Dim 6 Gluon Operators as Condensates
-Renormalization and Temperature dependence-

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HyungJoo Kim(金瀅周),
Su Houng Lee, Kenji Morita

Dept. of Physics
Yonsei university

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PART 1

Renormalization of dim 6 gluon operators

1. Introduction

- QCD vacuum and Condensates
- QCD sum rules
- Dimension 6 gluon operator
- Operator renormalization
- Methods for simple calculation

2. Feynman rules & Diagrams

3. Results

- Renormalization factor
- Scale invariant operators

Introduction

1. QCD vacuum and Condensate

According to QM, vacuum is not empty. It is filled with quantum fluctuations of all possible kinds of particles.

Especially, when interactions are strong(non-perturbative), fluctuation can condense into a non-vanishing vacuum exp. value of quark and gluon operators which is called as “condensate”.

ex) quark condensate $\langle :\bar{q}q: \rangle$ $\langle :\bar{q}q\bar{q}q: \rangle$ $\langle :\bar{q}\gamma^{\mu}q: \rangle \cdots$

gluon condensate $\left\langle :G^{a}_{\mu\nu}G^{a}_{\mu\nu}: \right\rangle$ $\left\langle :f^{abc}G^{a}_{\mu\alpha}G^{b}_{\alpha\beta}G^{c}_{\beta\mu}: \right\rangle \cdots$

These vacuum condensates can act as a medium and have important roles.

For example, quark masses receive additional mass from vacuum condensates and exceed the mass believed to be generated by the Higgs.

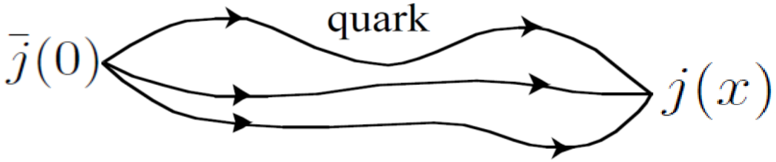
Introduction

2. QCD sum rule and Condensate

One approach to study non-perturbative physics is considering a correlation function(QCD sum rule) defined by

$$\begin{aligned}\Pi(q^2) &= i\int d^4 e^{iqx}\langle 0|T[j(x)\bar{j}(x)]|0\rangle = \Sigma_n C_n(q^2)\langle \hat{O}_n\rangle \\ &= C_0(q^2)\langle I\rangle + C_1(q^2)m_q\langle I\rangle + C_3(q^2)\langle \bar{q}q\rangle \\ &\quad + C_4^1(q^2)\langle G_{\mu\nu}G^{\mu\nu}\rangle + C_4^2(q^2)m_q\langle \bar{q}q\rangle + \ldots\end{aligned}$$

This represents an amplitude of particle(hadron) propagating from 0 to x. Current j(x) has the same quantum number with a particle under investigation.



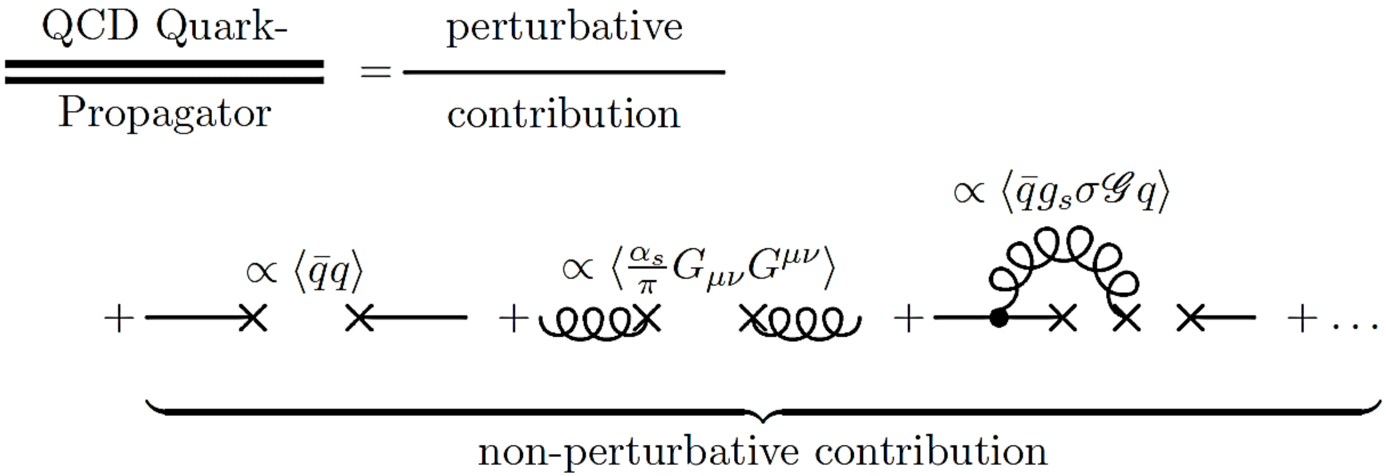
This is parameterized by vacuum condensates of quark and gluon fields.

