The 3-3-1 Models and Implication to Astroparticle Physics

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Outline

1. Motivation

Inflation

Inflation was suggested to solve the flatness and hoziron problems. The fundamental idea of inflation is that the Universe undergoes a period of acceleration expansion, defined as a period when $\ddot{a} > 0$, at early times. A plausible scenario for driving such an accelerated expansion is provided by scalar fields.

Matter-antimatter asymmetry

The baryon asymmetry of the Universe can be expressed as

$$Y_{\Delta B} \equiv \left. \frac{n_B - n_{\overline{B}}}{s} \right|_0 ,$$

where $n_B, n_{\overline{B}}$ are the number densities of baryons and antibaryons, s is the entropy density, $s = g_* (2\pi^2/45) T^3$ with g_* is the number of degrees of freedom in the plasma, and T is the temperature.
From Big-Bang Nucleosynthesis (BBN) and Wilkinson Microwave Anisotropy Probe (WMAP),

$$Y_{\Delta B}^{\mathrm{BBN}} = (8.10 \pm 0.85) \times 10^{-11}, \qquad Y_{\Delta B}^{\mathrm{CMB}} = (8.79 \pm 0.44) \times 10^{-11}.$$

All cosmological models agree that the Universe started with the same amount of baryon and anti-baryon
$\Rightarrow$ The baryon asymmetry must be generated dynamically.

Sakharov requires

- ▷ Baryon number violation
- ▷ C and CP violation
- ▷ Out of equilibrium dynamics

The candidate scenario is Leptogenesis. Singlet and heavy Majorana neutrinos N_i are introduced to provide mass to the light neutrinos via a seesaw mechanism. These heavy neutrinos can decay into lighter particles and create a lepton number asymmetry, which can be converted into a baryon asymmetry,

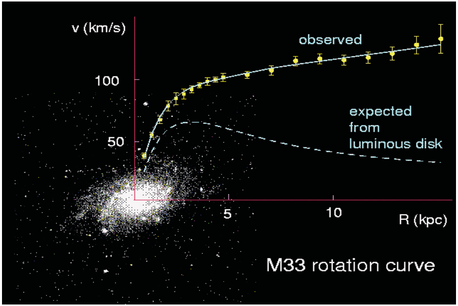
$$Y_{\Delta B} = \frac{C_{\rm sph}}{C_{\rm sph} - 1} Y_{\Delta L}.$$

Dark matter

Evidence for dark matter

$$\frac{v_{\rm rot}^2}{r} = \frac{GM(r)}{r^2} \Rightarrow v_{\rm rot} = \sqrt{\frac{GM(r)}{r}},$$

where r is the distance of the tracer star from the galactic center and $M(r)$ is the galactic mass enclosed within this distance.



DM should be non-baryonic and cold, electrically neutral, stable (the life time > the age of the Universe).

2. The $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1) Models

In general the fermion triplets in the 3-3-1 models are arranged as

$$\psi_{aL} = (\nu_{aL}, e_{aL}, F_{aL})^T \sim (1, 3, X_{\psi_a}),$$
$$Q_{\alpha L} = (d_{\alpha L}, -u_{\alpha L}, J_{\alpha L})^T \sim (3, 3^*, X_{Q_\alpha}),$$
$$Q_{3L} = (u_{3L}, d_{3L}, J_{3L})^T \sim (3, 3, X_{Q_3}),$$

where the index $a = 1, 2, 3$ and $\alpha = 1, 2$.
The electric charge operator is given by $Q = T_3 + \beta T_8 + XI$.
How to define X charges?
1-The electric charge is conserved requiring
$Q < \chi >= 0, \; Q < \eta >= 0, \; Q < \rho >= 0$
2-The Yukawa Lagrangian needed to generate mass to all quarks is invariant under the $U(1)_X$
3-The anomaly condition $\mathrm{Tr}[SU(3)_L]^2[U(1)_X] = 0$
4-The relations of X_{ψ_a} and the electric charge of leptonic particles are obtained by applying the electric charge operator on the lepton triplet.

The minimal 3-3-1 model: $F_{aL} = (e^c)_{aL}$, with $(e^c)_{aL} \equiv (e_{aR})^c$

$$q_{e_a} = -q_{e^c_a} \Rightarrow X_{\psi_a} = \frac{1}{4} + \frac{\beta}{4\sqrt{3}}.$$

All X charges of triplets and singlets can be expressed in a single parameter β. Using $q_{e_a} = -1$ then $\beta = -\sqrt{3}$. Therefore,

$$X_{\psi_a} = 0, \quad X_Q = -\frac{1}{3}, \quad X_{Q_3} = \frac{2}{3},$$

$$q_{u_a} = \frac{2}{3}, \quad q_{d_a} = -\frac{1}{3}, \quad q_{J_\alpha} = -\frac{4}{3}, \quad q_{J_3} = \frac{5}{3}.$$

The 3-3-1 model with neutral fermions: $F_{aL} = (N^c)_{aL}$, with $(N_{aR})^c \equiv (N^c)_{aL}$
We get $\beta = -1/\sqrt{3}$, and

$$X_{\psi_a} = -\frac{1}{3}, \quad X_Q = 0, \quad X_{Q_3} = \frac{1}{3},$$

$$q_{u_a} = \frac{2}{3}, \quad q_{d_a} = -\frac{1}{3}, \quad q_{J_\alpha} = -\frac{1}{3}, \quad q_{J_3} = \frac{2}{3}.$$

3. Investigation of Dark Matter in Minimal 3-3-1 Models

- There is no dark matter candidate in the original 3-3-1 model. One might introduce a Z_2 symmetry so that one scalar triplet of the theory is odd, while all other fields are even under the Z_2 symmetry. The odd particles act as inert fields. Therefore, the lightest and neutral inert particle is stable and can be a dark matter candidate.
- The minimal 3-3-1 model originally works with three scalar triplets $\rho = (\rho_1^+, \rho_2^0, \rho_3^{++})$, $\eta = (\eta_1^0, \eta_2^-, \eta_3^+)$, $\chi = (\chi_1^-, \chi_2^{--}, \chi_3^0)$.
- In order to enrich the inert scalar sector, one can consider the reduced 3-3-1 model by excluding η, or the simple 3-3-1 model by excluding ρ. The reduced 3-3-1 model gives large flavor-changing neutral currents as well as large ρ parameter.
- The simple 3-3-1 model with the replication of η or of χ, which are additional inert scalars, can provide realistic dark matter candidates.

