# New Avenues for Proton Decay

**L. Velasco-Sevilla**[1,2,3]

**Yonsei-Saga Workshop, Jan 20 2022**

[1]Korea Institute for Advanced Study, Seoul 02455, Korea

[2]CQUEST Sogang University

[3]Department of Physics and Technology, University of Bergen, PO Box 7803, 5020 Bergen, Norway

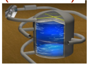
## Outline

- Motivation
- Introduction to GUT
- Basics of Proton Decay Computation
- Flavoured Channels
- B-L proton decay modes

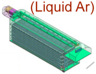
# Motivation

- The Standard Model of Particle Physics is great but we need to look for hints beyond it, especially because of the current LHC bounds on supersymmetric searches
- A new era of large underground detectors are scheduled to operate in the near future
- These detectors are both aimed at studying neutrino properties and as proton decay experiments.

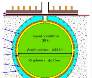
- Hyper-Kamiokande — Hyper-K — (Water C) — 2027



- DUNE — DEEP UNDERGROUND NEUTRINO EXPERIMENT — (Liquid Ar) — 2027



- JUNO — JUNO



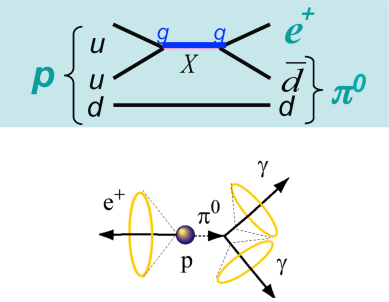
Current and future discovery/exclusion values (in units of $10^{33}$ years) will help us to exclude/constraint models

| Decay Mode | Current (90% CL) | Future (Discovery) | Future (90% CL) |
|---|---|---|---|
| $p \to K^+\bar{\nu}$ | 6.6 [6] | JUNO: 12 (20) [3]<br>DUNE: 30 (50) [3]<br>Hyper-K: 20 (30) [3] | JUNO: 19 (40) [1]<br>DUNE: 33 (65) [2]<br>Hyper-K: 32 (50) [3] |
| $p \to \pi^+\bar{\nu}$ | 0.39 [29] | | |
| $p \to e^+\pi^0$ | 16 [40] | DUNE: 15 (25) [3]<br>Hyper-K: 63 (100) [3] | DUNE: 20 (40) [3]<br>Hyper-K: 78 (130) [3] |
| $p \to \mu^+\pi^0$ | 7.7 [40] | Hyper-K: 69 [3] | Hyper-K: 77 [3] |
| $n \to K_S^0\bar{\nu}$ | 0.26 [25] | | |
| $n \to \pi^0\bar{\nu}$ | 1.1 [29] | | |
| $n \to e^+\pi^-$ | 5.3 [48] | Hyper-K: 13 [3] | Hyper-K: 20 [3] |
| $n \to \mu^+\pi^-$ | 3.5 [48] | Hyper-K: 11 [3] | Hyper-K: 18 [3] |

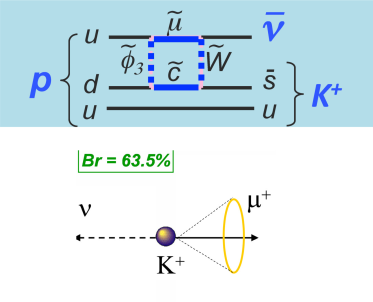
The two prominent decay modes are

$$p \to e^{+}\Pi^{0} \qquad\qquad p \to \bar{\nu}\, K^{+}$$

Leading for non-supersymmetric models



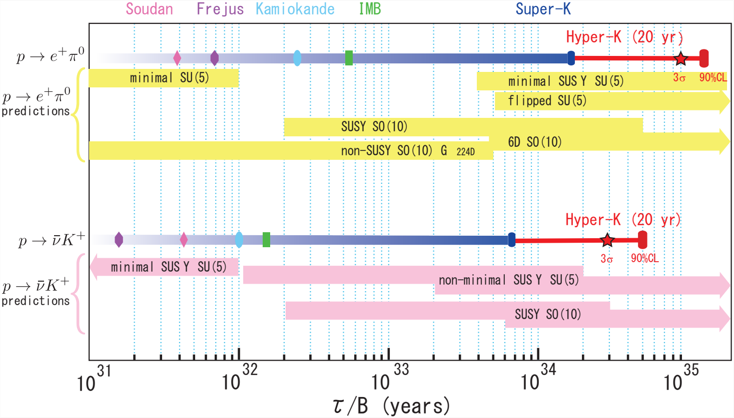

Leading for supersymmetric models



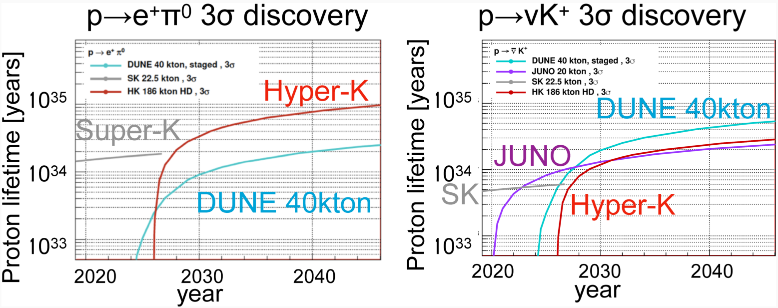

$$\Gamma(p \to e^{+}\pi^{0}) \sim \frac{g^{4}\, m_p^{5}}{M_X^{4}}$$

$$\Gamma(p \to \bar{\nu} K^{+}) \sim \frac{\tan\beta^{2}\, m_p^{5}}{m_{\tilde{q}}^{2}\, M_3^{2}}.$$

Thes two modes have been traditionally used to exclude and further construct models



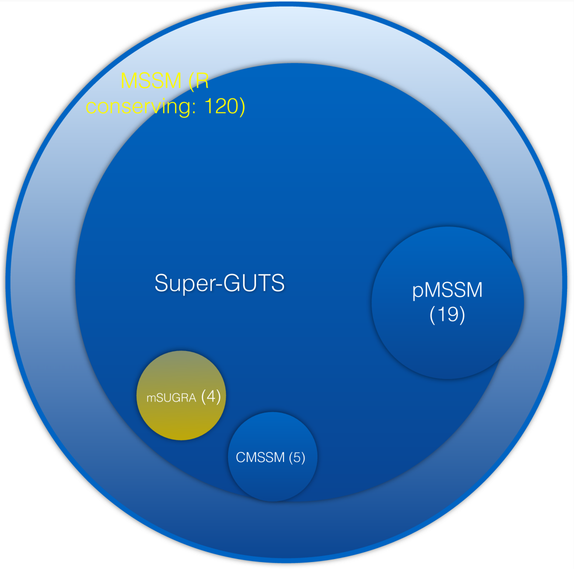
Being optimistic about the discovery we can have a better idea from the following sensitivity plots for discovery