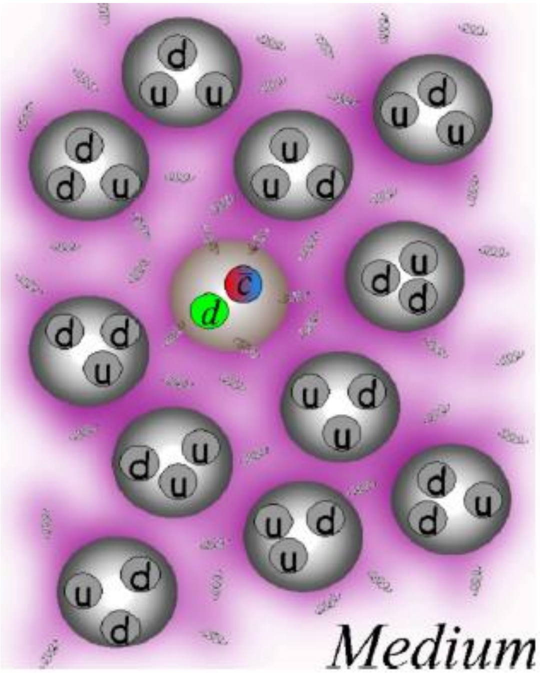
Introduction

3. Diagrams and Condensate

Condensates act as a medium, a particle can be annihilated by a virtual particle from the QCD vacuum and another real particle is created somewhere else.

-> Disconnected diagram





Introduction

4. Dimension6 Gluon operators

Dim 6 gauge invariant operators can be made of

"3 x $G_{\mu\nu}$" or "2 x D_μ and 2 x $G_{\mu\nu}$".

By index symmetry and Bianchi identity, 2 scalar* and 3 twist4 operators are independent.

$$\text{Scalar} \ : f^{abc} G^a_{\mu\nu} G^b_{\mu\alpha} G^c_{\nu\alpha},\ D_\mu G^a_{\mu\alpha} D_\nu G^a_{\nu\alpha}$$

$$\boxed{\text{Twist4} : \begin{pmatrix} O1 = D_\beta G^a_{\mu\nu} D_\alpha G^a_{\mu\nu} \\ O2 = D_\mu G^a_{\alpha\mu} D_\nu G^a_{\beta\nu} \\ O3 = D_\beta G^a_{\alpha\mu} D_\nu G^a_{\mu\nu} \end{pmatrix}}$$

These operators are **Scale dependent&Mixing** → **Renormalization** → **Non-mixing&Scaleinvariant operators**

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* S. Narison and R. Tarrach, Phys. Lett. B **125**, 217 (1983)

Introduction

5. Operator Renormalization

To get renormalization factor $O_{iR} = Z_{i,j} O^{0}_{jB}$
we considered the Green's ftns with 3 external fields.

$$\underset{\substack{\downarrow\\ \textit{renormalized}\\ \textit{operator}}}{\left\langle A^a_\mu A^b_\nu A^c_\lambda O_{iR} \right\rangle} = Z_{i,i} \underset{\substack{\downarrow\\ \textit{bare operator}}}{\left\langle A^a_\mu A^b_\nu A^c_\lambda O^0_{iB} \right\rangle} + \sum_{j \neq i} Z_{i,j} \underset{\substack{\downarrow\\ \textit{mixing}}}{\left\langle A^a_\mu A^b_\nu A^c_\lambda O^0_{jB} \right\rangle}$$

in the **SU(N) pure gauge theory + Dimensional regularization + Back ground field method****

L.F. Abbott, Nucl. Phys. **B185, 189 (1981)

Introduction

6. Back ground field method

For simple calculation, introduce new back ground field and impose it to retain gauge invariance of the effective action $\Gamma[\bar{A}] = -\mathrm{ilnZ[J]} - \mathrm{J}\cdot\bar{A}$.

$$Z[A] \rightarrow Z[A + \boxed{Q}]$$

$backgroundfield$

1PI diagrams satisfy the naive Ward identity. Even unphysical quantities like divergent counterterms take a gauge invariant form.

Calculations are simplified by gauge invariance.

Nucl. Pys. B185 (1981) 189

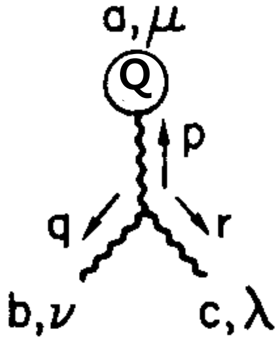
Introduction

6. Back ground field method

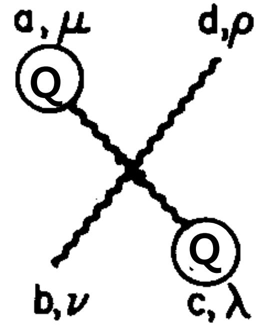
From S[A+Q], <AA>->ordinary propagator, <Q…>~interaction.