The simple 3-3-1 model

The model works well with two scalar triplets as

$$\eta = \begin{pmatrix} \frac{1}{\sqrt{2}}(u+S_1+iA_1) \\ \eta_2^- \\ \eta_3^+ \end{pmatrix} \sim (1,3,0),$$

$$\chi = \begin{pmatrix} \chi_1^- \\ \chi_2^{--} \\ \frac{1}{\sqrt{2}}(\omega+S_3+iA_3) \end{pmatrix} \sim (1,3,-1).$$

The scalar potential is given by

$$\begin{aligned} V_{\mathrm{simple}} &= \mu_1^2\eta^\dagger\eta + \mu_2^2\chi^\dagger\chi + \lambda_1(\eta^\dagger\eta)^2 + \lambda_2(\chi^\dagger\chi)^2 \\ &\quad +\lambda_3(\eta^\dagger\eta)(\chi^\dagger\chi) + \lambda_4(\eta^\dagger\chi)(\chi^\dagger\eta), \end{aligned}$$

where $\mu_{1,2}$ have dimension of mass, while $\lambda_{1,2,3,4}$ are dimensionless.

The Higgs sector contains
+ eight Goldstone bosons $G_Z \equiv A_1$, $G_{Z'} \equiv A_3$, $G_W^\pm \equiv \eta_2^\pm$, $G_Y^{\pm\pm} \equiv \chi_2^{\pm\pm}$ and $G_X^\pm \equiv c_\theta \chi_1^\pm - s_\theta \eta_3^\pm \simeq \chi_1^\pm$ ($t_\theta = \frac{u}{\omega} \to 0$ since $u \ll \omega$)
+ four massive scalars

$$\begin{aligned} h \simeq S_1, &\quad m_h^2 \simeq \frac{4\lambda_1\lambda_2 - \lambda_3^2}{2\lambda_2} u^2, \\ H \simeq S_3, &\quad m_H^2 \simeq 2\lambda_2 \omega^2, \\ H^\pm \simeq \eta_3^\pm, &\quad m_{H^\pm}^2 \simeq \frac{\lambda_4}{2}\omega^2. \end{aligned}$$

The gauge boson masses arise from the Lagrangian

$$\sum_{\Phi=\eta,\chi} (D_\mu \langle \Phi \rangle)^\dagger (D^\mu \langle \Phi \rangle),$$

where the covariant derivative is defined as

$$D_\mu = \partial_\mu + i g_s t_i G_{i\mu} + i g T_i A_{i\mu} + i g_X X B_\mu.$$

The gauge bosons with their masses are respectively given as

$$W^{\pm} \equiv \frac{A_1 \mp iA_2}{\sqrt{2}}, \quad m_W^2 = \frac{g^2}{4}u^2,$$
$$X^{\mp} \equiv \frac{A_4 \mp iA_5}{\sqrt{2}}, \quad m_X^2 = \frac{g^2}{4}(\omega^2 + u^2),$$
$$Y^{\mp\mp} \equiv \frac{A_6 \mp iA_7}{\sqrt{2}}, \quad m_Y^2 = \frac{g^2}{4}\omega^2,$$

$$A = s_W A_3 + c_W \left(-\sqrt{3}t_W A_8 + \sqrt{1-3t_W^2}B\right), \quad m_A = 0,$$
$$Z_1 \simeq c_W A_3 - s_W \left(-\sqrt{3}t_W A_8 + \sqrt{1-3t_W^2}B\right), \quad m_{Z_1}^2 \simeq \frac{g^2}{4c_W^2}u^2,$$
$$Z_2 \simeq \sqrt{1-3t_W^2}A_8 + \sqrt{3}t_W B, \quad m_{Z_2}^2 \simeq \frac{g^2 c_W^2}{3(1-4s_W^2)}\omega^2,$$

where $s_W = e/g = t/\sqrt{1+4t^2}$, with $t = g_X/g$.

The Yukawa Lagrangian is given by

$$\begin{aligned}\mathcal{L}_Y &= h^J_{33}\bar{Q}_{3L}\chi J_{3R} + h^J_{\alpha\beta}\bar{Q}_{\alpha L}\chi^* J_{\beta R} \\ &+h^u_{3a}\bar{Q}_{3L}\eta u_{aR} + \frac{h^u_{\alpha a}}{\Lambda}\bar{Q}_{\alpha L}\eta\chi u_{aR} \\ &+h^d_{\alpha a}\bar{Q}_{\alpha L}\eta^* d_{aR} + \frac{h^d_{3a}}{\Lambda}\bar{Q}_{3L}\eta^*\chi^* d_{aR} \\ &+h^e_{ab}\bar{\psi}^c_{aL}\psi_{bL}\eta + \frac{h'^e_{ab}}{\Lambda^2}(\bar{\psi}^c_{aL}\eta\chi)(\psi_{bL}\chi^*) \\ &+\frac{s^\nu_{ab}}{\Lambda}(\bar{\psi}^c_{aL}\eta^*)(\psi_{bL}\eta^*) + \mathrm{H.c.},\end{aligned}$$

where the $\Lambda \sim \omega$.

