

Dark photon search using  
 $B \rightarrow K A' A', A' \rightarrow l^+ l^-$  decay  
at Belle

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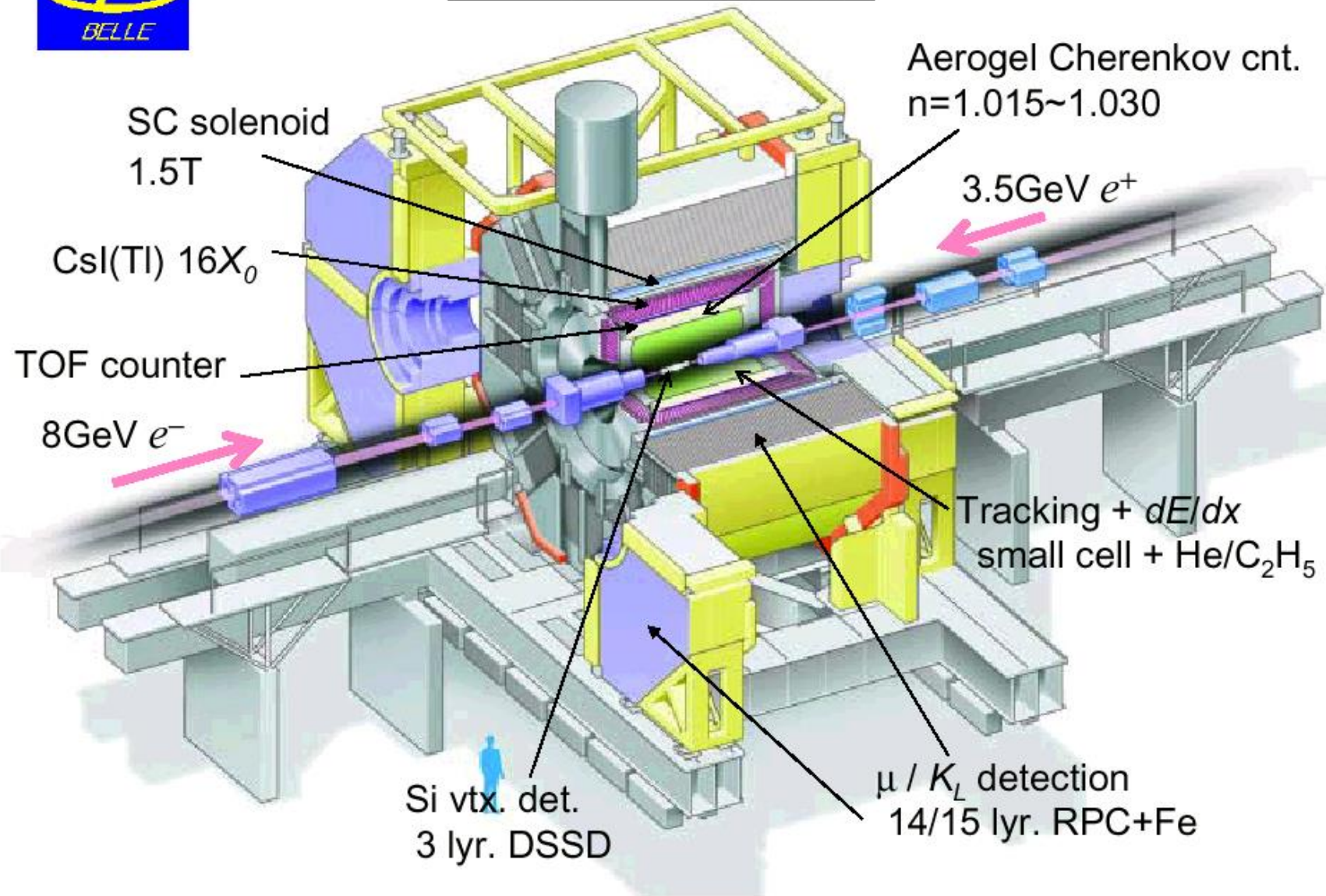


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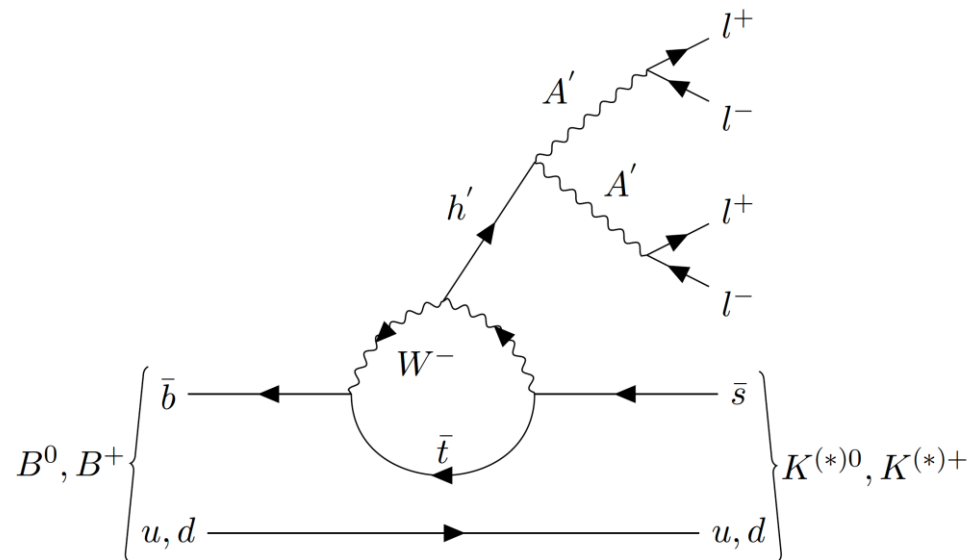
# Belle Detector



Electron : 8 GeV  
Positron : 3.5 GeV

On  $\Upsilon(4S) = 10.58$  GeV  
resonance mostly decay  
to  $B\bar{B}$  pair

# Theoretical Background



Multilepton signature of a hidden sector or in rare B decays.

Phys. Rev. D 83,054005, B. Batell. et al.

| <i>Final States</i>                           |   |  |  |
|---|---|--|--|
| $B^+ \rightarrow K^+ e^+ e^- e^+ e^+$         | $B^0 \rightarrow K^0 e^+ e^- e^+ e^+$         | $B^+ \rightarrow K^{*+} e^+ e^- e^+ e^+$         | $B^0 \rightarrow K^{*0} e^+ e^- e^+ e^+$         |
| $B^+ \rightarrow K^+ e^+ e^- \mu^+ \mu^+$     | $B^0 \rightarrow K^0 e^+ e^- \mu^+ \mu^+$     | $B^+ \rightarrow K^{*+} e^+ e^- \mu^+ \mu^+$     | $B^0 \rightarrow K^{*0} e^+ e^- \mu^+ \mu^+$     |
| $B^+ \rightarrow K^+ \mu^+ \mu^- \mu^+ \mu^+$ | $B^0 \rightarrow K^0 \mu^+ \mu^- \mu^+ \mu^+$ | $B^+ \rightarrow K^{*+} \mu^+ \mu^- \mu^+ \mu^+$ | $B^0 \rightarrow K^{*0} \mu^+ \mu^- \mu^+ \mu^+$ |

# Event Generation

- For Signal MC, for each final state decay and  $A'$  masses, 1M event generated.
- For  $A'$  masses, we generate 0.3 GeV  $\sim$  1.8 GeV region with 0.1 GeV gap.
- Using EVTGEN and GEANT3
- PHSP used for  $B \rightarrow KA'A'$ , PHOTOS additionally added to  $A' \rightarrow l\bar{l}$
- For Background MC, 10 streams of  $B\bar{B}$ , 6 streams of *continuum*, 50 streams of *rareB* and 20 streams of *ulv* was used to estimate backgrounds. Each stream corresponds to 772M  $B\bar{B}$  pairs.

# Particle selection

- $dr < 2 \text{ cm}, dz < 5 \text{ cm}$
- $e^\pm$  :  $\mathcal{L}_e > 0.9, \mathcal{L}_e > \mathcal{L}_\mu$ , Bremsstrahlung- $\gamma$  recon  $\angle_e^\gamma < 0.05 \text{ rad}$
- $\mu^\pm$  :  $\mathcal{L}_\mu > 0.9, \mathcal{L}_e < \mathcal{L}_\mu$
- $K^\pm$  :  $\mathcal{L}_{K/\pi} > 0.6, \mathcal{L}_{P/K} < 0.4$
- $\pi^\pm$  :  $\mathcal{L}_{K/\pi} < 0.4, \mathcal{L}_{P/\pi} < 0.4$
- $\gamma$  : Endcap : 150 MeV Barrel : 50 MeV
- $K_S^0$  : *nisksfinder* standard cut.
- $\pi^0$  :  $0.1 < M_\pi < 0.14 \text{ (GeV)}, P_\pi > 0.1 \text{ GeV}$

# Particle Selection cont'd

- $K^{*+} : K^{*+} \rightarrow K_S^0 \pi^+, K^+ \pi^0, 0.8 < M_{K^{*+}} < 1.0$  (GeV)
- $K^{*0} : K^{*0} \rightarrow K_S^0 \pi^0, K^+ \pi^-, 0.8 < M_{K^{*0}} < 1.0$  (GeV)
- $A' : A' \rightarrow e^+ e^-, \mu^+ \mu^-, \Delta M_{A'} < 0.1$  GeV
- Best  $A'$  pair selection based on least  $\Delta M_{A'}$
- When we select  $A' \rightarrow l_{1,3} l_{2,4}$ , we call  $A'_W \rightarrow l_{1,2} l_{4,3}$
- Best B selection based on least  $|\Delta E|$ .