All external lines are background field Q.

All internal lines are ordinary gauge field A.



$$g f_{abc}\left[g_{\mu\lambda}\left(p - r - \frac{1}{\alpha} q\right)_{\nu} + g_{\nu\lambda}(r-q)_{\mu} + g_{\mu\nu}\left(q - p + \frac{1}{\alpha} r\right)_{\lambda}\right]$$

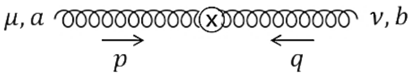


$$-ig^2\left[f_{abx} f_{xcd}\left(g_{\mu\lambda} g_{\nu\rho} - g_{\mu\rho} g_{\nu\lambda} + \frac{1}{\alpha} g_{\mu\nu} g_{\lambda\rho}\right) + f_{adx} f_{xbc}\left(g_{\mu\nu} g_{\lambda\rho} - g_{\mu\lambda} g_{\nu\rho} - \frac{1}{\alpha} g_{\mu\rho} g_{\nu\lambda}\right) + f_{acx} f_{xbd}\left(g_{\mu\nu} g_{\lambda\rho} - g_{\mu\rho} g_{\nu\lambda}\right)\right]$$

Feynman rules – operator insertion

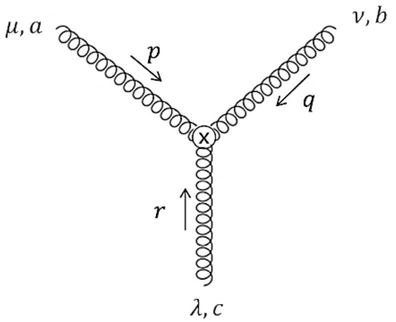
Operators are inserted with **zero momentum**.

2 gluon vertex :



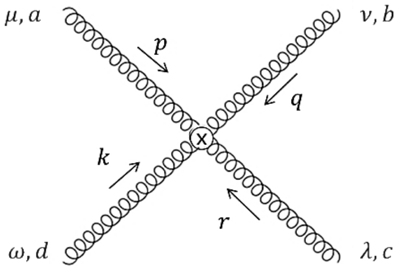
$$= \left\langle A^a_\mu(p) A^b_\nu(q) O_{\alpha\beta}(0) \right\rangle$$

3 gluon vertex :



$$= \left\langle A^a_\mu(p) A^b_\nu(q) A^c_\lambda(r) O_{\alpha\beta}(0) \right\rangle$$

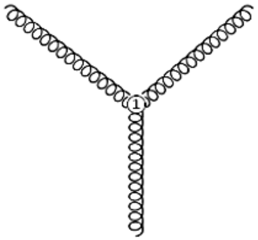
4 gluon vertex :



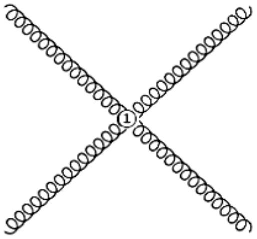
$$= \left\langle A^a_\mu(p) A^b_\nu(q) A^c_\lambda(r) A^d_\omega(k) O_{\alpha\beta}(0) \right\rangle$$

Feynman rules – for O1

$$4p^{\alpha}p^{\beta}\delta_{ab}(p^{2}g^{\mu\nu}-p^{\mu}p^{\nu})-\frac{1}{4}g^{\alpha\beta}(4p^{4}\delta_{ab}g^{\mu\nu}-4p^{2}p^{\mu}p^{\nu}\delta_{ab})$$