# Introduction to GUT

# +40 years of Grand Unified Theories

- Georgi- Glashow Model 1974, SU(5)
- 1980-1990 Matching to MSSM fields developed, use of numerical 1-loop beta functions for the CMSSM
- 1990-2000: Computation of 2-loops beta functions, matching at EW Scale developed
- 2000-2010: Computation of Higgs Observables, development of tools for probing SUSY at the LHC
- 2010: Adding of running above the GUT scale, use of supergravity: no- scale supergravity
- 2013: Code development of no-scale supergravity models
- 2015: Lattice calculation reduced to 30% uncertainty in hadronic parameters
- 2017: Lattice calculation reduced to 10% uncertainty in hadronic parameters
- 2019: Refinements in the theory and precision in calculations for PD


MSSM (R
conserving: 120)
Super-GUTS
pMSSM
(19)
mSUGRA (4)
CMSSM (5)

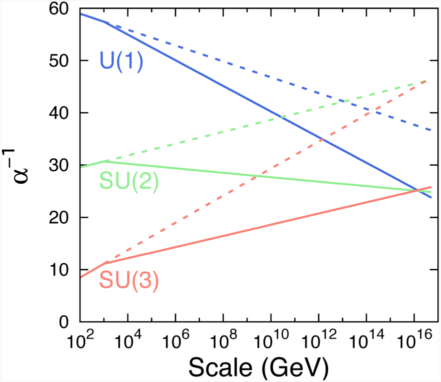
# Basic Motivations of GUTs

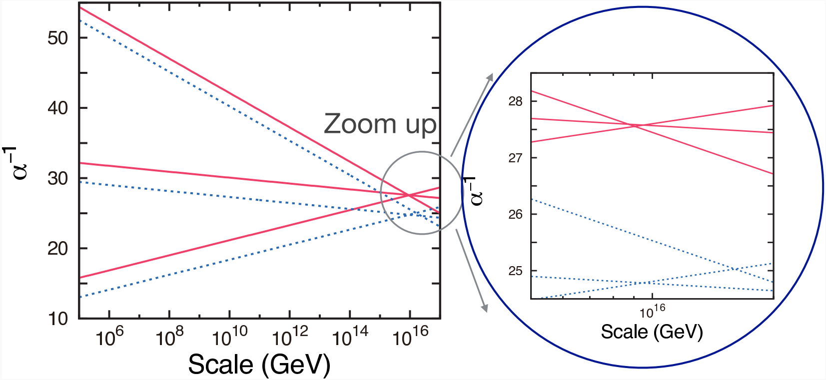
- Gauge Coupling Unification (one step in supersymmetry, multi-step in non-supersymmetric models)
- Sound predictions: Proton Decay, explain charge quantization, anomaly cancellation, neutrino masses, B-L
- Relations among the masses of quarks and leptons simply because they are in the same multiplet, e.g. in SU(5)

$$\bar{\mathbf{5}} = \begin{pmatrix} \overline{D}_1 \\ \overline{D}_2 \\ \overline{D}_3 \\ E \\ -N \end{pmatrix} \quad \mathbf{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \overline{U}_3 & -\overline{U}_2 & U^1 & D^1 \\ -\overline{U}_3 & 0 & \overline{U}_1 & U^2 & D^2 \\ \overline{U}_2 & -\overline{U}_1 & 0 & U^3 & D^3 \\ -U^1 & -U^2 & -U^3 & 0 & \overline{E} \\ -D^1 & -D^2 & -D^3 & \overline{E} & 0 \end{pmatrix} \tag{1}$$

In SO(10) all particles are in the **16** representation.

Supersymmetric (low-scale susy) vs. non-supersymmetric






| High – scale | Low – scale |
|---|---|
| $M_S = 10^2$ TeV | $M_S = 200$ GeV |
| $M_2 = 3$ TeV | $M_2 = 1$ TeV |
| $M_3/M_2 = 9$ | $M_3/M_2 = 3.5$ |

# Basics of Proton Decay Computation

# Non-Supersymmetric Models: Dimension-6 Operator

Example with the gauge group $\mathrm{SU}(4) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(2)$: In non-SUSY GUTs, proton decay is induced by gauge interactions. The relevant interactions are written as

$$\mathcal{L}_{\text{int}} = \frac{g_{\text{GUT}}}{\sqrt{2}} \left[ (\overline{Q})_{ar} \not{X}^{air} P_R (L^{\mathcal{C}})_i + (\overline{Q})_{ai} \not{X}^{air} P_L (L^{\mathcal{C}})_r + \epsilon_{ij}\epsilon_{rs}\epsilon_{abc} (\overline{Q^{\mathcal{C}}})^{ar} \not{X}^{bis} P_L Q^{cj} + \text{h.c.} \right] \tag{2}$$

where

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} , \qquad L = \begin{pmatrix} \nu \\ e^- \end{pmatrix} , \tag{3}$$

After integrating out the SO(10) gauge fields $X$, we obtain the dimension-six proton decay operator. The operator is expressed in a form that respects the intermediate gauge symmetry, $\mathrm{SU}(4) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(2)$:

$$\mathcal{L}_{\mathrm{eff}} = C(M_{\mathrm{GUT}}) \cdot \epsilon_{ij}\epsilon_{rs}\epsilon_{\alpha\beta\gamma\delta}(\overline{\Psi^{\mathcal{C}}})^{\alpha i} P_L \Psi^{\beta j} (\overline{\Psi^{\mathcal{C}}})^{\gamma r} P_R \Psi^{\delta s} \ , \tag{4}$$

where $\alpha, \beta, \ldots$ denote the SU(4) indices

$$\epsilon_{ij}\epsilon_{kl}\epsilon_{\alpha\beta\gamma\delta}(\overline{\Psi^{\mathcal{C}}})^{\alpha i} P_L \Psi^{\beta j} (\overline{\Psi^{\mathcal{C}}})^{\gamma k} P_L \Psi^{\delta l} = \epsilon_{rs}\epsilon_{tu}\epsilon_{\alpha\beta\gamma\delta}(\overline{\Psi^{\mathcal{C}}})^{\alpha r} P_R \Psi^{\beta s} (\overline{\Psi^{\mathcal{C}}})^{\gamma t} P_R \Psi^{\delta u} = 0 \, . \tag{5}$$

At tree level, the coefficient of the effective operator is evaluated as

$$C(M_{\mathrm{GUT}}) = \frac{g^2_{\mathrm{GUT}}}{2M_X^2} \ , \tag{6}$$

with $M_X$ the mass of the heavy gauge field $X$.

$$C(M_{\rm int}) = \left[\frac{\alpha_4(M_{\rm int})}{\alpha_{\rm GUT}}\right]^{-\frac{15}{4b_4}} \left[\frac{\alpha_{2L}(M_{\rm int})}{\alpha_{\rm GUT}}\right]^{-\frac{9}{4b_{2L}}} \left[\frac{\alpha_{2R}(M_{\rm int})}{\alpha_{\rm GUT}}\right]^{-\frac{9}{4b_{2R}}} C(M_{\rm GUT}) \ . \tag{7}$$

