_R\left(\boldsymbol{x}, \boldsymbol{y}\right)$: $\langle\hat{R}(\eta, \boldsymbol{x})\hat{R}(\eta, \boldsymbol{y})\rangle$

Main validity condition: $\eta_{\mathrm{c}} \ll \eta_v$

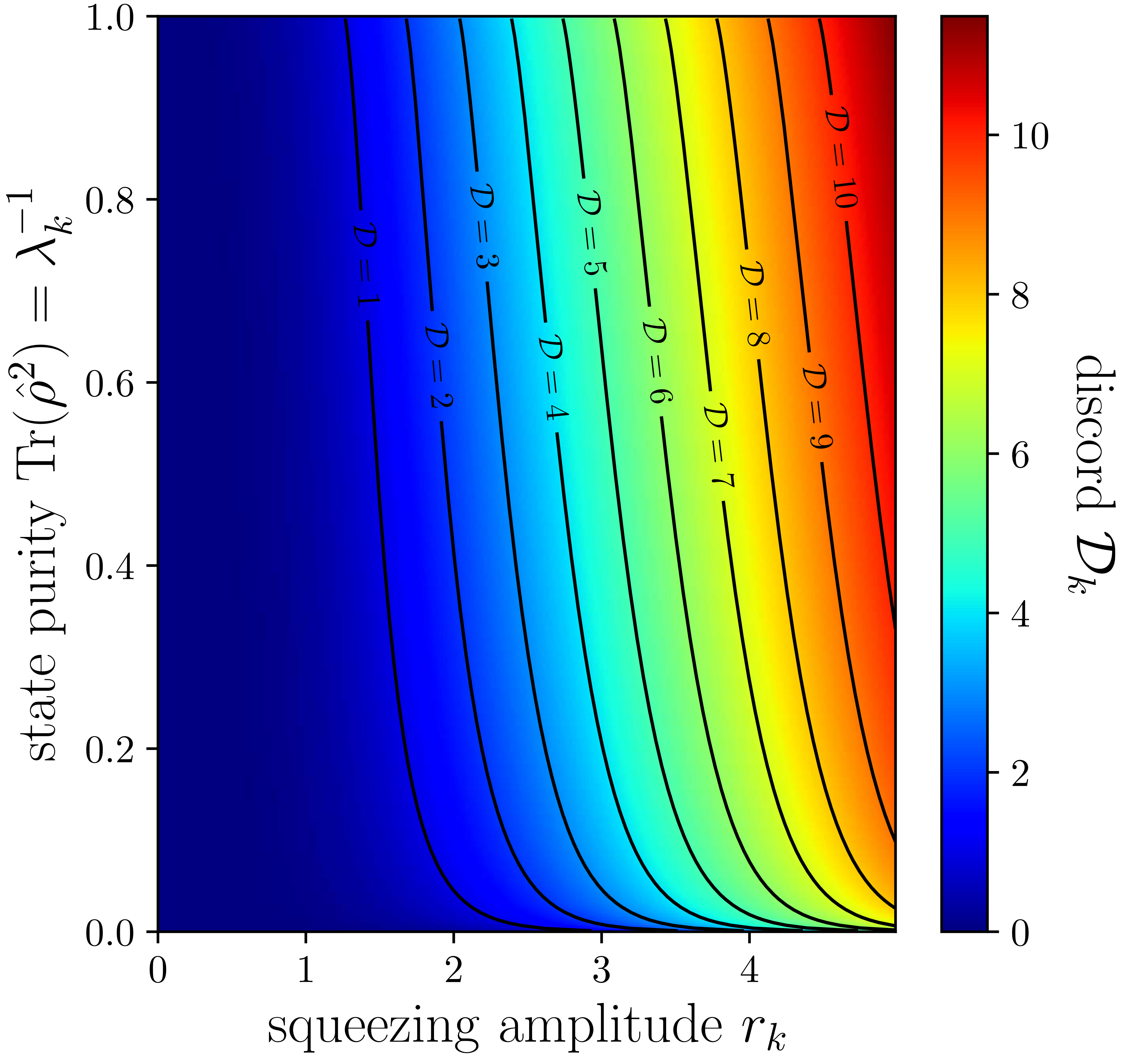
Most generic equation for quantum dynamical semi group!