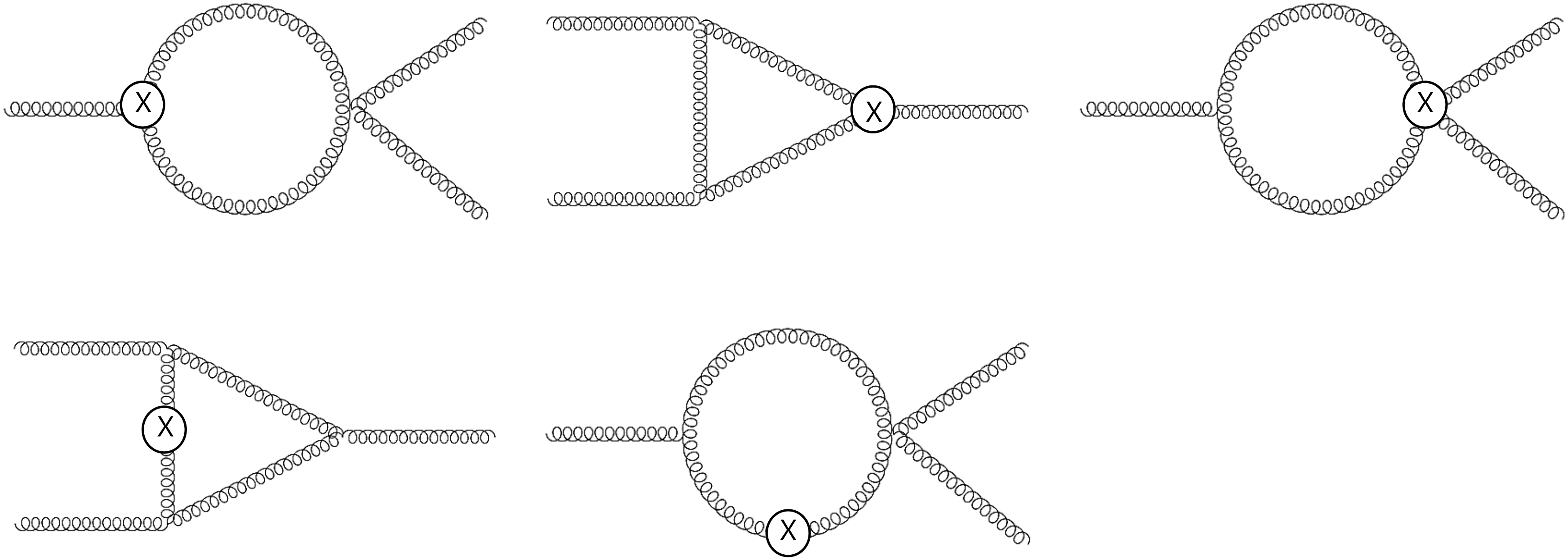
$$\begin{aligned}&-\frac{1}{2}igf_{abc}(2p^{\alpha}p^{\lambda}q^{\beta}g^{\mu\nu}-2p^{\alpha}p^{\nu}q^{\beta}g^{\lambda\mu}+2p^{\beta}q^{\alpha}(p^{\lambda}g^{\mu\nu}-p^{\nu}g^{\lambda\mu})-p^{\lambda}g^{\alpha\beta}g^{\mu\nu}p\cdot q+p^{\nu}g^{\alpha\beta}g^{\lambda\mu}p\cdot q+2p^{\alpha}p^{\lambda}r^{\beta}g^{\mu\nu}+4p^{\alpha}p^{\lambda}\\&r^{\mu}g^{\beta\nu}+4p^{\beta}p^{\lambda}r^{\mu}g^{\alpha\nu}-2p^{\alpha}p^{\nu}r^{\beta}g^{\lambda\mu}-2p^{\lambda}p^{\nu}r^{\mu}g^{\alpha\beta}+2p^{\beta}r^{\alpha}(p^{\lambda}g^{\mu\nu}-p^{\nu}g^{\lambda\mu})-p^{\lambda}g^{\alpha\beta}g^{\mu\nu}p\cdot r-4p^{\alpha}g^{\beta\nu}g^{\lambda\mu}p\cdot r\\&-4p^{\beta}g^{\alpha\nu}g^{\lambda\mu}p\cdot r+3p^{\nu}g^{\alpha\beta}g^{\lambda\mu}p\cdot r)+(\text{5 other terms from contraction order})\end{aligned}$$