At the intermediate scale, the $\mathrm{SU}(4) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(2)$ theory is matched onto the SM. The effective Lagrangian is written as

$$\mathcal{L}_{\rm eff} = \sum_{I=1}^{4} C_I \mathcal{O}_I \ , \tag{8}$$

with the effective operators given by

$$\begin{aligned} \mathcal{O}_1 &= \epsilon_{abc}\epsilon_{ij}(u_R^a d_R^b)(Q_L^{ci} L_L^j) \ , \\ \mathcal{O}_2 &= \epsilon_{abc}\epsilon_{ij}(Q_L^{ai} Q_L^{bj})(u_R^c e_R) \ , \\ \mathcal{O}_3 &= \epsilon_{abc}\epsilon_{ij}\epsilon_{kl}(Q_L^{ai} Q_L^{bk})(Q_L^{cl} L_L^j) \ , \\ \mathcal{O}_4 &= \epsilon_{abc}(u_R^a d_R^b)(u_R^c e_R) \ . \end{aligned} \tag{9}$$

We evaluate the coefficients $C_I$ as

$$\begin{aligned} C_1(M_{\rm int}) &= 4C(M_{\rm int}) \ , \\ C_2(M_{\rm int}) &= -4C(M_{\rm int}) \ , \\ C_3(M_{\rm int}) &= C_4(M_{\rm int}) = 0. \end{aligned} \tag{10}$$

$$C_1(m_Z) = \left[\frac{\alpha_3(m_Z)}{\alpha_3(M_{\rm int})}\right]^{-\frac{2}{b_3}} \left[\frac{\alpha_2(m_Z)}{\alpha_2(M_{\rm int})}\right]^{-\frac{9}{4b_2}} \left[\frac{\alpha_1(m_Z)}{\alpha_1(M_{\rm int})}\right]^{-\frac{11}{20b_1}} C_1(M_{\rm int}) \ , \quad (11)$$

$$C_2(m_Z) = \left[\frac{\alpha_3(m_Z)}{\alpha_3(M_{\rm int})}\right]^{-\frac{2}{b_3}} \left[\frac{\alpha_2(m_Z)}{\alpha_2(M_{\rm int})}\right]^{-\frac{9}{4b_2}} \left[\frac{\alpha_1(m_Z)}{\alpha_1(M_{\rm int})}\right]^{-\frac{23}{20b_1}} C_2(M_{\rm int}) \ . \quad (12)$$

The beta-function coefficients should be appropriately modified when the number of quark flavors changes. Below the electroweak scale, the QCD corrections are the dominant contribution. W can compute the Wilson coefficients at the hadronic scale $\mu_{\rm had}$ as

$$C_i(\mu_{\rm had}) = \left[\frac{\alpha_s(\mu_{\rm had})}{\alpha_s(m_b)}\right]^{\frac{6}{25}} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_Z)}\right]^{\frac{6}{23}} \left[\frac{\alpha_s(\mu_{\rm had}) + \frac{50\pi}{77}}{\alpha_s(m_b) + \frac{50\pi}{77}}\right]^{-\frac{173}{825}}$$

$$\left[\frac{\alpha_s(m_b) + \frac{23\pi}{29}}{\alpha_s(m_Z) + \frac{23\pi}{29}}\right]^{-\frac{430}{2001}} C_i(m_Z) \ , \quad (13)$$

with $i = 1, 2$.

Finally, the decay amplitude is given by

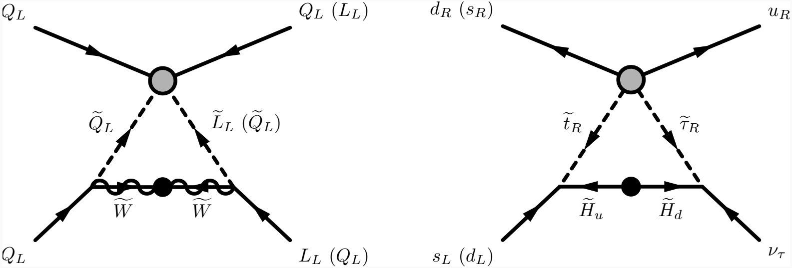
$$\Gamma(p \to e^+ \pi^0) \quad = \quad \frac{m_p}{32\pi}\left(1 - \frac{m_{\pi^0}^2}{m_p^2}\right)^2 \left[|\mathcal{A}_L(p \to e^+ \pi^0)|^2 + |\mathcal{A}_R(p \to e^+ \pi^0)|^2\right],$$

where $m_p$ and $m_{\pi^0}$ are the proton and pion masses respectively. The amplitude at EW is computed from

$$\begin{aligned}
\mathcal{A}_L(p \to e^+ \pi^0) &= C_{RL}((ud)_R u_L)(\mu = 2\mathrm{GeV})\langle \pi^0 | (\mathrm{ud})_\mathrm{R} \mathrm{u_R} | \mathrm{p}\rangle, \\
\mathcal{A}_R(p \to e^+ \pi^0) &= 2C_{LR}((ud)_L u_R)(\mu = 2\mathrm{GeV})\langle \pi^0 | (\mathrm{ud})_\mathrm{R} \mathrm{u_R} | \mathrm{p}\rangle, \qquad (14)
\end{aligned}$$

J. Ellis, J. Evans, N. Nagata, K. Olive and L.Velasco-Sevilla

1912.04888




controlled by the effective operators $\mathcal{O}^{5L}_{ijkl}$ and $\mathcal{O}^{5R}_{ijkl}$ are defined by

$$\mathcal{O}^{5L}_{ijkl} \equiv \int d^2\theta \ \frac{1}{2}\epsilon_{abc}(Q^a_i \cdot Q^b_j)(Q^c_k \cdot L_l) \ ,$$

$$\mathcal{O}^{5R}_{ijkl} \equiv \int d^2\theta \ \epsilon^{abc}\overline{U}_{ia}\overline{E}_j\overline{U}_{kb}\overline{D}_{lc} \ , \quad (15)$$