The simple 3-3-1 model with η replication

(called η'-model for shortcut)

An extra scalar triplet that replicates η is defined as

$$\eta' = \begin{pmatrix} \frac{1}{\sqrt{2}}(H_1' + iA_1') \\ \eta_2'^- \\ \eta_3'^+ \end{pmatrix} \sim (1,3,0).$$

The η' and η have the same gauge quantum numbers but η' is assigned as an odd field under the Z_2, $\eta' \to -\eta'$, so $<\eta'>= 0$.

The scalar potential includes the V_{simple} and the terms contained η',

$$\begin{aligned} V_{\eta'} &= \mu_{\eta'}^2 \eta'^\dagger \eta' + x_1(\eta'^\dagger \eta')^2 + x_2(\eta^\dagger \eta)(\eta'^\dagger \eta') + x_3(\chi^\dagger \chi)(\eta'^\dagger \eta') \\ &\quad + x_4(\eta^\dagger \eta')(\eta'^\dagger \eta) + x_5(\chi^\dagger \eta')(\eta'^\dagger \chi) + \frac{1}{2}[x_6(\eta'^\dagger \eta)^2 + H.c.]. \end{aligned}$$

Here, $\mu_{\eta'}$ has mass dimension, while x_i $(i = 1, 2, 3, ..., 6)$ are dimensionless.

The states H'_1, A'_1, $\eta'^{\pm}_2 \equiv H'^{\pm}_2$ and $\eta'^{\pm}_3 \equiv H'^{\pm}_3$ by themselves are physically inert particles with the corresponding masses as follows:

$$
\begin{aligned}
m^2_{H'_1} &= M^2_{\eta'} + \frac{1}{2}(x_4 + x_6)u^2, \quad m^2_{A'_1} = M^2_{\eta'} + \frac{1}{2}(x_4 - x_6)u^2, \\
m^2_{H'^{\pm}_2} &= M^2_{\eta'}, \quad m^2_{H'^{\pm}_3} = M^2_{\eta'} + \frac{1}{2}x_5\omega^2,
\end{aligned}
$$

where $M^2_{\eta'} \equiv \mu^2_{\eta'} + \frac{1}{2}x_2 u^2 + \frac{1}{2}x_3 w^2$. If H'_1 (or A'_1) is the lightest inert particle (LIP), it can be the dark matter candidate.

Due to the Z_2 symmetry, the inert scalars interact only with normal scalars and gauge bosons, not with fermions.
The interactions of the inert scalars with gauge bosons are given in

$$
\begin{aligned}
\mathcal{L}^{\mathrm{triple}}_{\mathrm{gauge}-\eta'} &= -ig[\eta'^{\dagger}(T_i A_{i\mu})\partial^{\mu}\eta'] + \mathrm{H.c.}, \\
\mathcal{L}^{\mathrm{quartic}}_{\mathrm{gauge}-\eta'} &= g^2[\eta'^{\dagger}(T_i A_{i\mu})^2\eta'].
\end{aligned}
$$

The simple 3-3-1 model with χ replication

(called χ'-model for shortcut)

The χ replication takes the form

$$\chi' = \begin{pmatrix} \chi_1'^{-} \\ \chi_2'^{--} \\ \frac{1}{\sqrt{2}}(H_3' + iA_3') \end{pmatrix} \sim (1,3,-1).$$

The χ' is assigned odd under the Z_2 symmetry that requires $<\chi'>=0$. The additional potential due to the χ' field is given as

$$\begin{aligned} V_{\chi'} &= \mu_{\chi'}^2 \chi'^\dagger \chi' + y_1(\chi'^\dagger\chi')^2 + y_2(\eta^\dagger\eta)(\chi'^\dagger\chi') + y_3(\chi^\dagger\chi)(\chi'^\dagger\chi') \\ &\quad + y_4(\eta^\dagger\chi')(\chi'^\dagger\eta) + y_5(\chi^\dagger\chi')(\chi'^\dagger\chi) + \frac{1}{2}[y_6(\chi'^\dagger\chi)^2 + \mathrm{H.c.}]. \end{aligned}$$

$$\begin{aligned} m_{H_3'}^2 &= M_{\chi'}^2 + \frac{1}{2}(y_5+y_6)\omega^2, \quad m_{A_3'}^2 = M_{\chi'}^2 + \frac{1}{2}(y_5-y_6)\omega^2, \\ m_{H_2'^{\pm\pm}}^2 &= M_{\chi'}^2, \quad m_{H_1'^{\pm}}^2 = M_{\chi'}^2 + \frac{1}{2}y_4 u^2, \end{aligned}$$

where $M_{\chi'}^2 \equiv \mu_{\chi'}^2 + \frac{1}{2}y_2u^2 + \frac{1}{2}y_3\omega^2$. If H_3' (or A_3') is the LIP, it can be the dark matter candidate.

Dark matter

The coupling λ_1 is constrained by the mass of the SM Higgs, $m_h = 125$ GeV. Fix $\lambda_2 = \lambda_3 = \lambda_4 = 0.1$.

In η'-model:

The inert particles are $H_1', A_1', H_2'^{\pm}, H_3'^{\pm}$. With the condition $x_6 < \mathrm{Min}\{0,\ -x_4,\ (w/u)^2 x_5 - x_4\}$, H_1' is the LIP $\Rightarrow$ H_1' is the DM candidate.

Fix

$$x_1 = 0.01,\ x_2 = 0.03,\ x_3 = 0.01,\ x_4 = 0.07,\ x_5 = 0.08,\ x_6 = -0.09.$$

$m_{H_1'}$ depends on $\mu_{\eta'}$ and ω.