# Signal Extraction

## Cuts

$$M_{BC} > 5.26$$

$$-0.2 < \Delta E < 0.1$$

$\Delta M_{A'}$  : 95% of signal cut

$$|E_{Asym}| < 0.8$$

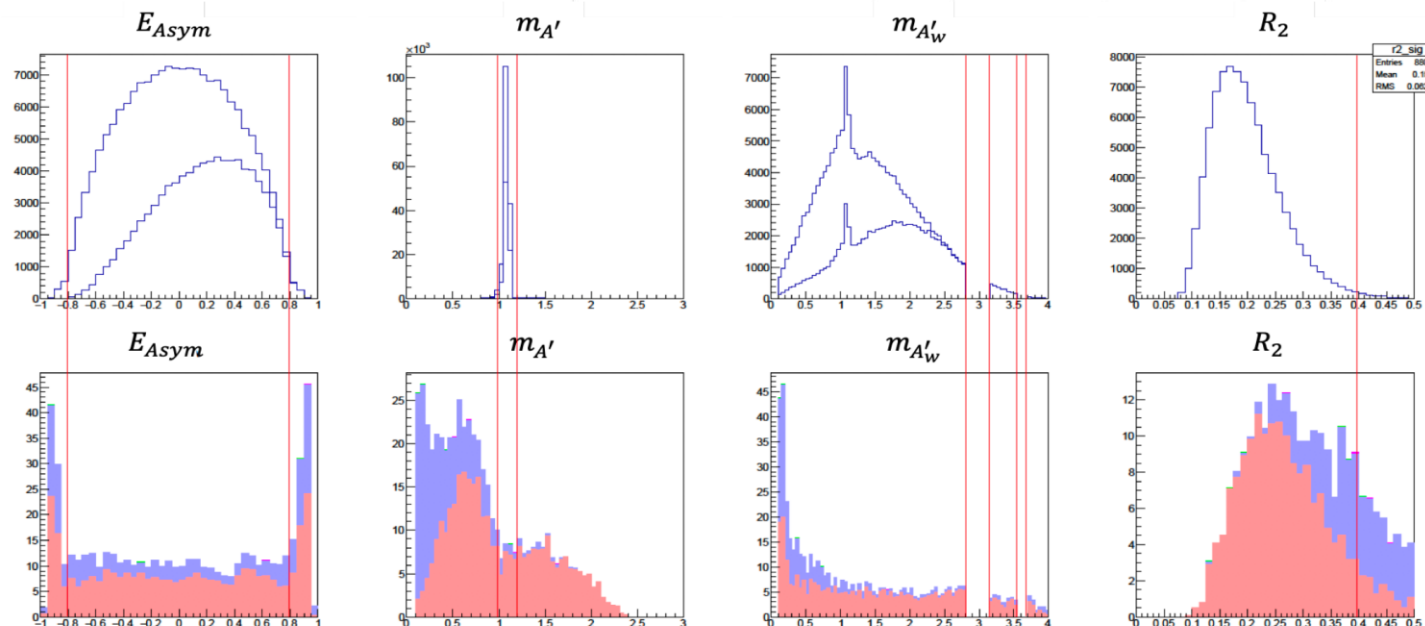
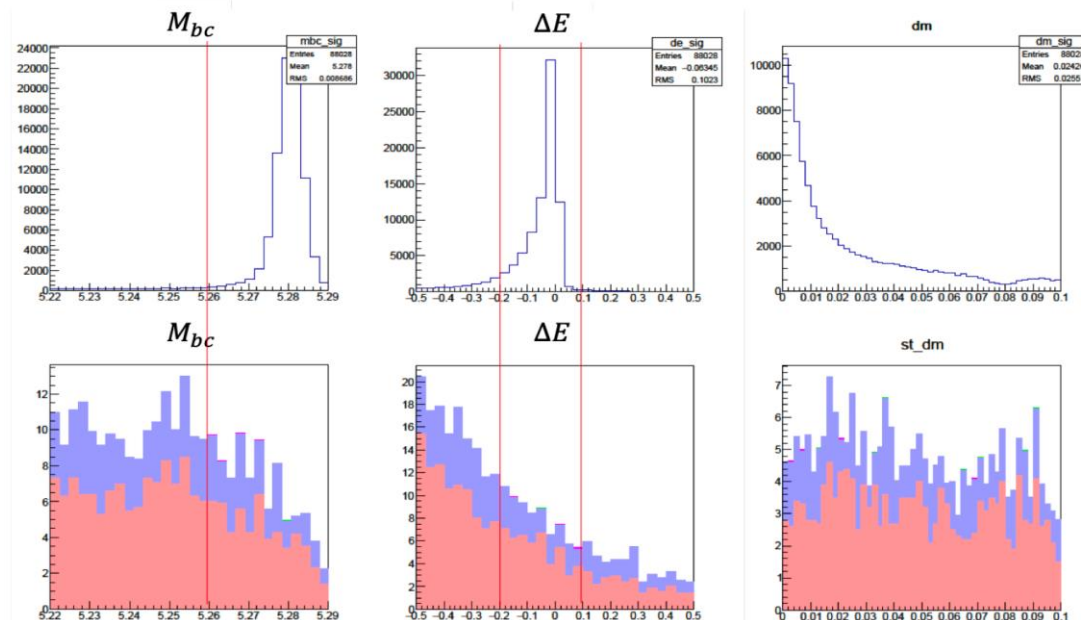
$$|M_{A'} - \overline{M_{A'}}| < 0.1$$

$$M_{A'} > 0.1$$

$$M_{l_1,2l_3} > 0.1$$

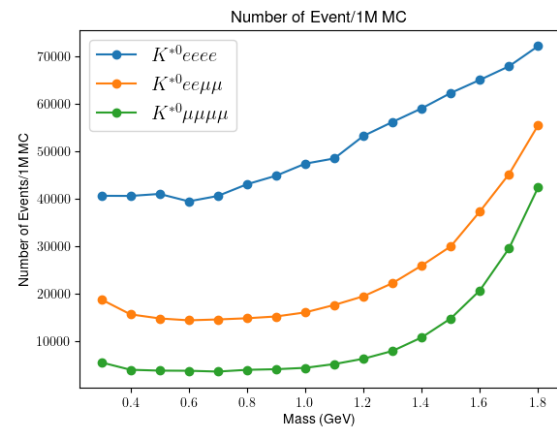
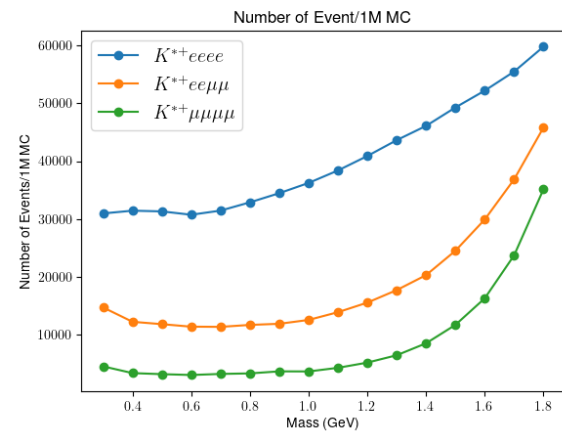
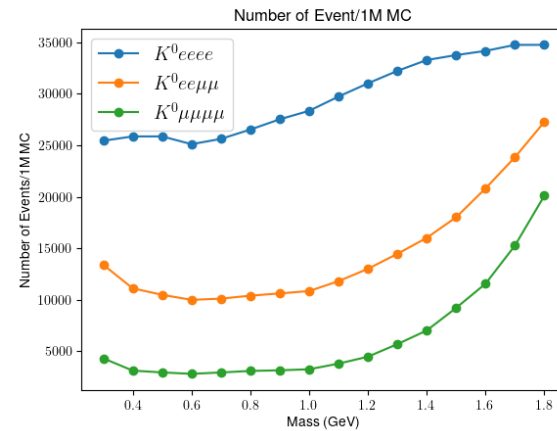
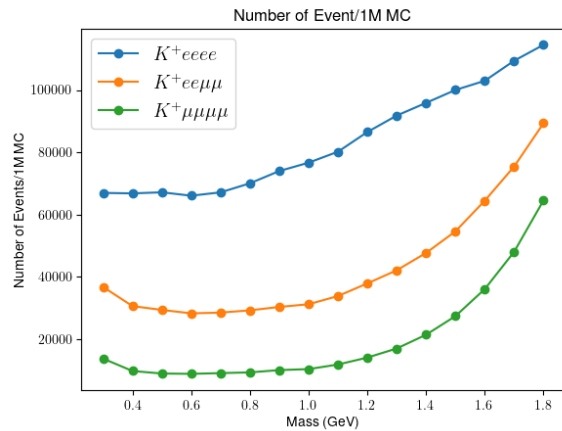
Veto  $J/\psi, \phi$  on  $M_{l_1,2l_3}$

$$R_2 < 0.4$$





# Number of event/1M MC vs Dark photon masses in SignalMC



Efficiency tend to increase as Mass increases.