The effective Lagrangian for this contribution is

$$\mathcal{L}_5^{\rm eff} = C_{5L}^{ijkl}\mathcal{O}_{ijkl}^{5L} + C_{5R}^{ijkl}\mathcal{O}_{ijkl}^{5R} \quad + \quad {\rm h.c.}\ , \tag{16}$$

where the effective operators $\mathcal{O}_{ijkl}^{5L}$ and $\mathcal{O}_{ijkl}^{5R}$ are defined by

$$\begin{aligned}\mathcal{O}_{ijkl}^{5L} &\equiv \int d^2\theta\ \frac{1}{2}\epsilon_{abc}(Q_i^a\cdot Q_j^b)(Q_k^c\cdot L_l)\ ,\\ \mathcal{O}_{ijkl}^{5R} &\equiv \int d^2\theta\ \epsilon^{abc}\overline{U}_{ia}\overline{E}_j\overline{U}_{kb}\overline{D}_{lc}\ ,\end{aligned} \tag{17}$$

and the Wilson coefficients $C_{5L}^{ijkl}$ and $C_{5R}^{ijkl}$ are given by

$$C_{5L}^{ijkl}(M_{\rm GUT}) = \frac{1}{M_{H_C}}h^{Q_iQ_j}h^{Q_kL_l},\quad C_{5R}^{ijkl}(M_{\rm GUT}) = \frac{1}{M_{H_C}}h^{U_iE_j}h^{U_kD_l}\ . \tag{18}$$

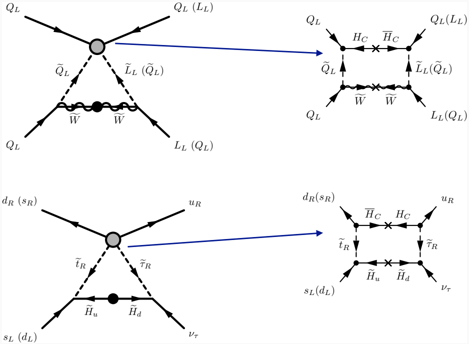
The leading-order RG evolutions of the $C_{5L}^{ijkl}$ and $C_{5R}^{ijkl}$ between $M_{\rm GUT}$ and the supersymmetry breaking scale are given by

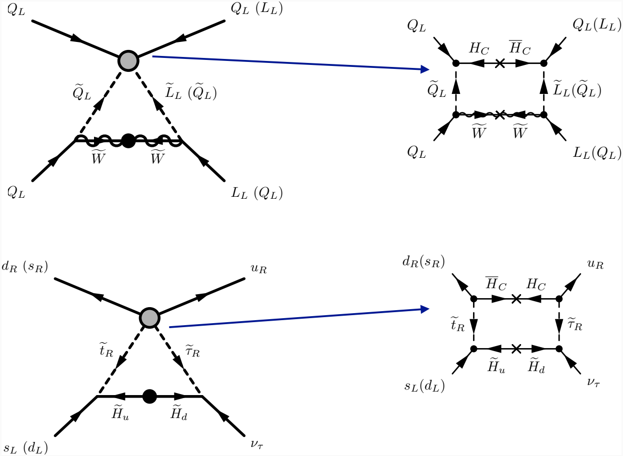
$$\begin{aligned}\overline{\beta}(C_{5L}^{ijkl}) \equiv (4\pi)^2\Lambda\frac{d}{d\Lambda}C_{5L}^{ijkl} &= \left(-8g_3^2-6g_2^2-\frac{2}{5}g_1^2\right)C_{5L}^{ijkl} + C_{5L}^{mjkl}\left(h_Dh_D^\dagger+h_Uh_U^\dagger\right)^i_{\ m}\\ &+C_{5L}^{imkl}\left(h_Dh_D^\dagger+h_Uh_U^\dagger\right)^j_{\ m} + C_{5L}^{ijml}\left(h_Dh_D^\dagger+h_Uh_U^\dagger\right)^k_{\ m} + C_{5L}^{ijkm}\left(h_E^\dagger h_E\right)^l_{\ m}\ ,\\ \overline{\beta}(C_{5R}^{ijkl})\equiv(4\pi)^2\Lambda\frac{d}{d\Lambda}C_{5R}^{ijkl} &= \left(-8g_3^2-\frac{12}{5}g_1^2\right)C_{5R}^{ijkl} + C_{5R}^{mjkl}\left(2\,h_U^\dagger h_U\right)^i_{\ m}\\ &+C_{5R}^{imkl}\left(2\,h_Eh_E^\dagger\right)^j_{\ m} + C_{5R}^{ijml}\left(2\,h_U^\dagger h_U\right)^k_{\ m} + C_{5R}^{ijkm}\left(2\,h_D^\dagger h_D\right)^l_{\ m}\ ,\end{aligned} \tag{19}$$

We write the effective Lagrangian for $p \to K^+ \bar{\nu}_i$ decay in the following form:

$$\begin{aligned}\mathcal{L}(p \to K^+ \bar{\nu}_i) =& C_{RL}(usd\nu_i)\left[\epsilon_{abc}(u_R^a s_R^b)(d_L^c \nu_i)\right] + C_{RL}(uds\nu_i)\left[\epsilon_{abc}(u_R^a d_R^b)(s_L^c \nu_i)\right] \\ &+ C_{LL}(usd\nu_i)\left[\epsilon_{abc}(u_L^a s_L^b)(d_L^c \nu_i)\right] + C_{LL}(uds\nu_i)\left[\epsilon_{abc}(u_L^a d_L^b)(s_L^c \nu_i)\right] .\end{aligned} \tag{20}$$

The operators $C_{LL}\,(usd\nu_k)$ and $C_{LL}\,(uds\nu_k)$ are mediated by Wino exchange, and $C_{RL}(usd\nu_\tau)$ and $C_{RL}(uds\nu_\tau)$ are mediated by the Higgsino.






$$
\begin{aligned}
C_{RL}(usd\nu_\tau) &= -V_{td} C_2^{\widetilde{H}}(m_Z)\ , & C_{RL}(uds\nu_\tau) &= -V_{ts} C_1^{\widetilde{H}}(m_Z)\ , \\
C_{LL}(usd\nu_k) &= \sum_{j=2,3} V_{j1} V_{j2} C_{jk}^{\widetilde{W}}(m_Z)\ , & C_{LL}(uds\nu_k) &= \sum_{j=2,3} V_{j1} V_{j2} C_{jk}^{\widetilde{W}}(m_Z)\ .
\end{aligned}
\tag{21}
$$

where

$$C_i^{\widetilde{H}} = \frac{y_t y_\tau}{(4\pi)^2} F(\mu, m^2_{\widetilde{t}_R}, m^2_{\tau_R})\, C_{5R}^{*331i} \,,$$
$$C_{jk}^{\widetilde{W}} = \frac{\alpha_2}{4\pi} \left[ F(M_2, m^2_{\widetilde{Q}_1}, m^2_{\widetilde{Q}_j}) + F(M_2, m^2_{\widetilde{Q}_j}, m^2_{\widetilde{L}_k}) \right] C_{5L}^{jj1k} \,. \tag{22}$$

Here $m_{\widetilde{t}_R}$, $m_{\widetilde{\tau}_R}$, $m_{\widetilde{Q}_j}$, and $m_{\widetilde{L}_k}$ are the masses of the right-handed stop, the right-handed stau, left-handed squarks, and left-handed sleptons, respectively, $\alpha_i \equiv g_i^2/(4\pi)$, and

$$F(M, m_1^2, m_2^2) \equiv \frac{M}{m_1^2 - m_2^2} \left[ \frac{m_1^2}{m_1^2 - M^2} \ln\left(\frac{m_1^2}{M^2}\right) - \frac{m_2^2}{m_2^2 - M^2} \ln\left(\frac{m_2^2}{M^2}\right) \right] . \tag{23}$$

Note that

$$C^{\tilde{H},\tilde{W}} \propto \frac{1}{M_{\overline{H}} M_S} \tag{24}$$

where

$$H = \begin{pmatrix} H_C^1 \\ H_C^2 \\ H_C^3 \\ H_u^+ \\ H_u^0 \end{pmatrix}, \quad H = \begin{pmatrix} \overline{H}_C^1 \\ \overline{H}_C^2 \\ \overline{H}_C^3 \\ \overline{H}_d^- \\ -\overline{H}_d^0 \end{pmatrix}, \tag{25}$$

where $H_C^i$ are the components color Higgs triplets and $H_{d,u}$ are the MSSM Higgs doublets.