In χ'-model:

The inert particles are $H_1'^{\pm}, H_2'^{\pm\pm}$, H_3', A_3'. If we assume that $y_6 < \mathrm{Min}\{0,\ -y_5,\ (u/w)^2 y_4 - y_5\}$, H_3' is the LIP $\Rightarrow$ H_3' is the DM candidate.

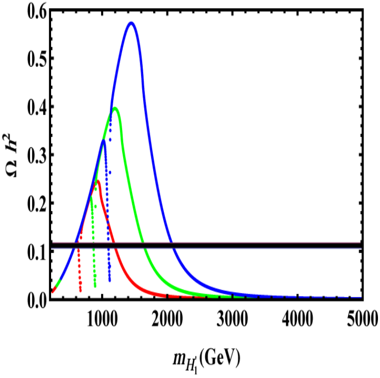
Fix

$$y_1 = 0.01,\ y_2 = 0.04,\ y_3 = 0.058,\ y_4 = 0.01,\ y_5 = 0.05,\ y_6 = -0.06.$$

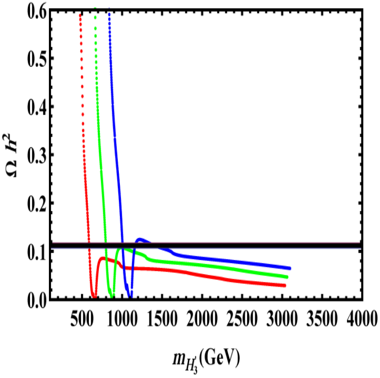
$m_{H_3'}$ depends on $\mu_{\chi'}$ and ω.

We figure out the relic density as a function of DM mass for $\omega = 3$ TeV (red), $\omega = 4$ TeV (green), and $\omega = 5$ TeV (blue). (The horizontal line is the WMAP limit on the relic density.)

η'-model



χ'-model



4. Inflation and Leptogenesis in the 3-3-1-1 Model

Motivation of the 3-3-1-1 model

- In the 3-3-1 model the lepton number of three components in a triplet are different, so the lepton number operator does not commute with the generators of the unitary group $SU(3)_L$. So, one constructed lepton number operator as the combination of T_3, T_8, and charged $\mathcal{L}$ with the relation $L = \alpha' T_3 + \beta' T_8 + \mathcal{L}I$. L is considered as a global symmetry.
- Since T_3, T_8 are gauged charges of the $SU(3)_L$ symmetry, $L, \mathcal{L}$ should be gauged or local generators.
  $\Rightarrow$ We extend the gauge group $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ to $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N$ (3-3-1-1), where $N = \mathcal{B} - \mathcal{L}$, $B = \mathcal{B}I$ so that the anomalies associated with $U(1)_N$ and with the usual $3-3-1$ symmetry obviously vanish.

- ▷ The Higgs scalar breaks the $U(1)_N$ symmetry can play a role of inflaton.
- ▷ The right-handed neutrinos not only solve the small masses of the observed neutrinos through a type I seesaw mechanism but also can be a source for the CP asymmetry.
- ▷ We apply the extension to the version with neutral fermions because there are some odd particles under the parity $P = (-1)^{3(B-L)+2s}$.

Review particles in the 3-3-1-1 model

Particle content

The fermion content of the 3-3-1-1 model which is anomaly free is given as

$$
\begin{aligned}
\psi_{aL} &= \begin{pmatrix} \nu_{aL} \\ e_{aL} \\ (N_{aR})^c \end{pmatrix} \sim (1, 3, -1/3, -2/3), \\
\nu_{aR} &\sim (1, 1, 0, -1), \quad e_{aR} \sim (1, 1, -1, -1), \\
Q_{\alpha L} &= \begin{pmatrix} d_{\alpha L} \\ -u_{\alpha L} \\ D_{\alpha L} \end{pmatrix} \sim (3, 3^*, 0, 0), \quad Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ U_L \end{pmatrix} \sim (3, 3, 1/3, 2/3), \\
u_{aR} &\sim (3, 1, 2/3, 1/3), \quad d_{aR} \sim (3, 1, -1/3, 1/3), \\
U_R &\sim (3, 1, 2/3, 4/3), \quad D_{\alpha R} \sim (3, 1, -1/3, -2/3),
\end{aligned}
$$

where the quantum numbers located in the parentheses are defined upon the gauge symmetries $(SU(3)_C,\ SU(3)_L,\ U(1)_X,\ U(1)_N)$, respectively.
The family indices are $a = 1, 2, 3$ and $\alpha = 1, 2$.
The N_{aR} are the neutral leptons and U, D_α are the exotic quarks.

To break the gauge symmetry and generate the masses in a correct way, the 3-3-1-1 model needs the following scalar multiplets with their VEVs conserving Q and P:

$$\begin{aligned}
\rho &= (\rho_1^+, \rho_2^0, \rho_3^+)^T \sim (1,3,2/3,1/3), \quad \langle\rho\rangle = \frac{1}{\sqrt{2}}(0,v,0)^T, \\
\eta &= (\eta_1^0, \eta_2^-, \eta_3^0)^T \sim (1,3,-1/3,1/3), \quad \langle\eta\rangle = \frac{1}{\sqrt{2}}(u,0,0)^T, \\
\chi &= (\chi_1^0, \chi_2^-, \chi_3^0)^T \sim (1,3,-1/3,-2/3), \quad \langle\chi\rangle = \frac{1}{\sqrt{2}}(0,0,\omega)^T, \\
\phi &\sim (1,1,0,2), \quad \langle\phi\rangle = \frac{1}{\sqrt{2}}\Lambda.
\end{aligned}$$