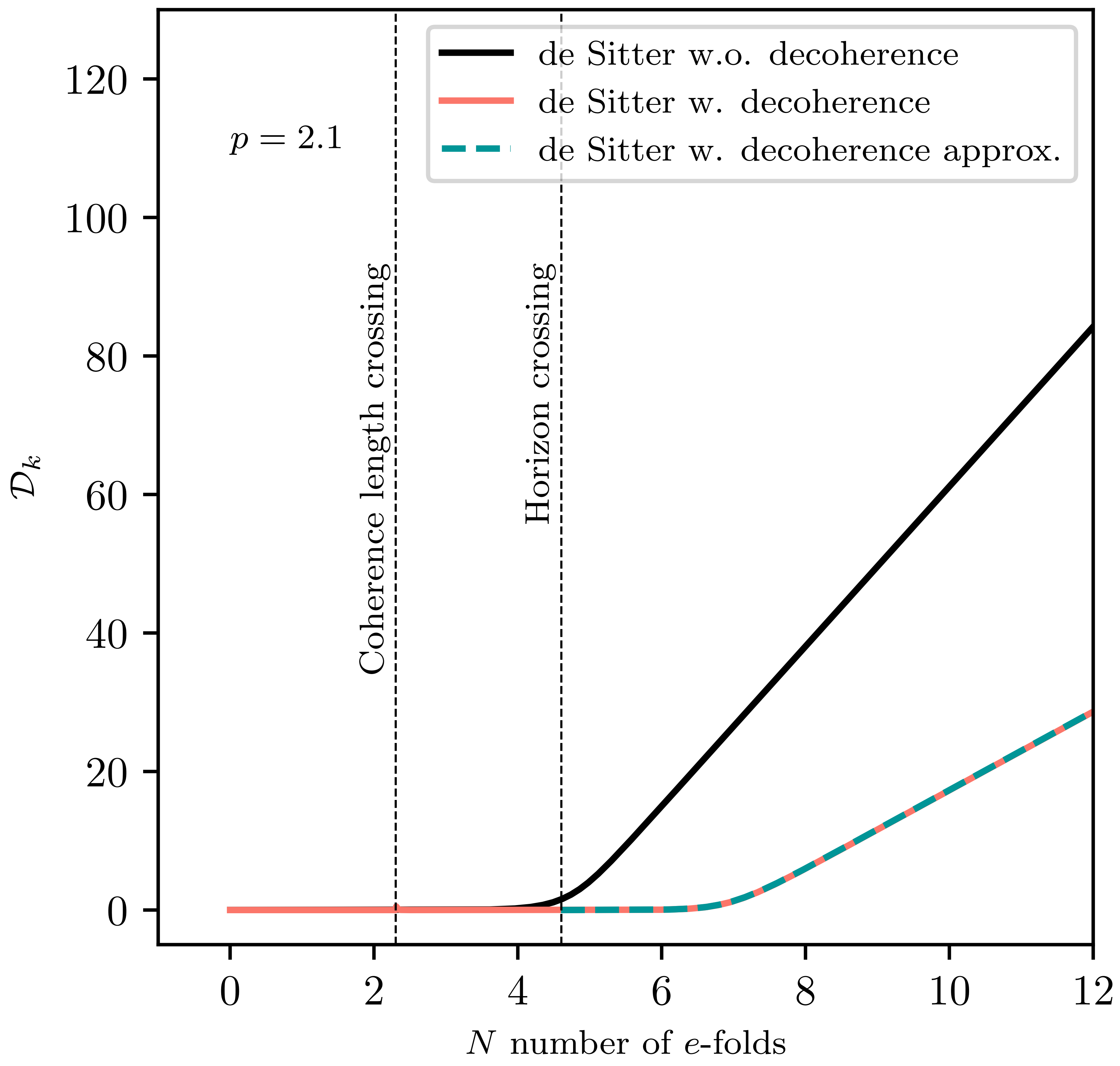
Decoherence



Role of decoherence