Finally, the decay amplitude is given by

$$\Gamma(p \to K^+\bar{\nu}) = \frac{m_p}{32\pi}\left(1 - \frac{m_K^2}{m_p^2}\right)^2 |\mathcal{A}(p \to K^+\bar{\nu}_i)|^2, \tag{26}$$

where $m_K$ is the mass of the kaon and the decay amplitude is given in terms of the Wilson Coefficients effective at the hadronic scale (2 GeV)

$$\begin{aligned} \mathcal{A}(p \to K^+\bar{\nu}_i) \quad &= \quad C_{RL}(usd\nu_i)\langle K^+|(us)_R\, d_L|p\rangle + C_{RL}(uds\nu_i)\langle K^+|(ud)_R\, s_L|p\rangle \\ &+ \quad C_{LL}(usd\nu_i)\langle K^+|(us)_L\, d_L|p\rangle + C_{LL}(uds\nu_i)\langle K^+|(ud)_L\, s_L|p\rangle. \end{aligned} \tag{27}$$

$$\mathcal{L}_6^{\text{eff}} = C^{ijkl}_{6(1)} \mathcal{O}^{6(1)}_{ijkl} + C^{ijkl}_{6(2)} \mathcal{O}^{6(2)}_{ijkl} + \text{h.c.}\ , \tag{28}$$

where

$$\mathcal{O}^{6(1)}_{ijkl} \equiv \int d^2\theta d^2\bar{\theta}\ \epsilon_{abc}\epsilon_{\alpha\beta}\left(\overline{U}^\dagger_i\right)^a \left(\overline{D}^\dagger_j\right)^b e^{-\frac{2}{3}g' B}\left(e^{2g_3 G} Q^\alpha_k\right)^c L^\beta_l\ , \tag{29}$$

$$\mathcal{O}^{6(2)}_{ijkl} \equiv \int d^2\theta d^2\bar{\theta} \epsilon_{abc}\epsilon_{\alpha\beta}\ Q^{a\alpha}_i Q^{b\beta}_j e^{\frac{2}{3}g' B}\left(e^{-2g_3 G}\overline{U}^\dagger_k\right)^c \overline{E}^\dagger_l\ , \tag{30}$$

and their Wilson coefficients are

$$C^{ijkl}_{6(1)} = -\frac{g_5^2}{M_X^2} e^{i\varphi_i}\delta^{ik}\delta^{jl}\ ,$$

$$C^{ijkl}_{6(2)} = -\frac{g_5^2}{M_X^2} e^{i\varphi_i}\delta^{ik}(V^*)^{jl}\ . \tag{31}$$

At the one-loop level, the RGEs of these coefficients can easily be solved. The coefficients are then matched at the electroweak scale onto the effective operators

$$\mathcal{L}(p \to \pi^0 l_i^+) = C_{RL}(udul_i)\big[\epsilon_{abc}(u_R^a d_R^b)(u_L^c l_{Li})\big] + C_{LR}(udul_i)\big[\epsilon_{abc}(u_L^a d_L^b)(u_R^c l_{Ri})\big] \ , \tag{32}$$

where

$$\begin{aligned} C_{RL}(udul_i) &= C_{6(1)}^{111i}(M_Z) \ , \\ C_{LR}(udul_i) &= V_{j1}\big[C_{6(2)}^{1j1i}(M_Z) + C_{6(2)}^{j11i}(M_Z)\big] \ . \end{aligned} \tag{33}$$

Using Nihei and Goto hep-ph/9808255 for the QCD RGEs. The partial decay width for $p \to e^+\pi^0$ is then given by

$$\Gamma(p \to \pi^0 e^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 \left[|\mathcal{A}_L(p \to \pi^0 e^+)|^2 + |\mathcal{A}_R(p \to \pi^0 e^+)|^2\right] , \quad (34)$$

with

$$\mathcal{A}_L(p \to \pi^0 e^+) = -\frac{g_5^2}{M_X^2} \cdot A_1 \cdot \langle \pi^0 | (ud)_R u_L | p \rangle ,$$
$$\mathcal{A}_R(p \to \pi^0 e^+) = -\frac{g_5^2}{M_X^2}(1 + |V_{ud}|^2) \cdot A_2 \cdot \langle \pi^0 | (ud)_R u_L | p \rangle , \quad (35)$$

where $A_1$ and $A_2$ are the renormalization factors:

$$A_1 = A_L \cdot \left[\frac{\alpha_3(M_{\rm SUSY})}{\alpha_3(M_{\rm GUT})}\right]^{\frac{4}{9}} \left[\frac{\alpha_2(M_{\rm SUSY})}{\alpha_2(M_{\rm GUT})}\right]^{-\frac{3}{2}} \left[\frac{\alpha_1(M_{\rm SUSY})}{\alpha_1(M_{\rm GUT})}\right]^{-\frac{1}{18}}$$
$$\times \left[\frac{\alpha_3(M_Z)}{\alpha_3(M_{\rm SUSY})}\right]^{\frac{2}{7}} \left[\frac{\alpha_2(M_Z)}{\alpha_2(M_{\rm SUSY})}\right]^{\frac{27}{38}} \left[\frac{\alpha_1(M_Z)}{\alpha_1(M_{\rm SUSY})}\right]^{-\frac{11}{82}} ,$$
$$A_2 = A_L \cdot \left[\frac{\alpha_3(M_{\rm SUSY})}{\alpha_3(M_{\rm GUT})}\right]^{\frac{4}{9}} \left[\frac{\alpha_2(M_{\rm SUSY})}{\alpha_2(M_{\rm GUT})}\right]^{-\frac{3}{2}} \left[\frac{\alpha_1(M_{\rm SUSY})}{\alpha_1(M_{\rm GUT})}\right]^{-\frac{23}{198}}$$
$$\times \left[\frac{\alpha_3(M_Z)}{\alpha_3(M_{\rm SUSY})}\right]^{\frac{2}{7}} \left[\frac{\alpha_2(M_Z)}{\alpha_2(M_{\rm SUSY})}\right]^{\frac{27}{38}} \left[\frac{\alpha_1(M_Z)}{\alpha_1(M_{\rm SUSY})}\right]^{-\frac{23}{82}} , \quad (36)$$

with $A_L = 1.25$ the long-distance QCD renormalization factor.