The gauge group $SU(3)_L \otimes U(1)_X \otimes U(1)_N$ is broken:

$$SU(3)_L \otimes U(1)_X \otimes U(1)_N \to U(1)_Q \otimes U(1)_{B-L}.$$

The Yukawa interactions and scalar potential are obtained as

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Yukawa}} &= h^{e}_{ab}\bar{\psi}_{aL}\rho e_{bR} + h^{\nu}_{ab}\bar{\psi}_{aL}\eta\nu_{bR} + h'^{\nu}_{ab}\bar{\nu}^{c}_{aR}\nu_{bR}\phi + h^{U}\bar{Q}_{3L}\chi U_{R} \\
&\quad + h^{D}_{\alpha\beta}\bar{Q}_{\alpha L}\chi^{*}D_{\beta R} + h^{u}_{a}\bar{Q}_{3L}\eta u_{aR} + h^{d}_{a}\bar{Q}_{3L}\rho d_{aR} \\
&\quad + h^{d}_{\alpha a}\bar{Q}_{\alpha L}\eta^{*}d_{aR} + h^{u}_{\alpha a}\bar{Q}_{\alpha L}\rho^{*}u_{aR} + H.c,
\end{aligned}
$$

$$
\begin{aligned}
V(\rho,\eta,\chi,\phi) &= \mu_1^2\rho^\dagger\rho + \mu_2^2\chi^\dagger\chi + \mu_3^2\eta^\dagger\eta + \lambda_1(\rho^\dagger\rho)^2 + \lambda_2(\chi^\dagger\chi)^2 \\
&\quad + \lambda_3(\eta^\dagger\eta)^2 + \lambda_4(\rho^\dagger\rho)(\chi^\dagger\chi) + \lambda_5(\rho^\dagger\rho)(\eta^\dagger\eta) \\
&\quad + \lambda_6(\chi^\dagger\chi)(\eta^\dagger\eta) + \lambda_7(\rho^\dagger\chi)(\chi^\dagger\rho) + \lambda_8(\rho^\dagger\eta)(\eta^\dagger\rho) \\
&\quad + \lambda_9(\chi^\dagger\eta)(\eta^\dagger\chi) + (f\epsilon^{mnp}\eta_m\rho_n\chi_p + H.c.) + \mu^2\phi^\dagger\phi \\
&\quad + \lambda(\phi^\dagger\phi)^2 + \lambda_{10}(\phi^\dagger\phi)(\rho^\dagger\rho) + \lambda_{11}(\phi^\dagger\phi)(\chi^\dagger\chi) + \lambda_{12}(\phi^\dagger\phi)(\eta^\dagger\eta).
\end{aligned}
$$

Scalar sector

We expand the neutral scalars around their VEVs such as

$$\rho = \begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}(v + S_2 + iA_2) \\ \rho_3^+ \end{pmatrix}; \quad \eta = \begin{pmatrix} \frac{1}{\sqrt{2}}(u + S_1 + iA_1) \\ \eta_2^- \\ \frac{1}{\sqrt{2}}(S_3' + iA_3') \end{pmatrix};$$

$$\chi = \begin{pmatrix} \frac{1}{\sqrt{2}}(S_1' + iA_1') \\ \chi_2^- \\ \frac{1}{\sqrt{2}}(\omega + S_3 + iA_3) \end{pmatrix}; \quad \phi \sim \frac{1}{\sqrt{2}}(\Lambda + S_4 + iA_4).$$

We assume that f, ω are the same order and $\Lambda \gg \omega \gg u, v$. The physical fields with respective masses can be written as:

For charged scalars,

$$H_4^- = \frac{v\chi_2^- + \omega\rho_3^-}{\sqrt{v^2+\omega^2}}, \qquad H_5^- = \frac{v\eta_2^- + u\rho_1^-}{\sqrt{u^2+v^2}},$$

$$G_Y^- = \frac{\omega\chi_2^- - v\rho_3^-}{\sqrt{v^2+\omega^2}}, G_W^- = \frac{u\eta_2^- - v\rho_1^-}{\sqrt{u^2+v^2}}.$$

The pseudoscalar A_4 is massless.

$$A = \frac{u^{-1}A_1 + v^{-1}A_2 + \omega^{-1}A_3}{\sqrt{u^{-2} + v^{-2} + \omega^{-2}}}.$$

$$G_Z = \frac{-uA_1 + vA_2}{\sqrt{u^2 + v^2}}, \qquad G_{Z'} = \frac{-\omega^{-1}(u^{-1}A_1 + v^{-1}A_2) + (u^{-2} + v^{-2})A_3}{\sqrt{(u^{-2} + v^{-2} + \omega^{-2})(u^{-2} + v^{-2})}},$$

$$G_X = \frac{\omega\chi_1 - u\eta_3^*}{\sqrt{u^2 + \omega^2}}, \qquad H' = \frac{u\chi_1^* + \omega\eta_3}{\sqrt{u^2 + \omega^2}}.$$

For neutral scalars,

$$H = \frac{uS_1 + vS_2}{\sqrt{u^2 + v^2}}, \qquad H_1 = \frac{-vS_1 + uS_2}{\sqrt{u^2 + v^2}}, \qquad H_2 = S_3, \qquad H_3 \simeq S_4.$$

H is identified as the SM Higgs boson.