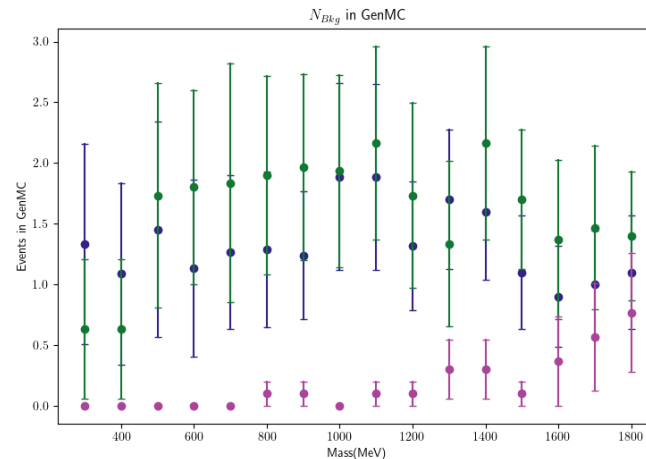
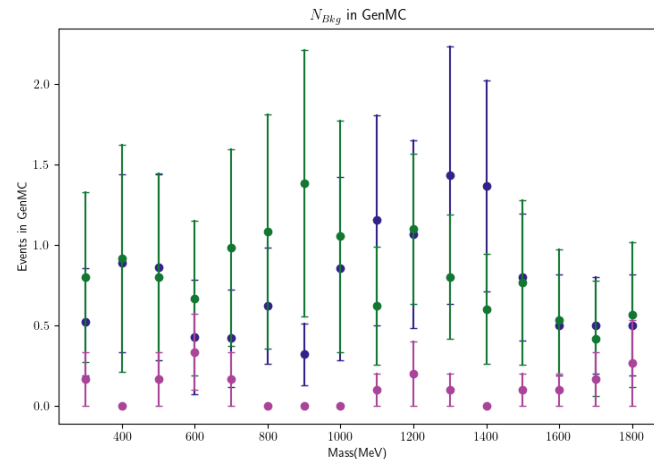
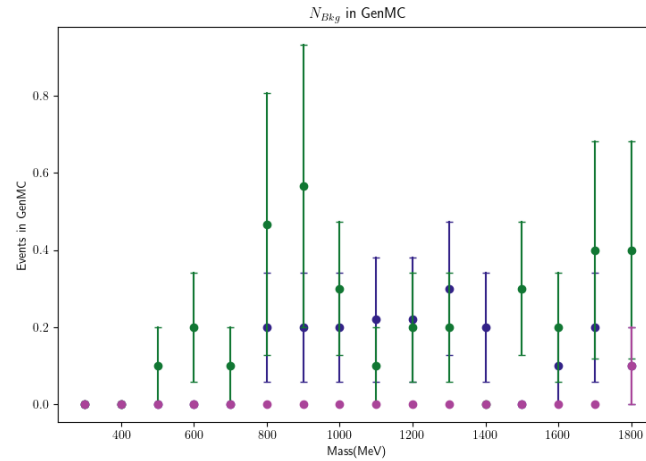
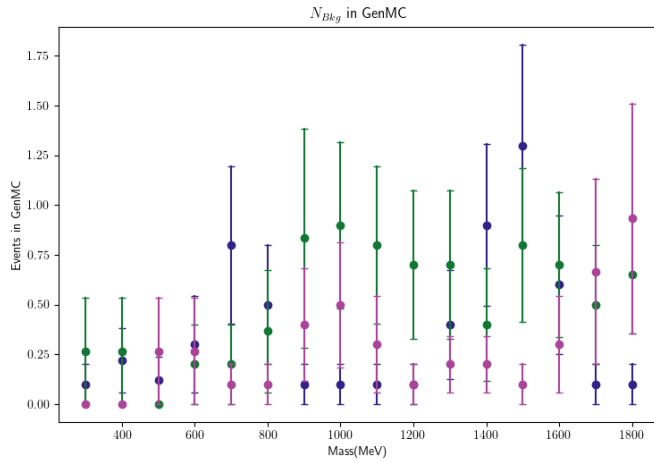
Minimum around 0.6 GeV

Possible for more higher mass region and 0.2 GeV for 4 electron subdecay mode.

( Currently 0.3 GeV ~ 1.8 GeV )

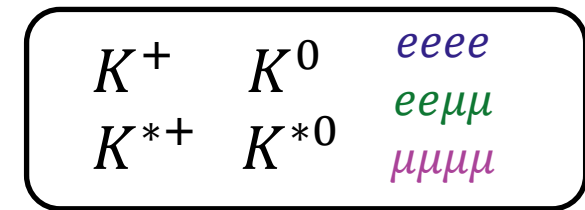
|          |          |                |
|----------|----------|----------------|
| $K^+$    | $K^0$    | $eeee$         |
| $K^{*+}$ | $K^{*0}$ | $ee\mu\mu$     |
|          |          | $\mu\mu\mu\mu$ |

# Number of event/1stream vs Dark photon masses in GenMC



This is number of background and its statistical error.

Most of case they are in  $O(1)$ ,  $4\mu$  decay have least background.



# $N_{BKG}$ and its statistical error

| Final State                                      | $m_{A'}$ | $N_{Bkg}$ | $\sigma_{Bkg}$ | Final State                                      | $m_{A'}$ | $N_{Bkg}$ | $\sigma_{Bkg}$ |
|--|----------|-----------|----------------|--|----------|-----------|----------------|
| $B^+ \rightarrow K^+ e^+ e^- e^+ e^-$            | 1.1      | 0.30      | 0.17           | $B^0 \rightarrow K^0 e^+ e^- e^+ e^-$            | 1.1      | 0.32      | 0.19           |
| $B^+ \rightarrow K^+ e^+ e^- \mu^+ \mu^-$        | 1.1      | 1.20      | 0.35           | $B^0 \rightarrow K^0 e^+ e^- \mu^+ \mu^-$        | 1.1      | 0.10      | 0.10           |
| $B^+ \rightarrow K^+ \mu^+ \mu^- \mu^+ \mu^-$    | 1.1      | 0.30      | 0.17           | $B^0 \rightarrow K^0 \mu^+ \mu^- \mu^+ \mu^-$    | 1.1      | 0.00      | 0.00           |
| $B^+ \rightarrow K^{*+} e^+ e^- e^+ e^-$         | 1.1      | 2.87      | 0.80           | $B^0 \rightarrow K^{*0} e^+ e^- e^+ e^-$         | 1.1      | 2.12      | 0.71           |
| $B^+ \rightarrow K^{*+} e^+ e^- \mu^+ \mu^-$     | 1.1      | 3.67      | 0.88           | $B^0 \rightarrow K^{*0} e^+ e^- \mu^+ \mu^-$     | 1.1      | 1.92      | 0.46           |
| $B^+ \rightarrow K^{*+} \mu^+ \mu^- \mu^+ \mu^-$ | 1.1      | 0.87      | 0.43           | $B^0 \rightarrow K^{*0} \mu^+ \mu^- \mu^+ \mu^-$ | 1.1      | 0.40      | 0.20           |

After cut, in most of mass region,  $N_{BKG} \sim O(1)$ , E.U.L of B.F  $O(10^{-8}) \sim O(10^{-6})$

# Control Sample Study

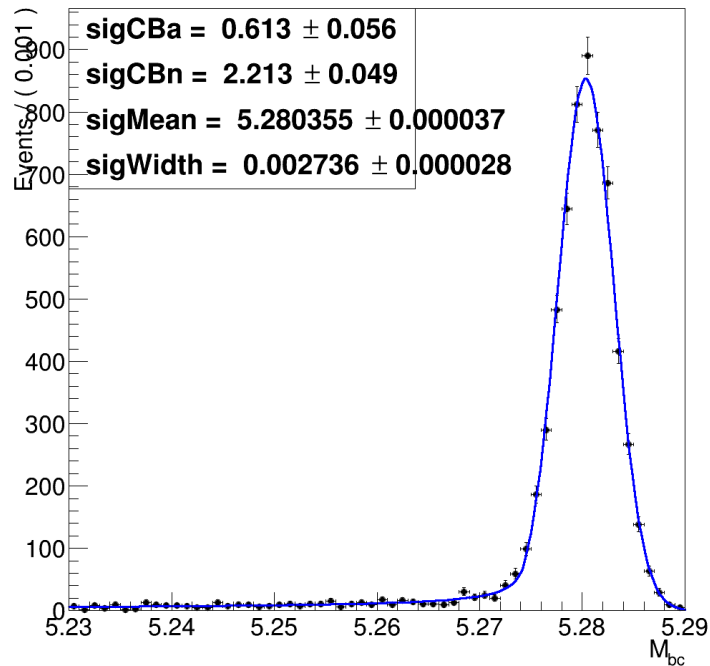
- To validate  $R_2$  cut.
- Control sample study using  $B \rightarrow D\bar{D}K$  was not successful
  - So many resonant state inside it.
- Control sample study now uses  $B^+ \rightarrow J/\psi\phi K^+, J/\psi \rightarrow l^+l^-, \phi \rightarrow K^+K^-$ 
  - Modified  $M_{BC}$  was used to solve peak shift.  $M_{BC} = M_{BC} - E_{beam} + 5.29$
  - Data/MC comparing result was consistent with our expectation.
  - Fitting was used.

# Control sample study

## Signal fit. SigMC/GenMC/Data

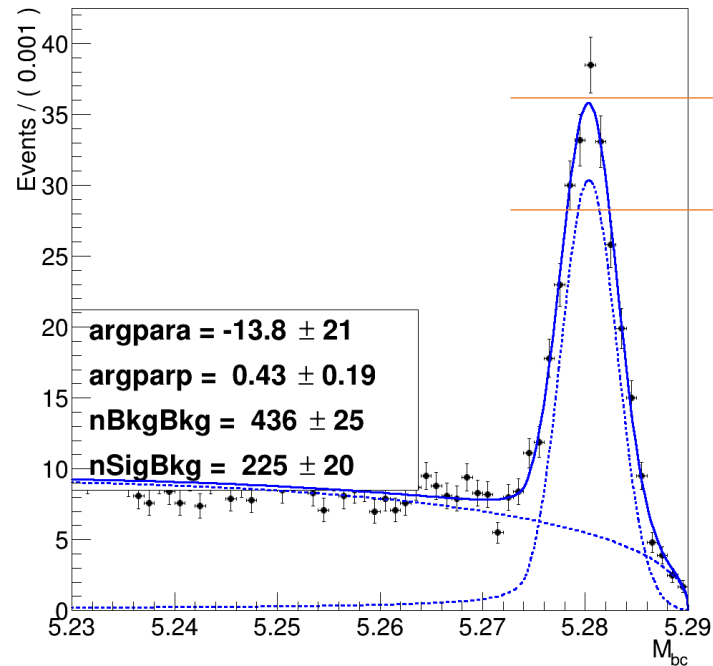
| Cuts                        | notes                 |
|-----------------------------|-----------------------|
| $M_{BC} > 5.22 \text{ GeV}$ | determined by 95% cut |
| $-0.05 < \Delta E < 0.05$   |                       |
| $3.0 < M_{J\psi} < 3.2$     |                       |
| $0.97 < M_{\phi} < 1.07$    |                       |
| $M_{(l,K)_i(l,K)_j} > 0.1$  |                       |
| $R_2 < 0.4$                 |                       |