Taking into account GUT threshold corrections

$$M_X = \left(\frac{2g_5}{\lambda'}\right)^{\frac{1}{3}} \left(M_X^2 M_\Sigma\right)^{\frac{1}{3}} \ , \tag{37}$$

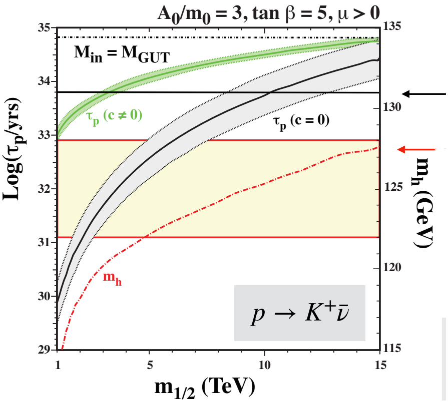
The error in the hadronic matrix element $\langle \pi^0 | (ud)_R u_L | p \rangle$ gives approximately 20% uncertainty in the decay rate. On the other hand, the uncertainty due to the error in the strong gauge coupling constant can be estimated in the same manner as before

$$\begin{aligned} \sigma_{\tau(p \to e^+ \pi^0)} &\simeq \tau(p \to e^+ \pi^0) \left(\frac{4\pi}{9}\right) \left(\frac{\Delta_{\alpha_s}}{\alpha_s(M_Z)^2}\right) \\ &= 0.11 \left(\frac{\Delta_{\alpha_s}}{0.0011}\right) \left(\frac{0.1181}{\alpha_s(M_Z)}\right)^2 \tau(p \to e^+ \pi^0) \ . \end{aligned} \tag{38}$$

$$\tau(p \to e^+ \pi^0) \simeq 1.8 \times 10^{35} \times \left(\frac{M_X}{10^{16}\ \text{GeV}}\right)^4 \ . \tag{39}$$

This expression shows that $p \to e^+ \pi^0$ can be probed at Hyper-Kamiokande if $M_X \lesssim 10^{16}$ GeV.

# CURRENT STATUS IN THE CMSSM

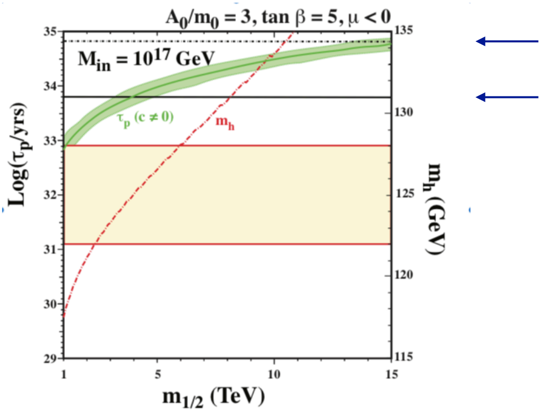



Expected Limit from DUNE

Current Limit

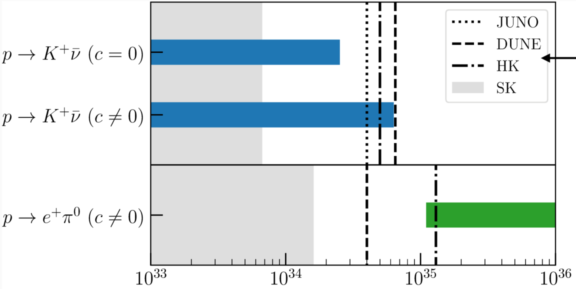
Shaded band shows where agrees with the experimental value of $m_h$ within the estimated $\pm 3\sigma$

# CURRENT STATUS SUPERGUT-SU(5)




Expected Limit from DUNE

Current Limit




90% CL (20 yrs)

In the region of sensitivity to Hyper-Kamiokande

## CMSSM

## super-GUT

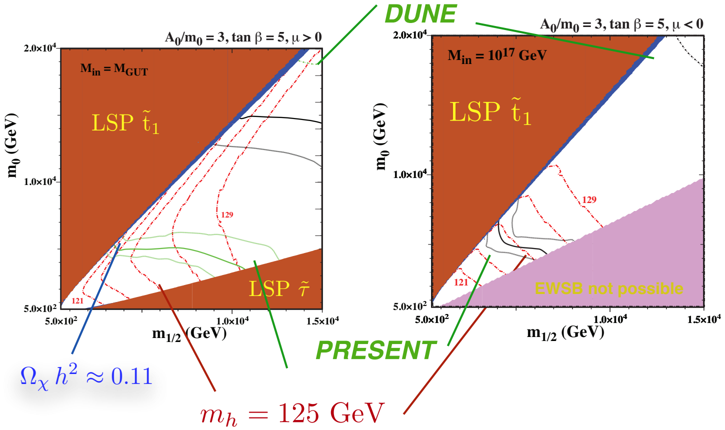
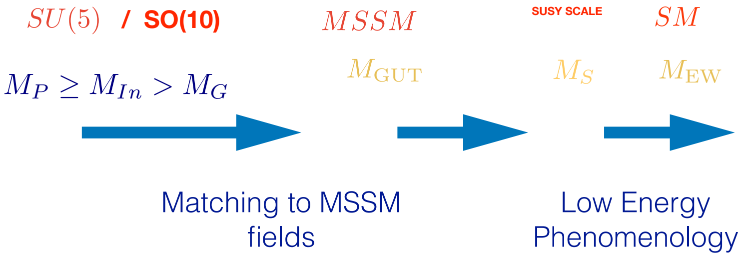


Table 1: Comparison of changes in hadronic matrix elements.