Gauge sector

$$W^{\pm}_\mu = \frac{A_{1\mu} \mp iA_{2\mu}}{\sqrt{2}},\ Y^{\mp}_\mu = \frac{A_{6\mu} \mp iA_{7\mu}}{\sqrt{2}},\ X^0_\mu = \frac{A_{4\mu} - iA_{5\mu}}{\sqrt{2}},$$

$$M^2_W = \frac{1}{4}g^2(u^2+v^2),\ M^2_Y = \frac{1}{4}g^2(v^2+\omega^2),\ M^2_X = \frac{1}{4}g^2(u^2+\omega^2).$$

$$M^2_\gamma = 0(\mathrm{exact!}),\ \mathrm{A}_\mu = \frac{\sqrt{3}}{\sqrt{3+4\mathrm{t}_1^2}}\left(\mathrm{t}_1\mathrm{A}_{3\mu} - \frac{\mathrm{t}_1}{\sqrt{3}}\mathrm{A}_{8\mu} + \mathrm{B}_\mu\right).$$

$$\begin{aligned}
Z^N_\mu &\simeq C_\mu, \quad m^2_{Z^N} \simeq 4g^2t_2^2\Lambda^2,\\
Z^1_\mu &\simeq \frac{\sqrt{3+t_1^2}}{\sqrt{3+4t_1^2}}A_{3\mu} + \frac{t_1(\sqrt{3}t_1A_{8\mu} - 3B_\mu)}{\sqrt{3+t_1^2}\sqrt{3+4t_1^2}}, \quad m^2_{Z^1} \simeq \frac{g^2(u^2+v^2)}{4c^2_W},\\
Z^2_\mu &\simeq \frac{\sqrt{3}}{\sqrt{3+t_1^2}}A_{8\mu} + \frac{t_1}{\sqrt{3+t_1^2}}B_\mu, \quad m^2_{Z^2} \simeq \frac{g^2c^2_W\omega^2}{(3-4s^2_W)}.
\end{aligned}$$

Note that we have set $t_1 \equiv g_X/g$, $t_2 \equiv g_N/g$.

Fermion sector

From the $\mathcal{L}_{\text{Yukawa}}$, we obtain the Dirac masses for all quarks and leptons. The right-handed neutrinos get Majorana masses in the form $-\frac{1}{2}\bar{\nu}^c_R m^M_\nu \nu_R + \text{H.c.}$, where

$$[m^M_\nu]_{ab} = -\sqrt{2} h'^\nu_{ab} \Lambda.$$

The observed neutrinos ($\sim \nu_L$) naturally get small masses via a type I seesaw mechanism,

$$m^{\text{eff}}_\nu = -m^D_\nu (m^M_\nu)^{-1} (m^D_\nu)^T \sim \frac{(h^\nu)^2}{h'^\nu} \frac{u^2}{\Lambda}.$$

The masses of the neutral fermions N_R can be generated via an effective operator invariant under the 3-3-1-1 symmetry

$$\frac{\lambda_{ab}}{M} \bar{\psi}^c_{aL} \psi_{bL} (\chi\chi)^* + \text{H.c},$$

$$[m_{N_R}]_{ab} = -\lambda_{ab} \frac{\omega^2}{M}.$$

Assume that $M \sim \omega$ then $m_{N_R} \sim \omega$.

In brief,

- ▷ After spontaneous symmetry breaking, there are
  - 9 goldstone bosons $A_4, G_Z, G_{Z'}, G_X, G_X^*, G_Y^\pm, G_W^\pm$,
  - 9 massive gauge bosons $Z^N, Z^1, Z^2, X^0, X^{0*}, Y^\pm, W^\pm$, and one massless γ,
  - 4 neutral Higgs bosons H, H_1, H_2, H_3, one massive pseudoscalar A, complex Higgs H', H'^*, 4 charged scalars $H_4^\pm, H_5^\pm$
- ▷ The mass of H_3, Z^N, ν_R is proportional to Λ.
  The mass of other new massive particles, $A, H_1, H_2, H_4^\pm, H_5^\pm, H', H'^*$, $Z_\mu^2, X_\mu^0, X_\mu^{0*}, Y_\mu^\pm$, U, D_α, N_R, is proportional to ω.
- ▷ In this model, $L(G_X, H'^*, H_4^-, G_Y^-, X^0, Y^-) = 1$ while the remaining Higgs and gauge bosons have zero lepton number.
- ▷ The Majorana masses of the right-handed neutrinos violate L with ± 2 units $\rightarrow$ The decay of Majorana right-handed neutrinos can generate the lepton asymmetry.

Generation of inflation in the 3-3-1-1 model

The scalar singlet ϕ is completely breaking $U(1)_N$. We expect that the VEV of ϕ is very high and consider the singlet scalar ϕ plays the role of inflaton field.

We identify the inflaton with the real part of the $B-L$ Higgs field, $\Phi = \sqrt{2}\mathcal{R}[\phi]$. In the leading- log approximation, we obtain

$$V(\Phi) = V_{\rm tree} + V_{\rm eff} \simeq \frac{\lambda}{4}(\Phi^4 + a'\Phi^4 \ln \frac{\Phi}{\Delta}),$$

where

$$a' = \frac{a + 72\lambda^2}{16\pi^2\lambda}, \qquad a = f(h'^{\nu}_{ii}, g_N, \lambda_{10,11,12}).$$

We can express the number of e-folds N, the spectral index n_s, the tensor to scalar ratio r (a canonical measure of gravity wave from inflation) and the running index α in terms of a', Δ, Φ. Experiments require $n_s \in (0.94, 0.98),\ r \in (0.001, 0.15),\ \alpha \in (-0.0314, 0.0046)$.