J. Martin, A Micheli, VV 2021





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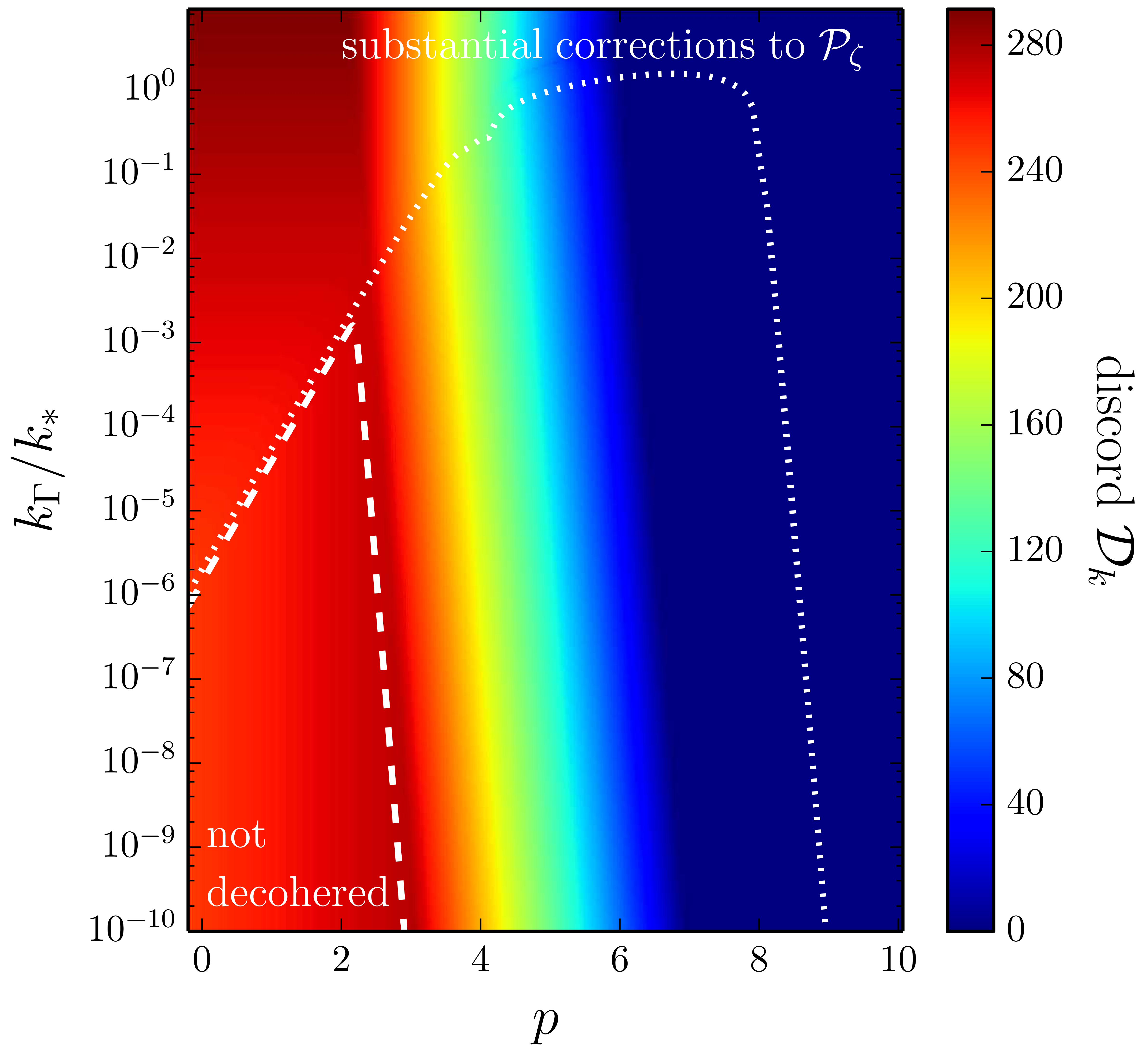
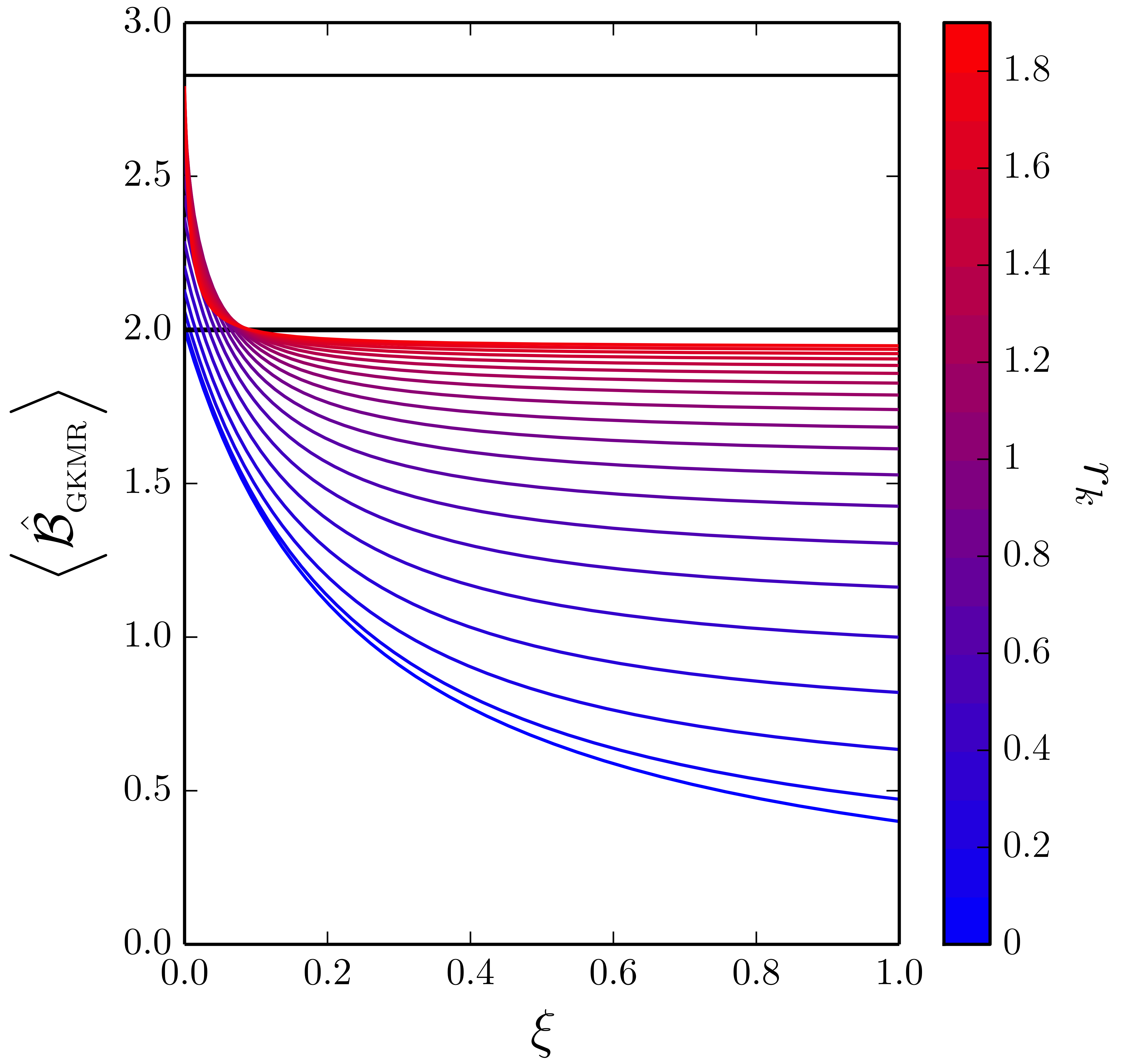