| Matrix element | (2015) $GeV^2$ | Error [%] | Net Error | (2017) $GeV^2$ | Stat.[%] | Sys. Error [%] | Net Error |
|---|---|---|---|---|---|---|---|
| $\langle\pi^0\|(ud)_R u_L\|p\rangle$ | $-0.103$ (23) (34) | 40 | 0.041 | -0.131 (4)(13) | 3.0 | 9.7 | 0.013 |
| $\langle\pi^0\|(ud)_L u_L\|p\rangle$ | 0.133 (29) (28) | 30 | 0.040 | 0.134 (5)(16) | 3.4 | 11.6 | 0.016 |
| $\langle\pi^+\|(du)_R d_L\|p\rangle$ | $-0.146$ (33) (48) | 40 | 0.058 | -0.186 (6)(18) | 3.0 | 9.7 | 0.019 |
| $\langle\pi^+\|(du)_L d_L\|p\rangle$ | 0.188 (41) (40) | 30 | 0.057 | 0.189 (6)(22) | 3.4 | 11.6 | 0.023 |
| $\langle K^0\|(us)_R u_L\|p\rangle$ | 0.098 (15) (12) | 20 | 0.019 | 0.103 (3)(11) | 2.8 | 10.4 | 0.011 |
| $\langle K^0\|(us)_L u_L\|p\rangle$ | 0.042 (13) (8) | 36 | 0.015 | 0.057 (2)(6) | 3.5 | 10.7 | 0.006 |
| $\langle K^+\|(us)_R d_L\|p\rangle$ | $-0.054$ (11) (9) | 26 | 0.014 | -0.049 (2)(5) | 3.7 | 10.9 | 0.006 |
| $\langle K^+\|(us)_L d_L\|p\rangle$ | 0.036 (12) (7) | 39 | 0.014 | 0.041(2)(5) | 4.4 | 13.1 | 0.006 |
| $\langle K^+\|(ud)_R s_L\|p\rangle$ | $-0.093$ (24) (18) | 32 | 0.030 | -0.134 (4)(14) | 3.2 | 10.3 | 0.014 |
| $\langle K^+\|(ud)_L s_L\|p\rangle$ | 0.111 (22) (16) | 25 | 0.027 | 0.139 (4)(15) | 3.0 | 10.9 | 0.016 |
| $\langle K^+\|(ds)_R u_L\|p\rangle$ | $-0.044$ (12) (5) | 30 | 0.013 | -0.054 (2)(6) | 3.6 | 10.6 | 0.006 |
| $\langle K^+\|(ds)_L u_L\|p\rangle$ | $-0.076$ (14) (9) | 22 | 0.017 | -0.098 (3)(10) | 2.8 | 10.3 | 0.010 |
| $\langle\eta\|(ud)_R u_L\|p\rangle$ | 0.015 (14) (17) | 147 | 0.022, | 0.006 (2)(3) | 30.0 | 42.1 | 0.003 |
| $\langle\eta\|(ud)_L u_L\|p\rangle$ | 0.088 (21) (16) | 30 | 0.026 | 0.113 (3)(12) | 3.1 | 10.2 | 0.012 |

# Before we finish, remember the interconnection of scales

***No-scale supergravity***




Wilson Coefficients have a different evolution from each scale to the other

We can summarize this section with the simple formula (given in years)

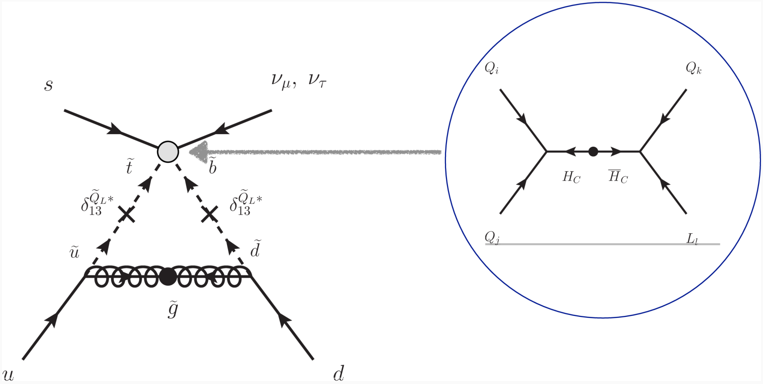
$$\tau \sim 4 \times 10^{35} \times \sin^4(2\beta) \times \left(\frac{0.1}{\overline{A}_R}\right)^2 \left(\frac{M_S}{100\,\mathrm{TeV}}\right)^2 \left(\frac{M_{HC}}{10^{16}\,\mathrm{GeV}}\right)^2, \tag{40}$$

where $M_{HC}$ is the colour-triplet Higgs mass, $\overline{A}_R$ is a hadronic parameter and $\beta$ is the usual angle determined by the ratio of the vacuum expectation values of the two Higgs doublets of the MSSM.

# Flavoured Channels

N. Nagata, S. Shirai 1312.7854*

# Enter flavour, enter more channels




*Thanks to N. Nagata for allowing me to use figures from his paper and presentations.

# Recall the MSSM content

| Names | Spin | $P_R$ | Gauge Eigenstates | Mass Eigenstates |
|---|---|---|---|---|
| Higgs bosons | 0 | +1 | $H_u^0\ H_d^0\ H_u^+\ H_d^-$ | $h^0\ H^0\ A^0\ H^\pm$ |
| squarks | 0 | −1 | $\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{d}_L\ \widetilde{d}_R$ | (same) |
| | | | $\widetilde{s}_L\ \widetilde{s}_R\ \widetilde{c}_L\ \widetilde{c}_R$ | (same) |
| | | | $\widetilde{t}_L\ \widetilde{t}_R\ \widetilde{b}_L\ \widetilde{b}_R$ | $\widetilde{t}_1\ \widetilde{t}_2\ \widetilde{b}_1\ \widetilde{b}_2$ |
| sleptons | 0 | −1 | $\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\nu}_e$ | (same) |
| | | | $\widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\nu}_\mu$ | (same) |
| | | | $\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\nu}_\tau$ | $\widetilde{\tau}_1\ \widetilde{\tau}_2\ \widetilde{\nu}_\tau$ |
| neutralinos | 1/2 | −1 | $\widetilde{B}^0\ \widetilde{W}^0\ \widetilde{H}_u^0\ \widetilde{H}_d^0$ | $\widetilde{N}_1\ \widetilde{N}_2\ \widetilde{N}_3\ \widetilde{N}_4$ |
| charginos | 1/2 | −1 | $\widetilde{W}^\pm\ \widetilde{H}_u^+\ \widetilde{H}_d^-$ | $\widetilde{C}_1^\pm\ \widetilde{C}_2^\pm$ |
| gluino | 1/2 | −1 | $\widetilde{g}$ | (same) |
| goldstino (gravitino) | 1/2 (3/2) | −1 | $\widetilde{G}$ | (same) |

$$m^2_{\tilde{f}} = m_0^2 \begin{pmatrix} 1+\Delta_1^{\tilde{f}} & \delta_{12}^{\tilde{f}} & \delta_{13}^{\tilde{f}} \\ \delta_{12}^{\tilde{f}*} & 1+\Delta_2^{\tilde{f}} & \delta_{23}^{\tilde{f}} \\ \delta_{13}^{\tilde{f}*} & \delta_{23}^{\tilde{f}*} & 1+\Delta_3^{\tilde{f}} \end{pmatrix}, \tag{41}$$

with $\tilde{f} = \widetilde{Q}_L, \widetilde{u}_R, \widetilde{d}_R, \widetilde{e}_R, \widetilde{L}_L$. In the minimal SU(5) GUT, there are relations among the sfermion mass matrices at the GUT scale:

$$m^2_{\tilde{Q}_L} = V_{QU} (m^2_{\tilde{u}_R})^t V_{QU}^\dagger = V_{QE} (m^2_{\tilde{e}_R})^t V_{QE}^\dagger \text{ and } m^2_{\tilde{d}_R} = V_{DL}^* (m^2_{\tilde{L}_L})^t V_{DL}^t, \tag{42}$$

where $V_{QU}$, $V_{QE}$ and $V_{DL}$ are the GUT "CKM" matrices.

