

Invariant Variation Problems

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M. A. Tavel's English translation of "Invariante Variationsprobleme," *Nachr. d. König. Gesellsch. d. Wiss. zu Göttingen, Math-phys. Klasse*, 235–257 (1918), which originally appeared in *Transport Theory and Statistical Physics*, **1** (3), 183–207 (1971).⁰

Abstract

The problems in variation here concerned are such as to admit a continuous group (in Lie's sense); the conclusions that emerge from the corresponding differential equations find their most general expression in the theorems formulated in Section 1 and proved in following sections. Concerning these differential equations that arise from problems of variation, far more precise statements can be made than about arbitrary differential equations admitting of a group, which are the subject of Lie's researches. What is to follow, therefore, represents a combination of the methods of the formal calculus of variations with those of Lie's group theory. For special groups and problems in variation, this combination of methods is not new; I may cite Hamel and Herglotz for special finite groups, Lorentz and his pupils (for instance Fokker), Weyl and Klein for special infinite groups.¹ Especially Klein's second Note and the present developments have been mutually influenced by each other, in which regard I may refer to the concluding remarks of Klein's Note.

§ 1. Preliminary Remarks and Formulation of Theorems

All functions occurring in the sequel are to be assumed analytic, or at least continuous and continuously differentiable a definite number of times, and unique in the interval considered.

By a "group of transformation," familiarly, is meant a system of transformations such that for