A RooPlot of " $M_{bc}$ "



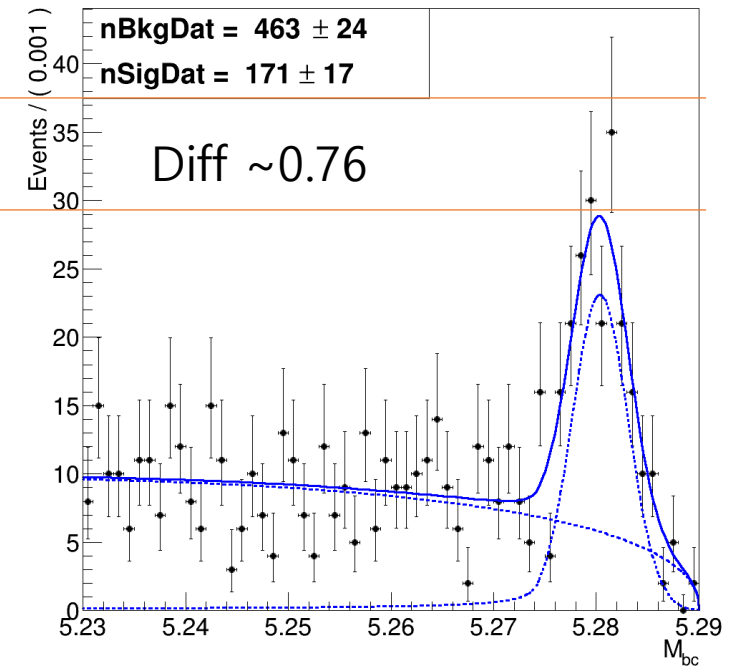
CB

A RooPlot of " $M_{bc}$ "



CB from SigMC + Argus

A RooPlot of " $M_{bc}$ "



# Control sample study result

|  |                              |  |                         |
|--|------------------------------|--|-------------------------|
| $R_{PID,/wR_2}$                                  | $0.9585 \pm 0.0392$          | $R_{PID,/woR_2}$                                 | $0.9578 \pm 0.0394$     |
| $N_{Sig,MC,/wR_2}$                               | $225.3 \pm 6.5$              | $N_{Sig,MC,/woR_2}$                              | $240.6 \pm 6.7$         |
| $N_{Sig,MC,/wR_2} \times R_{trk}$                | $216.0 \pm 6.2 \pm 8.8$      | $N_{Sig,MC,/woR_2} \times R_{trk}$               | $230.4 \pm 6.4 \pm 9.5$ |
| $N_{Sig,Data,/wR_2}$                             | $171 \pm 17$                 | $N_{Sig,Data,/woR_2}$                            | $178 \pm 17$            |
| $\frac{N_{Sig,MC,/wR_2}}{N_{Sig,MC,/woR_2}}$     | $0.9371 \pm 0.030 \pm 0.002$ | $\frac{N_{Sig,DATA,/wR_2}}{N_{Sig,DATA,/woR_2}}$ | $0.9607 \pm 0.093$      |
| $\frac{N_{Sig,Data,/wR_2}}{N_{Sig,Data,/woR_2}}$ | $1.025 \pm 0.100 \pm 0.002$  | $\frac{N_{Sig,Data,/wR_2}}{N_{Sig,MC,/wR_2}}$    | $0.7590 \pm 0.1035$     |

Comparison between  $R_2 < 0.4$  cut and No  $R_2$  cut. In compare to Data and MC they have almost same.

# Control sample study result

|                              | BF  | Notes                                |
|------------------------------|---|--------------------------------------|
| $BF_{PDG}$                   | $(5.0 \pm 0.4) \times 10^{-5}$            | BABR/CLE2                            |
| $BF_{MC}$                    | $(5.2) \times 10^{-5}$                    | DECAY.DEC                            |
| $BF_{MC}$                    | $(4.36 \pm 0.15 \pm 0.18) \times 10^{-5}$ |                                      |
| $BF_{DATA}$                  | $(4.36 \pm 0.44 \pm 0.18) \times 10^{-5}$ |                                      |
| $\frac{BF_{DATA}}{BF_{PDG}}$ | $(0.87 \pm 0.12)$                         | slight bigger than $1 \sigma$ from 1 |
| $\frac{BF_{DATA}}{BF_{MC}}$  | $(1.00 \pm 0.11 \pm 0.04)$                |                                      |

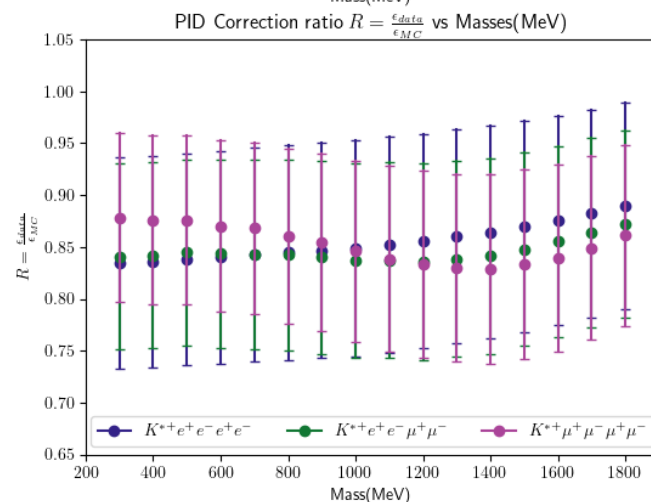
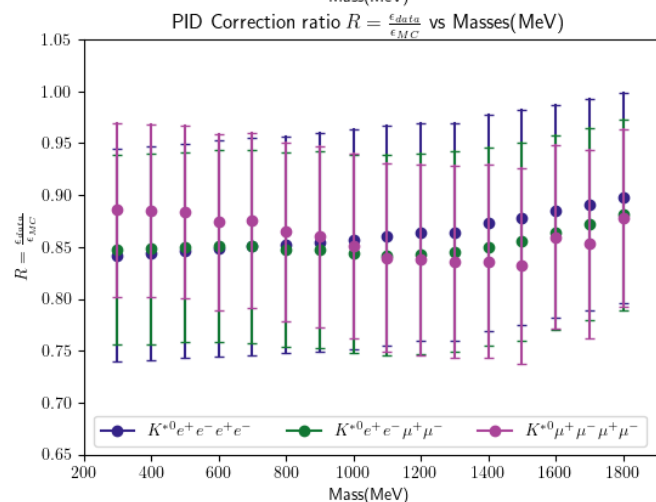
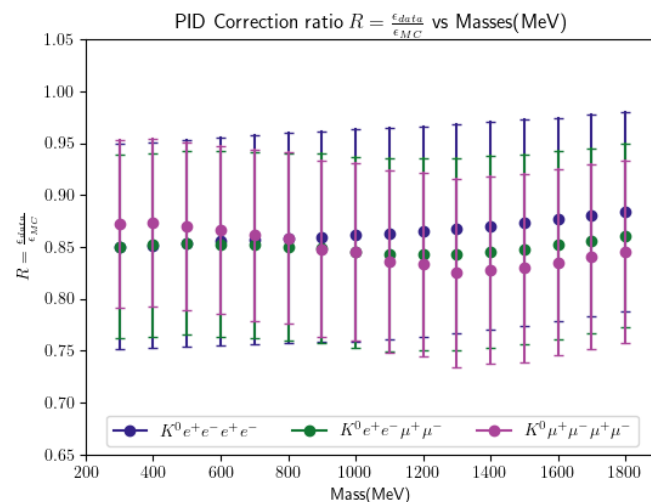
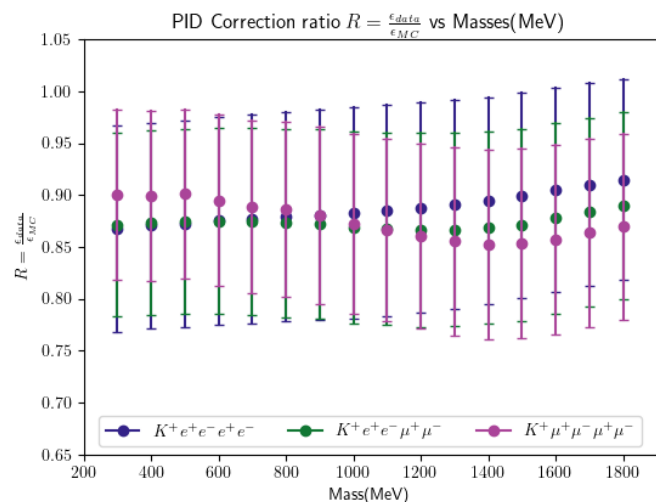
Difference between Data and MC are consistent with expected result.  
Slight lower than PDG value.

# Systematic study

- PID Correction is main source of systematic error.
- Systematics for tracking is relatively low (  $\sim 0.7\%$  in total) in compared to PID correction ( $\sim 10\%$ )



# PID Correction ratio $R$ vs Masses



$R$  is around 0.85,  
and its error is around 10%.