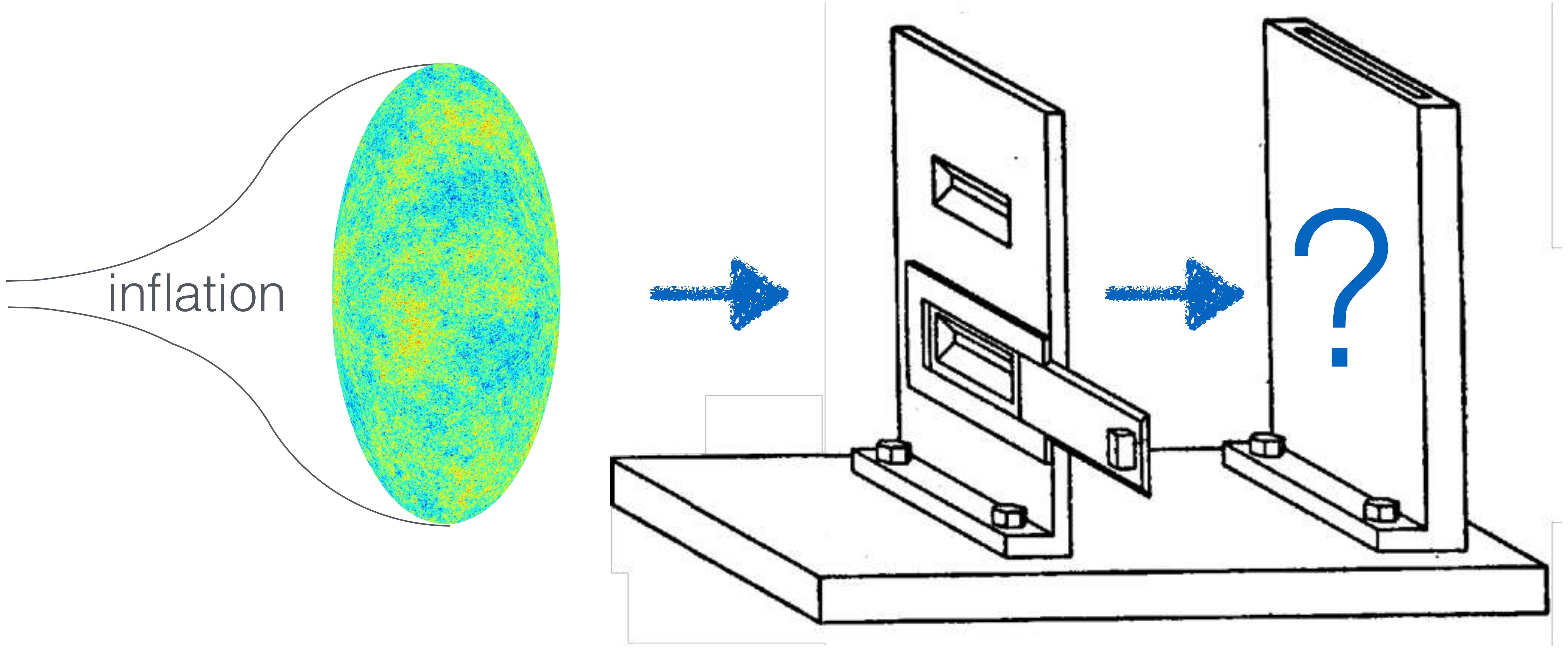


Role of decoherence





Thank you for your attention!