$$\begin{aligned}&-\frac{1}{4}g^{2}f_{abx}f_{cdx}(4k^{\alpha}p^{\beta}g^{\lambda\mu}g^{\nu\omega}+4r^{\alpha}p^{\beta}g^{\lambda\mu}g^{\nu\omega}+4k^{\alpha}q^{\beta}g^{\lambda\mu}g^{\nu\omega}+4r^{\alpha}q^{\beta}g^{\lambda\mu}g^{\nu\omega}+4k^{\alpha}g^{\beta\mu}q^{\lambda}g^{\nu\omega}+4r^{\alpha}g^{\beta\mu}q^{\lambda}g^{\nu\omega}\\&+4g^{\alpha\lambda}p^{\beta}k^{\mu}g^{\nu\omega}4g^{\alpha\lambda}q^{\beta}k^{\mu}g^{\nu\omega}-g^{\alpha\beta}p^{\lambda}k^{\mu}g^{\nu\omega}-2g^{\alpha\beta}q^{\lambda}k^{\mu}g^{\nu\omega}-g^{\alpha\beta}q^{\lambda}r^{\mu}g^{\nu\omega}-g^{\alpha\beta}g^{\lambda\mu}k\cdot pg^{\nu\omega}+8g^{\alpha\lambda}g^{\beta\mu}\\&k\cdot qg^{\nu\omega}-3g^{\alpha\beta}g^{\lambda\mu}k\cdot qg^{\nu\omega}-g^{\alpha\beta}g^{\lambda\mu}p\cdot rg^{\nu\omega}-g^{\alpha\beta}g^{\lambda\mu}q\cdot rg^{\nu\omega}-4g^{\alpha\lambda}p^{\beta}g^{\mu\omega}k^{\nu}-4g^{\alpha\lambda}q^{\beta}g^{\mu\omega}k^{\nu}+g^{\alpha\beta}p^{\lambda}g^{\mu\omega}\\&k^{\nu}+g^{\alpha\beta}q^{\lambda}g^{\mu\omega}k^{\nu}-2g^{\alpha\beta}p^{\lambda}g^{\mu\omega}p^{\nu}+2g^{\alpha\beta}g^{\lambda\mu}p^{\nu}p^{\omega}+4p^{\alpha}g^{\beta\nu}(p^{\lambda}g^{\mu\omega}-g^{\lambda\mu}p^{\omega})+4g^{\alpha\nu}p^{\beta}(p^{\lambda}g^{\mu\omega}-g^{\lambda\mu}p^{\omega})\\&-4k^{\alpha}g^{\beta\mu}g^{\lambda\nu}q^{\omega}-4r^{\alpha}g^{\beta\mu}g^{\lambda\nu}q^{\omega}+g^{\alpha\beta}g^{\lambda\nu}k^{\mu}q^{\omega}+g^{\alpha\beta}g^{\lambda\nu}r^{\mu}q^{\omega}-8g^{\alpha\lambda}g^{\beta\mu}k^{\nu}q^{\omega}+2g^{\alpha\beta}g^{\lambda\mu}k^{\nu}q^{\omega})+(\text{23 other terms})\end{aligned}$$

For each vertex, there are **(# of external gluon fields)!** ways of contraction. O2 and O3 are similar.

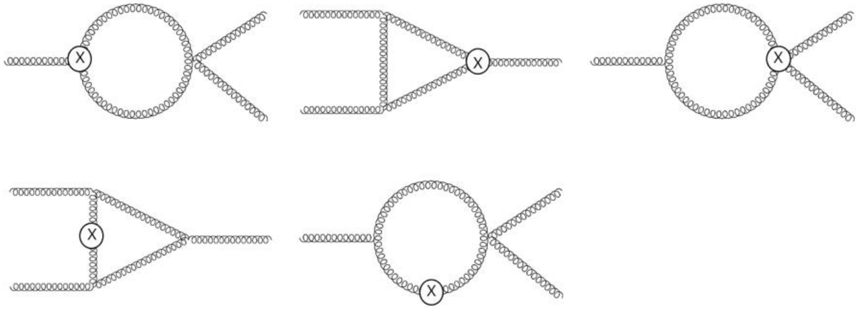
One loop Feynman Diagrams



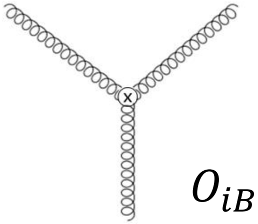
There are five diagrams contributing to the renormalization for each operators. ⓧ means the inserted operator with zero momentum.

One loop Calculation

Diagrammatically,

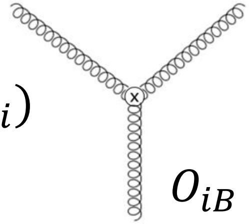


$= \; -\sum_i \delta_i$



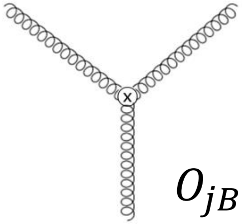
O_{iB}

$\langle A^a_\mu A^b_\nu A^c_\lambda O_{iR} \rangle = \; (1+\delta_i)$



O_{iB}

$+ \sum_{j \neq i} \delta_j$



O_{jB}

$$= Z_{i,i} \left\langle A^a_\mu A^b_\nu A^c_\lambda O^0_{iB} \right\rangle + \sum_{j \neq i} Z_{i,j} \left\langle A^a_\mu A^b_\nu A^c_\lambda O^0_{jB} \right\rangle$$

= Just one loop calculation

Result – renormalization factors

$O_{Ri} = Z_{i,j} O^0_{Bj}$ Renormalization factors

$$Z_{i,j} = \begin{pmatrix} 1 - \frac{3N\alpha_s}{4\pi\epsilon} & \frac{N\alpha_s}{12\pi\epsilon} & -\frac{2N\alpha_s}{3\pi\epsilon} \\ 0 & 1 - \frac{N\alpha_s}{3\pi\epsilon} & -\frac{N\alpha_s}{24\pi\epsilon} \\ 0 & -\frac{N\alpha_s}{6\pi\epsilon} & 1 - \frac{7N\alpha_s}{24\pi\epsilon} \end{pmatrix}$$