The more muon event have less  
error.

|          |          |                |
|----------|----------|----------------|
| $K^+$    | $K^0$    | $eeee$         |
| $K^{*0}$ | $K^{*+}$ | $ee\mu\mu$     |
|          |          | $\mu\mu\mu\mu$ |

# Expected Upper Limit of Branching Fraction using MC w.r.t $m_{A'}$ , $N_{Obs}$ ( $10^{-6}$ )

| Kpluseeee | 0        | 1        | 2        | 3        | 4       | 5       | Kpluseemm | 0        | 1        | 2        | 3       | 4       | 5       | Kplusmmmm | 0        | 1        | 2       | 3       | 4       | 5       |
|-----------|----------|----------|----------|----------|---------|---------|-----------|----------|----------|----------|---------|---------|---------|-----------|----------|----------|---------|---------|---------|---------|
| 300       | 0.053908 | 0.1005   | 0.13488  | 0.17526  | 0.20987 | 0.24248 | 300       | 0.094063 | 0.17245  | 0.23757  | 0.3039  | 0.37344 | 0.43253 | 300       | 0.25827  | 0.48025  | 0.64102 | 0.83706 | 0.99161 | 1.1441  |
| 400       | 0.052318 | 0.096877 | 0.13212  | 0.16937  | 0.20705 | 0.23964 | 400       | 0.11267  | 0.20656  | 0.28456  | 0.364   | 0.4473  | 0.51808 | 400       | 0.36312  | 0.6752   | 0.90124 | 1.1769  | 1.3942  | 1.6085  |
| 500       | 0.053257 | 0.099251 | 0.13336  | 0.17275  | 0.20775 | 0.2401  | 500       | 0.12476  | 0.23198  | 0.30964  | 0.40434 | 0.47899 | 0.55264 | 500       | 0.37111  | 0.68038  | 0.9373  | 1.199   | 1.4734  | 1.7065  |
| 600       | 0.051744 | 0.094567 | 0.13114  | 0.16728  | 0.20653 | 0.23932 | 600       | 0.12324  | 0.22879  | 0.31095  | 0.39935 | 0.48671 | 0.56315 | 600       | 0.37809  | 0.69316  | 0.95491 | 1.2215  | 1.501   | 1.7386  |
| 700       | 0.038764 | 0.077309 | 0.11782  | 0.15067  | 0.18659 | 0.22404 | 700       | 0.12234  | 0.22713  | 0.30869  | 0.39644 | 0.48316 | 0.55905 | 700       | 0.38526  | 0.7182   | 0.96394 | 1.2525  | 1.4998  | 1.7329  |
| 800       | 0.045028 | 0.082517 | 0.11896  | 0.15037  | 0.18975 | 0.22053 | 800       | 0.1149   | 0.20813  | 0.2928   | 0.37192 | 0.46313 | 0.53721 | 800       | 0.37667  | 0.70218  | 0.94245 | 1.2246  | 1.4664  | 1.6942  |
| 900       | 0.048086 | 0.089642 | 0.12031  | 0.15633  | 0.1872  | 0.21629 | 900       | 0.084142 | 0.16974  | 0.26021  | 0.33268 | 0.41147 | 0.4961  | 900       | 0.32715  | 0.59061  | 0.8367  | 1.0611  | 1.326   | 1.5388  |
| 1000      | 0.046301 | 0.086314 | 0.11585  | 0.15053  | 0.18025 | 0.20826 | 1000      | 0.078344 | 0.16144  | 0.2507   | 0.32192 | 0.39552 | 0.48099 | 1000      | 0.30426  | 0.55757  | 0.8038  | 1.0161  | 1.2821  | 1.4901  |
| 1100      | 0.044174 | 0.082349 | 0.11053  | 0.14361  | 0.17197 | 0.19869 | 1100      | 0.077615 | 0.15479  | 0.23592  | 0.30169 | 0.37361 | 0.44859 | 1100      | 0.28979  | 0.52961  | 0.73446 | 0.93681 | 1.1566  | 1.3403  |
| 1200      | 0.040818 | 0.076093 | 0.10213  | 0.1327   | 0.15891 | 0.1836  | 1200      | 0.074128 | 0.14394  | 0.21493  | 0.27376 | 0.34279 | 0.40516 | 1200      | 0.25747  | 0.47998  | 0.64421 | 0.83705 | 1.0023  | 1.1581  |
| 1300      | 0.035659 | 0.064376 | 0.091199 | 0.11566  | 0.14453 | 0.16772 | 1300      | 0.066709 | 0.12954  | 0.19342  | 0.24637 | 0.30849 | 0.36461 | 1300      | 0.20948  | 0.38891  | 0.52856 | 0.67882 | 0.82731 | 0.95725 |
| 1400      | 0.024822 | 0.051149 | 0.079432 | 0.102    | 0.12532 | 0.15239 | 1400      | 0.070283 | 0.12688  | 0.17975  | 0.22795 | 0.28486 | 0.33058 | 1400      | 0.16702  | 0.31008  | 0.42143 | 0.54124 | 0.65963 | 0.76323 |
| 1500      | 0.020235 | 0.043054 | 0.070178 | 0.091562 | 0.1128  | 0.13476 | 1500      | 0.047935 | 0.0956   | 0.1457   | 0.18633 | 0.23074 | 0.27705 | 1500      | 0.13374  | 0.24932  | 0.33462 | 0.43479 | 0.52065 | 0.60155 |
| 1600      | 0.027991 | 0.052656 | 0.077321 | 0.098106 | 0.12416 | 0.14453 | 1600      | 0.043055 | 0.083605 | 0.12484  | 0.15901 | 0.1991  | 0.23532 | 1600      | 0.096624 | 0.17659  | 0.24489 | 0.31236 | 0.38566 | 0.44688 |
| 1700      | 0.031521 | 0.058761 | 0.078866 | 0.10247  | 0.12271 | 0.14178 | 1700      | 0.041599 | 0.076232 | 0.1099   | 0.13892 | 0.17529 | 0.20374 | 1700      | 0.060336 | 0.11539  | 0.17137 | 0.21802 | 0.27462 | 0.32221 |
| 1800      | 0.029964 | 0.055858 | 0.074971 | 0.097413 | 0.11665 | 0.13478 | 1800      | 0.031814 | 0.060545 | 0.089762 | 0.11411 | 0.14414 | 0.16849 | 1800      | 0.036988 | 0.077192 | 0.12061 | 0.15484 | 0.18999 | 0.23204 |

They are in  $O(10^{-6}) \sim O(10^{-8})$

# Summary

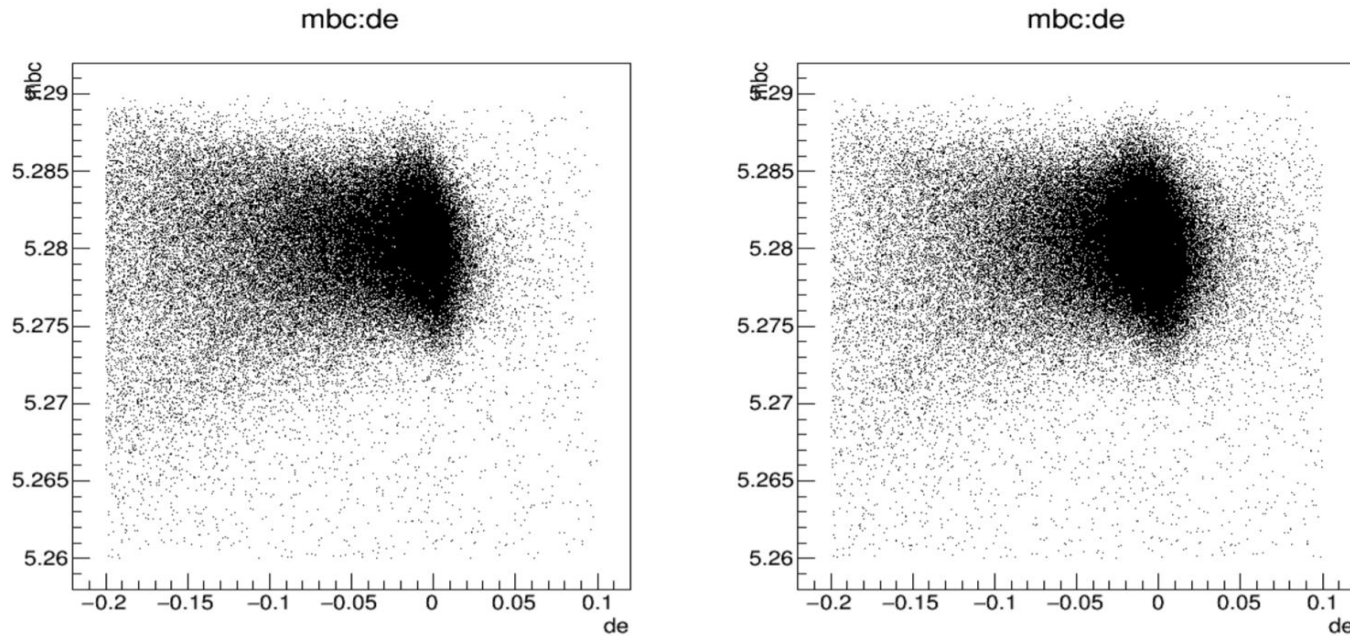
- Control sample study using  $B^+ \rightarrow J/\psi\phi K^+$  is consistent with our expectation.
- Systematic study was done. Main source of systematic error was PID correction.
- Expected upper limit of Branching fraction is estimated

# Backup

# Particle selection cont'd

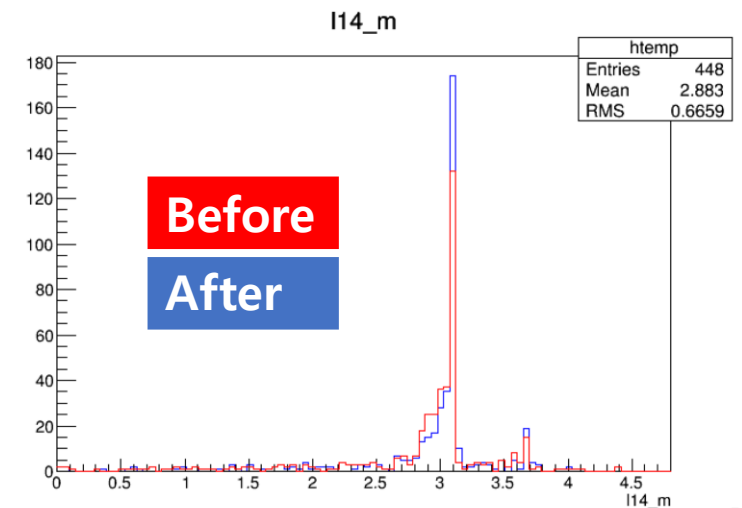
- Least  $\Delta M_{A'}$  for  $A'$  pair selection.
- Least  $|\Delta E|$  for best B selection.