We fix $N = 60$.

$$V'(\Phi) = 0 \Rightarrow < \Phi > \simeq 23.6 m_{\rm P},$$

$$m_\Phi = \sqrt{V''(\Phi)}\Big|_{\Phi=<\Phi>} \simeq 2.67 \times 10^{13}{\rm GeV}.$$

CP asymmetry

The Majorana neutrinos are defined as

$$\begin{aligned} \nu_{iM} &= \nu_{iR} + \nu_{iR}^c, \\ \nu_{iE} &= \nu_{iL} + \nu_{iL}^c, \\ N_i &= N_{iR} + N_{iR}^c. \end{aligned}$$

$\bar{e}_i \nu_{kM} H_5^-, \bar{N}_i H' \nu_{kM}$ interactions violate the lepton number
$\Longrightarrow \nu_{kM}$ can generate lepton asymmetry.

$$\varepsilon^{i(1)}_{\nu_{kM}} = \frac{\Gamma(\nu_{kM} \to e_i + H_5^+) - \Gamma(\nu_{kM} \to \overline{e}_i + H_5^-)}{2\Gamma_{\nu_{kM}}}$$
$$\simeq \frac{1}{8\pi C_0}\Big[\text{gauge transportation}\Big] s_\beta^2 \sum_l \mathrm{Im}[h^{\nu *}_{ik} h^{\nu}_{lk}]$$
$$+\frac{s_\beta^4}{8\pi C_0} \sum_j \sqrt{g_j}\left[1 - (1+g_j)log[1+1/g_j] + (1-g_j)^{-1}\right] \mathrm{Im}[(h^{\nu\dagger} h^{\nu})_{kj} h^{\nu *}_{ik} h^{\nu}_{ij}],$$

where $\Gamma_{\nu_{kM}}$ is the total decay rate of ν_{kM} at tree level,

$$g_j = \frac{m^2_{\nu_{jM}}}{m^2_{\nu_{kM}}}, \qquad C_0 = (2+s_\beta^2)\sum_i |h^\nu_{ik}|^2 = (2+s_\beta^2)(h^{\nu\dagger}h^\nu)_{kk}, \qquad t_\beta = v/u.$$

$$\varepsilon^{i(2)}_{\nu_{kM}} = \frac{\Gamma(\nu_{kM} \to N_i + H'^*) - \Gamma(\nu_{kM} \to N_i + H')}{2\Gamma_{\nu_{kM}}}$$
$$\simeq \frac{1}{8\pi C_0}\Big[\text{gauge transportation}\Big] \sum_l \mathrm{Im}[h^{\nu *}_{ik} h^{\nu}_{lk}]$$
$$+\frac{1}{8\pi C_0} \sum_j \sqrt{g_j}\left[1 - (1+g_j)log[1+1/g_j] + s_\beta^2(1-g_j)^{-1}\right] \mathrm{Im}[(h^{\nu\dagger} h^{\nu})_{kj} h^{\nu *}_{ik} h^{\nu}_{ij}],$$

Yukawa coupling matrix

h^ν matrix is required to be complex.

If we ignore

-the mixing between the charged lepton

-the mixing between heavy Majorana neutrinos,

the most general h^ν matrix is given by

$$h^\nu = \frac{\sqrt{2}}{u} Diag(\sqrt{m_{\nu_{1M}}}, \sqrt{m_{\nu_{2M}}}, \sqrt{m_{\nu_{3M}}}).R.Diag(\sqrt{m_{\nu_1}}, \sqrt{m_{\nu_2}}, \sqrt{m_{\nu_3}}).U^\dagger,$$

where R is an orthogonal matrix expressed in terms of arbitrary complex angles $\widehat{\theta}_1, \widehat{\theta}_2, \widehat{\theta}_3$ as following

$$R = \begin{pmatrix} \widehat{c}_2\widehat{c}_3 & -\widehat{c}_1\widehat{s}_3 - \widehat{s}_1\widehat{s}_2\widehat{c}_3 & \widehat{s}_1\widehat{s}_3 - \widehat{c}_1\widehat{s}_2\widehat{c}_3 \\ \widehat{c}_2\widehat{s}_3 & \widehat{c}_1\widehat{c}_3 - \widehat{s}_1\widehat{s}_2\widehat{s}_3 & -\widehat{s}_1\widehat{c}_3 - \widehat{c}_1\widehat{s}_2\widehat{s}_3 \\ \widehat{s}_2 & \widehat{s}_1\widehat{c}_2 & \widehat{c}_1\widehat{c}_2 \end{pmatrix},$$

where $\widehat{c}_i = \cos\widehat{\theta}_i, \widehat{s}_i = \sin\widehat{\theta}_i,\ i = 1, 2, 3.$

$$U = U_{\rm PMNS}.P,$$

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & e^{i\rho} \end{pmatrix},$$

where $c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}$,

$$\sin^2\theta_{23} \simeq 0.466, \quad \sin^2\theta_{12} \simeq 0.312, \quad \sin^2\theta_{13} \simeq 0.016.$$

$$\Delta m^2_{\nu_{12}} = m^2_{\nu_2} - m^2_{\nu_1} = 7.53\times 10^{-5}{\rm eV}^2,\ \Delta m^2_{\nu_{23}} = m^2_{\nu_3} - m^2_{\nu_2} = 2.44\times 10^{-3}{\rm eV}^2.$$

δ is unknown CP violating Dirac phase.
σ, ρ are the CP violating Majorana phases.
h^{ν}_{ab} are function of the phase δ, ρ, σ, the heavy majorana neutrinos masses and the complex angles. For simplicity, we assume $\widehat{\theta}_1 = \widehat{\theta}_2 = \widehat{\theta}_3 \equiv \widehat{\theta}$.

Thermal production

In the thermal scenario, the heavy Majorana neutrinos are produced in a thermal bath.

For the channel $\nu_{kM} \to e_i H_5^+,\ \overline{e}_i H_5^-$ the CP asymmetry depends on flavor because $L_i(e_i) = 1$. However, since $L(N_i) = 0, L(H') = -1$, the CP asymmetry due to the decay $\nu_{kM} \to N_i H'^*,\ N_i H'$ is considered flavor independent.

Assume that $\nu_{1M} \ll \nu_{2M}, \nu_{3M}$.

We consider the CP asymmetry due to the decay of the lightest heavy Majorana ν_{1M}.