Thus,

$$\begin{pmatrix} O_{1R} \\ O_{2R} \\ O_{3R} \end{pmatrix} = \begin{pmatrix} 1 - \frac{3N\alpha_s}{4\pi\epsilon} & \frac{N\alpha_s}{12\pi\epsilon} & -\frac{2N\alpha_s}{3\pi\epsilon} \\ 0 & 1 - \frac{N\alpha_s}{3\pi\epsilon} & -\frac{N\alpha_s}{24\pi\epsilon} \\ 0 & -\frac{N\alpha_s}{6\pi\epsilon} & 1 - \frac{7N\alpha_s}{24\pi\epsilon} \end{pmatrix} \begin{pmatrix} O^0_{1B} \\ O^0_{2B} \\ O^0_{3B} \end{pmatrix}$$

To remove Mixing → Diagonalization

Result – Non mixing new operator set

Non-mixing operator sets and New Renormalization factors

$$\begin{pmatrix} O'_{1R} \\ O'_{2R} \\ O'_{3R} \end{pmatrix} = \begin{pmatrix} 1+\frac{3N\alpha_s}{4\pi\epsilon} & 0 & 0 \\ 0 & 1-\frac{(15-\sqrt{17})N\alpha_s}{48\pi\epsilon} & 0 \\ 0 & 0 & 1-\frac{(15+\sqrt{17})N\alpha_s}{48\pi\epsilon} \end{pmatrix} \begin{pmatrix} O'^0_{1B} \\ O'^0_{2B} \\ O'^0_{3B} \end{pmatrix}$$

$$O'^0_{1B} = O^0_{1B}$$

$$O'^0_{2B} = \frac{-653+21\sqrt{17}}{424} O^0_{1B} + \frac{1-\sqrt{17}}{8} O^0_{2B} + O^0_{3B}$$

$$O'^0_{3B} = \frac{-653-21\sqrt{17}}{424} O^0_{1B} + \frac{1+\sqrt{17}}{8} O^0_{2B} + O^0_{3B}$$

Result – scale invariant operators

Scale invariant operator

Because of $Z_{\alpha_s} = 1 - \frac{11N\alpha_s}{12\pi\epsilon}$, appropriate powers of α_s can cancel the renormalization factor of the operator.

$$\phi_1 = \alpha_s^{-\frac{9}{11}} \langle O_1' \rangle$$

$$\phi_2 = \alpha_s^{-\frac{15-\sqrt{17}}{44}} \langle O_2' \rangle$$

$$\phi_3 = \alpha_s^{-\frac{15+\sqrt{17}}{44}} \langle O_3' \rangle$$

Particularly, in the pure gauge theory, only O1 is relevant because O2 and O3 become zero by EoM(DG=j). Combining with scalar's result, we can obtain two(scalar*+twist4) relevant dim 6 scale invariant non mixing operators.

$$\phi_{s1} = {\alpha_s}^{-\frac{7}{11}} \langle \frac{1}{4} g f^{abc} G^a_{\mu\nu} G^b_{\mu\alpha} G^c_{\nu\alpha} \rangle$$

$$\phi_1 = {\alpha_s}^{-\frac{9}{11}} \langle -2g f^{abc} G^a_{\mu\alpha} G^b_{\alpha\beta} G^c_{\beta\nu} \rangle$$

Summary

- We calculated the renormalization of the dimension 6 twist 4 gluon operators to one loop order in the pure gauge theory.

- Then, we found scale invariant operators by multiplication appropriate powers of coupling constant.

PART 2

Temperature dependence of dim 6 gluon operators

1. Field representations of operators
2. Temperature dependences
3. Application to Sumrule for J/Ψ mass

Field representations of gluon operators

Similar to the QED case, field strength tensor for QCD can be represented color E and B fields,

$$G^a_{\mu\nu} = \begin{bmatrix} 0 & E^a_x & E^a_y & E^a_z \\ -E^a_x & 0 & B^a_z & -B^a_y \\ -E^a_y & -B^a_z & 0 & B^a_x \\ -E^a_z & B^a_y & -B^a_x & 0 \end{bmatrix}$$