# Bremsstrahlung reconstruction



$M_{bc}$  vs  $\Delta E$  before, after bremsstrahlung reconstruction

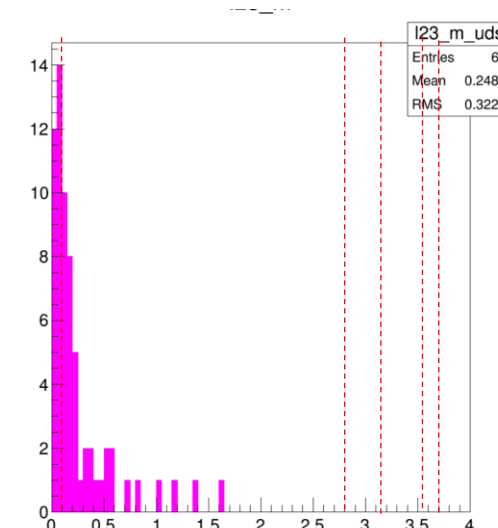
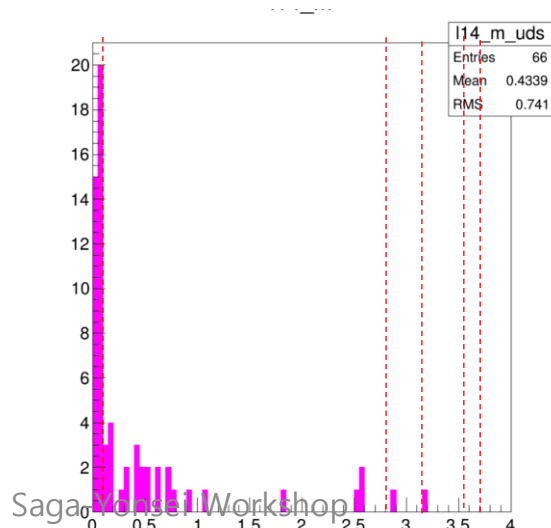
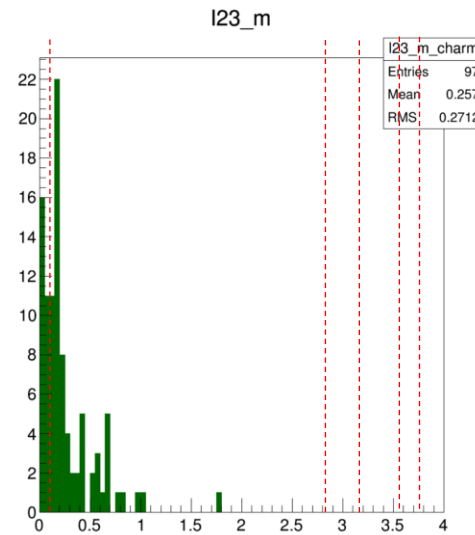
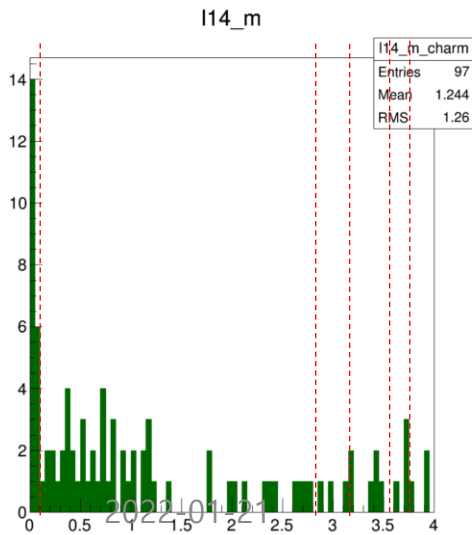
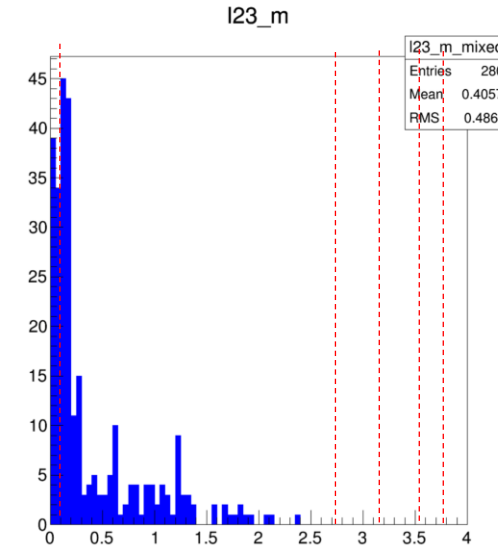
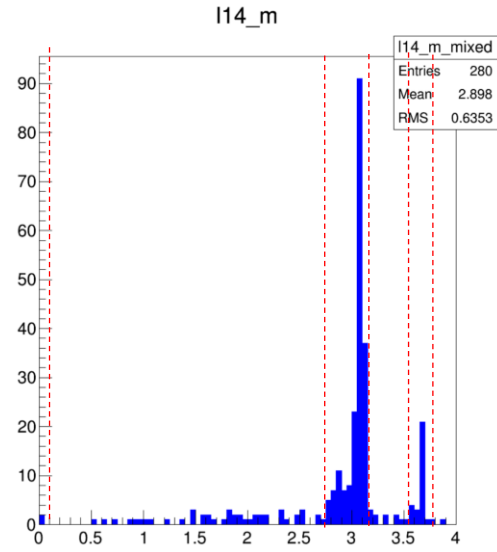
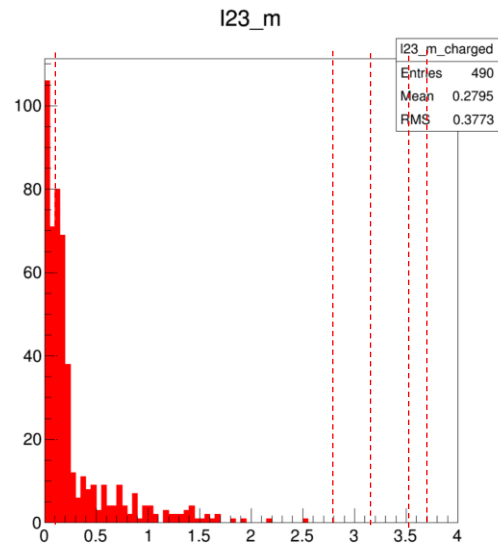
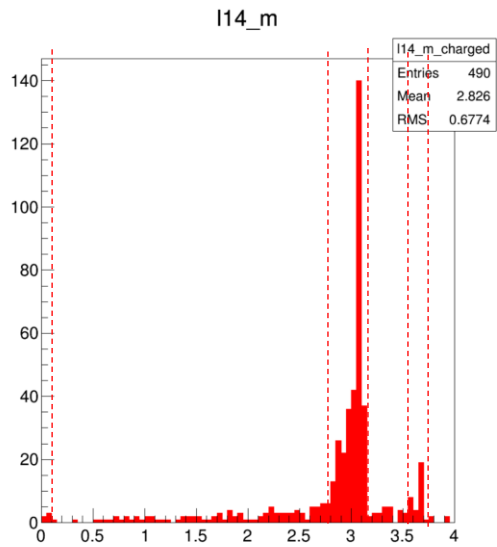
$\angle_e^\gamma < 0.05 \text{ rad}$  used bremsstrahlung reconstruction.  
Electron containing mode have Bremsstrahlung reconstruction process



Effect on  $J/\psi$ ,  $\psi(2S)$  background veto

# Low mass/cc veto

$J/\psi$  veto :  $2.8 < m_{A'_W} < 3.15$  GeV  
 $\psi(2S)$  veto :  $3.55 < m_{A'_W} < 3.7$  GeV  
 Low mass veto  
 $m_{A'} < 0.1$  GeV,  $m_{A'} < 0.1$  GeV



Charged  
 Mixed  
 Charm  
 Uds

# $\Delta m_{A'}$ cut determination

Background is almost flat.

Figure of merit punzi fluctuates hard due to lack of background. ( $O(1)$ )

Signal 95% cut applied to some points,

And interpolated it with dark photon mass

| Final States                     | 0.6 GeV | 1.1 GeV | 1.6 GeV | Final States                     | 0.6 GeV | 1.1 GeV | 1.6 GeV |
|----------------------------------|---------|---------|---------|----------------------------------|---------|---------|---------|
| $K^0 e^+ e^- e^+ e^-$            | 0.062   | 0.068   | 0.078   | $K^+ e^+ e^- e^+ e^-$            | 0.060   | 0.068   | 0.080   |
| $K^0 e^+ e^- \mu^+ \mu^-$        | 0.056   | 0.064   | 0.074   | $K^+ e^+ e^- \mu^+ \mu^-$        | 0.054   | 0.062   | 0.074   |
| $K^0 \mu^+ \mu^- \mu^+ \mu^-$    | 0.016   | 0.020   | 0.030   | $K^+ \mu^+ \mu^- \mu^+ \mu^-$    | 0.014   | 0.020   | 0.030   |
| $K^{*0} e^+ e^- e^+ e^-$         | 0.062   | 0.068   | 0.078   | $K^{*+} e^+ e^- e^+ e^-$         | 0.064   | 0.068   | 0.078   |
| $K^{*0} e^+ e^- \mu^+ \mu^-$     | 0.058   | 0.064   | 0.076   | $K^{*+} e^+ e^- \mu^+ \mu^-$     | 0.056   | 0.062   | 0.072   |
| $K^{*0} \mu^+ \mu^- \mu^+ \mu^-$ | 0.016   | 0.024   | 0.030   | $K^{*+} \mu^+ \mu^- \mu^+ \mu^-$ | 0.020   | 0.028   | 0.030   |