The baryon asymmetry is related to the lepton asymmetry as

$$Y_{\Delta B} = -\frac{8}{15}\Big(\sum_{i=1,2,3} Y^i_{\Delta L} + Y^0_{\Delta L}\Big).$$

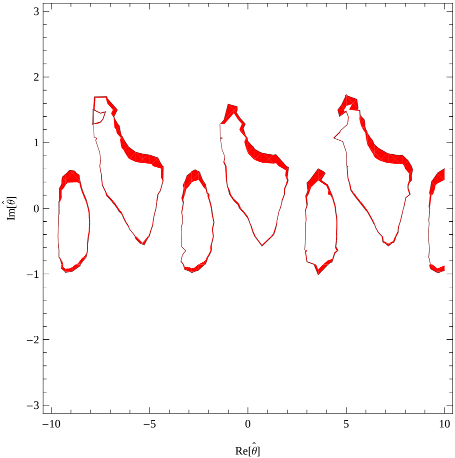


Figure: Contour plot of $Y_{\Delta B}$ in the region $5 \times 10^{-11} < Y_{\Delta B} < 10^{-10}$ on the plane of the complex angle $\widehat{\theta}$ for $\delta = 4.3$ rad, $\sigma = -1.5$ rad, $\rho = -1$ rad, $m_{H_3} = 2.67 \times 10^{13}$ GeV, $< \phi >= 23.6 m_{\mathrm{P}}$, $m_{\nu_{2M}} = m_{\nu_{3M}} = 10^3 m_{\nu_{1M}}$, $m_{\nu_{1M}} = 10^9$ GeV, $m_{\nu_1} = 0.01$ eV.

Non-thermal production

The heavy Majorana neutrinos are produced through the direct non-thermal decay of the heavy H_3 Higgs boson.
The total CP asymmetry is the summation of all flavor CP asymmetry,

$$\varepsilon_{\nu_{kM}} = \sum_i (\varepsilon^{i(1)}_{\nu_{kM}} + \varepsilon^{i(2)}_{\nu_{kM}}) = \frac{\sum_{j\neq k} B_j \mathrm{Im}[[(h^{\nu\dagger} h^{\nu})_{kj}]^2]}{(h^{\nu\dagger} h^{\nu})_{kk}}, \qquad B_j \simeq -\frac{11}{160\pi\sqrt{g_j}}.$$

The lepton asymmetry is related with the CP asymmetry through

$$Y_{\Delta L} = \frac{3}{2}\varepsilon_{\nu_{kM}} \times Br_k \times \frac{T_R}{m_{H_3}}, \qquad T_R = \left(\frac{90}{\pi^2 g^*}\right)^{\frac{1}{4}} (\Gamma_{H_3} m_{\mathrm{P}})^{\frac{1}{2}},\ g^* = 106.75,$$

Br_k denotes the branching ratio of the decay channel $H_3 \to \nu_{kM}\nu_{kM}$.
Assumed that $m_{\nu_{1M}} \ll m_{H_3} < m_{\nu_{2M}} \sim m_{\nu_{3M}}$, $m_{H_3} < m_{Z^N}$ and $\Gamma(H_3 \to hh) \ll \Gamma(H_3 \to \nu_{1M}\nu_{1M})$ when $\lambda_{10;11;12}$ are negligibly small, therefore,

$$Y_{\Delta L} \simeq \frac{3}{2}\varepsilon_{\nu_{1M}} \times \frac{T_R}{m_{H_3}}.$$

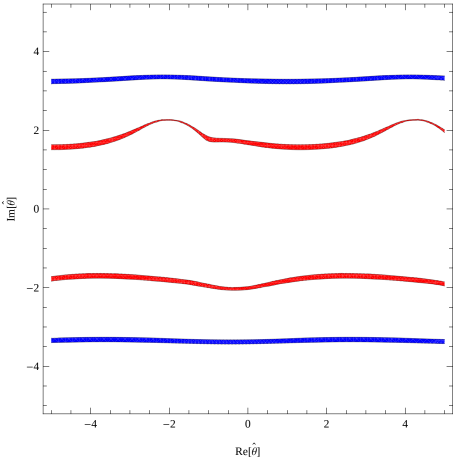


Figure: Contour plot of $Y_{\Delta B}$ in the region $(5 \times 10^{-11} < Y_{\Delta B} < 10^{-10})$ on the plane of the complex angle $\widehat{\theta}$ for δ=4.3 rad, σ=-1.5 rad, ρ=-1 rad, m_{ν_1}=0.01 eV, $m_{H_3} = 2.67 \times 10^{13}$ GeV, $< \phi >= 23.6 m_{\mathrm{P}}$, $m_{\nu_{2M}} = m_{\nu_{3M}} = 10^{14}$ GeV, $m_{\nu_{1M}} = 10^{11}$ GeV (red) and $m_{\nu_{1M}} = 10^{9}$ GeV (blue).

5. Summary

We have considered two versions of the 3-3-1 model:

- **The minimal 3-3-1 model** behaved as the simple 3-3-1 model with two scalar triplets η and χ has been reviewed. The original simple 3-3-1 model does not contain dark matter. By introducing an odd Higgs triplet (η' or χ') under a Z_2 symmetry while all other fields are even, the simple 3-3-1 model with the replication of η or of χ can provide the dark matter candidate.
- **The 3-3-1 model with neutral fermion** is extended to the 3-3-1-1 model in order to generate inflation as well as explain the baryon asymmetry of the Universe. The $U(1)_N$, where $N = \mathcal{B} - \mathcal{L}$ is considered as a gauged charge, is broken by the singlet ϕ at GUT scale. ϕ can play role of inflaton.
  The model contains the heavy Majorana neutrinos, which can be produced in a thermal bath or by decay of the Higgs singlet H_3, real part of ϕ. Both thermal and non-thermal productions have been calculated. The baryon asymmetry is in agreement with experimental result in both cases with different choice of the complex angle $\widehat{\theta}$.

Thanks you for your attention!