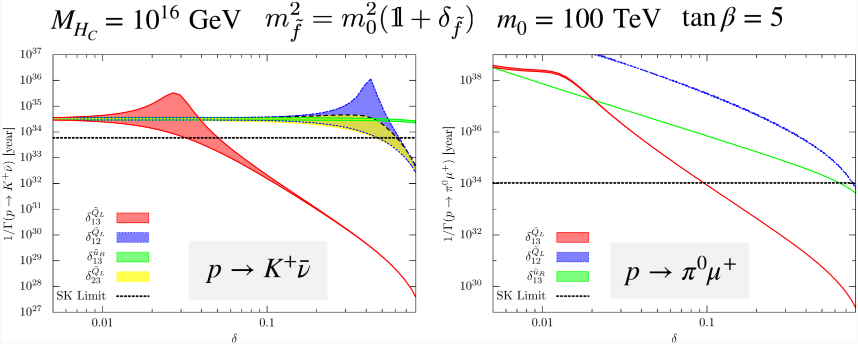
When the flavor violation is small but sizable, *e.g.*, $\delta_{13}^{\widetilde{Q}_L} \sim 0.1$, the contribution is evaluated as

$$\begin{aligned} C_{LL}(uds\nu_\mu) &\simeq -\frac{4}{3} \frac{\alpha_2\alpha_3}{\sin 2\beta} \frac{m_t m_s}{M_{H_C} m_W^2} \frac{M_{\widetilde{g}}}{m_0^2} \, e^{i\varphi_3} (V_{ud} V_{cs} V_{cs}^*) \big(\delta_{13}^{\widetilde{Q}_L *}\big)^2 \, , \\ C_{LL}(uds\nu_\tau) &\simeq -\frac{4}{3} \frac{\alpha_2\alpha_3}{\sin 2\beta} \frac{m_t m_b}{M_{H_C} m_W^2} \frac{M_{\widetilde{g}}}{m_0^2} \, e^{i\varphi_3} (V_{ud} V_{cs} V_{cb}^*) \big(\delta_{13}^{\widetilde{Q}_L *}\big)^2 \, . \end{aligned} \tag{43}$$

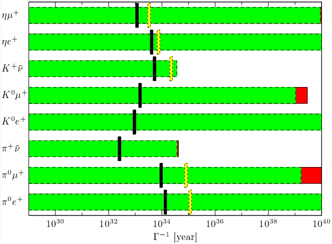
In particular gluino exchange becomes important when :

$$\left|\delta_{13}^{\widetilde{Q}_L}\right| \gtrsim 2 \times 10^{-3} \times \left(\frac{1}{\sin 2\beta}\left|\frac{\mu_H}{M_{\widetilde{g}}}\right|\right)^{\frac{1}{2}} . \tag{44}$$

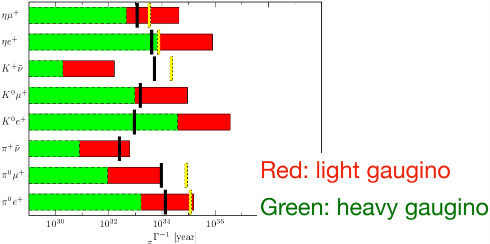
Then proton decay rates are enhanced



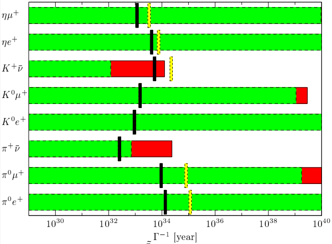
The scale of supersymmetry can be probed as the decay rates depend on this scale



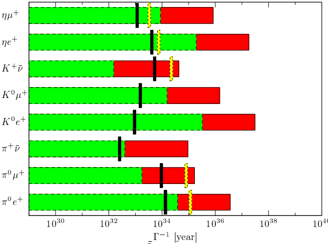

(a) Minimal FV




(b) $\delta_{13}^{\tilde{Q}_L} = 0.1$



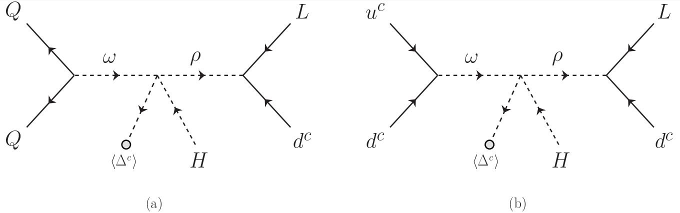

(c) $\delta_{23}^{\tilde{Q}_L} = 0.5$




(d) $\delta_{12}^{\tilde{Q}_L} = 0.5$

# B-L proton decay modes

K. S. Babu and R.N. Mohapatra 1203.5544




Effective baryon number violating $d = 7$ operators induced by the symmetric Yukawa couplings of $10_H$ and $\overline{126}_H$ of $SO(10)$. Here the SM quantum numbers of the various fields are $\omega(3, 1, -1/3)$, $\rho(3, 2, 1/6)$, and $H(1, 2, 1/2)$.

## Summary and Outlook

- Optimistic view: Supergravity and supersymmetry are very rich frameworks which are predictive and allow for the majority of computed observables to be in agreement with measured quantities
- Pessimistic view: PD, EDMs and flavor observables are a quicker way to exclude models!
- Upcoming proton decay experiments will be ruling some models and
- Make accessible flavoured channels
- Need to complete our machinery to studied in a unified way EDM, proton decay and flavour-violation

# Suggested problem

Consider the flavoured decay mode $p \to \bar{\nu} K^+$ mediated by the gluino as depicted in the figure of pg. 38. Take into account the two Wilson Coefficients of Eqs. (43) and set the Wilson Coefficients $C_{RL} = 0$. Then use the formula for the decay amplitude of Eq. (26). Can you then find a set of values that satisfy Eq. (44) and such that the flavour decaying mode of the diagram in Fig. of page 38 renders a smaller life time than the current limit (check the value in the table of pg. 4) ? For $M_{\overline{H}}$ use $10^{16}$ GeV, for the masses of the quarks at $M_Z$ you can use

$$\{m_t, m_b, m_c, m_s, m_d, m_u\} = \{171.4, 2.87, 0.64, 0.055, 0.0028, 0.0015\} \text{ GeV}. \quad (45)$$

For the CKM matrix values you can use the current values given by the Particle Data Group. For the gauge coupling values at $M_Z$ you can use $g_1^2 = 0.128$, $g_2^2 = 0.425$, $g_3^2 = 1.5$ .