Dimension 4 scalar and twist2 operators -> E^2 and B^2

Ex) $G^a_{\mu\nu} G^a_{\mu\nu} = 2(B^2 - E^2)$

$$G_2 = -\frac{2}{3}(E^2 + B^2),$$

Dimension 6 -> 'BBB' and 'BEE' in the pure gauge theory

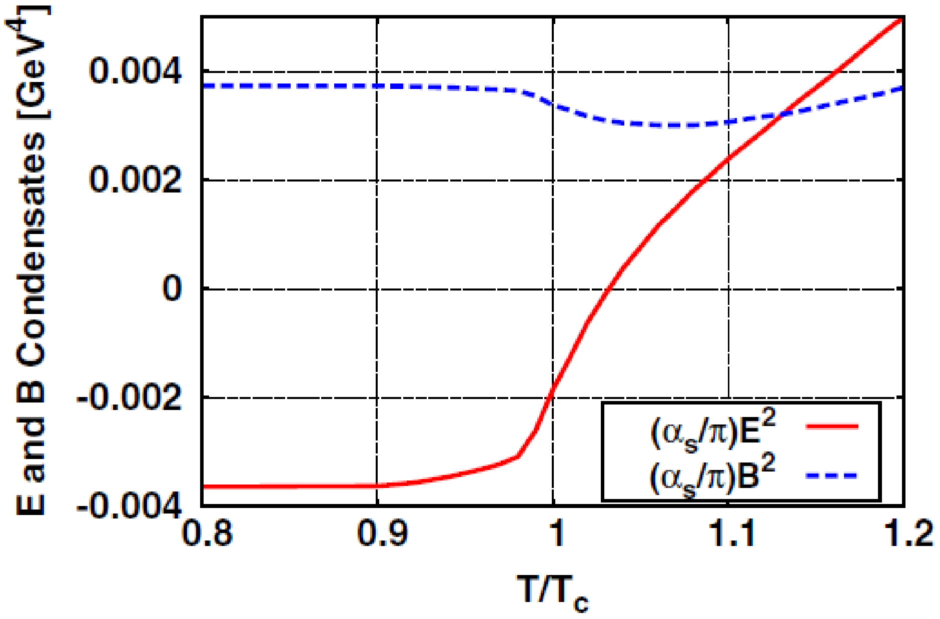
Ex) $f^{abc} G^3 = f^{abc} G^a{}_{\mu\alpha} G^{b\alpha\beta} G^c{}_{\beta\nu}$

$$Tr(f^{abc} G^3) = f^{abc} G^3_{00} - f^{abc} G^3_{ii} = -B^a \cdot (B^b \times B^c) + 3B^a \cdot (E^b \times E^c)$$

Temperature dependence

From Lattice QCD, we know the temperature dependence of dimension 4 gluon operators. That is, E^2 and B^2.

E^2 has a rapid phase transition near Tc.



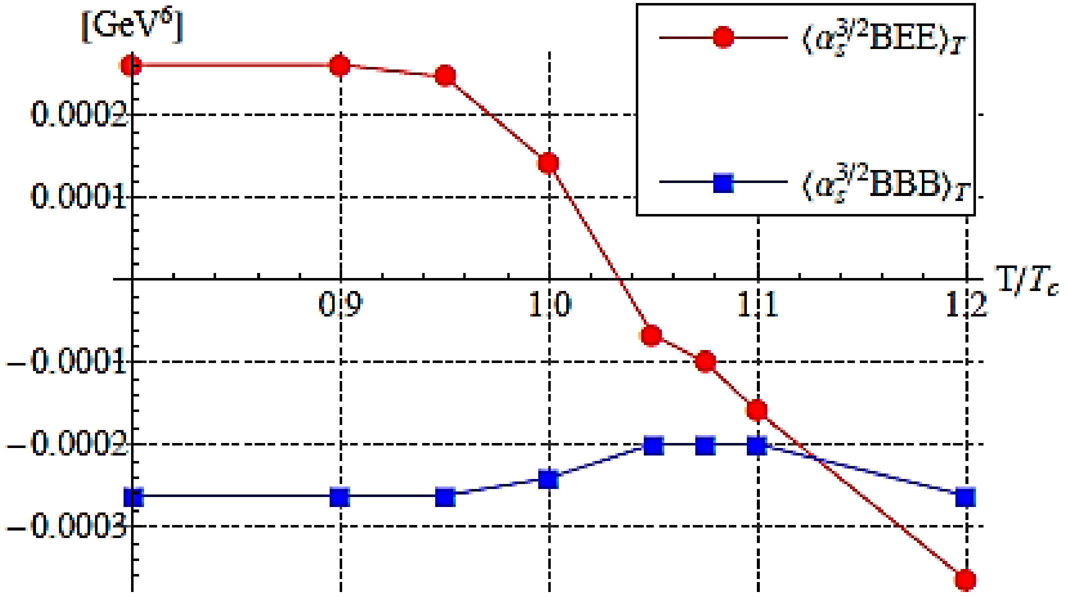
Unfortunately, we don’t know the exact temperature dependence of dimension 6 gluon operators in the SU(3).