# PID Correction

|                        | Mass<br>(GeV) | $K^+$  |            | $K^0$  |            | $K^{*+}$ |            | $K^{*0}$ |            |
|------------------------|---------------|--------|------------|--------|------------|----------|------------|----------|------------|
|                        |               | $R$    | $\sigma_R$ | $R$    | $\sigma_R$ | $R$      | $\sigma_R$ | $R$      | $\sigma_R$ |
| $e^+e^-e^+e^-$         | 0.6           | 0.8751 | 0.1004     | 0.8553 | 0.1004     | 0.8402   | 0.1026     | 0.8483   | 0.1041     |
|                        | 1.1           | 0.8849 | 0.1021     | 0.8628 | 0.1018     | 0.8519   | 0.1042     | 0.8609   | 0.1059     |
|                        | 1.6           | 0.9049 | 0.0983     | 0.8765 | 0.0978     | 0.8757   | 0.1009     | 0.8844   | 0.1024     |
| $e^+e^-\mu^+\mu^-$     | 0.6           | 0.8750 | 0.0896     | 0.8526 | 0.0895     | 0.8434   | 0.0908     | 0.8509   | 0.0925     |
|                        | 1.1           | 0.8673 | 0.0928     | 0.8427 | 0.0931     | 0.8375   | 0.0943     | 0.8422   | 0.0965     |
|                        | 1.6           | 0.8777 | 0.0918     | 0.8520 | 0.0906     | 0.8553   | 0.0918     | 0.8637   | 0.0935     |
| $\mu^+\mu^-\mu^+\mu^-$ | 0.6           | 0.8949 | 0.0825     | 0.8663 | 0.0811     | 0.8699   | 0.0824     | 0.8741   | 0.0848     |
|                        | 1.1           | 0.8661 | 0.0881     | 0.8357 | 0.0882     | 0.8385   | 0.0892     | 0.8396   | 0.0906     |
|                        | 1.6           | 0.8568 | 0.0912     | 0.8352 | 0.0895     | 0.8395   | 0.0901     | 0.8596   | 0.0881     |

Table. PID correction ratio(  $R = \frac{\epsilon_{DATA}}{\epsilon_{MC}}$  ) and its uncertainty

| K0eeee | 0       | 1       | 2       | 3       | 4       | 5       |
|--------|---------|---------|---------|---------|---------|---------|
| 300    | 0.15686 | 0.29167 | 0.38932 | 0.50838 | 0.60224 | 0.69485 |
| 400    | 0.1541  | 0.28653 | 0.38246 | 0.49942 | 0.59163 | 0.68261 |
| 500    | 0.1537  | 0.2858  | 0.38147 | 0.49814 | 0.59011 | 0.68085 |
| 600    | 0.158   | 0.29379 | 0.39214 | 0.51207 | 0.60661 | 0.69989 |
| 700    | 0.15462 | 0.28751 | 0.38376 | 0.50112 | 0.59364 | 0.68493 |
| 800    | 0.14181 | 0.26328 | 0.35783 | 0.45955 | 0.56008 | 0.64804 |
| 900    | 0.13602 | 0.25253 | 0.34321 | 0.44078 | 0.5372  | 0.62157 |
| 1000   | 0.13232 | 0.24565 | 0.33387 | 0.42878 | 0.52258 | 0.60465 |
| 1100   | 0.12537 | 0.23215 | 0.31662 | 0.40587 | 0.49618 | 0.57428 |
| 1200   | 0.11988 | 0.22198 | 0.30274 | 0.38808 | 0.47443 | 0.5491  |
| 1300   | 0.11317 | 0.20682 | 0.28682 | 0.36584 | 0.45169 | 0.52339 |
| 1400   | 0.11153 | 0.20706 | 0.28141 | 0.36141 | 0.44047 | 0.50965 |
| 1500   | 0.1151  | 0.21403 | 0.28567 | 0.37304 | 0.44192 | 0.50987 |
| 1600   | 0.11059 | 0.20615 | 0.27669 | 0.35952 | 0.43051 | 0.49741 |
| 1700   | 0.10558 | 0.19602 | 0.26641 | 0.34214 | 0.41699 | 0.48247 |
| 1800   | 0.1078  | 0.20095 | 0.26971 | 0.35045 | 0.41965 | 0.48486 |

| K0eemm | 0       | 1       | 2       | 3       | 4       | 5       |
|--------|---------|---------|---------|---------|---------|---------|
| 300    | 0.29801 | 0.55413 | 0.73964 | 0.96583 | 1.1442  | 1.3201  |
| 400    | 0.35852 | 0.66664 | 0.88981 | 1.1619  | 1.3765  | 1.5881  |
| 500    | 0.37022 | 0.69017 | 0.92632 | 1.2036  | 1.4413  | 1.6652  |
| 600    | 0.37915 | 0.7039  | 0.95667 | 1.2286  | 1.4974  | 1.7326  |
| 700    | 0.38456 | 0.7169  | 0.9622  | 1.2502  | 1.4971  | 1.7297  |
| 800    | 0.33932 | 0.6154  | 0.88069 | 1.1136  | 1.402   | 1.6287  |
| 900    | 0.3116  | 0.58237 | 0.84859 | 1.0755  | 1.3599  | 1.5822  |
| 1000   | 0.34501 | 0.63053 | 0.87441 | 1.1153  | 1.377   | 1.5956  |
| 1100   | 0.3328  | 0.62041 | 0.83269 | 1.082   | 1.2956  | 1.4969  |
| 1200   | 0.29479 | 0.54728 | 0.74381 | 0.95526 | 1.1642  | 1.3471  |
| 1300   | 0.26509 | 0.49215 | 0.66888 | 0.85903 | 1.0469  | 1.2114  |
| 1400   | 0.25071 | 0.46618 | 0.62225 | 0.81255 | 0.96257 | 1.1106  |
| 1500   | 0.20709 | 0.37847 | 0.52486 | 0.66946 | 0.82657 | 0.95778 |
| 1600   | 0.1821  | 0.33807 | 0.45947 | 0.59009 | 0.71918 | 0.83212 |
| 1700   | 0.15096 | 0.27252 | 0.38607 | 0.4896  | 0.61184 | 0.71003 |
| 1800   | 0.13116 | 0.23678 | 0.33544 | 0.4254  | 0.5316  | 0.61691 |

| K0mmmm | 0       | 1       | 2       | 3       | 4       | 5       |
|--------|---------|---------|---------|---------|---------|---------|
| 300    | 0.90272 | 1.6786  | 2.2405  | 2.9257  | 3.4659  | 3.9988  |
| 400    | 1.2459  | 2.3166  | 3.0922  | 4.0379  | 4.7834  | 5.5189  |
| 500    | 1.326   | 2.4656  | 3.291   | 4.2975  | 5.0909  | 5.8737  |
| 600    | 1.3972  | 2.598   | 3.4677  | 4.5282  | 5.3642  | 6.1891  |
| 700    | 1.341   | 2.4936  | 3.3284  | 4.3463  | 5.1488  | 5.9405  |
| 800    | 1.2805  | 2.381   | 3.178   | 4.15    | 4.9162  | 5.6721  |
| 900    | 1.2717  | 2.3646  | 3.1562  | 4.1214  | 4.8823  | 5.6331  |
| 1000   | 1.236   | 2.2982  | 3.0676  | 4.0057  | 4.7453  | 5.475   |
| 1100   | 1.0696  | 1.9888  | 2.6546  | 3.4664  | 4.1064  | 4.7379  |
| 1200   | 0.91116 | 1.6942  | 2.2614  | 2.953   | 3.4983  | 4.0362  |
| 1300   | 0.72329 | 1.3449  | 1.7952  | 2.3442  | 2.777   | 3.204   |
| 1400   | 0.58428 | 1.0864  | 1.4501  | 1.8936  | 2.2433  | 2.5882  |
| 1500   | 0.44415 | 0.82586 | 1.1023  | 1.4395  | 1.7052  | 1.9674  |
| 1600   | 0.3514  | 0.65341 | 0.87216 | 1.1389  | 1.3492  | 1.5566  |
| 1700   | 0.26452 | 0.49186 | 0.65653 | 0.85731 | 1.0156  | 1.1718  |
| 1800   | 0.19496 | 0.36345 | 0.48781 | 0.63383 | 0.75899 | 0.87693 |

| Kstarpluseeee | 0        | 1        | 2       | 3       | 4       | 5       |
|---------------|----------|----------|---------|---------|---------|---------|
| 300           | 0.1058   | 0.19514  | 0.28247 | 0.35734 | 0.45116 | 0.52452 |
| 400           | 0.081914 | 0.16775  | 0.25947 | 0.33305 | 0.41006 | 0.49737 |
| 500           | 0.083506 | 0.16996  | 0.26132 | 0.33501 | 0.41311 | 0.49956 |
| 600           | 0.11186  | 0.20224  | 0.28714 | 0.36354 | 0.45592 | 0.52933 |
| 700           | 0.10948  | 0.19754  | 0.28026 | 0.3547  | 0.44472 | 0.51624 |
| 800           | 0.09286  | 0.17504  | 0.25815 | 0.3278  | 0.41508 | 0.48334 |
| 900           | 0.10166  | 0.18607  | 0.259   | 0.32972 | 0.40839 | 0.47336 |
| 1000          | 0.07167  | 0.14502  | 0.22339 | 0.28584 | 0.3529  | 0.42667 |
| 1100          | 0.058728 | 0.12337  | 0.19826 | 0.25738 | 0.31532 | 0.38233 |
| 1200          | 0.05674  | 0.1179   | 0.18864 | 0.24354 | 0.29807 | 0.3666  |
| 1300          | 0.046034 | 0.099283 | 0.16318 | 0.21471 | 0.26521 | 0.31571 |
| 1400          | 0.044643 | 0.095109 | 0.15593 | 0.20413 | 0.25201 | 0.29988 |
| 1500          | 0.053214 | 0.10613  | 0.16175 | 0.20684 | 0.25615 | 0.30756 |
| 1600          | 0.060633 | 0.11111  | 0.16018 | 0.20249 | 0.2555  | 0.29696 |
| 1700          | 0.056601 | 0.10372  | 0.14953 | 0.18902 | 0.23851 | 0.27721 |
| 1800          | 0.052106 | 0.095486 | 0.13766 | 0.17401 | 0.21957 | 0.2552  |

| Kstarpluseemm | 0        | 1       | 2       | 3       | 4       | 5       |
|---------------|----------|---------|---------|---------|---------|---------|
| 300           | 0.18486  | 0.36867 | 0.56189 | 0.71855 | 0.88983 | 1.0684  |
| 400           | 0.20465  | 0.42437 | 0.66041 | 0.84874 | 1.0421  | 1.2693  |
| 500           | 0.2292   | 0.45711 | 0.69667 | 0.8909  | 1.1033  | 1.3247  |
| 600           | 0.26125  | 0.49962 | 0.74202 | 0.94402 | 1.1891  | 1.3952  |
| 700           | 0.21357  | 0.44607 | 0.7029  | 0.9043  | 1.1071  | 1.3585  |
| 800           | 0.20089  | 0.41884 | 0.66962 | 0.86526 | 1.0596  | 1.2985  |
| 900           | 0.17747  | 0.37956 | 0.62181 | 0.81612 | 1.0066  | 1.197   |
| 1000          | 0.18955  | 0.39633 | 0.63141 | 0.81481 | 0.99697 | 1.2284  |
| 1100          | 0.22214  | 0.41873 | 0.61754 | 0.78414 | 0.99295 | 1.1562  |
| 1200          | 0.15113  | 0.31518 | 0.50508 | 0.65421 | 0.80136 | 0.97834 |
| 1300          | 0.15389  | 0.30692 | 0.46777 | 0.59819 | 0.74078 | 0.88946 |
| 1400          | 0.15267  | 0.28719 | 0.42172 | 0.53509 | 0.67717 | 0.78827 |
| 1500          | 0.11239  | 0.22229 | 0.33592 | 0.42906 | 0.53338 | 0.63708 |
| 1600          | 0.10633  | 0.19602 | 0.28471 | 0.3598  | 0.45453 | 0.52911 |
| 1700          | 0.091119 | 0.16442 | 0.23326 | 0.29563 | 0.37015 | 0.42968 |
| 1800          | 0.066446 | 0.12418 | 0.18095 | 0.22933 | 0.28997 | 0.33739 |

| Kstarplummmmm | 0        | 1       | 2       | 3       | 4       | 5       |
|---------------|----------|---------|---------|---------|---------|---------|
| 300           | 0.78402  | 1.4663  | 1.9715  | 2.5423  | 3.0803  | 3.5625  |
| 400           | 1.1066   | 2.0576  | 2.7465  | 3.5864  | 4.2486  | 4.9019  |
| 500           | 1.1244   | 2.103   | 2.8276  | 3.6462  | 4.4178  | 5.1094  |
| 600           | 1.1431   | 2.0823  | 2.9073  | 3.6975  | 4.5822  | 5.3177  |
| 700           | 1.1211   | 2.0967  | 2.819   | 3.6352  | 4.4045  | 5.094   |
| 800           | 1.1448   | 2.1287  | 2.8414  | 3.7103  | 4.3954  | 5.0712  |
| 900           | 1.04     | 1.9338  | 2.5812  | 3.3706  | 3.9929  | 4.6069  |
| 1000          | 1.0551   | 1.9618  | 2.6186  | 3.4195  | 4.0508  | 4.6737  |
| 1100          | 0.88378  | 1.6475  | 2.2113  | 2.8732  | 3.4406  | 3.9752  |
| 1200          | 0.71523  | 1.3278  | 1.8047  | 2.3177  | 2.8247  | 3.2683  |
| 1300          | 0.58962  | 1.0992  | 1.4753  | 1.9169  | 2.2954  | 2.6521  |
| 1400          | 0.45735  | 0.85041 | 1.1351  | 1.4822  | 1.7559  | 2.0259  |
| 1500          | 0.32252  | 0.60123 | 0.80696 | 1.0485  | 1.2556  | 1.4507  |
| 1600          | 0.22943  | 0.4277  | 0.57405 | 0.74588 | 0.89317 | 1.032   |
| 1700          | 0.15311  | 0.28635 | 0.38501 | 0.49647 | 0.60154 | 0.69571 |
| 1800          | 0.099706 | 0.18279 | 0.25182 | 0.32213 | 0.39584 | 0.45848 |

| Kstar0eeee | 0        | 1        | 2       | 3       | 4       | 5       |
|------------|----------|----------|---------|---------|---------|---------|
| 300        | 0.055387 | 0.11834  | 0.19326 | 0.25302 | 0.3116  | 0.37177 |
| 400        | 0.060857 | 0.12688  | 0.20286 | 0.26212 | 0.32099 | 0.39298 |
| 500        | 0.052225 | 0.1127   | 0.18573 | 0.24463 | 0.30235 | 0.36007 |
| 600        | 0.061078 | 0.12786  | 0.20563 | 0.26671 | 0.32657 | 0.39701 |
| 700        | 0.056443 | 0.1196   | 0.1942  | 0.25301 | 0.31142 | 0.37339 |
| 800        | 0.05275  | 0.11181  | 0.18202 | 0.23737 | 0.29235 | 0.34993 |
| 900        | 0.051208 | 0.10846  | 0.17567 | 0.22901 | 0.28164 | 0.33818 |
| 1000       | 0.037251 | 0.085577 | 0.13558 | 0.19465 | 0.24398 | 0.29331 |
| 1100       | 0.036241 | 0.083255 | 0.1319  | 0.18936 | 0.23736 | 0.28535 |
| 1200       | 0.041493 | 0.088322 | 0.14434 | 0.1885  | 0.23236 | 0.27741 |
| 1300       | 0.033541 | 0.074909 | 0.12075 | 0.16715 | 0.20851 | 0.24932 |
| 1400       | 0.033311 | 0.072966 | 0.11844 | 0.16074 | 0.19986 | 0.23846 |
| 1500       | 0.037883 | 0.079007 | 0.12661 | 0.16399 | 0.20088 | 0.24524 |
| 1600       | 0.039111 | 0.080592 | 0.12515 | 0.16071 | 0.19745 | 0.24012 |
| 1700       | 0.035393 | 0.073943 | 0.11678 | 0.15059 | 0.18396 | 0.22611 |
| 1800       | 0.032007 | 0.066751 | 0.10697 | 0.13855 | 0.16972 | 0.2072  |

| Kstar0eemm | 0        | 1        | 2       | 3       | 4       | 5       |
|------------|----------|----------|---------|---------|---------|---------|
| 300        | 0.16942  | 0.32088  | 0.4749  | 0.60239 | 0.76326 | 0.8899  |
| 400        | 0.20315  | 0.38475  | 0.56943 | 0.72231 | 0.9152  | 1.067   |
| 500        | 0.12898  | 0.28832  | 0.46174 | 0.64492 | 0.80534 | 0.96359 |
| 600        | 0.12903  | 0.29032  | 0.46385 | 0.65406 | 0.81868 | 0.9822  |
| 700        | 0.12525  | 0.28346  | 0.45156 | 0.64274 | 0.80425 | 0.96575 |
| 800        | 0.12038  | 0.27547  | 0.43598 | 0.62795 | 0.78737 | 0.9468  |
| 900        | 0.11318  | 0.26337  | 0.41568 | 0.60501 | 0.7605  | 0.91598 |
| 1000       | 0.10961  | 0.25241  | 0.40024 | 0.57824 | 0.72707 | 0.87389 |
| 1100       | 0.08994  | 0.21567  | 0.3359  | 0.5066  | 0.64243 | 0.77642 |
| 1200       | 0.098842 | 0.22094  | 0.35384 | 0.49421 | 0.61714 | 0.73841 |
| 1300       | 0.10077  | 0.21531  | 0.35159 | 0.46033 | 0.5669  | 0.67637 |
| 1400       | 0.060575 | 0.14526  | 0.22623 | 0.3412  | 0.43268 | 0.52292 |
| 1500       | 0.063762 | 0.1424   | 0.22954 | 0.31775 | 0.39639 | 0.47396 |
| 1600       | 0.058327 | 0.12426  | 0.20372 | 0.2667  | 0.32925 | 0.3918  |
| 1700       | 0.045647 | 0.098901 | 0.16322 | 0.21475 | 0.26593 | 0.31676 |
| 1800       | 0.03792  | 0.081137 | 0.13356 | 0.1751  | 0.21637 | 0.25707 |

| Kstar0mmmm | 0       | 1       | 2       | 3      | 4       | 5       |
|------------|---------|---------|---------|--------|---------|---------|
| 300        | 0.69305 | 1.2887  | 1.7201  | 2.2462 | 2.6609  | 3.07    |
| 400        | 0.96409 | 1.7927  | 2.3928  | 3.1246 | 3.7015  | 4.2706  |
| 500        | 1.0111  | 1.8801  | 2.5094  | 3.2769 | 3.8819  | 4.4788  |
| 600        | 1.0296  | 1.9145  | 2.5555  | 3.337  | 3.9531  | 4.561   |
| 700        | 1.0772  | 2.003   | 2.6736  | 3.4913 | 4.1359  | 4.7718  |
| 800        | 0.9652  | 1.7993  | 2.415   | 3.1379 | 3.7575  | 4.3414  |
| 900        | 0.93979 | 1.7519  | 2.3514  | 3.0553 | 3.6586  | 4.2271  |
| 1000       | 0.90949 | 1.6911  | 2.2573  | 2.9476 | 3.4919  | 4.0288  |
| 1100       | 0.76141 | 1.4194  | 1.9051  | 2.4754 | 2.9642  | 3.4248  |
| 1200       | 0.62719 | 1.1692  | 1.5693  | 2.039  | 2.4417  | 2.8211  |
| 1300       | 0.45699 | 0.83519 | 1.1582  | 1.4773 | 1.824   | 2.1136  |
| 1400       | 0.37279 | 0.6813  | 0.94483 | 1.2051 | 1.4879  | 1.7241  |
| 1500       | 0.66884 | 1.2468  | 1.6735  | 2.1744 | 2.6038  | 3.0084  |
| 1600       | 0.49824 | 0.90252 | 1.2696  | 1.6127 | 2.0083  | 2.3295  |
| 1700       | 0.3706  | 0.69263 | 1.0093  | 1.2791 | 1.6173  | 1.8818  |
| 1800       | 0.13165 | 0.26039 | 0.3935  | 0.5026 | 0.62479 | 0.74626 |