Temperature dependence

Instead of Lattice QCD, we assumed that fields are isotropic and the angular correlations can be neglected. Then, we estimate the temperature dependences of BEE and BBB from the E^2 and B^2 as follows,

$$\langle\alpha_s^{3/2}BEE\rangle_T = \langle\alpha_s^{3/2}BEE\rangle_0 \frac{\langle\frac{\alpha_s}{\pi}B^2\rangle_T{}^{1/2}\langle\frac{\alpha_s}{\pi}E^2\rangle_T}{\langle\frac{\alpha_s}{\pi}B^2\rangle_0{}^{1/2}\langle\frac{\alpha_s}{\pi}E^2\rangle_0}$$

$$\langle\alpha_s^{3/2}BBB\rangle_T = \langle\alpha_s^{3/2}BBB\rangle_0 \frac{\langle\frac{\alpha_s}{\pi}B^2\rangle_T{}^{3/2}}{\langle\frac{\alpha_s}{\pi}B^2\rangle_0{}^{3/2}}.$$

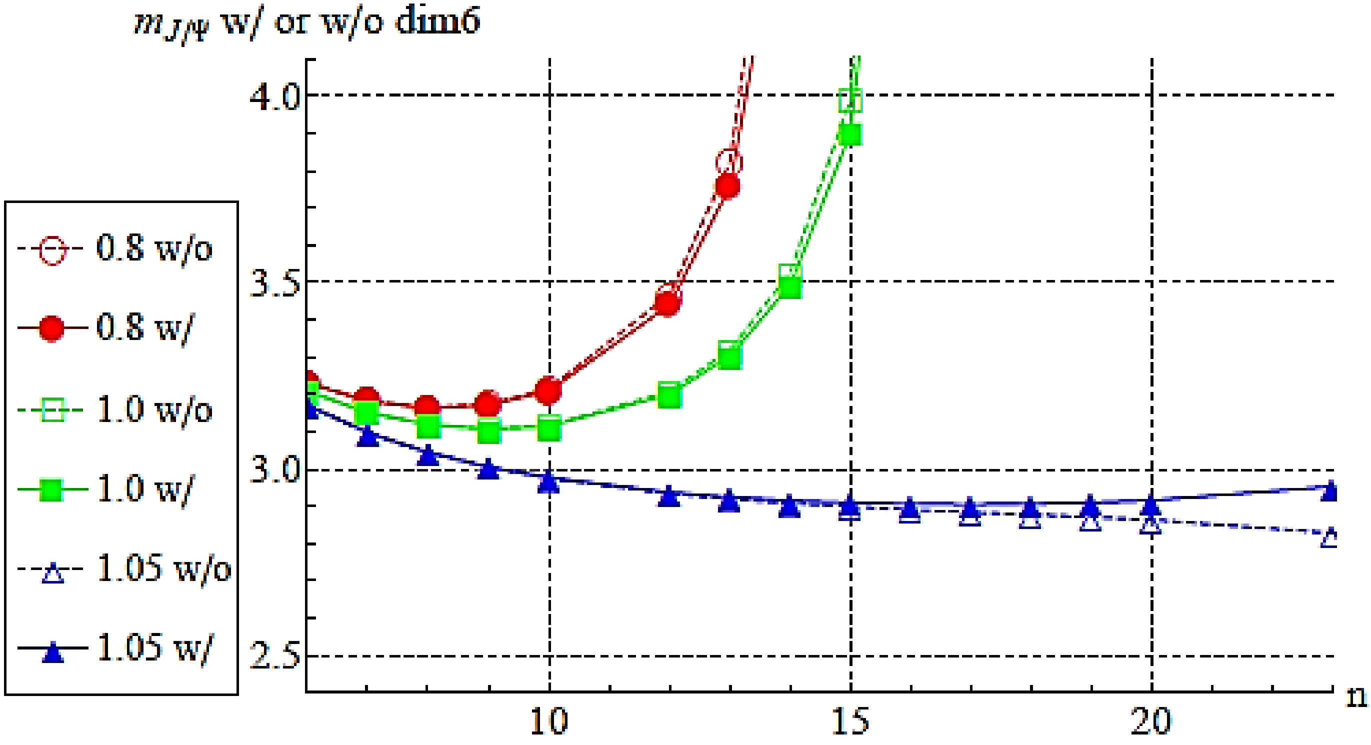


Application to SumRule for J/Ψ mass

We adopted our temperature dependence of dim 6 gluon operators to SumRule for $J/\Psi(= c\bar{c})$.

$$M_n(Q_0^2) = \frac{1}{n!}\left(-\frac{d}{dQ^2}\right)^n \Pi(Q^2)|_{Q^2=Q_0^2}$$

$$m_{J/\psi}^2 = \frac{M_{n-1}}{M_n} - 4m_c^2$$



Summary

- We have introduced temperature dependence of the dimension 6 gluon operators based on the temperature dependence of the dim 4 electric and magnetic condensates extracted from lattice QCD.

- Then, we improved the previous QCD sum rules for the J/Ψ mass near Tc based on dimension 4 operators, by including the temperature dependent dimension 6 operators.

- We find that the addition of dim 6 condensates extends the stability in the sum rule up to slightly higher temperature of 1.05Tc.

Thank you for listening